Complex No. $\to z = a + ib$

Real Part, Img. Part

Real

$z = a$,

Purely Real

$z = 2$

Imaginary

i

$\sqrt{-}$

$a = 0$

$z = ib$

Purely imaginary

* Imaginary no.

$\sqrt{-ve} \to i \to$ iota

$\boxed{i = \sqrt{-1}}$

e.g: $\sqrt{-2} = \sqrt{2 \times (-1)} = \sqrt{2} \times \sqrt{-1}$

$= \sqrt{2}\, i$

e.g: $\sqrt{-4} = \sqrt{4 \times (-1)} = 2i$

$i^0 = 1$, $i^1 = i$, $i^2 = i \times i$

$= \sqrt{-1} \times \sqrt{-1} = -1$

$i^3 = i^2 \times i = (-1) i = -i$

$i^4 = i^2 \times i^2 = (-1) \times (-1) = 1$

$5^0 = 1$

$4n \to i^{4n} = 1$

$4n+1 \to i^{4n+1} = i^{4n} \times i$

$= 1 \times i$

$= i$

$i^{4n} = 1$

$i^{4n} = (i^4)^n$

$= (1)^n$

$= 1$

$4n+2 \to i^{4n+2} = i^{4n} \times i^2$

$= -1$

$4n+3 \to i^{4n+3} = i^{4n} \times i^3$

$= 1 \times (-i)$

$= -i$

Q. $i^{41} + \dfrac{1}{i^{71}} = i^{4 \times 10 + 1} + \dfrac{1}{i^{71}}$

$= i$

Q. $i^{19} = i^{4 \times 4 + 3}$

$= i^3 = -i$

$i^7 = i^{4\times 2-1} = i^1$

$i^{4\times 1+3}$

$4)71($

$4m-1$

$4m+3$

$\frac{1}{i^{71}} = \frac{1}{i^3}$

$= \frac{1}{-i} = \frac{i^{4}}{-i} = -i^3 = -(-i) = i$

$\frac{1}{-i} \times \frac{i}{i} = \frac{i}{-i^2} = \frac{i}{-(-1)} = \frac{i}{+1} = i$

$1, i, -1, -i$

$i^{-9} = \frac{1}{i^9} = \frac{1}{i^{4\times 2+1}} = \frac{1}{i} \times \frac{i}{i}$

$= \frac{i}{i^2}$

$= \frac{i}{-1} = -i$

Q.

method 1 $\frac{1}{i} = \frac{1}{i} \times \frac{i}{i} = \frac{i}{i^2} = \frac{i}{-1} = -i$ ✓

method 2

$\frac{1}{i} = \frac{i^{43}}{\cancel{i}} = i^3 = -i$

$\frac{i^{43}}{\cancel{i}}$ ✓

$i^{93} = i^{4x}$

Q. $i^{-5} = \frac{1}{i^5} = \frac{1}{i^{4\times1+1}} = \frac{1}{i} = \frac{i^{43}}{\cancel{i}} = i^3 = -i$

$(-i)^3 = -(-i)$
$= i$

$a^3 + b^3 + 3ab(a+b)$

Q. $i^{-7} = \frac{1}{i^7} = \frac{1}{i^{4\times1+3}} = \frac{1}{i^3} = \frac{i^4}{i^3} = i$

$(a+b)^3 =$

Q. $\left[i^{18} + \left(\frac{1}{i}\right)^{25} \right]^3 = 2(1-i)$

L.H.S $= \left(i^{2} + \frac{1}{i^{25}}\right)^3$

$= \left(-1 + \frac{1}{i}\right)^3$

$= (-1 - i)^3$

$= (-1)^3 + (-i)^3 + 3(-1)(-i)(-1-i)$

$= -1 + i + 3i(-1-i)$

$= -1 + i - 3i - 3i^2$

$= -1 + i - 3i + 3 = 2 - 2i$

* $z = a+ib$, $(a, b) \in R$

$Re(z)$ $\quad$ $Im(z)$

Q. $z = 5+9i$

$Re(z) = 5$

$Im(z) = 9$

$z = a+ib$

* Purely Real $\quad$ $Im(z) = 0$

$b = 0$

eg: 2, 3, 4, $\frac{7}{5}$

$z = a$

* Purely imag. $\quad$ $Re(z) = 0$

$a = 0$

eg: $2i$, $3i$, $13i$, $-7i$ $\quad$ $z = ib$

* conjugate of a complex no.

$z = a+ib = ib+a$

$\bar{z} = a-ib = -ib+a$

eg: $z = 6i-5$

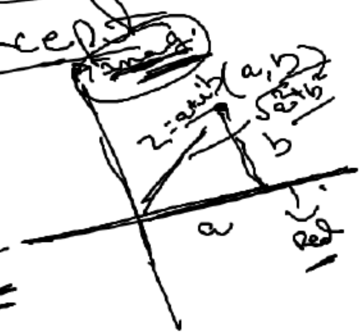
~~$\bar{z} = 6i+5$~~

$\bar{z} = -6i-5$

* Modulus concept

$z = a+ib$

$|z| = \sqrt{a^2+b^2}$



* Equality of Complex no. —

$z_1 = a_1 + ib_1$

$z_2 = a_2 + ib_2$

$z_1 = x + 2i$

$z_2 = 3 + yi$

$x = 3, \; y = 2$

$z_1 = z_2$

$z_1 = 4^2 + 13i$

$z_2 = 4^2 + 13i$

Algebra of Complex No. —

$z_1 = 2 + 3i$

$z_2 = 3 + 5i$

$z_1 + z_2 = (2+3) + (3i + 5i)$

$= 5 + 8i$

$z_2 - z_1 = (3-2) + (5i - 3i)$

$= 1 + 2i$

* Multiplication —

$(a_1 + ib_1)(a_2 + ib_2)$

$= a_1 a_2 + i a_1 b_2 + i a_2 b_1 + i^2 b_1 b_2$

$i^2 = -1$

* Division → Rationalisation

$a + b = b + a$

$z_1 + z_2 = z_2 + z_1$

$z + 0 = z$

Additive Inverse — Additive Identity

$z_1 + z_2 = 0$ $\quad z_2 = -z$

Multiplicative Id. → 1

$z \times 1 = z$

$a + ib \longrightarrow$ Add. inverse

$-a - ib$

$(a-b)(a+b) = a^2 - b^2$

$(a+ib)(a-ib) = |z|^2$

z

$= a^2 - (ib)^2$

$= a^2 - i^2 b^2$

$= a^2 + b^2$

$z_1 = 3+4i$

$z_1 z_2 = 1$

$\boxed{z_2 = \frac{1}{z_1}}$

$= \frac{1}{3+4i} \times \frac{3-4i}{3-4i}$

$= \frac{3-4i}{3^2+4^2}$

$= \frac{3}{5} - \frac{4}{5} i$

Complex No. (Practice)

$(2+3\sqrt{3}i)$

1. (vi).

$(5+\sqrt{-3})(5-\sqrt{-3})$

$= (5+\sqrt{3}i)(5-\sqrt{3}i)$

$= 5^2 + (\sqrt{3})^2$

$= 25 + 3$

$= 28$

1. (v).

$(1-i)^2(1+i) - (3-4i)^2$

$= (1+1)(1-i) - (9+16i^2-24i)$

$= 2(1-i) - (-7-24i)$

$= 2 - 2i + 7 + 24i$

$= 9 + 22i$

2 (iv) $\left(-2-\frac{1}{3}i\right)^3 =$

$\boxed{(a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2}$

$= (-2)^3 + \left(-\frac{1}{3}i\right)^3 + \cancel{3}(-2)^2\left(-\frac{1}{\cancel{3}}i\right)$

$+ \cancel{3}(-2)\left(-\frac{1}{\cancel{3}}i\right)^2$

$\frac{5}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$\frac{5\sqrt{2}}{2}$

$= -8 - \frac{1}{27}(-i) \; \cancel{8} - \frac{\cancel{12}^{\,4}}{\cancel{3}} i$

$+ \cancel{2}\frac{2}{3}$

$= \cancel{-8}\,\frac{-22}{3} - 4i + \frac{i}{27}$

$= -\frac{22}{3} - \frac{107}{27} i$

$z = a + ib$

$\bar{z} = a - ib$

$z \cdot \bar{z} = |z|^2$

2. (vii) $(2+i)^{-2} = \frac{1}{(2+i)^2}$

$\frac{1}{2-\sqrt{2}} \times \frac{2+\sqrt{2}}{2+\sqrt{2}}$

$= \frac{1}{4+i^2+4i}$

$= \frac{1}{3+4i} \times \frac{3-4i}{3-4i} = \frac{3-4i}{9+16}$

Q. 3 (vi)

$$\frac{(3-2i)(2+3i)}{(1+2i)(2-i)}$$

$$= \frac{6-4i+9i-6i^2}{2+4i-i-2i^2}$$

$$= \frac{12+5i}{4+3i} \times \frac{4-3i}{4-3i}$$

$$= \frac{48+20i-36i-15i^2}{16+9}$$

$$= \frac{63-16i}{25}.$$

5.(ii)

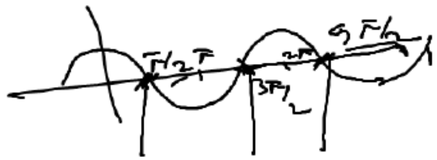
$$\frac{\sqrt7+i\sqrt3}{\sqrt7-i\sqrt3} + \frac{\sqrt7-i\sqrt3}{\sqrt7+i\sqrt3}$$

$$= \frac{7-3+\cancel{2\sqrt{21}i}+7-3-\cancel{2\sqrt{21}i}}{7+3}$$

$$= \frac{8}{10} = \frac{4}{5}.$$

6.

$$\frac{1+i\cos\theta}{1-2i\cos\theta}$$



$$\cos\theta = 0$$

$$\theta = (2n+1)\frac{\pi}{2}, \quad n \in I$$

odd no. $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}.$

Q(ii)

$$z = \frac{1}{4+3i} \times \frac{4-3i}{4-3i} = \frac{4-3i}{25}$$

$$= \frac{4}{25} - \frac{3}{25}i$$

$$\bar{z} = ?$$

$$\frac{4}{25} + \frac{3}{25}i$$

Q. 11(ii) $(1-\sqrt{3}i)\cdot z = 1$

$$z = \frac{1}{1-\sqrt{3}i}$$

$$= \frac{1}{1-\sqrt{3}i} \times \frac{1+\sqrt{3}i}{1+\sqrt{3}i}$$

$$= \frac{1+\sqrt{3}i}{1+3} = \frac{1+\sqrt{3}i}{4}$$

14. $$(x+iy) = \frac{a+ib}{a-ib} \times \frac{a+ib}{a+ib}$$

$$x+iy = \frac{a^2+iab+iab+i^2b^2}{a^2+b^2}$$

$$x+iy = \frac{(a^2-b^2)+2abi}{a^2+b^2}$$

$$x = \frac{a^2-b^2}{a^2+b^2}, \quad y = \frac{2ab}{a^2+b^2}$$

11(iv) $$\frac{(1+i)(1+2i)}{(1+3i)} \times z = 1$$

$$z = \frac{1+3i}{(1+i)(1+2i)}$$

$$= \frac{1+3i}{1+i+2i+2i^2}$$

$$= \frac{1+3i}{-1+3i} \times \frac{-1-3i}{-1-3i}$$

$$= \frac{-1-3i-3i-9i^2}{1+9}$$

$$= \frac{\cancel{8}\,4 - \cancel{6}\,3 i}{\cancel{10}\,5}$$

$$= \frac{4}{5} - \frac{3}{5}i$$