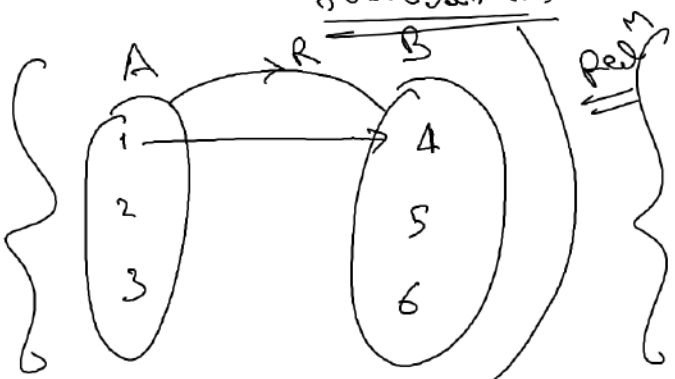
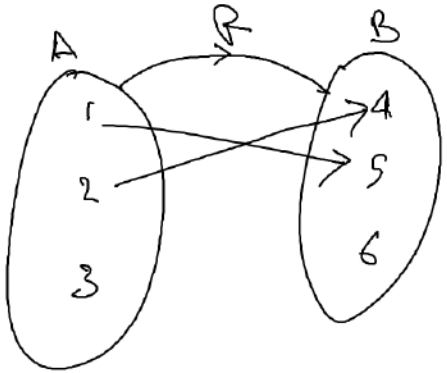
Functions

Rel



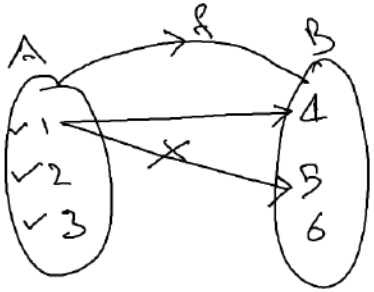
Rel



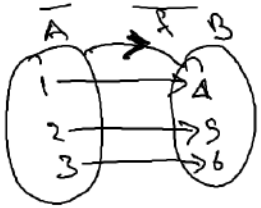
Func

It is a type of Relation in each element of set A is associated with unique element of set B

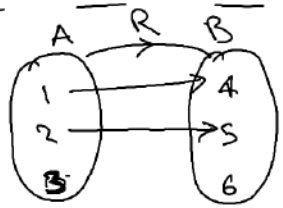
$f : A \to B$



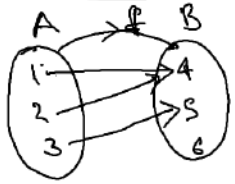
(i)



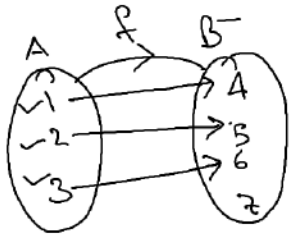
(ii)



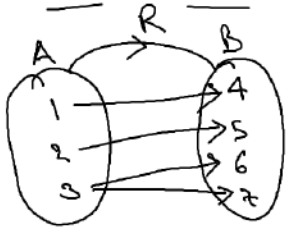
(iii)



(iv)



(v)



* Domain & Range of a function

Domain (x) → which Real no. we can put to get function

$y = f(x)$

$y = f(x) = x^2 - 5x + 6$

$f(2) = 2^2 - 5 \times 2 + 6$

$= 0$

value of func^n at $x = 2$ → y

Range (values of func^n) → y

$y = f(x)$

Num. (2) (-2)

$\sqrt{-ve}$

$|2| = 2$

$|-2| = 2$

- $f(x) = x + 1$ → Lin. func^n
- $f(x) = x^2$ → quad. func^n
- $f(x) = x^3$ → cubic func^n
- $f(x) = \frac{1}{x}$ → Reciprocal func^n
- $f(x) = e^x$ → exponential func^n
- $f(x) = \log x$ → logarithmic func^n
- $f(x) = |x|$ → Modulus func^n
- $f(x) = \sqrt{x}$ → Sq. root func^n

Q. $y = x + 1$

$f(x) = x + 1$

find with domain.

$f(x) = x + 1$

Dom $\in R$

Q. $y = \sqrt{x^2 - 5x + 6}$

$x^2 - 5x + 6 \geq 0$

$(x-2)(x-3) \geq 0$

+ve, 2 -ve, 3 +ve

$x \in (-\infty, 2] \cup [3, \infty)$

Dom.

Q. $y = \frac{1}{x}$

$f(x) = \frac{1}{x}$

$x \neq 0$

Dom $= R - \{0\}$

$\frac{1}{5}$

$\frac{1}{0} = \infty$ (not def.)

$\frac{0}{K} = 0$ (Real)

Q. $y = \frac{1}{\sqrt{x-5}}$

find Dom.

$x - 5 \geq 0$

$x > 5$

$x \in (5, \infty)$

Dom $= (5, \infty)$

$\frac{1}{\sqrt{0}} = \frac{1}{0} = \infty$

R X

Q. $y = \sqrt{x-3}$

find Domain.

$x - 3 \geq 0$

$x \geq 3$

$x \in [3, \infty)$

Dom $= [3, \infty)$

$\sqrt{0} = 0 \to R$

$\sqrt{+ve} = R$

$\sqrt{-ve} \to \not\in$

$x - 3 = 0$

$x = 3$

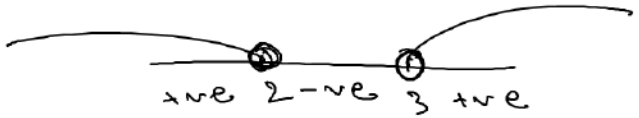
Q. $y = \frac{1}{\sqrt{x^2-5x+6}}$

Hello

$x^2 - 5x + 6 > 0$

$(x-2)(x-3) > 0$

$x \in (-\infty, 2) \cup (3, \infty)$

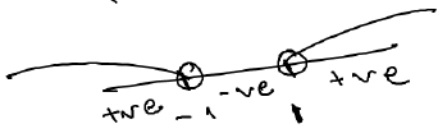


Q. $y = \frac{1}{\sqrt{x^2-1}}$

$x^2 - 1 \geq 0$

$(x-1)(x+1) \geq 0$

$x \in (-\infty, -1) \cup (1, \infty)$



Find Domain $g(x) \neq 0$

Q.1. $f(x) = \frac{x-3}{2-x}$ $\left(\frac{p(x)}{g(x)}\right)$

$2 - x \neq 0 \quad x \neq 2$

$x \in R - \{2\}$ → Domain

Q.2. $f(x) = \sqrt{\frac{x-5}{3-x}}$

$x-5=0 \quad x=5$

$3-x=0 \quad x=3$

$x \in (3,5]$

$\frac{x-5}{3-x} \geq 0$

-ve 3 +ve 5 -ve

Q.3. $f(x) = \frac{x^2-9}{x-3} =$ defined

for $f(x)$ to be

$x-3 \neq 0$

$x \neq 3$

$x \in R - \{3\}$

Q.4. $f(x) = \frac{ax-b}{cx-d}$

$cx-d \neq 0 \Rightarrow cx \neq d$

$\Rightarrow x \neq \frac{d}{c}$

$x \in R - \{\frac{d}{c}\}$

Q.5. $f(x) = \frac{x^3-8}{x^2-1}$

$\sqrt{(x-5)(3-x)}$

Q.6. $f(x) = \frac{x^2-2x+1}{x^2-8x+12}$

Q.7. $f(x) = \sqrt{x^2-7x+12}$

$x^2-7x+12 \geq 0$

$(x-4)(x-3) \geq 0$

+ve 3 -ve 4 +ve

$x \in (-\infty, 3] \cup [4, \infty)$

Q.8. $f(x) = \frac{1}{\sqrt{(x-3)(x-4)(x-5)}}$

$(x-3)(x-4)(x-5) > 0$

-ve 3 +ve 4 -ve 5 +ve

$x \in (3,4) \cup (5,\infty)$

Q. $f(x) = \frac{1}{\sqrt{(x-3)^2(x-4)}}$

$(x-3)^2(x-4) > 0$

-ve 3 -ve 4 +ve

$x \in (4, \infty)$

Q. $f(x) = \sqrt{\frac{(x-3)^2(x-5)}{(x-8)^3(x-2)}}$

$x \in (2,3] \cup [3,5] \cup (8,\infty)$

-ve 2 +ve 3 +ve 5 -ve 8 +ve

$y = \frac{1}{\sqrt{x-3}}$

$\sqrt{0} = 0 \rightarrow R_f$

$\sqrt{+ve} \rightarrow R$

for $f(x)$ to be def.

$x - 3 > 0$

$x > 3$

$x \in (3, \infty)$

Dom $= (3, \infty)$

$y = \sqrt{x-3}$

for $f(x)$ to be def

$x - 3 \geq 0$

$x \geq 3$

Dom $= [3, \infty)$

Q. $f(x) = \sqrt{\frac{(x-3)(x-5)}{(x-7)(x-9)}}$

$\frac{(x-3)(x-5)}{(x-7)(x-9)} \geq 0$

$x \in (-\infty, 3] \cup [5, 7) \cup (9, \infty)$

$x \in [3, 5] \cup [7, 9)$

$(-\infty, 3] \cup [3, 5] \cup [7, 9)$

+ve 3 -ve 5 +ve 7 -ve 9 +ve

Q. $f(x) = \sqrt{\frac{(x-3)^2}{x-4}}$

$(x-3)^2 \geq 0$

$x - 4 > 0$

$x > 4$

$x \in (4, \infty)$

Q. $f(x) = \sqrt{(x-3)^2(x-5)^3(x-7)}$

+ve 3 +ve 5 -ve 7 +ve

Range

$x \in$ what → Domain

$y \in$ what → Range

eg:

$y = \frac{1}{x}$ → $x = \frac{1}{y}$, $D = R - \{0\}$, Range $= R - \{0\}$

$A \cap (B - C)$
$= (B - C) \cap A$
$= (B \cap C') \cap A$
$= (B - A') \cap (C' - A)$
$= (B \cap A) \cap (C' \cap A)$

eg:

$y = \frac{1}{x-3}$, $D = R - \{3\}$

$x - 3 = \frac{1}{y}$

$x = \frac{1}{y} + 3 = \frac{1+3y}{y}$

$x = \frac{1+3y}{y}$

for x to be real

$y \neq 0$

$y \in R - \{0\}$

Q. $y = \frac{1}{x-5}$

$x - 5 \neq 0$

$x \neq 5$

$x \in R - \{5\}$

Q. $y = \frac{x-3}{2-x}$

for y to be def.

$2 - x \neq 0$

$x \neq 2$

$x \in R - \{2\}$

Q. $y = \frac{3x-2}{x+2}$

$x + 2 \neq 0$ $x \neq -2$

$x \in R - \{-2\}$

Q. $y = \sqrt{3x-5}$

$3x - 5 \geq 0$

$x \geq \frac{5}{3}$

$x \in [\frac{5}{3}, \infty)$

$-\infty$
0
$-ve$

0
-0
$\sqrt{-ve}$

$y = \frac{1}{x-5}$

for y to be real

$x - 5 \neq 0$

$x \neq 5$

$x \in R - \{5\}$

$y = \sqrt{x-5}$

$x - 5 \geq 0$

$x \geq 5$

$y = \frac{1}{\sqrt{x-5}}$

$x - 5 > 0$

$x \in (5, \infty)$

$x \in (-\infty, 5) \cup (5, \infty)$

Q. $A \cap (B - C) = (A \cap B) - (A \cap C)$

Q. $y = \frac{1}{x-5}$

$x - 5 = \frac{1}{y}$

$x = \frac{1}{y} + 5$

$x = \frac{1+5y}{y}$

$y \neq 0$

$y \in R - \{0\}$

$y = \frac{x-3}{2-x}$

$2y - xy = x - 3$

$2y + 3 = x + xy$

$2y + 3 = x(1+y)$

$x = \frac{2y+3}{1+y}$

$y + 1 \neq 0$

$y \neq -1$

$y \in R - \{-1\}$

$y = 3x$

Q. $y = \frac{3x-2}{x+2}$

$xy + 2y = 3x - 2$

$xy - 3x = -2 - 2y$

$x(y-3) = -2 - 2y$

$x = \frac{-2-2y}{y-3}$

Q. $y = \sqrt{3x-5}$

Dom.

y to be real

$3x - 5 \geq 0$

$3x \geq 5$

$x \geq \frac{5}{3}$

$x \in [\frac{5}{3}, \infty)$

Range

$y^2 = 3x - 5$

$x = \frac{y^2+5}{3}$

x to be real

$y \in R$

$y \geq 0$

$y \in (0, \infty)$

Q. $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$

$x \in (A-B) \cup (B-A) \Rightarrow x \in (A-B)$ or $x \in (B-A)$

$\Rightarrow (x \in A$ and $x \notin B)$ or $(x \in B$ and $x \in A)$

$\rightarrow (x \in A$ or $x \in B)$ or $(x \in B$ or $x \in A)$

$\Rightarrow x \in (A \cup B)$

Q. $y = x^2 + 2$

Q. $y = \frac{1}{\sqrt{\quad}}$

Q. $f(x) = \frac{x-2}{x-3}$

find dom & Range of $f(x)$

$y = \frac{x-2}{x-3}$

For y to be real

$x - 3 \neq 0$

$x \neq 3$

$x \in R - \{3\}$

$\rightarrow$ dom

$y(x-3) = x-2$

$yx - 3y = x - 2$

$yx - x = 3y - 2$

$x(y-1) = 3y-2$

$x = \frac{3y-2}{y-1}$

For x to be real

$y - 1 \neq 0$

$y \neq 1$

$y \in R - \{1\}$

Range

Q. $y = \frac{1}{\sqrt{x-2}}$ → find dom & range

For y to be real

$x - 2 > 0$

$x > 2$

$x \in (2, \infty)$

$y^2 = \frac{1}{x-2}$

$x - 2 = \frac{1}{y^2}$

$x = \frac{1}{y^2} + 2$

$x = \frac{1 + 2y^2}{y^2}$

For x to be real

$y^2 \neq 0$

$y \neq 0$

$y \in R - \{0\}$

$y \in (0, \infty)$

Q. $y = \sqrt{x-3}$, find dom & range

For y to be real

$x - 3 \geq 0$

$x \geq 3$

$x \in [3, \infty)$

$y = \frac{1}{\sqrt{x-3}}$

$x - 3 > 0$

$x > 3$

$x \in (3, \infty)$

$y = \sqrt{x-3}$

$y^2 = x - 3$

$x = y^2 + 3$

$y \geq 0$

$y \in R$

$y \in (0, \infty)$

Q. $f(x) = \frac{1}{x-5}$

find dom

For y to be real

$x - 5 \neq 0$

$x \neq 5$

$x \in R - \{5\}$

$y = \frac{1}{x-5} \Rightarrow yx - 5y = 1$

$x = \frac{1+5y}{y}$

Q. $f(x) = \frac{3x-2}{x+2}$

find dom

$y = \frac{3x-2}{x+2}$ ....

$yx + 2y = 3x - 2$

$yx - 3x = -2 - 2y$

$x(y-3) = -2-2y$

$x = \frac{-2-2y}{y-3}$

$y - 3 \neq 0$

$y \neq 3$

$y \in R - \{3\}$

For x to be real

$y \neq 0$

$y \in R - \{0\}$

Q. $y = \frac{1}{\sqrt{2x-3}}$

$\sqrt{}$ ve $\sqrt{}$ve $\sqrt{0}$

$2x-3 > 0$

$\sqrt{2x-3} > 0$

$\frac{1}{\sqrt{2x-3}} > 0$

Range $\in (0, \infty)$

$\Rightarrow y > 0$, $y^2 = \frac{1}{2x-3}$

$2x-3 = \frac{1}{y^2}$

$2x = \frac{1+3y^2}{y^2}$

$y \neq 0$

$y \in R - \{0\}$

$2x-3 > 0$

$x > \frac{3}{2}$

$x \in (\frac{3}{2}, \infty)$

0

$y \in (0, \infty)$

Q. $y = \sqrt{3x-5}$

for y to be real

$3x-5 \geq 0$

$3x \geq 5$

$x \geq 5/3$

$x \in [\frac{5}{3}, \infty)$

$y = \sqrt{3x-5}$ $y \geq 0$

$y^2 = 3x-5$

$x = \frac{y^2+5}{3}$

for x to be real

$y \in R$

0

$y \in [0, \infty)$

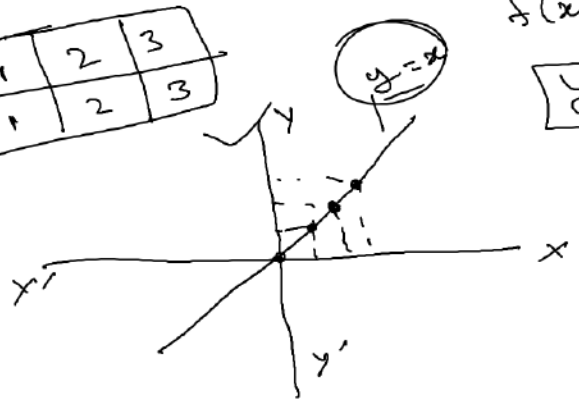
Some imp. function & its graphs

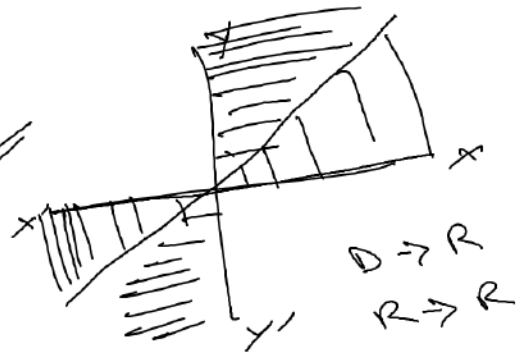
1. Identity func^n –

$f(x) = x$

$y = x$ ✓

x	1	2	3
y	1	2	3

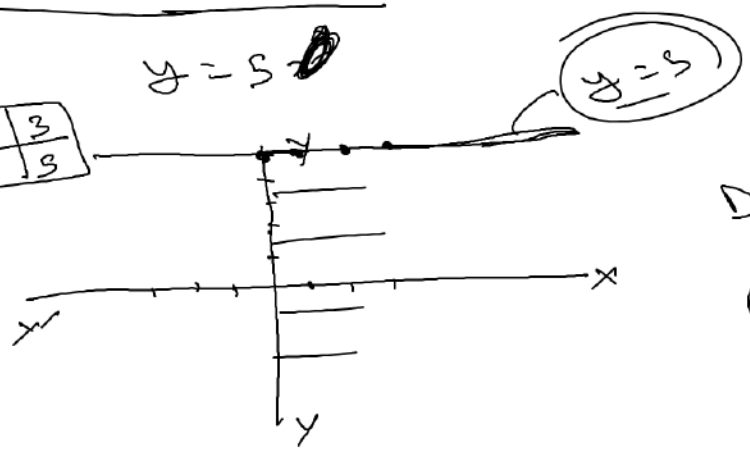




2. Constant function → Parallel to x-axis

$y = 5$

x	1	2	3
y	5	5	5



D → R

Range → {5}

$y = -5$

D → R

R → {-5}

3. Modulus function — $y = |x|$

$|2| = 2$

$|-2| = 2$

$|3.5| = 3.5$

$|-3.5| = 3.5$

$y = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

eg:

$|0|$, $|1|$, $|2|$, $|3|$

$|-2|$, $|-3|$, $|-4|$

$-(-4) = 4$

$-(-2) = 2$

$-(-3) = 3$

x	0	1	2	-1	-2
y	0	1	2	1	2

$y = |x|$

$y = x$ ($x \geq 0$), $y = -x$ ($x < 0$)

$y = |x|$

$D \rightarrow R$

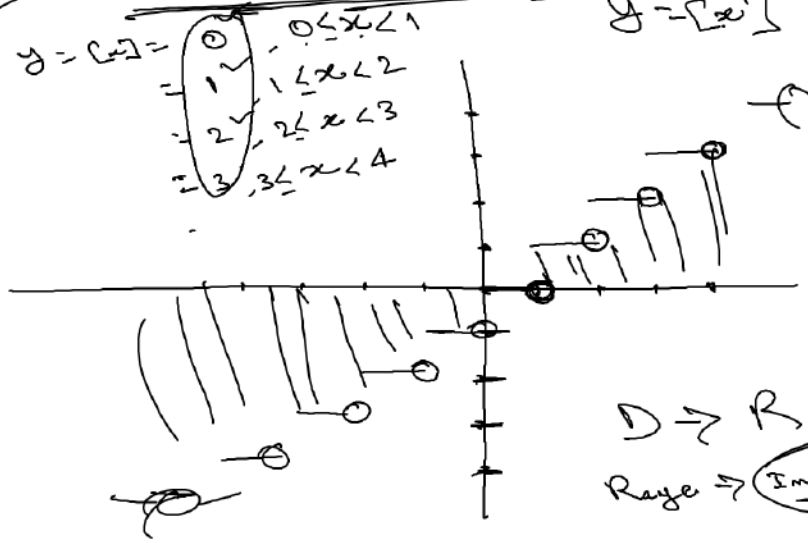
$(-\infty, \infty)$

$R \rightarrow [0, \infty)$

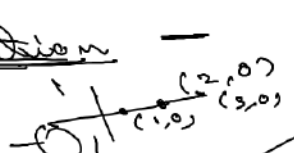
4. Greatest Integer function

$y = [x]$ (box x)

$y = [x] = \begin{cases} 0, & 0 \le x < 1 \\ 1, & 1 \le x < 2 \\ 2, & 2 \le x < 3 \\ 3, & 3 \le x < 4 \end{cases}$



$y = [x]$



$D \to R$

Range $\Rightarrow$ Integer

$|2| = 2$

$|-2| = 2$

$[0] = 0$

$[2.1] = 2$

$[2.9] = 2$

$[2.99] = 2$

$[2] = 2$

Equal or just less integer

$[-3] = -3$

$[-3.5] = -4$

5. Fractional Part function

$y = \{x\}$

$= x - [x]$

$D \to R$

Range $\to [0, 1)$

$\{2.3\} = 0.3$

$\{3.5\} = 0.5$

$\{0.99\} = 0.99$

$\{2.1\} = 0.1$

$\{x\} = x - [x]$

$\{5.9\} = 5.9 - [5.9]$

$= 5.9 - 5$

$= 0.9$

Signum func

$y = \frac{x}{|x|} = \begin{cases} 1 \\ 0 \\ -1 \end{cases}$

$|x| = x$

$x \geq 0$

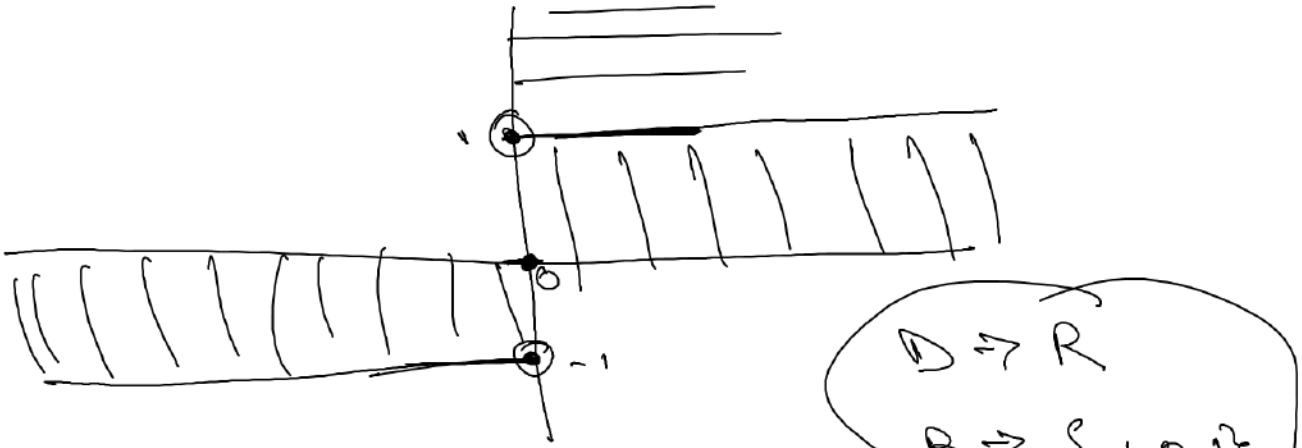
$x = 0$

$x < 0$

$|x| = -x$

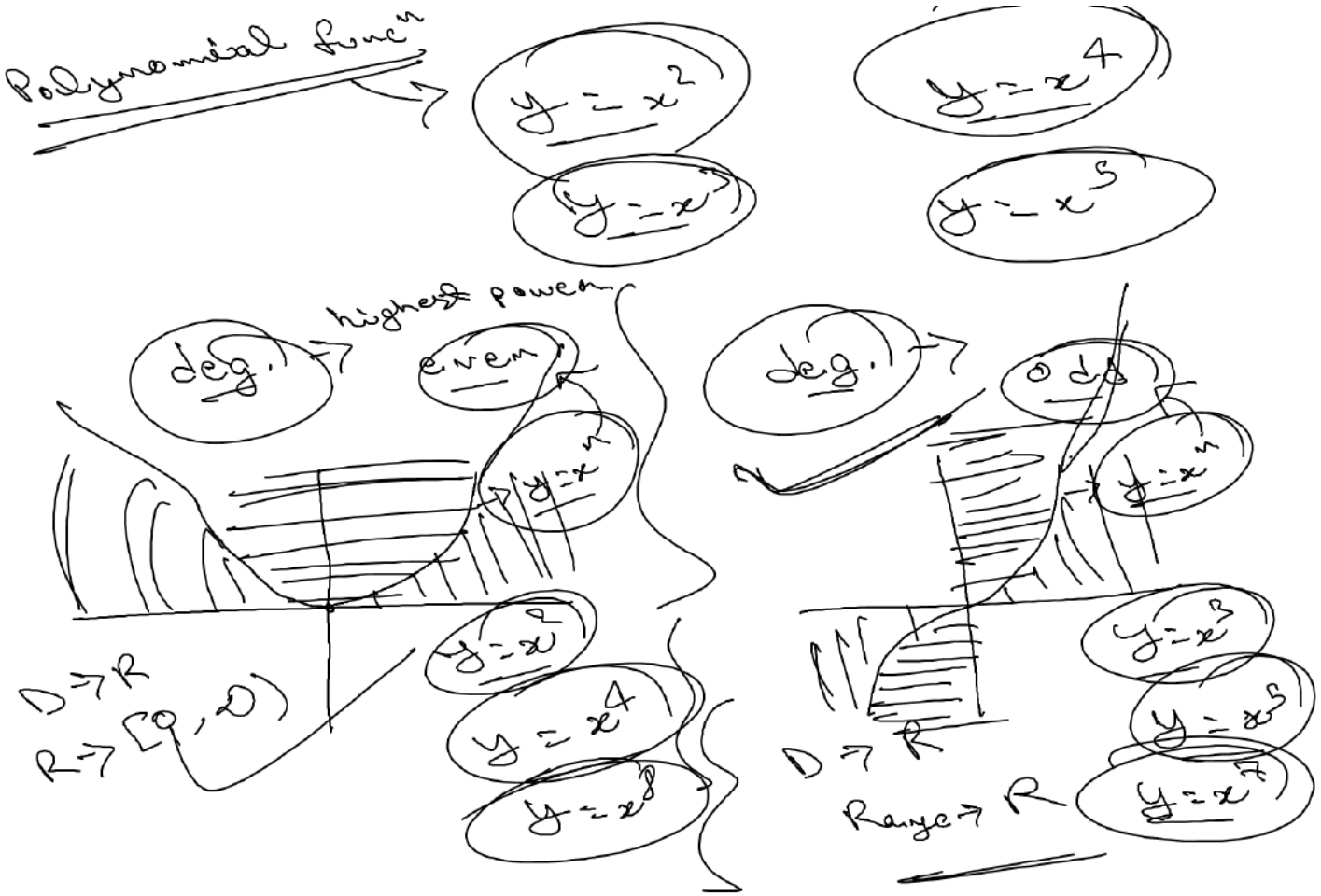
$\frac{x}{x}$, $\frac{x}{-x}$

1 , -1



$D \to R$

$R \to \{-1, 0, 1\}$


Polynomial fun^n
$y=x^2$
$y=x^3$
$y=x^4$
$y=x^5$
highest power
deg.
even
$y=x^n$
$y=x^8$
$y=x^4$
$y=x^8$
D → R
R → [0, ∞)
deg.
odd
$y=x^n$
$y=x^3$
$y=x^5$
$y=x^7$
D → R
Range → R

Operations on functions

eg: $f(x) = x^2$, $g(x) = 2x+1$