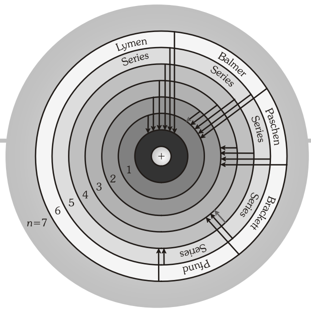


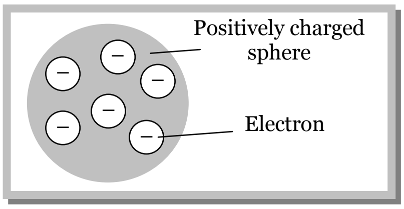
Atomic Structure

Important Atomic Models

(1) **Thomson's model**

J.J. Thomson gave the first idea regarding structure of atom. According to this model.

(i) An atom is a solid sphere in which entire and positive charge and it's mass is uniformly distributed and in which negative charge (*i.e.* electron) are embedded like seeds in watermelon.



Success and failure

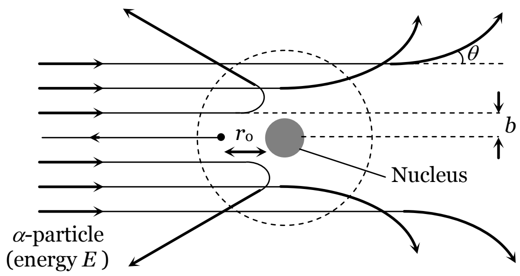
Explained successfully the phenomenon of thermionic emission, photoelectric emission and ionization.

The model fail to explain the scattering of α- particles and it cannot explain the origin of spectral lines observed in the spectrum of hydrogen and other atoms.

(2) **Rutherford's model**

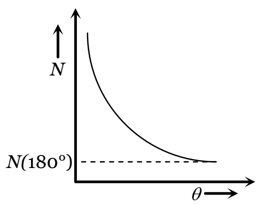
Rutherford's α-particle scattering experiment

Rutherford performed experiments on the scattering of alpha particles by extremely thin gold foils and made the following observations



Number of scattered particles :

$$N \propto \frac{1}{\sin^4(\theta/2)}$$



(i) Most of the α-particles pass through the foil straight away undeflected.

(ii) Some of them are deflected through small angles.

(iii) A few α-particles (1 in 1000) are deflected through the angle more than 90°.

(iv) A few α -particles (very few) returned back *i.e.* deflected by 180°.

(v) Distance of closest approach (Nuclear dimension)

The minimum distance from the nucleus up to which the α-particle approach, is called the distance of closest approach (r_0). From figure $r_0 = \frac{1}{4\pi\varepsilon_0}.\frac{Ze^2}{E}$; $E = \frac{1}{2}mv^2 =$ K.E. of α-particle

(vi) Impact parameter (b) : The perpendicular distance of the velocity vector ($\vec{v}$) of the α-particle from the centre of the nucleus when it is far away from the nucleus is known as impact parameter. It is given as

$$b = \frac{Ze^2 \cot(\theta/2)}{4\pi\varepsilon_0\left(\frac{1}{2}mv^2\right)} \Rightarrow b \propto \cot(\theta/2)$$

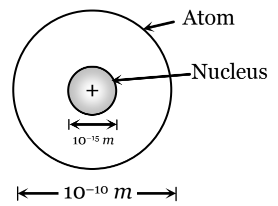
Note : ❑ If t is the thickness of the foil and N is the number of α-particles scattered in a particular direction (θ = constant), it was observed that $\frac{N}{t}$ = constant $\Rightarrow \frac{N_1}{N_2} = \frac{t_1}{t_2}$.

After Rutherford's scattering of α-particles experiment, following conclusions were made as regard as atomic structure :

(a) Most of the mass and all of the charge of an atom concentrated in a very small region is called atomic nucleus.

(b) Nucleus is positively charged and it's size is of the order of 10^{-15} $m \approx 1$ *Fermi*.

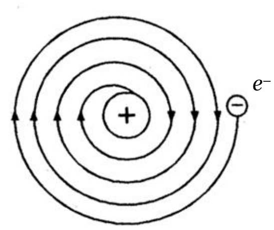
(c) In an atom there is maximum empty space and the electrons revolve around the nucleus in the same way as the planets revolve around the sun.



Size of the nucleus = 1 Fermi = 10^{-15} m
Size of the atom 1 Å = 10^{-10} m

Draw backs

(i) Stability of atom : It could not explain stability of atom because according to classical electrodynamic theory an accelerated charged particle should continuously radiate energy. Thus an electron moving in an circular path around the nucleus should also radiate energy and thus move into smaller and smaller orbits of gradually decreasing radius and it should ultimately fall into nucleus.



Instability of atom

(ii) According to this model the spectrum of atom must be continuous where as practically it is a line spectrum.

(iii) It did not explain the distribution of electrons outside the nucleus.

(3) **Bohr's model**

Bohr proposed a model for hydrogen atom which is also applicable for some lighter atoms in which a single electron revolves around a stationary nucleus of positive charge *Ze* (called hydrogen like atom)

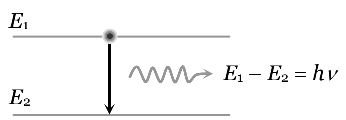
Bohr's model is based on the following postulates.

(i) The electron can revolve only in certain discrete non-radiating orbits, called stationary orbits, for which total angular momentum of the revolving electrons is an integral multiple of $\frac{h}{2\pi}$ $(= \hbar)$

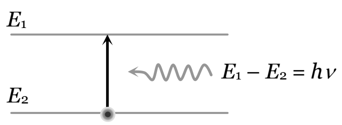
i.e. $L = n\left(\frac{h}{2\pi}\right) = mvr$; where n = 1, 2, 3, ...= Principal quantum number

(ii) The radiation of energy occurs only when an electron jumps from one permitted orbit to another.

When electron jumps from higher energy orbit (E_1) to lower energy orbit (E_2) then difference of energies of these orbits *i.e.* $E_1 - E_2$ emits in the form of photon. But if electron goes from E_2 to E_1 it absorbs the same amount of energy.



Emission



Absorption

Note : ❑ According to Bohr theory the momentum of an e^- revolving in second orbit of H_2 atom will be $\frac{h}{\pi}$

❑ For an electron in the n^{th} orbit of hydrogen atom in Bohr model, circumference of orbit $= n\lambda$; where λ = de-Broglie wavelength.

Bohr's Orbits (For Hydrogen and H_2-Like Atoms).

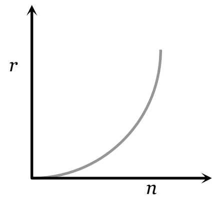
(1) **Radius of orbit**

For an electron around a stationary nucleus the electrostatics force of attraction provides the necessary centripetal force

i.e. $\frac{1}{4\pi\varepsilon_0}\frac{(Ze)e}{r^2} = \frac{mv^2}{r}$ .... (i) also $mvr = \frac{nh}{2\pi}$ ....(ii)

From equation (i) and (ii) radius of n^{th} orbit

$$r_n = \frac{n^2h^2}{4\pi^2kZme^2} = \frac{n^2h^2\varepsilon_0}{\pi mZe^2} = 0.53\frac{n^2}{Z}\text{Å} \qquad \left[\text{where } k = \frac{1}{4\pi\varepsilon_0}\right]$$



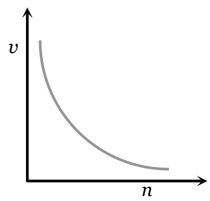
$$\Rightarrow \boldsymbol{r_n \propto \frac{n^2}{Z}}$$

Note : ❑ The radius of the innermost orbit (n = 1) hydrogen atom (z = 1) is called Bohr's radius a_0 *i.e.* $a_0 = 0.53\,\text{Å}$.

(2) **Speed of electron**

From the above relations, speed of electron in n^{th} orbit can be calculated as

$$v_n = \frac{2\pi kZe^2}{nh} = \frac{Ze^2}{2\varepsilon_0 nh} = \left(\frac{c}{137}\right).\frac{Z}{n} = 2.2\times 10^6\frac{Z}{n}\,m/sec$$



where (c = speed of light 3×10^8 m/s)

Note : ❑ The ratio of speed of an electron in ground state in Bohr's first orbit of hydrogen atom to velocity of light in air is equal to $\frac{e^2}{2\varepsilon_0 ch} = \frac{1}{137}$ (where c = speed of light in air)

(3) **Some other quantities**

For the revolution of electron in n^{th} orbit, some other quantities are given in the following table

Quantity	**Formula**	**Dependency on *n* and *Z***
(1) Angular speed	$\omega_n = \frac{v_n}{r_n} = \frac{\pi mz^2e^4}{2\varepsilon_0^2n^3h^3}$	$\omega_n \propto \frac{Z^2}{n^3}$
(2) Frequency	$\nu_n = \frac{\omega_n}{2\pi} = \frac{mz^2e^4}{4\varepsilon_0^2n^3h^3}$	$\nu_n \propto \frac{Z^2}{n^3}$
(3) Time period	$T_n = \frac{1}{\nu_n} = \frac{4\varepsilon_0^2n^3h^3}{mz^2e^4}$	$T_n \propto \frac{n^3}{Z^2}$
(4) Angular momentum	$L_n = mv_nr_n = n\left(\frac{h}{2\pi}\right)$	$L_n \propto n$
(5) Corresponding current	$i_n = e\nu_n = \frac{mz^2e^5}{4\varepsilon_0^2n^3h^3}$	$i_n \propto \frac{Z^2}{n^3}$
(6) Magnetic moment	$M_n = i_nA = i_n(\pi r_n^2)$	$M_n \propto n$

	(where $\mu_0 = \frac{eh}{4\pi m}$ = Bohr magneton)	
(7) Magnetic field	$B = \frac{\mu_0 i_n}{2r_n} = \frac{\pi m^2 z^3 e^7 \mu_0}{8\varepsilon_0^3 n^5 h^5}$	$B \propto \frac{Z^3}{n^5}$

(4) **Energy**

(i) **Potential energy :** An electron possesses some potential energy because it is found in the field of nucleus potential energy of electron in n^{th} orbit of radius r_n is given by $U = k.\frac{(Ze)(-e)}{r_n} = -\frac{kZe^2}{r_n}$

(ii) **Kinetic energy :** Electron posses kinetic energy because of it's motion. Closer orbits have greater kinetic energy than outer ones.

As we know $\frac{mv^2}{r_n} = \frac{k.(Ze)(e)}{r_n^2} \Rightarrow$ Kinetic energy $K = \frac{kZe^2}{2r_n} = \frac{|U|}{2}$

(iii) **Total energy :** Total energy (E) is the sum of potential energy and kinetic energy *i.e.* $E = K + U$

$\Rightarrow E = -\frac{kZe^2}{2r_n}$ also $r_n = \frac{n^2h^2\varepsilon_0}{\pi mze^2}$. Hence $E = -\left(\frac{me^4}{8\varepsilon_0^2 h^2}\right).\frac{z^2}{n^2} = -\left(\frac{me^4}{8\varepsilon_0^2 ch^3}\right)ch\frac{z^2}{n^2} = -Rch\frac{Z^2}{n^2} = -13.6\frac{Z^2}{n^2}eV$

where $R = \frac{me^4}{8\varepsilon_0^2 ch^3}$ = Rydberg's constant = 1.09×10^7 *per metre*

Note : ❑ Each Bohr orbit has a definite energy

❑ For hydrogen atom ($Z = 1$) $\Rightarrow E_n = -\frac{13.6}{n^2}eV$

❑ The state with $n = 1$ has the lowest (most negative) energy. For hydrogen atom it is $E_1 = -13.6$ *eV*.

❑ Rch = Rydberg's energy $\simeq 2.17 \times 10^{-18} J \simeq 31.6\, eV$.

❑ $E = -K = \frac{U}{2}$.

(iv) **Ionisation energy and potential :** The energy required to ionise an atom is called ionisation energy. It is the energy required to make the electron jump from the present orbit to the infinite orbit.

Hence $E_{ionisation} = E_\infty - E_n = 0 - \left(-13.6\frac{Z^2}{n^2}\right) = +\frac{13.6Z^2}{n^2}eV$

For H_2-atom in the ground state $E_{ionisation} = \frac{+13.6(1)^2}{n^2} = 13.6\, eV$

The potential through which an electron need to be accelerated so that it acquires energy equal to the ionisation energy is called ionisation potential. $V_{ionisation} = \frac{E_{ionisation}}{e}$

(v) **Excitation energy and potential :** When the electron is given energy from external source, it jumps to higher energy level. This phenomenon is called excitation.

The minimum energy required to excite an atom is called excitation energy of the particular excited state and corresponding potential is called exciting potential.

$$E_{Excitation} = E_{Final} - E_{Initial} \text{ and } V_{Excitation} = \frac{E_{excitation}}{e}$$

(vi) **Binding energy (B.E.) :** Binding energy of a system is defined as the energy released when it's constituents are brought from infinity to form the system. It may also be defined as the energy needed to separate it's constituents to large distances. If an electron and a proton are initially at rest and brought from large distances to form a hydrogen atom, 13.6 eV energy will be released. The binding energy of a hydrogen atom is therefore 13.6 eV.

Note : ❑ For hydrogen atom principle quantum number $n = \sqrt{\frac{13.6}{(\text{B.E.})}}$.

(5) **Energy level diagram**

The diagrammatic description of the energy of the electron in different orbits around the nucleus is called energy level diagram.

Energy level diagram of hydrogen/hydrogen like atom

Principle quantum number	Orbit	Excited state	Energy for H_2 – atom	Energy for H_2 – like atom	Ionisation energy from this level (for H_2 – atom)
$n = \infty$	Infinite	Infinite	$E_\infty = 0\ eV$	$0\ eV$	$0\ eV$
$n = 4$	Fourth	Third	$E_4 = -0.85\ eV$	$-0.85\ Z^2$	$+0.85\ eV$
$n = 3$	Third	Second	$E_3 = -1.51\ eV$	$-1.51\ Z^2$	$+1.51\ eV$
$n = 2$	Second	First	$E_2 = -3.4\ eV$	$-3.4\ Z^2$	$+3.4\ eV$
$n = 1$	First	Ground	$E_1 = -13.6\ eV$	$-13.6\ Z^2$	$+13.6\ eV$

Note : ❑ In hydrogen atom excitation energy to excite electron from ground state to first excited state will be $-3.4-(-13.6) = 10.2\ eV$.

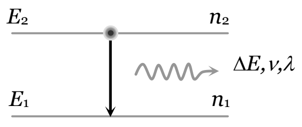
and from ground state to second excited state it is [$-1.51-(-13.6) = 12.09\ eV$].

❑ In an H_2 atom when e^- makes a transition from an excited state to the ground state it's kinetic energy increases while potential and total energy decreases.

(6) **Transition of electron**

When an electron makes transition from higher energy level having energy $E_2(n_2)$ to a lower energy level having energy E_1 (n_1) then a photon of frequency ν is emitted

(i) **Energy of emitted radiation**



Emission

$$\Delta E = E_2 - E_1 = \frac{-Rc\,h\,Z^2}{n_2^2} - \left(-\frac{Rch\,Z^2}{n_1^2}\right) = 13.6Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

(ii) **Frequency of emitted radiation**

$$\Delta E = h\nu \Rightarrow \nu = \frac{\Delta E}{h} = \frac{E_2 - E_1}{h} = Rc\,Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

(iii) **Wave number/wavelength**

Wave number is the number of waves in unit length $\bar{\nu} = \frac{1}{\lambda} = \frac{\nu}{c}$

$$\Rightarrow \frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) = \frac{13.6Z^2}{hc}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

(iv) **Number of spectral lines** : If an electron jumps from higher energy orbit to lower energy orbit it emits raidations with various spectral lines.

If electron falls from orbit n_2 to n_1 then the number of spectral lines emitted is given by

$$N_E = \frac{(n_2 - n_1 + 1)(n_2 - n_1)}{2}$$

If electron falls from n^{th} orbit to ground state (*i.e.* $n_2 = n$ and $n_1 = 1$) then number of spectral lines emitted $\boldsymbol{N_E = \frac{n(n-1)}{2}}$

Note : ❑ Absorption spectrum is obtained only for the transition from lowest energy level to higher energy levels. Hence the number of absorption spectral lines will be $(n - 1)$.

(v) **Recoiling of an atom** : Due to the transition of electron, photon is emitted and the atom is recoiled

Recoil momentum of atom = momentum of photon $= \frac{h}{\lambda} = hRZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

Also recoil energy of atom $= \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2}$ (where m = mass of recoil atom)

(7) **Drawbacks of Bohr's atomic model**

(i) It is valid only for one electron atoms, *e.g.* : H, He^+, Li^{+2}, Na^{+1} *etc.*

(ii) Orbits were taken as circular but according to Sommerfield these are elliptical.

(iii) Intensity of spectral lines could not be explained.

(iv) Nucleus was taken as stationary but it also rotates on its own axis.

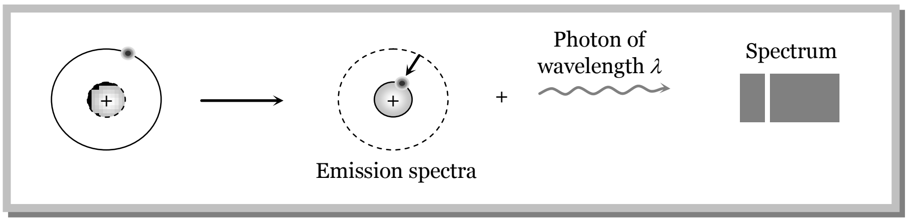
(v) It could not be explained the minute structure in spectrum line.

(vi) This does not explain the Zeeman effect (splitting up of spectral lines in magnetic field) and Stark effect (splitting up in electric field)

(vii) This does not explain the doublets in the spectrum of some of the atoms like sodium (5890$Å$ & 5896$Å$)

Hydrogen Spectrum and Spectral Series

When hydrogen atom is excited, it returns to its normal unexcited (or ground state) state by emitting the energy it had absorbed earlier. This energy is given out by the atom in the form of radiations of different wavelengths as the electron jumps down from a higher to a lower orbit. Transition from different orbits cause different wavelengths, these constitute spectral series which are characteristic of the atom emitting them. When observed through a spectroscope, these radiations are imaged as sharp and straight vertical lines of a single colour.



Spectral series

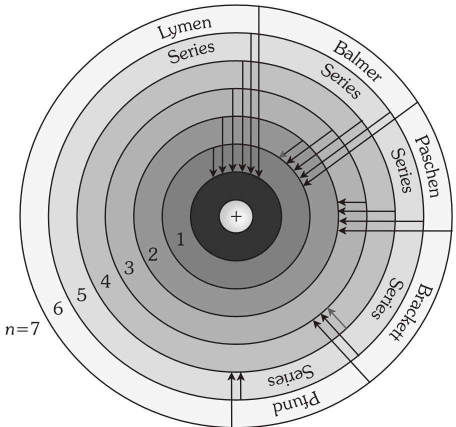
The spectral lines arising from the transition of electron forms a spectra series.

(i) Mainly there are five series and each series is named after it's discover as Lymen series, Balmer series, Paschen series, Bracket series and Pfund series.

(ii) According to the Bohr's theory the wavelength of the radiations emitted from hydrogen atom is given by

$$\frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

where n_2 = outer orbit (electron jumps from this orbit), n_1 = inner orbit (electron falls in this orbit)



(iii) First line of the series is called first member, for this line wavelength is maximum (λ_{max})

(iv) Last line of the series ($n_2 = \infty$) is called series limit, for this line wavelength is minimum (λ_{min})

Spectral series	Transition	Wavelength (λ) $= \frac{n_1^2 n_2^2}{(n_2^2 - n_1^2)R} = \frac{n_1^2}{\left(1 - \frac{n_1^2}{n_2^2}\right)R}$		$\frac{\lambda_{max}}{\lambda_{min}} = \frac{(n+1)^2}{(2n+1)}$	Region
		Maximum wavelength $(n_1 = n$ **and** $n_2 = n+1)$ $\lambda_{max} = \frac{n^2(n+1)^2}{(2n+1)R}$	**Minimum wavelength** $(n_2 = \infty, n_1 = n)$ $\lambda_{min} = \frac{n^2}{R}$		
1. Lymen series	$n_2 = 2, 3, 4 \ldots\infty$ $n_1 = 1$	$\lambda_{max} = \frac{(1)^2(1+1)^2}{(2\times1+1)R} = \frac{4}{3R}$	$n_1 = n = 1$ $\lambda_{min} = \frac{1}{R}$	$\frac{4}{3}$	Ultraviolet region
2.Balmer series	$n_2 = 3, 4, 5 \ldots\infty$ $n_1 = 2$	$n_1 = n = 2$, $n_2 = 2 + 1 = 3$ $\lambda_{max} = \frac{36}{5R}$	$\lambda_{min} = \frac{4}{R}$	$\frac{9}{5}$	Visible region
3. Paschen series	$n_2 = 4, 5, 6 \ldots\infty$ $n_1 = 3$	$n_1 = n = 3$, $n_2 = 3 + 1 = 4$ $\lambda_{max} = \frac{144}{7R}$	$n_1 = n = 3$ $\lambda_{min} = \frac{9}{R}$	$\frac{16}{7}$	Infrared region
4. Bracket series	$n_2 = 5, 6, 7 \ldots \infty$ $n_1 = 4$	$n_1 = n = 4$, $n_2 = 4 + 1 = 5$ $\lambda_{max} = \frac{400}{9R}$	$n_1 = n = 4$ $\lambda_{min} = \frac{16}{R}$	$\frac{25}{9}$	Infrared region
5. Pfund series	$n_2 = 6, 7, 8 \ldots \infty$ $n_1 = 5$	$n_1 = \lambda = 5$, $n_2 = 5 + 1 = 6$ $\lambda_{max} = \frac{900}{11R}$	$\lambda_{min} = \frac{25}{R}$	$\frac{36}{11}$	Infrared region

Quantum Numbers.

An atom contains large number of shells and subshells. These are distinguished from one another on the basis of their size, shape and orientation (direction) in space. The parameters are expressed in terms of different numbers called quantum number.

Quantum numbers may be defined as a set of four number with the help of which we can get complete information about all the electrons in an atom. It tells us the address of the electron *i.e.* location, energy, the type of orbital occupied and orientation of that orbital.

(1) **Principal Quantum number (n) :** This quantum number determines the main energy level or shell in which the electron is present. The average distance of the electron from the nucleus and the energy of the electron depends on it. $E_n \propto \frac{1}{n^2}$ and $r_n \propto n^2$ (in H-atom)

The principal quantum number takes whole number values, n = 1, 2, 3, 4,..... ∞

(2) **Orbital quantum number (l) or azimuthal quantum number (l)**

This represents the number of subshells present in the main shell. These subsidiary orbits within a shell will be denoted as 1, 2, 3, 4 ... or s, p, d, f ... This tells the shape of the subshells.

The orbital angular momentum of the electron is given as $L = \sqrt{l(l+1)}\frac{h}{2\pi}$ (for a particular value of n).

For a given value of n the possible values of l are l = 0, 1, 2, ... upto (n – 1)

(3) **Magnetic quantum number (m_l) :** An electron due to it's angular motion around the nucleus generates an electric field. This electric field is expected to produce a magnetic field. Under the influence of external magnetic field, the electrons of a subshell can orient themselves in certain preferred regions of space around the nucleus called orbitals.

The magnetic quantum number determines the number of preferred orientations of the electron present in a subshell.

The angular momentum quantum number m can assume all integral value between $-l$ to $+l$ including zero. Thus m_l can be – 1, 0, + 1 for $l = 1$. Total values of m_l associated with a particular value of l is given by $(2l + 1)$.

(4) **Spin (magnetic) quantum number (m_s) :** An electron in atom not only revolves around the nucleus but also spins about its own axis. Since an electron can spin either in clockwise direction or in anticlockwise direction. Therefore for any particular value of magnetic quantum number, spin quantum number can have two values, *i.e.* $m_s = \frac{1}{2}$ (Spin up) or $m_s = -\frac{1}{2}$ (Spin down)

This quantum number helps to explain the magnetic properties of the substance.

Electronic Configurations of Atoms

The distribution of electrons in different orbitals of an atom is called the electronic configuration of the atom. The filling of electrons in orbitals is governed by the following rules.

(1) **Pauli's exclusion principle**

"It states that no two electrons in an atom can have all the four quantum number (n, l, m_l and m_s) the same."

It means each quantum state of an electron must have a different set of quantum numbers n, l, m_l and m_s. This principle sets an upper limit on the number of electrons that can occupy a shell.

N_{max} in one shell = $2n^2$; Thus N_{max} in K, L, M, N .... shells are 2, 8, 18, 32,

Note : ❑ The maximum number of electrons in a subshell with orbital quantum number l is $2(2l + 1)$.

(2) **Aufbau principle**

Electrons enter the orbitals of lowest energy first.

As a general rule, a new electron enters an empty orbital for which $(n + l)$ is minimum. In case the value $(n + l)$ is equal for two orbitals, the one with lower value of n is filled first.

Thus the electrons are filled in subshells in the following order (memorize)

1*s*, 2*s*, 2*p*, 3*s*, 3*p*, 4*s*, 3*d*, 4*p*, 5*s*, 4*d*, 5*p*, 6*s*, 4*f*, 5*d*, 6*p*, 7*s*, 5*f*, 6*d*, 7*p*, ...

(3) **Hund's Rule**

When electrons are added to a subshell where more than one orbital of the same energy is available, their spins remain parallel. They occupy different orbitals until each one of them has at least one electron. Pairing starts only when all orbitals are filled up.

Pairing takes place only after filling 3, 5 and 7 electrons in *p*, *d* and *f* orbitals, respectively.

Concepts

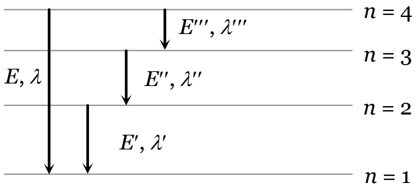
☞ *With the increase in principal quantum number the energy difference between the two successive energy level decreases, while wavelength of spectral line increases.*

$E' > E'' > E'''$

$\lambda' < \lambda'' < \lambda'''$

$E = E' + E'' + E'''$

$$\frac{1}{\lambda} = \frac{1}{\lambda'} + \frac{1}{\lambda''} + \frac{1}{\lambda'''}$$



☞ ***Rydberg constant is different for different elements***

R($=1.09 \times 10^{7}$ m^{-1}) is the value of Rydberg constant when the nucleus is considered to be infinitely massive as compared to the revolving electron. In other words, the nucleus is considered to be stationary.

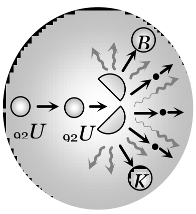
In case, the nucleus is not infinitely massive or stationary, then the value of Rydberg constant is given as $R' = \dfrac{R}{1 + \dfrac{m}{M}}$

where m is the mass of electron and M is the mass of nucleus.

☞ ***Atomic spectrum is a line spectrum***

Each atom has it's own characteristic allowed orbits depending upon the electronic configuration. Therefore photons emitted during transition of electrons from one allowed orbit to inner allowed orbit are of some definite energy only. They do not have a continuous graduation of energy. Therefore the spectrum of the emitted light has only some definite lines and therefore atomic spectrum is line spectrum.

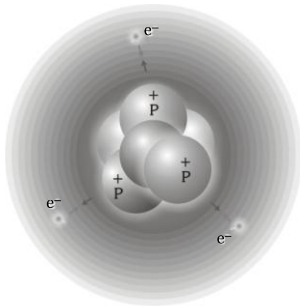
☞ *Just as dots of light of only three colours combine to form almost every conceivable colour on T.V. screen, only about 100 distinct kinds of atoms combine to form all the materials in the universe.*



Nuclear Physics & Radioactivity

Rutherford's α-scattering experiment established that the mass of atom is concentrated with small positively charged region at the centre which is called 'nucleus'.

Nuclei are made up of proton and neutron. The number of protons in a nucleus (called the atomic number or proton number) is represented by the symbol Z. The number of neutrons (neutron number) is represented by N. The total number of neutrons and protons in a nucleus is called it's mass number A so $A = Z + N$.



Neutrons and proton, when described collectively are called ***nucleons.***

Nucleus contains two types of particles : Protons and neutrons

Nuclides are represented as $_ZX^A$; where X denotes the chemical symbol of the element.

Neutron

Neutron is a fundamental particle which is essential constituent of all nuclei except that of hydrogen atom. It was discovered by Chadwick.

(1) The charge of neutron : It is neutral

(2) The mass of neutron : 1.6750×10^{-27} *kg*

(3) It's spin angular momentum : $\frac{1}{2} \times \left(\frac{h}{2\pi}\right) J\text{ - }s$

(4) It's magnetic moment : 9.57×10^{-27} *J/Tesla*

(5) It's half life : 12 minutes

(6) Penetration power : High

(7) Types : Neutrons are of two types slow neutron and fast neutron, both are fully capable of penetrating a nucleus and causing artificial disintegration.

> A free neutron outside the nucleus is unstable and decays into proton and electron.
>
> $$_0n^1 \rightarrow \underset{\text{Proton}}{_1H^1} + \underset{\text{Electron}}{_{-1}\beta^0} + \underset{\text{Antinutrino}}{\bar{\nu}}$$

Thermal neutrons

Fast neutrons can be converted into slow neutrons by certain materials called moderator's (Paraffin wax, heavy water, graphite) when fast moving neutrons pass through a moderator, they collide with the molecules of the moderator, as a result of this, the energy of moving neutron decreases while that of the molecules of the moderator increases. After sometime they both attains same energy. The neutrons are then in thermal equilibrium with the molecules of the moderator and are called thermal neutrons.

Note : ❑ Energy of thermal neutron is about 0.025 *eV* and speed is about 2.2 *km/s*.

Nucleus

(1) **Different types of nuclei**

The nuclei have been classified on the basis of the number of protons (atomic number) or the total number of nucleons (mass number) as follows

(i) **Isotopes :** The atoms of element having same atomic number but different mass number are called isotopes. All isotopes have the same chemical properties. The isotopes of some elements are the following

$$_1H^1,\ _1H^2,\ _1H^3 \quad _8O^{16},\ _8O^{17},\ _8O^{18} \quad _2He^3,\ _2He^4 \quad _{17}Cl^{35},\ _{17}Cl^{37} \quad _{92}U^{235},\ _{92}U^{238}$$

(ii) **Isobars :** The nuclei which have the same mass number (A) but different atomic number (Z) are called isobars. Isobars occupy different positions in periodic table so all isobars have different chemical properties. Some of the examples of isobars are

$$_1H^3 \text{ and } _2He^3, \quad _6C^{14} \text{ and } _7N^{14}, \ _8O^{17} \text{ and } _9F^{17}$$

(iii) **Isotones :** The nuclei having equal number of neutrons are called isotones. For them both the atomic number (Z) and mass number (A) are different, but the value of ($A - Z$) is same. Some examples are

$$_4Be^9 \text{ and } _5B^{10},\ _6C^{13} \text{ and } _7N^{14},\ _8O^{18} \text{ and } _9F^{19},\ _3Li^7 \text{ and } _4Be^8,\ _1H^3 \text{ and } _2He^4$$

(iv) **Mirror nuclei :** Nuclei having the same mass number A but with the proton number (Z) and neutron number ($A - Z$) interchanged (or whose atomic number differ by 1) are called mirror nuclei for example.

$$_1H^3 \text{ and } _2He^3, \ _3Li^7 \text{ and } _4Be^7$$

(2) **Size of nucleus**

(i) Nuclear radius : Experimental results indicates that the nuclear radius is proportional to $A^{1/3}$, where A is the mass number of nucleus *i.e.* $R \propto A^{1/3} \Rightarrow R = R_0 A^{1/3}$, where $R_0 = 1.2 \times 10^{-15}\ m = 1.2\ fm$.

Note : ❑ Heavier nuclei are bigger in size than lighter nuclei.

(ii) Nuclear volume : The volume of nucleus is given by $V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi R_0^3 A \Rightarrow V \propto A$

(iii) Nuclear density : Mass per unit volume of a nucleus is called nuclear density.

$$\text{Nuclear density}(\rho) = \frac{\text{Mass of nucleus}}{\text{Volume of nucleus}} = \frac{mA}{\frac{4}{3}\pi (R_0 A^{1/3})^3}$$

where m = Average of mass of a nucleon (= mass of proton + mass of neutron = $1.66 \times 10^{-27}\ kg$)

and mA = Mass of nucleus

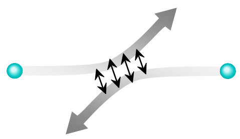
$$\Rightarrow \rho = \frac{3m}{4\pi R_0^3} = 2.38 \times 10^{17}\ kg/m^3$$

Note : ❑ ρ is independent of A, it means ρ is same of all atoms.

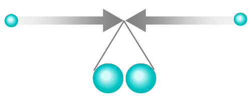
❑ Density of a nucleus is maximum at it's centre and decreases as we move outwards from the nucleus.

(3) **Nuclear force**

Forces that keep the nucleons bound in the nucleus are called nuclear forces.



At low speeds, electromagnetic repulsion prevents the collision of nuclei



At high speeds, nuclei come close enough for the strong force to bind them together.

(i) Nuclear forces are short range forces. These do not exist at large distances greater than $10^{-15}\ m$.

(ii) Nuclear forces are the strongest forces in nature.

(iii) These are attractive force and causes stability of the nucleus.

(iv) These forces are charge independent.

(v) Nuclear forces are non-central force.

Nuclear forces are exchange forces

According to scientist Yukawa the nuclear force between the two nucleons is the result of the exchange of particles called mesons between the nucleons.

π - mesons are of three types – Positive π meson (π^+), negative π meson (π^-), neutral π *meson* (π^0)

The force between neutron and proton is due to exchange of charged meson between them *i.e.*

$$p \rightarrow \pi^+ + n, \quad n \rightarrow p + \pi^-$$

The forces between a pair of neutrons or a pair of protons are the result of the exchange of neutral meson (π^0) between them *i.e.* $p \rightarrow p' + \pi^0$ and $n \rightarrow n' + \pi^0$

Thus exchange of π meson between nucleons keeps the nucleons bound together. It is responsible for the nuclear forces.

Dog-Bone analogy

The above interactions can be explained with the dog bone analogy according to which we consider the two interacting nucleons to be two dogs having a common bone clenched in between their teeth very firmly. Each one of these dogs wants to take the bone and hence they cannot be separated easily. They seem to be bound to each other with a strong attractive force (which is the bone) though the dogs themselves are strong enemies. The meson plays the same role of the common bone in between two nucleons.

(4) **Atomic mass unit (*amu*)**

The unit in which atomic and nuclear masses are measured is called atomic mass unit (*amu*)

1 *amu* (or 1*u*) = $\frac{1}{12}th$ of mass of ${}_6C^{12}$ atom = $1.66 \times 10^{-27}\ kg$

Masses of electron, proton and neutrons

Mass of electron (m_e) = $9.1 \times 10^{-31}\ kg$ = 0.0005486 *amu*, Mass of proton (m_p) = $1.6726 \times 10^{-27}\ kg$ = 1.007276 *amu*

Mass of neutron (m_n) = $1.6750 \times 10^{-27}\ kg$ = 1.00865 *amu*, Mass of hydrogen atom ($m_e + m_p$) = $1.6729 \times 10^{-27}\ kg$ = 1.0078 *amu*

Mass-energy equivalence

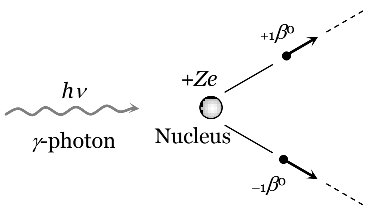
According to Einstein, mass and energy are inter convertible. The Einstein's mass energy relationship is given by $E = mc^2$

If $m = 1\ amu$, $c = 3 \times 10^8\ m/sec$ then $E = 931\ MeV$ *i.e.* 1 *amu* is equivalent to 931 *MeV* or **1 *amu* (or 1 *u*) = 931 *MeV***

(5) **Pair production and pair-annihilation**

When an energetic γ-ray photon falls on a heavy substance. It is absorbed by some nucleus of the substance and an electron and a positron are produced. This phenomenon is called pair production and may be represented by the following equation

$$\underset{(\gamma-\text{photon})}{h\nu} = \underset{(\text{Positron})}{{}_1\beta^0} + \underset{(\text{Electron})}{{}_{-1}\beta^0}$$



The rest-mass energy of each of positron and electron is

$$E_0 = m_0c^2 = (9.1 \times 10^{-31}\ kg) \times (3.0 \times 10^8\ m/s)^2$$

$$= 8.2 \times 10^{-14}\ J = \mathbf{0.51\ MeV}$$

Hence, for pair-production it is essential that the energy of γ-photon must be at least $2 \times 0.51 = 1.02\ MeV$. If the energy of γ-photon is less than this, it would cause photo-electric effect or Compton effect on striking the matter.

The converse phenomenon pair-annihilation is also possible. Whenever an electron and a positron come very close to each other, they annihilate each other by combining together and two γ-photons (energy) are produced. This phenomenon is called pair annihilation and is represented by the following equation.

$$\underset{(\text{Positron})}{{}_{+1}\beta^0} + \underset{(\text{Electron})}{{}_{-1}\beta^0} = \underset{(\gamma\text{-photon})}{h\nu} + \underset{(\gamma\text{-photon})}{h\nu}$$

(6) **Nuclear stability**

Among about 1500 known nuclides, less than 260 are stable. The others are unstable that decay to form other nuclides by emitting α, β-particles and γ - EM waves. (This process is called radioactivity). The stability of nucleus is determined by many factors. Few such factors are given below :

(i) Neutron-proton ratio $\left(\frac{N}{Z}\text{ Ratio}\right)$

The chemical properties of an atom are governed entirely by the number of protons (Z) in the nucleus, the stability of an atom appears to depend on both the number of protons and the number of neutrons.

For lighter nuclei, the greatest stability is achieved when the number of protons and neutrons are approximately equal ($N \approx Z$) *i.e.* $\frac{N}{Z} = 1$

Heavy nuclei are stable only when they have more neutrons than protons. Thus heavy nuclei are neutron rich compared to lighter nuclei (for heavy nuclei, more is the number of protons in the nucleus, greater is the electrical repulsive force between them. Therefore more neutrons are added to provide the strong attractive forces necessary to keep the nucleus stable.)

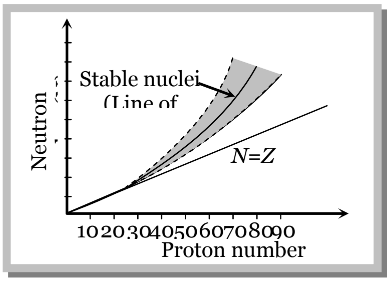
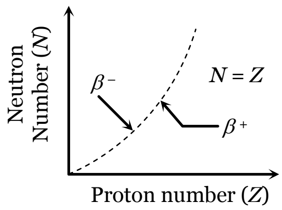


Figure shows a plot of *N* verses *Z* for the stable nuclei. For mass number upto about *A* = 40. For larger value of *Z* the nuclear force is unable to hold the nucleus together against the electrical repulsion of the protons unless the number of neutrons exceeds the number of protons. At *Bi* (*Z* = 83, *A* = 209), the neutron excess in *N* – *Z* = 43. There are no stable nuclides with *Z* > 83.

Note : ❑ The nuclide $_{83}Bi^{209}$ is the heaviest stable nucleus.

❑ A nuclide above the line of stability *i.e.* having excess neutrons, decay through β^- emission (neutron changes into proton). Thus increasing atomic number *Z* and decreasing neutron number *N*. In β^- emission, $\frac{N}{Z}$ ratio decreases.

A nuclide below the line of stability have excess number of protons. It decays by β^+ emission, results in decreasing *Z* and increasing *N*. In β^+ emission, the $\frac{N}{Z}$ ratio increases.



(ii) Even or odd numbers of *Z* or *N* : The stability of a nuclide is also determined by the consideration whether it contains an even or odd number of protons and neutrons.

It is found that an even-even nucleus (even *Z* and even *N*) is more stable (60% of stable nuclide have even *Z* and even *N*).

An even-odd nucleus (even *Z* and odd *N*) or odd-even nuclide (odd *Z* and even *N*) is found to be lesser sable while the odd-odd nucleus is found to be less stable.

Only five stable odd-odd nuclides are known : $_1H^2$, $_3Li^6$, $_5Be^{10}$, $_7N^{14}$ and $_{75}Ta^{180}$

(iii) Binding energy per nucleon : The stability of a nucleus is determined by value of it's binding energy per nucleon. In general higher the value of binding energy per nucleon, more stable the nucleus is

Mass Defect and Binding Energy.

(1) **Mass defect (Δm)**

It is found that the mass of a nucleus is always less than the sum of masses of it's constituent nucleons in free state. This difference in masses is called mass defect. Hence mass defect

Δm = Sum of masses of nucleons – Mass of nucleus

$$= \{Zm_p + (A-Z)m_n\} - M = \{Zm_p + Zm_e + (A-Z)m_z\} - M'$$

where m_p = Mass of proton, m_n = Mass of each neutron, m_e = Mass of each electron

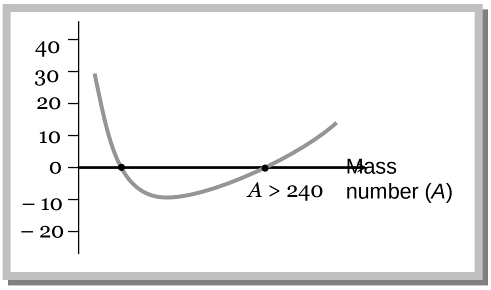
M = Mass of nucleus, Z = Atomic number, A = Mass number, M' = Mass of atom as a whole.

Note : ❑ The mass of a typical nucleus is about 1% less than the sum of masses of nucleons.

(2) **Packing fraction**

Mass defect per nucleon is called packing fraction

Packing fraction $(f) = \dfrac{\Delta m}{A} = \dfrac{M-A}{A}$ where M = Mass of nucleus, A = Mass number



Packing fraction measures the stability of a nucleus. Smaller the value of packing fraction, larger is the stability of the nucleus.

(i) Packing fraction may be of positive, negative or zero value.

(iii) At $A = 16, f \to$ Zero

(3) **Binding energy (B.E.)**

The neutrons and protons in a stable nucleus are held together by nuclear forces and energy is needed to pull them infinitely apart (or the same energy is released during the formation of the nucleus). This energy is called the binding energy of the nucleus.

or

The binding energy of a nucleus may be defined as the energy equivalent to the mass defect of the nucleus.

If Δm is mass defect then according to Einstein's mass energy relation

Binding energy = $\Delta m \cdot c^2 = [\{m_p Z + m_n(A-Z)\} - M]\cdot c^2$

(This binding energy is expressed in *joule*, because Δm is measured in *kg*)

If Δm is measured in *amu* then binding energy = Δm *amu* = $[\{m_p Z + m_n(A-Z)\} - M]$ *amu* = $\Delta m \times 931$ *MeV*

(4) **Binding energy per nucleon**

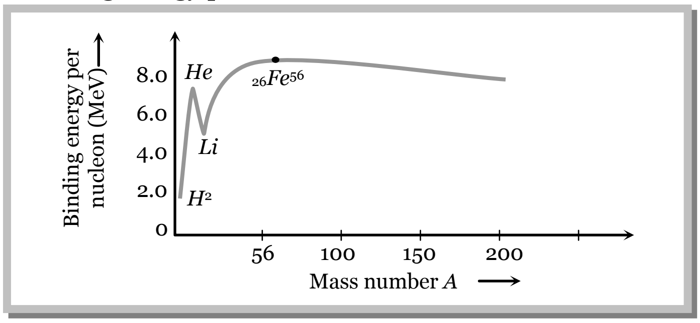
The average energy required to release a nucleon from the nucleus is called binding energy per nucleon.

$$\text{Binding energy per nucleon} = \frac{\text{Total binding energy}}{\text{Mass number } (i.e.\text{ total number of nucleons})} = \frac{\Delta m \times 931}{A}\,\frac{MeV}{Nucleon}$$

Binding energy per nucleon $\propto$ Stability of nucleus

Binding Energy Curve.

It is the graph between binding energy per nucleon and total number of nucleons (*i.e.* mass number A)



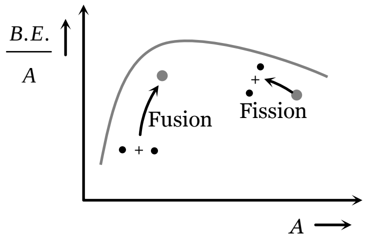
(1) Some nuclei with mass number $A < 20$ have large binding energy per nucleon than their neighbour nuclei. For example $_2He^4$, $_4Be^8$, $_6C^{12}$, $_8O^{16}$ and $_{10}Ne^{20}$. These nuclei are more stable than their neighbours.

(2) The binding energy per nucleon is maximum for nuclei of mass number A = 56 $({}_{26}Fe^{56})$. It's value is 8.8 *MeV* per nucleon.

(3) For nuclei having A > 56, binding energy per nucleon gradually decreases for uranium (A = 238), the value of binding energy per nucleon drops to 7.5 *MeV*.

Note : ❑ When a heavy nucleus splits up into lighter nuclei, then binding energy per nucleon of lighter nuclei is more than that of the original heavy nucleus. Thus a large amount of energy is liberated in this process (nuclear fission).

❑ When two very light nuclei combines to form a relatively heavy nucleus, then binding energy per nucleon increases. Thus, energy is released in this process (nuclear fusion).



Nuclear Reactions.

The process by which the identity of a nucleus is changed when it is bombarded by an energetic particle is called nuclear reaction. The general expression for the nuclear reaction is as follows.

$$\underset{\text{(Parent nucleus)}}{X} + \underset{\text{(Incident particle)}}{a} \longrightarrow \underset{\text{(Compound nucleus)}}{C} \longrightarrow \underset{\text{(Compound nucleus)}}{Y} + \underset{\text{(Product particles)}}{b} + \underset{\text{(Energy)}}{Q}$$

Here X and a are known as reactants and Y and b are known as products. This reaction is known as (a, b) reaction and can be represented as $X(a, b)\,Y$

(1) ***Q* value or energy of nuclear reaction**

The energy absorbed or released during nuclear reaction is known as Q-value of nuclear reaction.

Q-value = (Mass of reactants – mass of products)c^2 *Joules*

= (Mass of reactants – mass of products) *amu*

If Q < 0, The nuclear reaction is known as endothermic. (The energy is absorbed in the reaction)

If Q > 0, The nuclear reaction is known as exothermic (The energy is released in the reaction)

(2) **Law of conservation in nuclear reactions**

(i) Conservation of mass number and charge number : In the following nuclear reaction

$$ {}_2He^4 + {}_7N^{14} \rightarrow {}_8O^{17} + {}_1H^1 $$

	Before the reaction	After the reaction
Mass number (A) →	4 +14 = 18	17 + 1 = 18
Charge number (Z) →	2 + 7 = 9	8 + 1 = 9

(ii) Conservation of momentum : Linear momentum/angular momentum of particles before the reaction is equal to the linear/angular momentum of the particles after the reaction. That is $\Sigma p = 0$

(iii) Conservation of energy : Total energy before the reaction is equal to total energy after the reaction. Term Q is added to balance the total energy of the reaction.

(3) **Common nuclear reactions**

The nuclear reactions lead to artificial transmutation of nuclei. Rutherford was the first to carry out artificial transmutation of nitrogen to oxygen in the year 1919.

$$ {}_2He^4 + {}_7N^{14} \rightarrow {}_9F^{18} \rightarrow {}_8O^{17} + {}_1H^1 $$

It is called (α, p) reaction. Some other nuclear reactions are given as follows.

(p, n) reaction $\Rightarrow$ ${}_1H^1 + {}_5B^{11} \rightarrow {}_6C^{12} \rightarrow {}_6C^{11} + {}_0n^1$

(p, α) reaction $\Rightarrow$ ${}_1H^1 + {}_3Li^{11} \rightarrow {}_4Be^8 \rightarrow {}_2He^4 + {}_2He^4$

(p, γ) reaction $\Rightarrow$ ${}_1H^1 + {}_6C^{12} \rightarrow {}_7N^{13} \rightarrow {}_7N^{13} + \gamma$

(n, p) reaction $\Rightarrow$ ${}_0n^1 + {}_7N^{14} \rightarrow {}_7N^{15} \rightarrow {}_6C^{14} + {}_1H^1$

(γ, n) reaction $\Rightarrow$ $\gamma + {}_1H^2 \rightarrow {}_1H^1 + {}_0n^1$

Nuclear Fission and Fusion

Nuclear fission

The process of splitting of a heavy nucleus into two lighter nuclei of comparable masses (after bombardment with a energetic particle) with liberation of energy is called nuclear fission.

The phenomenon of nuclear fission was discovered by scientist Ottohann and F. Strassman and was explained by N. Bohr and J.A. Wheeler on the basis of liquid drop model of nucleus.

(1) **Fission reaction of U^{235}**

(i) Nuclear reaction :

$$ {}_{92}U^{235} + {}_0n^1 \rightarrow \underset{\text{(unstable nucleus)}}{{}_{92}U^{236}} \rightarrow {}_{56}Ba^{141} + {}_{36}Kr^{92} + 3{}_0n^1 + Q$$

(ii) The energy released in U^{235} fission is about 200 MeV or 0.8 MeV per nucleon.

(iii) By fission of ${}_{92}U^{235}$, on an average 2.5 neutrons are liberated. These neutrons are called fast neutrons and their energy is about 2 MeV (for each). These fast neutrons can escape from the reaction so as to proceed the chain reaction they are need to slow down.

(iv) Fission of U^{235} occurs by slow neutrons only (of energy about 1eV) or even by thermal neutrons (of energy about 0.025 eV).

(v) 50 kg of U^{235} on fission will release $\approx 4 \times 10^{15}$ J of energy. This is equivalence to 20,000 tones of TNT explosion. The nuclear bomb dropped at Hiroshima had this much explosion power.

(vi) The mass of the compound nucleus must be greater than the sum of masses of fission products.

(vii) The $\dfrac{\text{Binding energy}}{\text{A}}$ of compound nucleus must be less than that of the fission products.

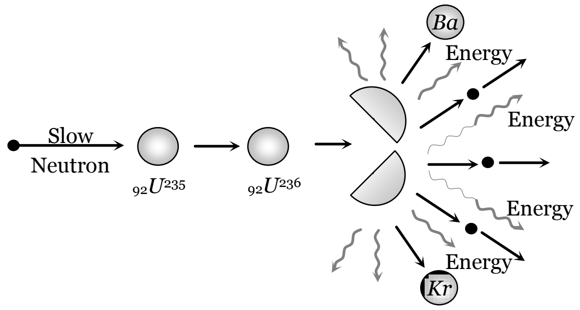
(viii) It may be pointed out that it is not necessary that in each fission of uranium, the two fragments ${}_{56}Ba$ and ${}_{36}Kr$ are formed but they may be any stable isotopes of middle weight atoms.

Same other U^{235} fission reactions are

$$ {}_{92}U^{235} + {}_0n^1 \rightarrow {}_{54}Xe^{140} + {}_{38}Sr^{94} + 2{}_0n^1$$
$$\rightarrow {}_{57}La^{148} + {}_{35}Br^{85} + 3\,{}_0n^1$$
$$\rightarrow \text{Many more}$$

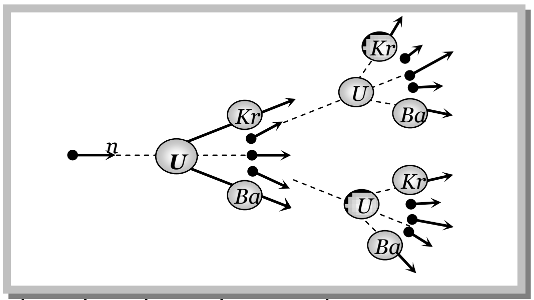
(ix) The neutrons released during the fission process are called prompt neutrons.

(x) Most of energy released appears in the form of kinetic energy of fission fragments.



(2) **Chain reaction**

In nuclear fission, three neutrons are produced along with the release of large energy. Under favourable conditions, these neutrons can cause further fission of other nuclei, producing large number of neutrons. Thus a chain of nuclear fissions is established which continues until the whole of the uranium is consumed.



In the chain reaction, the number of nuclei undergoing fission increases very fast. So, the energy produced takes a tremendous magnitude very soon.

Difficulties in chain reaction

(i) Absorption of neutrons by U^{238}, the major part in natural uranium is the isotope U^{238} (99.3%), the isotope U^{235} is very little (0.7%). It is found that U^{238} is fissionable with fast neutrons, whereas U^{235} is fissionable with slow neutrons. Due to the large percentage of U^{238}, there is more possibility of collision of neutrons with U^{238}. It is found that the neutrons get slowed on coliding with U^{238}, as a result of it further fission of U^{238} is not possible (Because they are slow and they are absorbed by U^{238}). This stops the chain reaction.

Removal : (i) To sustain chain reaction ${}_{92}U^{235}$ is separated from the ordinary uranium. Uranium so obtained $\left({}_{92}U^{235}\right)$ is known as enriched uranium, which is fissionable with the fast and slow neutrons and hence chain reaction can be sustained.

(ii) If neutrons are slowed down by any method to an energy of about 0.3 *eV*, then the probability of their absorption by U^{238} becomes very low, while the probability of their fissioning U^{235} becomes high. This job is done by moderators. Which reduce the speed of neutron rapidly graphite and heavy water are the example of moderators.

(iii) Critical size : The neutrons emitted during fission are very fast and they travel a large distance before being slowed down. If the size of the fissionable material is small, the neutrons emitted will escape the fissionable material before they are slowed down. Hence chain reaction cannot be sustained.

Removal : The size of the fissionable material should be large than a critical size.

The chain reaction once started will remain steady, accelerate or retard depending upon, a factor called neutron reproduction factor (k). It is defined as follows.

$$k = \frac{\text{Rate of production of neutrons}}{\text{Rate of loss of neutrons}}$$

→ If $k = 1$, the chain reaction will be steady. The size of the fissionable material used is said to be the critical size and it's mass, the critical mass.

→ If $k > 1$, the chain reaction accelerates, resulting in an explosion. The size of the material in this case is super critical. (Atom bomb)

→ If $k < 1$, the chain reaction gradually comes to a halt. The size of the material used us said to be sub-critical.

Types of chain reaction : Chain reactions are of following two types

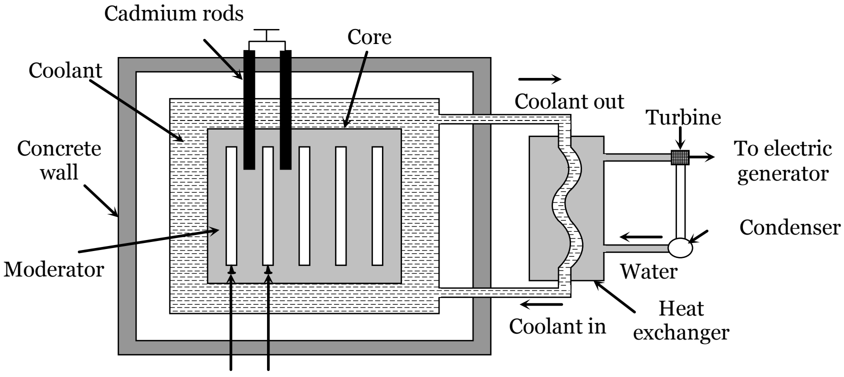
Controlled chain reaction	Uncontrolled chain reaction
Controlled by artificial method	No control over this type of nuclear reaction
All neurons are absorbed except one	More than one neutron takes part into reaction
It's rate is slow	Fast rate

Reproduction factor $k = 1$	Reproduction factor $k > 1$
Energy liberated in this type of reaction is always less than explosive energy	A large amount of energy is liberated in this type of reaction
Chain reaction is the principle of nuclear reactors	Uncontrolled chain reaction is the principle of atom bomb.

Note : ❑ The energy released in the explosion of an atom bomb is equal to the energy released by 2000 tonn of TNT and the temperature at the place of explosion is of the order of 10^7 ^oC.

Nuclear Reactor

A nuclear reactor is a device in which nuclear fission can be carried out through a sustained and a controlled chain reaction. It is also called an atomic pile. It is thus a source of controlled energy which is utilised for many useful purposes.



(1) **Parts of nuclear reactor**

(i) **Fissionable material (Fuel) :** The fissionable material used in the reactor is called the fuel of the reactor. Uranium isotope (U^{235}) Thorium isotope (Th^{232}) and Plutonium isotopes (Pu^{239}, Pu^{240} and Pu^{241}) are the most commonly used fuels in the reactor.

(ii) **Moderator :** Moderator is used to slow down the fast moving neutrons. Most commonly used moderators are graphite and heavy water (D_2O).

(iii) **Control Material :** Control material is used to control the chain reaction and to maintain a stable rate of reaction. This material controls the number of neutrons available for the fission. For example, cadmium rods are inserted into the core of the reactor because they can absorb the neutrons. The neutrons available for fission are controlled by moving the cadmium rods in or out of the core of the reactor.

(iv) **Coolant :** Coolant is a cooling material which removes the heat generated due to fission in the reactor. Commonly used coolants are water, CO_2 nitrogen *etc.*

(v) **Protective shield :** A protective shield in the form a concrete thick wall surrounds the core of the reactor to save the persons working around the reactor from the hazardous radiations.

Note : ❑ It may be noted that Plutonium is the best fuel as compared to other fissionable material. It is because fission in Plutonium can be initiated by both slow and fast neutrons. Moreover it can be obtained from U^{238}.

❑ Nuclear reactor is firstly devised by fermi. ❑ Apsara was the first Indian nuclear reactor.

(2) **Uses of nuclear reactor**

(i) In electric power generation.

(ii) To produce radioactive isotopes for their use in medical science, agriculture and industry.

(iii) In manufacturing of PU^{239} which is used in atom bomb.

(iv) They are used to produce neutron beam of high intensity which is used in the treatment of cancer and nuclear research.

Note : ❑ A type of reactor that can produce more fissile fuel than it consumes is the breeder reactor.

Nuclear fusion

In nuclear fusion two or more than two lighter nuclei combine to form a single heavy nucleus. The mass of single nucleus so formed is less than the sum of the masses of parent nuclei. This difference in mass results in the release of tremendous amount of energy

$${}_1H^2 + {}_1H^2 \rightarrow {}_1H^3 + {}_1H^1 + 4\,MeV$$

$${}_1H^3 + {}_1H^2 \rightarrow {}_2He^4 + {}_0n^1 + 17.6\,MeV$$

or $${}_1H^2 + {}_1H^2 \rightarrow {}_2He^4 + 24\,MeV$$

For fusion high pressure ($\approx 10^6$ *atm*) and high temperature (of the order of 10^7 *K* to 10^8 *K*) is required and so the reaction is called thermonuclear reaction.

Fusion energy is greater then fission energy fission of one uranium atom releases about 200 *MeV* of energy. But the fusion of a deutron (${}_1H^2$) and triton (${}_1H^3$) releases about 17.6 *MeV* of energy. However the energy released per nucleon in fission is about 0.85 *MeV* but that in fusion is 4.4 *MeV*. So for the same mass of the fuel, the energy released in fusion is much larger than in fission.

Plasma : The temperature of the order of 10^8 *K* required for thermonuclear reactions leads to the complete ionisation of the atom of light elements. The combination of base nuclei and electron cloud is called plasma. The enormous gravitational field of the sun confines the plasma in the interior of the sun.

The main problem to carryout nuclear fusion in the laboratory is to contain the plasma at a temperature of $10^8 K$. No solid container can tolerate this much temperature. If this problem of containing plasma is solved, then the large quantity of deuterium present in sea water would be able to serve as in-exhaustible source of energy.

Note : ❑ To achieve fusion in laboratory a device is used to confine the plasma, called **Tokamak**.

Stellar Energy

Stellar energy is the energy obtained continuously from the sun and the stars. Sun radiates energy at the rate of about 10^{26} *joules* per *second*.

Scientist Hans Bethe suggested that the fusion of hydrogen to form helium (thermo nuclear reaction) is continuously taking place in the sun (or in the other stars) and it is the source of sun's (star's) energy.

The stellar energy is explained by two cycles

Proton-proton cycle	Carbon-nitrogen cycle
${}_1H^1 + {}_1H^1 \rightarrow {}_1H^2 + {}_1e^0 + Q_1$	${}_1H^1 + {}_6C^{12} \rightarrow {}_7N^{13} + Q_1$
${}_1H^2 + {}_1H^1 \rightarrow {}_2He^3 + Q_2$	${}_7N^{13} \rightarrow {}_6C^{13} + {}_{+1}e^0$
${}_2He^3 + {}_2He^3 \rightarrow {}_2He^4 + 2\,{}_1H^1 + Q_3$	${}_1H^1 + {}_6C^{13} \rightarrow {}_7N^{14} + Q_2$
$4\,{}_1H^1 \rightarrow {}_2He^4 + 2\,{}_{+1}e^0 + 2\gamma + 26.7\,MeV$	${}_1H^1 + {}_7N^{14} \rightarrow {}_8O^{15} + Q_3$
	${}_8O^{15} \rightarrow {}_7N^{15} + {}_1e^0 + Q_4$
	${}_1H^1 + {}_7N^{15} \rightarrow {}_6C^{12} + {}_2He^4$
	$4\,{}_1H^1 \rightarrow {}_2He^4 + 2\,{}_1e^0 + 24.7\,MeV$

About 90% of the mass of the sun consists of hydrogen and helium.

Nuclear Bomb Based on uncontrolled nuclear reactions.

Atom bomb	Hydrogen bomb
Based on fission process it involves the fission of U^{235}	Based on fusion process. Mixture of deutron and tritium is used in it
In this critical size is important	There is no limit to critical size
Explosion is possible at normal temperature and pressure	High temperature and pressure are required
Less energy is released compared to hydrogen bomb	More energy is released as compared to atom bomb so it is more dangerous than atom bomb

Concepts

☞ *A test tube full of base nuclei will weight heavier than the earth.*

☞ *The nucleus of hydrogen contains only one proton. Therefore we may say that the proton is the nucleus of hydrogen atom.*

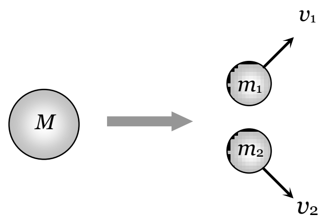
☞ *If the relative abundance of isotopes in an element has a ratio $n_1 : n_2$ whose atomic masses are m_1 and m_2 then atomic mass of the element is* $M = \frac{n_1 m_1 + n_2 m_2}{n_1 + n_2}$

Examples

Example: 1 A heavy nucleus at rest breaks into two fragments which fly off with velocities in the ratio 8 : 1. The ratio of radii of the fragments is

(a) 1 : 2 (b) 1 : 4 (c) 4 : 1 (d) 2 : 1

Solution : (a)



By conservation of momentum $m_1v_1 = m_2v_2$

$\Rightarrow \frac{v_1}{v_2} = \frac{8}{1} = \frac{m_2}{m_1}$ ...... (i)

Also from $r \propto A^{1/3} \Rightarrow \frac{r_1}{r_2} = \left(\frac{A_1}{A_2}\right)^{1/3} = \left(\frac{1}{8}\right)^{1/3} = \frac{1}{2}$

Example: 2 The ratio of radii of nuclei $_{13}^{27}Al$ and $_{52}^{125}Te$ is approximately **[J & K CET 2000]**

(a) 6 : 10 (b) 13 : 52 (c) 40 : 177 (d) 14 : 7

Solution : (a) By using $r \propto A^{1/3} \Rightarrow \frac{r_1}{r_2} = \left(\frac{A_1}{A_2}\right)^{1/3} = \left(\frac{27}{125}\right)^{1/3} = \frac{8}{5} = \frac{6}{10}$

Example: 3 If Avogadro's number is 6×10^{23} then the number of protons, neutrons and electrons in 14 *g* of $_6C^{14}$ are respectively

(a) $36 \times 10^{23}, 48 \times 10^{23}, 36 \times 10^{23}$ (b) $36 \times 10^{23}, 36 \times 10^{23}, 36 \times 10^{21}$

(c) $48 \times 10^{23}, 36 \times 10^{23}, 48 \times 10^{21}$ (d) $48 \times 10^{23}, 48 \times 10^{23}, 36 \times 10^{21}$

Solution : (a) Since the number of protons, neutrons and electrons in an atom of $_6C^{14}$ are 6, 8 and 6 respectively. As 14 *gm* of $_6C^{14}$ contains 6×10^{23} atoms, therefore the numbers of protons, neutrons and electrons in 14 *gm* of $_6C^{14}$ are $6 \times 6 \times 10^{23} = 36 \times 10^{23}$, $8 \times 6 \times 10^{23} = 48 \times 10^{23}$, $6 \times 6 \times 10^{23} = 36 \times 10^{23}$.

Example: 4 Two Cu^{64} nuclei touch each other. The electrostatics repulsive energy of the system will be

(a) 0.788 *MeV* (b) 7.88 *MeV* (c) 126.15 *MeV* (d) 788 *MeV*

Solution : (c) Radius of each nucleus $R = R_0(A)^{1/3} = 1.2\,(64)^{1/3} = 4.8\ fm$

Distance between two nuclei (r) = 2R

So potential energy $U = \frac{k \cdot q^2}{r} = \frac{9 \times 10^9 \times (1.6 \times 10^{-19} \times 29)^2}{2 \times 4.8 \times 10^{-15} \times 1.6 \times 10^{-19}} = 126.15\ MeV$.

Example: 5 When $_{92}U^{235}$ undergoes fission. 0.1% of its original mass is changed into energy. How much energy is released if 1 *kg* of $_{92}U^{235}$ undergoes fission **[MP PET 1994; MP PMT/PET 1998; BHU 2001; BVP 2003]**

(a) $9 \times 10^{10}\ J$ (b) $9 \times 10^{11}\ J$ (c) $9 \times 10^{12}\ J$ (d) $9 \times 10^{13}\ J$

Solution : (d) By using $E = \Delta m \cdot c^2 \Rightarrow E = \left(\frac{0.1}{100} \times 1\right)(3 \times 10^8)^2 = 9 \times 10^{13}\ J$

Example: 6 1 *g* of hydrogen is converted into 0.993 *g* of helium in a thermonuclear reaction. The energy released is **[EAMCET (Med.) 1995; CPMT 1999]**

(a) $63 \times 10^7\ J$ (b) $63 \times 10^{10}\ J$ (c) $63 \times 10^{14}\ J$ (d) $63 \times 10^{20}\ J$

Solution : (b) $\Delta m = 1 - 0.993 = 0.007\ gm$

$\therefore E = \Delta mc^2 = 0.007 \times 10^{-3} \times (3 \times 10^8)^2 = 63 \times 10^{10}\ J$

Example: 7 The binding energy per nucleon of deuteron $({}_1^2H)$ and helium nucleus $({}_2^4He)$ is 1.1 *MeV* and 7 *MeV* respectively. If two deuteron nuclei react to form a single helium nucleus, then the energy released is

[MP PMT 1992; Roorkee 1994; IIT-JEE 1996; AIIMS 1997; Haryana PMT 2000; Pb PMT 2001; CPMT 2001; AIEEE 2004]

(a) 13.9 *MeV* (b) 26.9 *MeV* (c) 23.6 *MeV* (d) 19.2 *MeV*

Solution : (c) ${}_1H^2 + {}_1H^2 \rightarrow {}_2He^4 + Q$

Total binding energy of helium nucleus = 4 × 7 = 28 *MeV*

Total binding energy of each deutron = 2 × 1.1 = 2.2 *MeV*

Hence energy released = 28 – 2 × 2.2 = 23.6 *MeV*

Example: 8 The masses of neutron and proton are 1.0087 *amu* and 1.0073 *amu* respectively. If the neutrons and protons combine to form a helium nucleus (alpha particles) of mass 4.0015 *amu*. The binding energy of the helium nucleus will be [1 *amu*= 931 *MeV*] **[CPMT 1986; MP PMT 1995; CBSE 2003]**

(a) 28.4 *MeV* (b) 20.8 *MeV* (c) 27.3 *MeV* (d) 14.2 *MeV*

Solution : (a) Helium nucleus consist of two neutrons and two protons.

So binding energy $E = \Delta m$ *amu* $= \Delta m \times 931$ *MeV*

$\Rightarrow E = (2 \times m_p + 2m_n - M) \times 931\ MeV = (2 \times 1.0073 + 2 \times 1.0087 - 4.0015) \times 931 = 28.4\ MeV$

Example: 9 A atomic power reactor furnace can deliver 300 *MW*. The energy released due to fission of each of uranium atom U^{238} is 170 *MeV*. The number of uranium atoms fissioned per hour will be

(a) 5×10^{15} (b) 10×10^{20} (c) 40×10^{21} (d) 30×10^{25}

Solution : (c) By using $P = \frac{W}{t} = \frac{n \times E}{t}$ where n = Number of uranium atom fissioned and E = Energy released due to each fission so $300 \times 10^6 = \frac{n \times 170 \times 10^6 \times 1.6 \times 10^{-19}}{3600}$ $\Rightarrow n = 40 \times 10^{21}$

Example: 10 The binding energy per nucleon of O^{16} is 7.97 *MeV* and that of O^{17} is 7.75 *MeV*. The energy (in *MeV*) required to remove a neutron from O^{17} is **[IIT-JEE 1995]**

(a) 3.52 (b) 3.64 (c) 4.23 (d) 7.86

Solution : (c) $O^{17} \rightarrow O^{16} + {}_0n^1$

$\therefore$ Energy required = Binding of O^{17} – binding energy of O^{16} = 17 × 7.75 – 16 × 7.97 = 4.23 *MeV*

Example: 11 A gamma ray photon creates an electron-positron pair. If the rest mass energy of an electron is 0.5 *MeV* and the total kinetic energy of the electron-positron pair is 0.78 *MeV*, then the energy of the gamma ray photon must be **[MP PMT 1991]**

(a) 0.78 *MeV* (b) 1.78 *MeV* (c) 1.28 *MeV* (d) 0.28 *MeV*

Solution : (b) Energy of γ-rays photon = 0.5 + 0.5 +0.78 = 1.78 *MeV*

Example: 12 What is the mass of one Curie of U^{234} **[MNR 1985]**

(a) 3.7×10^{10} *gm* (b) 2.348×10^{23} *gm* (c) 1.48×10^{-11} *gm* (d) 6.25×10^{-34} *gm*

Solution : (c) 1 *curie* = 3.71×10^{10} disintegration/*sec* and mass of 6.02×10^{23} atoms of U^{234} = 234 *gm*

$\therefore$ Mass of 3.71×10^{10} atoms $= \frac{234 \times 3.71 \times 10^{10}}{6.02 \times 10^{23}} = 1.48 \times 10^{-11}\ gm$

Example: 13 In the nuclear fusion reaction ${}_1^2H + {}_1^3H \rightarrow {}_2^4He + n,$ given that the repulsive potential energy between the two nuclei is $-7.7 \times 10^{-14}\ J$, the temperature at which the gases must be heated to initiate the reaction is nearly [Boltzmann's constant $k = 1.38 \times 10^{-23}\ J/K$] **[AIEEE 2003]**

(a) $10^9\ K$ (b) $10^7\ K$ (c) $10^5\ K$ (d) $10^3\ K$

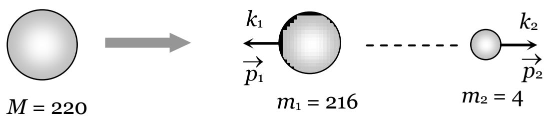
Solution : (a) Kinetic energy of molecules of a gas at a temperature *T* is 3/2 *kT*

$\therefore$ To initiate the reaction $\frac{3}{2}kT = 7.7 \times 10^{-14}\ J \Rightarrow T = 3.7 \times 10^9\ K.$

Example: 14 A nucleus with mass number 220 initially at rest emits an α-particle. If the *Q* value of the reaction is 5.5 *MeV*. Calculate the kinetic energy of the α-particle **[IIT-JEE (Screening) 2003]**

(a) 4.4 *MeV* (b) 5.4 *MeV* (c) 5.6 *MeV* (d) 6.5 *MeV*

Solution : (b)



Q-value of the reaction is 5.5 *eV* *i.e.* $k_1 + k_2 = 5.5\ MeV$ ......(i)

By conservation of linear momentum $p_1 = p_2 \Rightarrow \sqrt{2(216)k_1} = \sqrt{2(4)k_2} \Rightarrow k_2 = 54\ k_1$ ......(ii)

On solving equation (i) and (ii) we get $k_2 = 5.4\ MeV$.

***Example*: 15** Let m_p be the mass of a proton, m_n the mass of a neutron, M_1 the mass of a $^{20}_{10}Ne$ nucleus and M_2 the mass of a $^{40}_{20}Ca$ nucleus. Then **[IIT 1998; DPMT 2000]**

(a) $M_2 = 2M_1$ (b) $M_2 > 2M_1$ (c) $M_2 < 2M_1$ (d) $M_1 < 10(m_n + m_p)$

Solution : (c, d) Due to mass defect (which is finally responsible for the binding energy of the nucleus), mass of a nucleus is always less then the sum of masses of it's constituent particles $^{20}_{10}Ne$ is made up of 10 protons plus 10 neutrons. Therefore, mass of $^{20}_{10}Ne$ nucleus $M_1 < 10\,(m_p + m_n)$

Also heavier the nucleus, more is he mass defect thus $20\,(m_n + m_p) - M_2 > 10(m_p + m_n) - M_1$

or $10\,(m_p + m_n) > M_2 - M_1$

$\Rightarrow$ $M_2 < M_1 + 10\,(m_p + m_n) \Rightarrow M_2 < M_1 + M_1 \Rightarrow M_2 < 2M_1$

Tricky example: 1

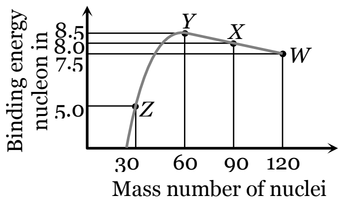
Binding energy per nucleon *vs* mass number curve for nuclei is shown in the figure. *W*, *X*, *Y* and *Z* are four nuclei indicated on the curve. The process that would release energy is **[IIT-JEE 1999]**

(a) $Y \to 2Z$

(b) $W \to X + Z$

(c) $W \to 2Y$

(d) $X \to Y + Z$



Solution : (c) Energy is released in a process when total binding energy of the nucleus (= binding energy per nucleon × number of nucleon) is increased or we can say, when total binding energy of products is more than the reactants. By calculation we can see that only in case of option (c) this happens.

Given $W \to 2Y$

Binding energy of reactants = 120 × 7.5 = 900 *MeV*

and binding energy of products = 2 (60 × 8.5) = 1020 *MeV* > 900 *MeV*

Radioactivity

The phenomenon of spontaneous emission of radiatons by heavy elements is called radioactivity. The elements which shows this phenomenon are called radioactive elements.

(1) Radioactivity was discovered by Henery Becquerel in uranium salt in the year 1896.

(2) After the discovery of radioactivity in uranium, Piere Curie and Madame Curie discovered a new radioactive element called radium (which is 10^6 times more radioactive than uranium)

(3) Some examples of radio active substances are : Uranium, Radium, Thorium, Polonium, Neptunium *etc.*

(4) Radioactivity of a sample cannot be controlled by any physical (pressure, temperature, electric or magnetic field) or chemical changes.

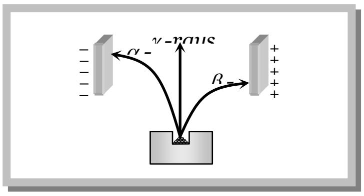
(5) All the elements with atomic number (Z) > 82 are naturally radioactive.

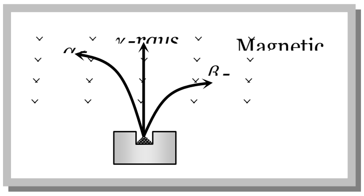
(6) The conversion of lighter elements into radioactive elements by the bombardment of fast moving particles is called artificial or induced radioactivity.

(7) Radioactivity is a nuclear event and not atomic. Hence electronic configuration of atom don't have any relationship with radioactivity.

Nuclear radiatons

According to Rutherford's experiment when a sample of radioactive substance is put in a lead box and allow the emission of radiation through a small hole only. When the radiation enters into the external electric field, they splits into three parts





(i) Radiations which deflects towards negative plate are called α-rays (stream of positively charged particles)

(ii) Radiations which deflects towards positive plate are called β particles (stream of negatively charged particles)

(iii) Radiations which are undeflected called γ-*rays*. (E.M. waves or photons)

Note : ❑ Exactly same results were obtained when these radiations were subjected to magnetic field.

❑ No radioactive substance emits both α and β particles simultaneously. Also γ-rays are emitted after the emission of α or β-particles.

❑ β-particles are not orbital electrons they come from nucleus. The neutron in the nucleus decays into proton and an electron. This electron is emitted out of the nucleus in the form of β-rays.

Properties of α, β and γ-rays

Features	α- particles	β - particles	γ - rays
1. Identity	Helium nucleus or doubly ionised helium atom ($_2He^4$)	Fast moving electron $(-\beta^0$ or $\beta^-)$	Photons (E.M. waves)
2. Charge	+ 2*e*	− *e*	Zero
3. Mass 4 m_p (m_p = mass of proton = 1.87×10^{-27}	4 m_p	m_e	Massless
4. Speed	$\approx 10^7$ *m/s*	1% to 99% of speed of light	Speed of light
5. Range of kinetic energy	4 *MeV* to 9 *MeV*	All possible values between a minimum certain value to 1.2 *MeV*	Between a minimum value to 2.23 *MeV*
6. Penetration power (γ, β, α)	1 (Stopped by a paper)	100 (100 times of α)	10,000 (100 times of β upto 30 *cm* of iron (or *Pb*) sheet
7. Ionisation power ($\alpha > \beta > \gamma$)	10,000	100	1
8. Effect of electric or magnetic field	Deflected	Deflected	Not deflected
9. Energy spectrum	Line and discrete	Continuous	Line and discrete
10. Mutual interaction with matter	Produces heat	Produces heat	Produces, photo-electric effect, Compton effect, pair production
11. Equation of decay	$_Z X^A \xrightarrow{\alpha-decay}$ $_{Z-2}Y^{A-4} + {_2He^4}$	$_Z X^A \to {_{Z+1}Y^A} + {_{-1}e^0} + \bar{\nu}$ $_Z X^A \xrightarrow{n_\beta} {_{Z'}X^A}$	$_Z X^A \to {_Z X^a} + \gamma$

	$_ZX^A \xrightarrow{n_\alpha} {}_{Z'}Y^{A'}$ $\Rightarrow \boldsymbol{n_\alpha = \frac{A'-A}{4}}$	$\Rightarrow \boldsymbol{n_\beta = (2n_\alpha - Z + Z')}$	

Radioactive Disintegration

(1) **Law of radioactive disintegration**

According to Rutherford and Soddy law for radioactive decay is as follows.

"At any instant the rate of decay of radioactive atoms is proportional to the number of atoms present at that instant"

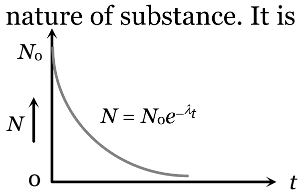
i.e. $-\frac{dN}{dt} \propto N \quad \Rightarrow \frac{dN}{dt} = -\lambda N$. It can be proved that $\boldsymbol{N = N_0 e^{-\lambda t}}$

This equation can also be written in terms of mass *i.e.* $M = M_0 e^{-\lambda t}$

where N = Number of atoms remains undecayed after time t, N_0 = Number of atoms present initially (*i.e.* at t = 0), M = Mass of radioactive nuclei at time t, M_0 = Mass of radioactive nuclei at time t = 0, $N_0 - N$ = Number of disintegrated nucleus in time t

$\frac{dN}{dt}$ = rate of decay, λ = Decay constant or disintegration constant or radioactivity constant or Rutherford Soddy's constant or the probability of decay per unit time of a nucleus.

Note : ❑ λ depends only on the nature of substance. It is independent of time and any physical or chemical changes.



(2) **Activity**

It is defined as the rate of disintegration (or count rate) of the substance (or the number of atoms of any material decaying per second) *i.e.* $A = -\frac{dN}{dt} = \lambda N = \lambda N_0 e^{-\lambda t} = A_0 e^{-\lambda t}$

where A_0 = Activity of t = 0, A = Activity after time t

Units of activity (Radioactivity)

It's units are *Becqueral* (*Bq*), Curie (*Ci*) and Rutherford (*Rd*)

1 *Becquerel* = 1 *disintegration/sec*, 1 *Rutherford* = 10^6 *dis/sec*, 1 *Curie* = 3.7×10^{11} *dis/sec*

Note : ❑ Activity per *gm* of a substance is known as specific activity. The specific activity of 1 *gm* of radium – 226 is 1 *Curie*.

❑ 1 *millicurie* = 37 *Rutherford*

❑ The activity of a radioactive substance decreases as the number of undecayed nuclei decreases with time.

❑ Activity $\propto \frac{1}{\text{Half life}}$

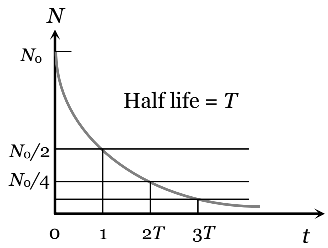
(3) **Half life ($T_{1/2}$)**

Time interval in which the mass of a radioactive substance or the number of it's atom reduces to half of it's initial value is called the half life of the substance.

i.e. if $N = \frac{N_0}{2}$ then $t = T_{1/2}$

Hence from $N = N_0 e^{-\lambda t}$

$$\frac{N_0}{2} = N_0 e^{-\lambda (T_{1/2})} \Rightarrow T_{1/2} = \frac{\log_e 2}{\lambda} = \frac{0.693}{\lambda}$$



Time (t)	Number of undecayed atoms (N) (N_0 = Number of initial atoms)	Remaining fraction of active atoms (N/N_0) probability of survival	Fraction of atoms decayed ($N_0 - N$) /N_0 probability of decay
t = 0	N_0	1 (100%)	0
$t = T_{1/2}$	$\frac{N_0}{2}$	$\frac{1}{2}$ (50%)	$\frac{1}{2}$ (50%)

$t = 2(T_{1/2})$	$\frac{1}{2} \times \frac{N_0}{2} = \frac{N_0}{(2)^2}$	$\frac{1}{4}$ (25%)	$\frac{3}{4}$ (75%)
$t = 3(T_{1/2})$	$\frac{1}{2} \times \frac{N_0}{(2)} = \frac{N_0}{(2)^3}$	$\frac{1}{8}$ (12.5%)	$\frac{7}{8}$ (87.5%)
$t = 10\ (T_{1/2})$	$\frac{N_0}{(2)^{10}}$	$\left(\frac{1}{2}\right)^{10} \approx 0.1\%$	$\approx 99.9\%$
$t = n\ (N_{1/2})$	$\frac{N}{(2)^2}$	$\left(\frac{1}{2}\right)^n$	$\left\{1 - \left(\frac{1}{2}\right)^n\right\}$

Useful relation

After n half-lives, number of undecayed atoms $N = N_0\left(\frac{1}{2}\right)^n = N_0\left(\frac{1}{2}\right)^{t/T_{1/2}}$

(4) **Mean (or average) life (τ)**

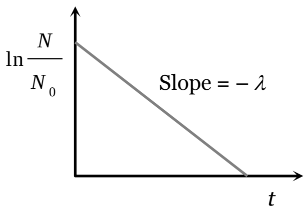
The time for which a radioactive material remains active is defined as mean (average) life of that material.

Other definitions

(i) It is defined as the sum of lives of all atoms divided by the total number of atoms

$$i.e.\ \tau = \frac{\text{Sum of the lives of all the atoms}}{\text{Total number of atoms}} = \frac{1}{\lambda}$$

(ii) From $N = N_0 e^{-\lambda t} \Rightarrow \frac{\ln \frac{N}{N_0}}{t} = -\lambda$ slope of the line shown in the graph



i.e. the magnitude of inverse of slope of $\ln\frac{N}{N_0}$ *vs* t curve is known as mean life (τ).

(iii) From $N = N_0 e^{-\lambda t}$

If $t = \frac{1}{\lambda} = \tau \Rightarrow N = N_0 e^{-1} = N_0\left(\frac{1}{e}\right) = 0.37\, N_0 = 37\%$ of N_0.

i.e. mean life is the time interval in which number of undecayed atoms (N) becomes $\frac{1}{e}$ times or 0.37 times or 37% of original number of atoms. or

It is the time in which number of decayed atoms ($N_0 - N$) becomes $\left(1 - \frac{1}{e}\right)$ times or 0.63 times or 63% of original number of atoms.

(iv) From $T_{1/2} = \frac{0.693}{\lambda} \Rightarrow \frac{1}{\lambda} = \tau = \frac{1}{0.693}.(t_{1/2}) = 1.44\,(T_{1/2})$

i.e. mean life is about 44% more than that of half life. Which gives us $\tau > T_{(1/2)}$

Note : ❑ Half life and mean life of a substance doesn't change with time or with pressure, temperature *etc.*

Radioactive Series

If the isotope that results from a radioactive decay is itself radioactive then it will also decay and so on.

The sequence of decays is known as radioactive decay series. Most of the radio-nuclides found in nature are members of four radioactive series. These are as follows

Mass number	Series (Nature)	Parent	Stable and product	Integer n	Number of lost particles
$4n$	Thorium (natural)	$_{90}Th^{232}$	$_{82}Pb^{208}$	52	$\alpha = 6, \beta = 4$
$4n + 1$	Neptunium (Artificial)	$_{93}Np^{237}$	$_{83}Bi^{209}$	52	$\alpha = 8, \beta = 5$

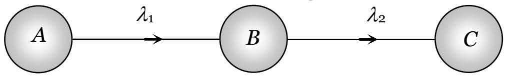
$4n + 2$	Uranium (Natural)	${}_{92}U^{238}$	${}_{82}Pb^{206}$	51	$\alpha = 8, \beta = 6$
$4n + 3$	Actinium (Natural)	${}_{89}Ac^{227}$	${}_{82}Pb^{207}$	51	$\alpha = 7, \beta = 4$

Note : ❑ The $4n + 1$ series starts from ${}_{94}PU^{241}$ but commonly known as neptunium series because neptunium is the longest lived member of the series.

❑ The $4n + 3$ series actually starts from ${}_{92}U^{235}$.

Successive Disintegration and Radioactive Equilibrium

Suppose a radioactive element A disintegrates to form another radioactive element B which intern disintegrates to still another element C; such decays are called successive disintegration.



Rate of disintegration of $A = \frac{dN_1}{dt} = -\lambda_1 N_1$ (which is also the rate of formation of B)

Rate of disintegration of $B = \frac{dN_2}{dt} = -\lambda_2 N_2$

$\therefore$ Net rate of formation of B = Rate of disintegration of A – Rate of disintegration of B

$= \lambda_1 N_1 - \lambda_2 N_2$

Equilibrium

In radioactive equilibrium, the rate of decay of any radioactive product is just equal to it's rate of production from the previous member.

i.e. $\lambda_1 N_1 = \lambda_2 N_2$ $\Rightarrow$ $\frac{\lambda_1}{\lambda_2} = \frac{N_2}{N_2} = \frac{\tau_2}{\tau_1} = \frac{(T_{1/2})}{(T_{1/2})_1}$

Note : ❑ In successive disintegration if N_0 is the initial number of nuclei of A at $t = 0$ then number of nuclei of product B at time t is given by $N_2 = \frac{\lambda_1 N_0}{(\lambda_2 - \lambda_1)}(e^{-\lambda_1 t} - e^{-\lambda_2 t})$ where $\lambda_1\lambda_2$ – decay constant of A and B.

Uses of radioactive isotopes

(1) **In medicine**

(i) For testing blood-chromium - 51 (ii) For testing blood circu

(iii) For detecting brain tumor- Radio mercury - 203

(iv) For detecting fault in thyroid gland - Radio iodine - 131

(v) For cancer - cobalt – 60 (vi) For blood - Gold - 189

(vii) For skin diseases - Phospohorous - 31

(2) **In Archaeology**

(i) For determining age of archaeological sample (carbon dating) C^{14}

(ii) For determining age of meteorites - K^{40} (iii) For determining age of earth-Lead isotopes

(3) **In agriculture**

(i) For protecting potato crop from earthworm- CO^{60} (ii) For artificial rains - AgI (iii) As fertilizers - P^{32}

(4) **As tracers** - (Tracer) : Very small quantity of radioisotopes present in a mixture is known as tracer

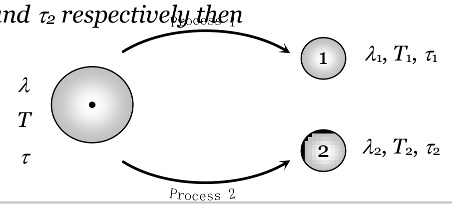
(i) Tracer technique is used for studying biochemical reaction in tracer and animals.

(5) **In industries**

(i) For detecting leakage in oil or water pipe lines (ii) For determining the age of planets.

Concept

☞ *If a nuclide can decay simultaneously by two different process which have decay constant λ_1 and λ_2, half life T_1 and T_2 and mean lives τ_1 and τ_2 respectively then*



$\Rightarrow \lambda = \lambda_1 + \lambda_2$

$\Rightarrow T = \frac{T_1 T_2}{T_1 + T_2}$

$\Rightarrow \tau = \frac{\tau_1 \tau_2}{\tau_1 + \tau_2}$

***Example*: 16** When ${}_{90}Th^{228}$ transforms to ${}_{83}Bi^{212}$, then the number of the emitted α–and β–particles is, respectively **[MP PET 2002]**

(a) $8\alpha, 7\beta$ (b) $4\alpha, 7\beta$ (c) $4\alpha, 4\beta$ (d) $4\alpha, 1\beta$

Solution : (d) $_{Z=90}Th^{A=228} \rightarrow {}_{Z'=83}Bi^{A'=212}$

Number of α-particles emitted $n_\alpha = \frac{A-A'}{4} = \frac{228-212}{4} = 4$

Number of β-particles emitted $n_\beta = 2n_\alpha - Z + Z' = 2\times 4 - 90 + 83 = 1.$

***Example*: 17** A radioactive substance decays to 1/16th of its initial activity in 40 days. The half-life of the radioactive substance expressed in days is

(a) 2.5 (b) 5 (c) 10 (d) 20

Solution : (c) By using $N = N_0\left(\frac{1}{2}\right)^{t/T_{1/2}} \Rightarrow \frac{N}{N_0} = \frac{1}{16} = \left(\frac{1}{2}\right)^{40/T_{1/2}} \Rightarrow T_{1/2} = 10$ days.

***Example*: 18** A sample of radioactive element has a mass of 10 *gm* at an instant *t* = 0. The approximate mass of this element in the sample after two mean lives is **[CBSE PMT 2003]**

(a) 2.50 *gm* (b) 3.70 *gm* (c) 6.30 *gm* (d) 1.35 *gm*

Solution : (d) By using $M = M_0e^{-\lambda t} \Rightarrow M = 10e^{-\lambda(2\tau)} = 10e^{-\lambda\left(\frac{2}{\lambda}\right)} = 10\left(\frac{1}{e}\right)^2 = 1.359\ gm$

***Example*: 19** The half-life of ^{215}At is 100 μs. The time taken for the radioactivity of a sample of ^{215}At to decay to 1/16th of its initial value is **[IIT-JEE (Screening) 2002]**

(a) 400 μs (b) 6.3 μs (c) 40 μs (d) 300 μs

Solution : (a) By using $N = N_0\left(\frac{1}{2}\right)^n \Rightarrow \frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/T_{1/2}} \Rightarrow \frac{1}{16} = \left(\frac{1}{2}\right)^{t/100} \Rightarrow t = 400\ \mu sec.$

***Example*: 20** The mean lives of a radioactive substance for α and β emissions are 1620 years and 405 years respectively. After how much time will the activity be reduced to one fourth **[RPET 1999]**

(a) 405 year (b) 1620 year (c) 449 year (d) None of these

Solution : (c) $\lambda_\alpha = \frac{1}{1620}$ *per year* and $\lambda_\beta = \frac{1}{405}$ *per year* and it is given that the fraction of the remained activity $\frac{A}{A_0} = \frac{1}{4}$

Total decay constant $\lambda = \lambda_\alpha + \lambda_\beta = \frac{1}{1620} + \frac{1}{405} = \frac{1}{324}$ *per year*

We know that $A = A_0e^{-\lambda t} \Rightarrow t = \frac{1}{\lambda}\log_e\frac{A_0}{A} \Rightarrow t = \frac{1}{\lambda}\log_e 4 = \frac{2}{\lambda}\log_e 2$ = 324 × 2 × 0.693 = 449 *years*.

***Example*: 21** At any instant the ratio of the amount of radioactive substances is 2 : 1. If their half lives be respectively 12 and 16 hours, then after two days, what will be the ratio of the substances

(a) 1 : 1 (b) 2 : 1 (c) 1 : 2 (d) 1 : 4

Solution : (a) By using $N = N_0\left(\frac{1}{2}\right)^n \Rightarrow \frac{N_1}{N_2} = \frac{(N_0)_1}{(N_0)_2}\times\frac{(1/2)^{n_1}}{(1/2)^{n_2}} = \frac{2}{1}\times\frac{\left(\frac{1}{2}\right)^{\frac{2\times 24}{12}}}{\left(\frac{1}{2}\right)^{\frac{2\times 24}{16}}} = \frac{1}{1}$

***Example*: 22** From a newly formed radioactive substance (Half-life 2 *hours*), the intensity of radiation is 64 times the permissible safe level. The minimum time after which work can be done safely from this source is **[IIT 1983; SCRA 1996]**

(a) 6 *hours* (b) 12 *hours* (c) 24 *hours* (d) 128 *hours*

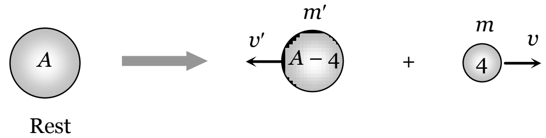
Solution : (b) By using $A = A_0\left(\frac{1}{2}\right)^n \Rightarrow \frac{A}{A_0} = \frac{1}{64} = \left(\frac{1}{2}\right)^0 = \left(\frac{1}{2}\right)^n \Rightarrow n = 6$

$\Rightarrow \frac{t}{T_{1/2}} = 6 \Rightarrow t = 6\times 2 = 12$ *hours*.

***Example*: 23** nucleus of mass number *A*, originally at rest, emits an α-particle with speed *v*. The daughter nucleus recoils with a speed **[DCE 2000; AIIMS 2004]**

(a) $2v/(A+4)$ (b) $4v/(A+4)$ (c) $4v/(A-4)$ (d) $2v/(A-4)$

Solution : (c)



According to conservation of momentum $4v = (A-4)v' \Rightarrow v' = \frac{4v}{A-4}$.

***Example*: 24** The counting rate observed from a radioactive source at t = 0 second was 1600 counts per second and at t = 8 seconds it was 100 counts per second. The counting rate observed as counts per second at t = 6 seconds will be **[MP PET 1996; UPSEAT 2000]**

(a) 400 (b) 300 (c) 200 (d) 150

Solution : (c) By using $A = A_0\left(\frac{1}{2}\right)^n \Rightarrow 100 = 1600\left(\frac{1}{2}\right)^{8/T_{1/2}} \Rightarrow \frac{1}{16} = \left(\frac{1}{2}\right)^{8/T_{1/2}} \Rightarrow T_{1/2} = 2\, sec$

Again by using the same relation the count rate at t = 6 *sec* will be $A = 1600\left(\frac{1}{2}\right)^{6/2} = 200$.

***Example*: 25** The kinetic energy of a neutron beam is 0.0837 *eV*. The half-life of neutrons is 693*s* and the mass of neutrons is 1.675×10^{-27} *kg*. The fraction of decay in travelling a distance of 40*m* will be

(a) 10^{-3} (b) 10^{-4} (c) 10^{-5} (d) 10^{-6}

Solution : (c) $v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2\times0.0837\times1.6\times10^{-19}}{1.675\times10^{-27}}} = 4\times10^3\ m/sec$

$\therefore$ Time taken by neutrons to travel a distance of 40 *m* $\Delta t' = \frac{40}{4\times10^3} = 10^{-2}\, sec$

$\therefore \frac{dN}{dt} = \lambda N \Rightarrow \frac{dN}{N} = \lambda\, dt$

$\therefore$ Fraction of neutrons decayed in Δt *sec* in $\frac{\Delta N}{N} = \lambda\,\Delta t = \frac{0.693}{T}\Delta t = \frac{0.693}{693}\times10^{-2} = 10^{-5}$

***Example*: 26** The fraction of atoms of radioactive element that decays in 6 days is 7/8. The fraction that decays in 10 days will be

(a) 77/80 (b) 71/80 (c) 31/32 (d) 15/16

Solution : (c) By using $N = N_0\left(\frac{1}{2}\right)^{t/T_{1/2}} \Rightarrow t = \frac{T_{1/2}\log_e\left(\frac{N_0}{N}\right)}{\log_e(2)} \Rightarrow t \propto \log_e\frac{N_0}{N} \Rightarrow \frac{t_1}{t_2} = \frac{\left(\log_e\frac{N_0}{N}\right)_1}{\left(\log_e\frac{N_0}{N}\right)_2}$

Hence $\frac{6}{10} = \frac{\log_e(8/1)}{\log_e(N_0/N)} \Rightarrow \log_e\frac{N_0}{N} = \frac{10}{6}\log_e(8) = \log_e 32 \Rightarrow \frac{N_0}{N} = 32$.

So fraction that decays $= 1 - \frac{1}{32} = \frac{31}{32}$.

Tricky example: 2

Half-life of a substance is 20 minutes. What is the time between 33% decay and 67% decay **[AIIMS 200**

(a) 40 minutes (b) 20 minutes (c) 30 minutes (d) 25 minutes

Solution : (b) Let N_0 be the number of nuclei at beginning

$\therefore$ Number of undecayed nuclei after 33% decay = 0.67 N_0

and number of undecayed nuclei after 67% of decay = 0.33 N_0

$\because$ 0.33 $N_0 \simeq \frac{0.67 N_0}{2}$ and in the half-life time the number of undecayed nuclei becomes half.

Example

***Example*: 1** The ratio of areas within the electron orbits for the first excited state to the ground state for hydrogen atom is

(a) 16 : 1 (b) 18 : 1 (c) 4 : 1 (d) 2 : 1

Solution : (a) For a hydrogen atom

Radius $r \propto n^2 \Rightarrow \frac{r_1^2}{r_2^2} = \frac{n_1^4}{n_2^4} \Rightarrow \frac{\pi r_1^2}{\pi r_2^2} = \frac{n_1^4}{n_2^4} \Rightarrow \frac{A_1}{A_2} = \frac{n_1^4}{n_2^4} = \frac{2^4}{1^4} = 16 \Rightarrow \frac{A_1}{A_2} = \frac{16}{1}$

***Example*: 2** The electric potential between a proton and an electron is given by $V = V_0 \ln \frac{r}{r_0}$, where r_0 is a constant. Assuming Bohr's model to be applicable, write variation of r_n with n, n being the principal quantum number

[IIT-JEE (Screening) 2003]

(a) $r_n \propto n$ (b) $r_n \propto 1/n$ (c) $r_n \propto n^2$ (d) $r_n \propto 1/n^2$

Solution : (a) Potential energy $U = eV = eV_0 \ln \frac{r}{r_0}$

$\therefore$ Force $F = -\left|\frac{dU}{dr}\right| = \frac{eV_0}{r}$. The force will provide the necessary centripetal force. Hence

$\frac{mv^2}{r} = \frac{eV_0}{r} \Rightarrow v = \sqrt{\frac{eV_0}{m}}$ .....(i) and $mvr = \frac{nh}{2\pi}$ .....(ii)

Dividing equation (ii) by (i) we have $mr = \left(\frac{nh}{2\pi}\right)\sqrt{\frac{m}{eV_0}}$ or $r \propto n$

***Example*: 3** The innermost orbit of the hydrogen atom has a diameter 1.06 $Å$. The diameter of tenth orbit is

[UPSEAT 2002]

(a) 5.3 $Å$ (b) 10.6 $Å$ (c) 53 $Å$ (d) 106 $Å$

Solution : (d) Using $r \propto n^2 \Rightarrow \frac{r_2}{r_1} = \left(\frac{n_2}{n_1}\right)^2$ or $\frac{d_2}{d_1} = \left(\frac{n_2}{n_1}\right)^2 \Rightarrow \frac{d_2}{1.06} = \left(\frac{10}{1}\right)^2 \Rightarrow d = 106\,Å$

***Example*: 4** Energy of the electron in n^{th} orbit of hydrogen atom is given by $E_n = -\frac{13.6}{n^2} eV$. The amount of energy needed to transfer electron from first orbit to third orbit is

(a) 13.6 eV (b) 3.4 eV (c) 12.09 eV (d) 1.51 eV

Solution : (c) Using $E = -\frac{13.6}{n^2} eV$

For $n = 1$, $E_1 = \frac{-13.6}{1^2} = -13.6\,eV$ and for $n = 3$ $E_3 = -\frac{13.6}{3^2} = -1.51\,eV$

So required energy $= E_3 - E_1 = -1.51 - (-13.6) = 12.09\,eV$

***Example*: 5** If the binding energy of the electron in a hydrogen atom is 13.6 eV, the energy required to remove the electron from the first excited state of Li^{++} is **[AIEEE 2003]**

(a) 122.4 eV (b) 30.6 eV (c) 13.6 eV (d) 3.4 eV

Solution : (b) Using $E_n = -\frac{13.6 \times Z^2}{n^2} eV$

For first excited state $n = 2$ and for $Li^{++}, Z = 3$

$\therefore$ $E = -\frac{13.6}{2^2} \times 3^2 = -\frac{13.6 \times 9}{4} = -30.6\,eV$. Hence, remove the electron from the first excited state of Li^{++} be 30.6 eV

***Example*: 6** The ratio of the wavelengths for 2 → 1 transition in Li^{++}, He^+ and H is **[UPSEAT 2003]**

(a) 1 : 2 : 3 (b) 1 : 4 : 9 (c) 4 : 9 : 36 (d) 3 : 2 : 1

Solution : (c) Using $\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \Rightarrow \lambda \propto \frac{1}{Z^2} \Rightarrow \lambda_{Li} : \lambda_{He^+} : \lambda_H = \frac{1}{9} : \frac{1}{4} : \frac{1}{1} = 4 : 9 : 36$

***Example*: 7** Energy E of a hydrogen atom with principal quantum number n is given by $E = \frac{-13.6}{n^2} eV$. The energy of a photon ejected when the electron jumps $n = 3$ state to $n = 2$ state of hydrogen is approximately

[CBSE PMT/PDT Screening 2004]

(a) 1.9 eV (b) 1.5 eV (c) 0.85 eV (d) 3.4 eV

Solution : (a) $\Delta E = 13.6\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = 13.6 \times \frac{5}{36} = 1.9\, eV$

***Example*: 8** In the Bohr model of the hydrogen atom, let R, v and E represent the radius of the orbit, the speed of electron and the total energy of the electron respectively. Which of the following quantity is proportional to the quantum number n **[KCET 2002]**

(a) R/E (b) E/v (c) RE (d) vR

Solution : (d) Rydberg constant $R = \frac{\varepsilon_0 n^2 h^2}{\pi m Z e^2}$

Velocity $v = \frac{Ze^2}{2\varepsilon_0 nh}$ and energy $E = -\frac{mZ^2e^4}{8\varepsilon_0^2 n^2 h^2}$

Now, it is clear from above expressions $R.v \propto n$

***Example*: 9** The energy of hydrogen atom in nth orbit is E_n, then the energy in nth orbit of singly ionised helium atom will be

(a) $4E_n$ (b) $E_n/4$ (c) $2E_n$ (d) $E_n/2$

Solution : (a) By using $E = -\frac{13.6\, Z^2}{n^2} \Rightarrow \frac{E_H}{E_{He}} = \left(\frac{Z_H}{Z_{He}}\right)^2 = \left(\frac{1}{2}\right)^2 \Rightarrow E_{He} = 4E_n$.

***Example*: 10** The wavelength of radiation emitted is λ_0 when an electron jumps from the third to the second orbit of hydrogen atom. For the electron jump from the fourth to the second orbit of the hydrogen atom, the wavelength of radiation emitted will be **[SCRA 1998; MP PET 2001]**

(a) $\frac{16}{25}\lambda_0$ (b) $\frac{20}{27}\lambda_0$ (c) $\frac{27}{20}\lambda_0$ (d) $\frac{25}{16}\lambda_0$

Solution : (b) Wavelength of radiation in hydrogen atom is given by

$$\frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] \Rightarrow \frac{1}{\lambda_0} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right] = R\left[\frac{1}{4} - \frac{1}{9}\right] = \frac{5}{36}R \quad .....(i)$$

and $$\frac{1}{\lambda'} = R\left[\frac{1}{2^2} - \frac{1}{4^2}\right] = R\left[\frac{1}{4} - \frac{1}{16}\right] = \frac{3R}{16} \quad .....(ii)$$

From equation (i) and (ii) $\frac{\lambda'}{\lambda} = \frac{5R}{36} \times \frac{16}{3R} = \frac{20}{27} \Rightarrow \lambda' = \frac{20}{27}\lambda_0$

***Example*: 11** If scattering particles are 56 for 90^o angle then this will be at 60^o angle **[RPMT 2000]**

(a) 224 (b) 256 (c) 98 (d) 108

Solution : (a) Using Scattering formula

$$N \propto \frac{1}{\sin^4(\theta/2)} \Rightarrow \frac{N_2}{N_1} = \left[\frac{\sin\left(\frac{\theta_1}{2}\right)}{\sin\left(\frac{\theta_2}{2}\right)}\right]^4 \Rightarrow \frac{N_2}{N_1} = \left[\frac{\sin\left(\frac{90°}{2}\right)}{\sin\left(\frac{60°}{2}\right)}\right]^4 = \left[\frac{\sin 45°}{\sin 30°}\right]^4 = 4 \Rightarrow N_2 = 4N_1 = 4 \times 56 = 224$$

***Example*: 12** When an electron in hydrogen atom is excited, from its 4th to 5th stationary orbit, the change in angular momentum of electron is (Planck's constant: $h = 6.6 \times 10^{-34}$ J–s) **[AFMC 2000]**

(a) 4.16×10^{-34} J- s (b) 3.32×10^{-34} J-s (c) 1.05×10^{-34} J-s (d) 2.08×10^{-34} J-s

Solution : (c) Change in angular momentum

$$\Delta L = L_2 - L_1 = \frac{n_2 h}{2\pi} - \frac{n_1 h}{2\pi} \Rightarrow \Delta L = \frac{h}{2\pi}(n_2 - n_1) = \frac{6.6 \times 10^{-34}}{2 \times 3.14}(5-4) = 1.05 \times 10^{-34}\ J\text{-}s$$

***Example*: 13** In hydrogen atom, if the difference in the energy of the electron in n = 2 and n = 3 orbits is E, the ionization energy of hydrogen atom is

(a) 13.2 E (b) 7.2 E (c) 5.6 E (d) 3.2 E

Solution : (b) Energy difference between n = 2 and n = 3; $E = K\left(\frac{1}{2^2}-\frac{1}{3^2}\right) = K\left(\frac{1}{4}-\frac{1}{9}\right) = \frac{5}{36}K$ .....(i)

Ionization energy of hydrogen atom $n_1 = 1$ and $n_2 = \infty$; $E' = K\left(\frac{1}{1^2}-\frac{1}{\infty^2}\right) = K$ .....(ii)

From equation (i) and (ii) $E' = \frac{36}{5}E = 7.2\,E$

***Example*: 14** In Bohr model of hydrogen atom, the ratio of periods of revolution of an electron in n = 2 and n = 1 orbits is **[EAMCET (Engg.) 2000]**

(a) 2 : 1 (b) 4 : 1 (c) 8 : 1 (d) 16 : 1

Solution : (c) According to Bohr model time period of electron $T \propto n^3 \Rightarrow \frac{T_2}{T_1} = \frac{n_2^3}{n_1^3} = \frac{2^3}{1^3} = \frac{8}{1} \Rightarrow T_2 = 8T_1$.

***Example*: 15** A double charged lithium atom is equivalent to hydrogen whose atomic number is 3. The wavelength of required radiation for emitting electron from first to third Bohr orbit in Li^{++} will be (Ionisation energy of hydrogen atom is 13.6 eV)

(a) 182.51 $Å$ (b) 177.17 $Å$ (c) 142.25 $Å$ (d) 113.74 $Å$

Solution : (d) Energy of a electron in nth orbit of a hydrogen like atom is given by

$E_n = -13.6\frac{Z^2}{n^2}eV$, and Z = 3 for Li

Required energy for said transition

$$\Delta E = E_3 - E_1 = 13.6Z^2\left(\frac{1}{1^2}-\frac{1}{3^2}\right) = 13.6\times 3^2\left[\frac{8}{9}\right] = 108.8\,eV = 108.8\times 1.6\times 10^{-19}\ J$$

Now using $\Delta E = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{\Delta E} \Rightarrow \lambda = \frac{6.6\times 10^{-34}\times 3\times 10^8}{108.8\times 1.6\times 10^{-19}} = 0.11374\times 10^{-7}\ m \Rightarrow \lambda = 113.74\ Å$

***Example*: 16** The absorption transition between two energy states of hydrogen atom are 3. The emission transitions between these states will be

(a) 3 (b) 4 (c) 5 (d) 6

Solution : (d) Number of absorption lines = $(n-1) \Rightarrow 3 = (n-1) \Rightarrow n = 4$

Hence number of emitted lines $= \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$

***Example*: 17** The energy levels of a certain atom for 1st, 2nd and 3rd levels are E, $4E/3$ and $2E$ respectively. A photon of wavelength λ is emitted for a transition $3 \to 1$. What will be the wavelength of emissions for transition $2 \to 1$

[CPMT 1996]

(a) $\lambda/3$ (b) $4\lambda/3$ (c) $3\lambda/4$ (d) 3λ

Solution : (d) For transition $3 \to 1$ $\Delta E = 2E - E = \frac{hc}{\lambda} \Rightarrow E = \frac{hc}{\lambda}$ .....(i)

For transition $2 \to 1$ $\frac{4E}{3} - E = \frac{hc}{\lambda'} \Rightarrow E = \frac{3hc}{\lambda'}$ .....(ii)

From equation (i) and (ii) $\lambda' = 3\lambda$

***Example*: 18** Hydrogen atom emits blue light when it changes from n = 4 energy level to n = 2 level. Which colour of light would the atom emit when it changes from n = 5 level to n = 2 level **[KCET 1993]**

(a) Red (b) Yellow (c) Green (d) Violet

Solution : (d) In the transition from orbits $5 \to 2$ more energy will be liberated as compared to transition from $4 \to 2$. So emitted photon would be of violet light.

***Example*: 19** A single electron orbits a stationary nucleus of charge $+Ze$, where Z is a constant. It requires 47.2 eV to excited electron from second Bohr orbit to third Bohr orbit. Find the value of Z **[IIT-JEE 1981]**

(a) 2 (b) 5 (c) 3 (d) 4

Solution : (b) Excitation energy of hydrogen like atom for $n_2 \to n_1$

$$\Delta E = 13.6Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)eV \Rightarrow 47.2 = 13.6Z^2\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = 13.6 \times \frac{5}{36}Z^2 \Rightarrow Z^2 = \frac{47.2 \times 36}{13.6 \times 5} = 24.98 \simeq 25$$

$\Rightarrow Z = 5$

***Example*: 20** The first member of the Paschen series in hydrogen spectrum is of wavelength 18,800 Å. The short wavelength limit of Paschen series is **[EAMCET (Med.) 2000]**

(a) 1215 Å (b) 6560 Å (c) 8225 Å (d) 12850 Å

Solution : (c) First member of Paschen series mean it's $\lambda_{max} = \frac{144}{7R}$

Short wavelength of Paschen series means $\lambda_{min} = \frac{9}{R}$

Hence $\frac{\lambda_{max}}{\lambda_{min}} = \frac{16}{7} \Rightarrow \lambda_{min} = \frac{7}{16} \times \lambda_{max} = \frac{7}{16} \times 18{,}800 = 8225 \mathring{A}$.

***Example*: 21** Ratio of the wavelengths of first line of Lyman series and first line of Balmer series is

[EAMCET (Engg.) 1995; MP PMT 1997]

(a) 1 : 3 (b) 27 : 5 (c) 5 : 27 (d) 4 : 9

Solution : (c) For Lyman series $\frac{1}{\lambda_{L_1}} = R\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = \frac{3R}{4}$ .....(i)

For Balmer series $\frac{1}{\lambda_{B_1}} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right] = \frac{5R}{36}$ .....(ii)

From equation (i) and (ii) $\frac{\lambda_{L_1}}{\lambda_{B_1}} = \frac{5}{27}$.

***Example*: 22** The third line of Balmer series of an ion equivalent to hydrogen atom has wavelength of 108.5 nm. The ground state energy of an electron of this ion will be **[RPET 1997]**

(a) 3.4 eV (b) 13.6 eV (c) 54.4 eV (d) 122.4 eV

Solution : (c) Using $\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \Rightarrow \frac{1}{108.5 \times 10^{-9}} = 1.1 \times 10^7 \times Z^2\left(\frac{1}{2^2} - \frac{1}{5^2}\right)$

$$\Rightarrow \frac{1}{108.5 \times 10^{-9}} = 1.1 \times 10^7 \times Z^2 \times \frac{21}{100} \Rightarrow Z^2 = \frac{100}{108.5 \times 10^{-9} \times 1.1 \times 10^{-7} \times 21} = 4 \Rightarrow Z = 2$$

Now Energy in ground state $E = -13.6Z^2\ eV = -13.6 \times 2^2\ eV = -54.4\ eV$

***Example*: 23** Hydrogen (H), deuterium (D), singly ionized helium (He^+) and doubly ionized lithium (Li^{++}) all have one electron around the nucleus. Consider $n = 2$ to $n = 1$ transition. The wavelengths of emitted radiations are $\lambda_1, \lambda_2, \lambda_3$ and λ_4 respectively. Then approximately **[KCET 1994]**

(a) $\lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$ (b) $4\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$ (c) $\lambda_1 = 2\lambda_2 = 2\sqrt{2}\lambda_3 = 3\sqrt{2}\lambda_4$ (d) $\lambda_1 = \lambda_2 = 2\lambda_3 = 3\lambda_4$

Solution : (a) Using $\Delta E \propto Z^2$ ($\because$ n_1 and n_2 are same)

$$\Rightarrow \frac{hc}{\lambda} \propto Z^2 \Rightarrow \lambda Z^2 = \text{constant} \Rightarrow \lambda_1 Z_1^2 = \lambda_2 Z_2^2 = \lambda_3 Z_3^2 = \lambda_4 Z^4 \Rightarrow \lambda_1 \times 1 = \lambda_2 \times 1^2 = \lambda_3 \times 2^2 = \lambda_4 \times 3^3$$

$\Rightarrow \lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$.

***Example*: 24** Hydrogen atom in its ground state is excited by radiation of wavelength 975 $\mathring{A}$. How many lines will be there in the emission spectrum

(a) 2 (b) 4 (c) 6 (d) 8

Solution : (c) Using $\frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] \Rightarrow \frac{1}{975 \times 10^{-10}} = 1.097 \times 10^7 \left(\frac{1}{1^2} - \frac{1}{n^2}\right) \Rightarrow n = 4$

Now number of spectral lines $N = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$.

***Example*: 25** A photon of energy 12.4 *eV* is completely absorbed by a hydrogen atom initially in the ground state so that it is excited. The quantum number of the excited state is

(a) $n = 1$ (b) $n = 3$ (c) $n = 4$ (d) $n = \infty$

Solution : (c) Let electron absorbing the photon energy reaches to the excited state n. Then using energy conservation

$\Rightarrow -\frac{13.6}{n^2} = -13.6 + 12.4 \Rightarrow -\frac{13.6}{n^2} = -1.2 \Rightarrow n^2 = \frac{13.6}{1.2} = 12 \Rightarrow n = 3.46 \simeq 4$

***Example*: 26** The wave number of the energy emitted when electron comes from fourth orbit to second orbit in hydrogen is 20,397 cm^{-1}. The wave number of the energy for the same transition in He^+ is

(a) 5,099 cm^{-1} (b) 20,497 cm^{-1} (c) 40,994 cm^{-1} (d) 81,998 cm^{-1}

Solution : (d) Using $\frac{1}{\lambda} = \bar{\nu} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \Rightarrow \bar{\nu} \propto Z^2 \Rightarrow \frac{\bar{\nu}_2}{\bar{\nu}_1} = \left(\frac{Z_2}{Z_1}\right)^2 = \left(\frac{Z}{1}\right)^2 = 4 \Rightarrow \bar{\nu}_2 = \bar{\nu} \times 4 = 81588\ cm^{-1}$.

***Example*: 27** In an atom, the two electrons move round the nucleus in circular orbits of radii R and $4R$. the ratio of the time taken by them to complete one revolution is

(a) 1/4 (b) 4/1 (c) 8/1 (d) 1/8

Solution : (d) Time period $T \propto \frac{n^3}{Z^2}$

For a given atom (Z = constant) So $T \propto n^3$ .....(i) and radius $R \propto n^2$ .....(ii)

$\therefore$ From equation (i) and (ii) $T \propto R^{3/2} \Rightarrow \frac{T_1}{T_2} = \left(\frac{R_1}{R_2}\right)^{3/2} = \left(\frac{R}{4R}\right)^{3/2} = \frac{1}{8}$.

***Example*: 28** Ionisation energy for hydrogen atom in the ground state is E. What is the ionisation energy of Li^{++} atom in the 2nd excited state

(a) E (b) $3E$ (c) $6E$ (d) $9E$

Solution : (a) Ionisation energy of atom in nth state $E_n = \frac{Z^2}{n^2}$

For hydrogen atom in ground state (n = 1) and Z = 1 $\Rightarrow E = E_0$ .....(i)

For Li^{++} atom in 2nd excited state n = 3 and Z = 3, hence $E' = \frac{E_0}{3^2} \times 3^2 = E_0$ .....(ii)

From equation (i) and (ii) $E' = E$.

***Example*: 29** An electron jumps from n = 4 to n = 1 state in H-atom. The recoil momentum of H-atom (in eV/C) is

(a) 12.75 (b) 6.75 (c) 14.45 (d) 0.85

Solution : (a) The H-atom before the transition was at rest. Therefore from conservation of momentum

Photon momentum = Recoil momentum of H-atom or $P_{recoil} = \frac{h\nu}{c} = \frac{E_4 - E_1}{c} = \frac{-0.85eV - (-13.6\ eV)}{c} = 12.75 \frac{eV}{c}$

***Example*: 30** If elements with principal quantum number $n > 4$ were not allowed in nature, the number of possible elements would be

[IIT-JEE 1983; CBSE PMT 1991, 93; MP PET 1999; RPET 1993, 2001; RPMT 1999, 2003; J & K CET 2004]

(a) 60 (b) 32 (c) 4 (d) 64

Solution : (a) Maximum value of n = 4

So possible (maximum) no. of elements

$N = 2\times1^2 + 2\times2^2 + 2\times3^2 + 2\times4^2 = 2+8+18+32 = 60$.

Tricky example: 1

If the atom ${}_{100}Fm^{257}$ follows the Bohr model and the radius of ${}_{100}Fm^{257}$ is n times the Bohr radius, then find n

[IIT-JEE (Screening) 2003]

(a) 100 (b) 200 (c) 4 (d) 1/4

Solution : (d) $(r_m) = \left(\frac{m^2}{Z}\right)(0.53\,Å) = (n\times 0.53\,Å) \Rightarrow \frac{m^2}{Z} = n$

m = 5 for ${}_{100}Fm^{257}$ (the outermost shell) and z = 100

$\therefore \quad n = \frac{(5)^2}{100} = \frac{1}{4}$

Tricky example: 2

An energy of 24.6 *eV* is required to remove one of the electrons from a neutral helium atom. The energy (in *eV*) required to remove both the electrons from a neutral helium atom is

(a) 79.0 (b) 51.8 (c) 49.2 (d) 38.2

Solution : (a) After the removal of first electron remaining atom will be hydrogen like atom.

So energy required to remove second electron from the atom $E = 13.6\times\frac{2^2}{1} = 54.4\,eV$

$\therefore$ Total energy required = 24.6 + 54.4 = 79 *eV*