CHAPTER 1

ELECTRIC CHARGES AND FIELD

1.1 FRICTIONAL ELECTRICITY

1. *What is frictional electricity ? When is a body said to be electrified or charged ?*

Frictional electricity. If a glass rod is rubbed with a silk cloth, or a fountain-pen with a coat-sleeve, it is able to attract small pieces of paper, straw, lint, light feathers, etc. Similarly, a plastic comb passed through dry hair can attract such light objects. In all these examples, we can say that the rubbed substance has become *electrified* or *electrically charged*. It is because of friction that the substances get charged on rubbing.

The property of rubbed substances due to which they attract light objects is called ***electricity****. The electricity developed by rubbing or friction is called* ***frictional*** *or* ***static electricity****. The rubbed substances which show this property of attraction are said to have become* ***electrified*** *or* ***electrically charged****.*

2. *Give a historical view of frictional electricity. From where did the term electricity get its origin ?*

Historical view of frictional electricity. In 600 B.C., *Thales of Miletus*, one of the founders of Greek science, first noticed that if a piece of amber is rubbed with a woollen cloth, it then acquires the property of attracting light feathers, dust, lint, pieces of leaves, etc.

In 1600 A.D., *William Gillbert*, the personal doctor to *Queen Elizabeth* – I of England, made a systematic study of the substances that behave like amber. In his book *De Magnete* (on the magnet), he introduced the name *electrica* for such substances. In fact, the Greek name for amber is *elektron* which is the origin of all such words : electricity, electric force, electric charge and electron.

For Your Knowledge

- Amber is a yellow resinous (gum like) substance found on the shores of the Baltic sea.
- Both electric and magnetic phenomena can be derived from charged particles. Magnetism arises from charges in motion. The charged particles in motion exert both electric and magnetic forces on each other. Hence electricity and magnetism are studied together as *electromagnetism*.

1.2 ELECTRIC CHARGE

3. *What is electric charge ? Is it a scalar or vector quantity ? Name its SI unit.*

Electric charge. Electric charge is an intrinsic property of the elementary particles like electrons, protons, etc., of which all the objects are made up of. It is because of these electric charges that various objects exert strong electric forces of attraction or repulsion on each other.

Electric charge *is an intrinsic property of elementary particles of matter which gives rise to electric force between various objects.*

Electric charge is a scalar quantity. Its SI unit is *coulomb* (C). A proton has a positive charge $(+e)$ and an electron has a negative charge $(-e)$, where

$$e = 1.6 \times 10^{-19} \text{ coulomb}$$

Large-scale matter that consists of equal number of electrons and protons is *electrically neutral*. If there is an excess of electrons, the body has a negative charge and an excess of protons results in a positive charge.

1.3 ELECTROSTATICS

4. *What is electrostatics ? Mention some of its important applications.*

Electrostatics. *Electrostatics is the study of electric charges at rest.* Here we study the forces, fields and potentials associated with static charges.

Applications of electrostatics. The attraction and repulsion between charged bodies have many industrial applications. Some of these are as follows :

1. In electrostatic loudspeaker.
2. In electrostatic spraying of paints and powder coating.
3. In flyash collection in chimneys.
4. In a Xerox copying machine.
5. In the design of a cathode-ray tube used in television and radar.

1.4 TWO KINDS OF ELECTRIC CHARGES

5. *How will you show experimentally that (i) there are only two kinds of electric charges and (ii) like charges repel and unlike charges attract each other ?*

Two kinds of electric charges. About 100 years ago, *Charles Du Fay* of France showed that electric charges on various objects are of only two kinds. The following simple experiments prove this fact.

EXPERIMENT 1

(*i*) Rub a glass rod with silk and suspend it from a rigid support by means of a silk thread. Bring another similarly charged rod near it. The two rods repel each other [Fig. 1.1(*a*)].

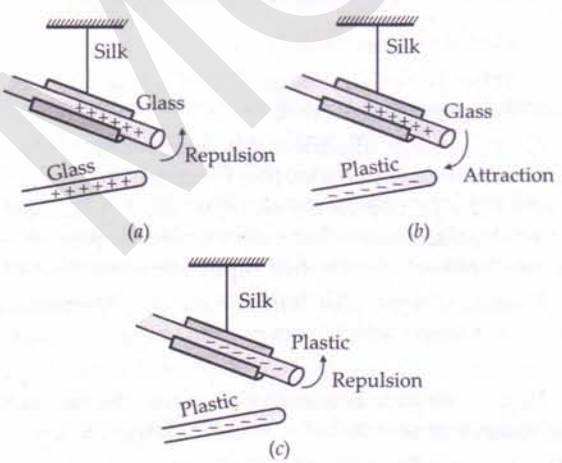


Fig. 1.1 Like charges repel and unlike charges attract each other.

(*ii*) Bring a plastic rod rubbed with wool near the charged glass rod. The two rods attract each other [Fig. 1.1(*b*)].

(*iii*) Now rub a plastic rod with wool and suspend it from a rigid support. Bring another similarly charged plastic rod near it. There will be a repulsion between the two rods [Fig. 1.1(*c*)].

EXPERIMENT 2. If a glass rod, rubbed with silk, is made to touch two small pith balls (or polystyrene balls) which are suspended by silk threads, then the two balls repel each other, as shown in Fig. 1.2(*a*). Similarly, two pith balls touched with a plastic rod rubbed with fur are found to repel each other [Fig. 1.2(*b*)]. But it is seen that a pith ball touched with glass rod attracts another pith ball touched with a plastic rod [Fig. 1.2(*c*)].

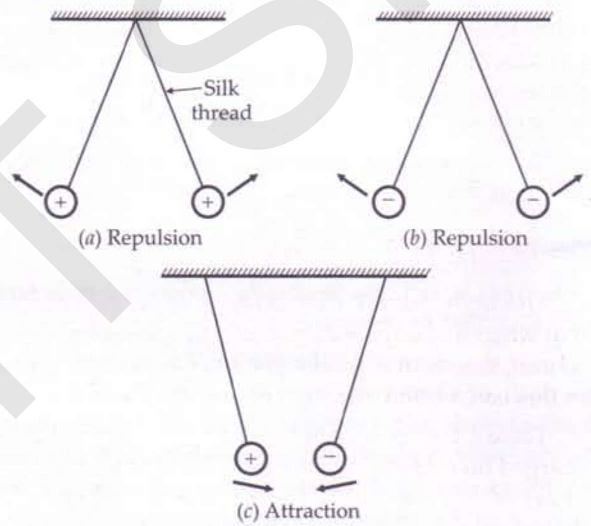


Fig. 1.2 Like charges repel and unlike charges attract.

From the above experiments, we note that the charge produced on a glass rod is different from the charge produced on a plastic rod. Also the charge produced on a pith ball touched with a glass rod is different from the charge produced on pith ball touched with a plastic rod. We can conclude that :

1. *There are only two kinds of electric charges – positive and negative.*
2. *Like charges repel and unlike charges attract each other.*

The statement 2 is known as the ***fundamental law of electrostatics***.

The above experiments also demonstrate that the charges are transferred from the rods to the pith balls on contact. We say that the pith balls have been electrified or charged by contact. *This property which distinguishes the two kinds of charges is called the* ***polarity of charge***.

6. *What are vitreous and resinous charges ? What was wrong with this nomenclature ?*

Vitreous and resinous charges. *Charles Du Fay* used the terms vitreous and resinous for the *two* kinds of charges.

1. *The charge developed on glass rod when rubbed with silk was called* ***vitreous charge*** (Latin *virtum* = glass).
2. *The charge developed on amber when rubbed with wool was called* ***resinous charge*** (amber is a resin).

But later on, these terms were found to be misleading. For example, a ground glass rod develops resinous electricity while a highly polished ebonite rod develops vitreous electricity.

7. *What are positive and negative charges ? What is the nature of charge on an electron in this convention ?*

Positive and negative charges. *Benjamin Franklin* (1706-1790), an American pioneer of electrostatics introduced the present-day convention by replacing the terms vitreous and resinous by positive and negative, respectively. According to this convention :

1. *The charge developed on a glass rod when rubbed with silk is called* ***positive charge****.*
2. *The charge developed on a plastic rod when rubbed with wool is called* ***negative charge****.*

The above convention is consistent with the fact that when two opposite kinds of charges are brought in contact, they tend to cancel each other's effect. According to this convention, *the charge on an electron is negative.*

Table 1.1 gives a list of the pairs of objects which get charged on rubbing against each other. On rubbing, an object of column I will acquire positive charge while that of column II will acquire negative charge.

Table 1.1 Two kinds of charges developed on rubbing

Column I (Positive charge)	Column II (Negative charge)
Glass rod	Silk cloth
Flannel or cat skin	Ebonite rod
Woollen cloth	Amber rod
Woollen coat	Plastic seat
Woollen carpet	Rubber shoes

Obviously, any two charged objects belonging to the same column will repel each other while those of two different columns will attract each other.

For Your Knowledge

- Benjamine's choice of positive and negative charges is purely conventional one. However, it is unfortunate that the charge on an electron (which is so important to physical and chemical properties of materials) turns out to be negative in this convention. It would have been more convenient if electrons were assigned positive charge. But in science, sometimes we have to live with the historical conventions.
- Different substances can be arranged in a series in such a way that if any two of them are rubbed together, then the one occurring earlier in the series acquires a positive charge while the other occurring later acquires a negative charge :

1.	Fur	2.	Flannel	3.	Sealing wax
4.	Glass	5.	Cotton	6.	Paper
7.	Silk	8.	Human body	9.	Wood
10.	Metals	11.	Rubber	12.	Resin
13.	Amber	14.	Sulphur	15.	Ebonite
16.	Guta parcha				

Thus glass acquires a positive charge when rubbed with silk but it acquires negative charge when rubbed with flannel.

1.5 ELECTRONIC THEORY OF FRICTIONAL ELECTRICITY

8. *Describe the electronic theory of frictional electricity. Are the frictional forces electric in origin ?*

Electronic theory of frictional electricity. All matter is made of atoms. An atom consists of a small central nucleus containing protons and neutrons, around which revolve a number of electrons. In any piece of matter, the positive proton charges and the negative electron charges cancel each other and so the matter in bulk is electrically neutral.

The electrons of the outer shell of an atom are loosely bound to the nucleus. The energy required to remove an electron from the surface of a material is called its 'work function'. When two different bodies are rubbed against each other, electrons are transferred from the material with lower work function to the material with higher work function. For example, when a glass rod is rubbed with a silk cloth, some electrons are transferred from glass rod to silk. The glass rod develops a positive charge due to deficiency of electrons while the silk cloth develops an equal negative charge due to excess of electrons. The combined total charge of the glass rod and silk cloth is still zero, as it was before rubbing *i.e.*, electric charge is conserved during rubbing.

Electric origin of frictional forces. The only way by which an electron can be pulled away from an atom is to exert a strong electric force on it. As electrons are actually transferred from one body to another during rubbing, so frictional forces must have an electric origin.

For Your Knowledge

- The cause of charging is the actual transfer of electrons from one material to another during rubbing. Protons are not transferred during rubbing.
- The material with lower work function loses electrons and becomes positively charged.
- As an electron has a finite mass, therefore, there always occurs some change in mass during charging. The mass of a positively charged body slightly decreases due to loss of some electrons. The mass of a negatively charged body slightly increases due to gain in some electrons.

1.6 CONDUCTORS AND INSULATORS

9. *How do the conductors differ from the insulators ? Why cannot we electrify a metal rod by rubbing it while holding it in our hand ? How can we charge it ?*

Conductors. *The substances through which electric charges can flow easily are called conductors.* They contain a large number of free electrons which make them good conductor of electricity. Metals, human and animal bodies, graphite, acids, alkalies, etc. are conductors.

Insulators. *The substances through which electric charges cannot flow easily are called insulators.* In the atoms of such substances, electrons of the outer shell are tightly bound to the nucleus. Due to the absence of free charge carriers, these substances offer high resistance to the flow of electricity through them. Most of the non-metals like glass, diamond, porcelain, plastic, nylon, wood, mica, etc. are insulators.

An important difference between conductors and insulators is that when some charge is transferred to a conductor, it readily gets distributed over its entire surface. On the other hand, if some charge is put on an insulator, it stays at the same place. We shall discuss this distinguishing feature in the next chapter.

A metal rod held in hand and rubbed with wool does not develop any charge. This is because the human body is a good conductor of electricity, so any charge developed on the metal rod is transferred to the earth through the human body. We can electrify the rod by providing it a plastic or a rubber handle and rubbing it without touching its metal part.

10. *What is meant by earthing or grounding in household circuits ? What is its importance ?*

Earthing and safety. When a charged body is brought in contact with the earth (through a connecting conductor), its entire charge passes to the ground in the form of a momentary current. *This process in which a body shares its charges with the earth is called* ***grounding*** *or* ***earthing***.

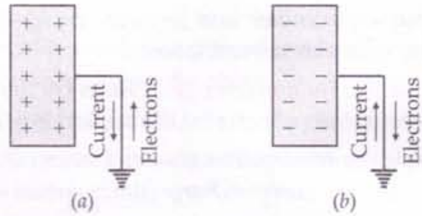


Fig. 1.3 (*a*) Positively charge (*b*) Negatively charge, earthed body.

The electricity from the mains is supplied to our houses using a three-core wiring : ***live, neutral*** and ***earth wires***. The live wire red in colour brings in the current. The black neutral wire is the return wire. The green earth wire is connected to a thick metal plate buried deep into the earth. The metallic bodies of the electric appliances such as electric iron, refrigerator, TV, etc. are connected to the earth wire. When any fault occurs or live wire touches the metallic body, the charge flows to the earth and the person who happens to touch the body of the appliance does not receive any shock.

1.7 ELECTROSTATIC INDUCTION

11. *What is meant by electrostatic induction ?*

Electrostatic induction. As shown in Fig. 1.4, hold a conducting rod *AB* over an insulating stand. Bring a positively charged glass rod near its end *A*. The free electrons of the conducting rod get attracted towards the end *A* while the end *B* becomes electron deficient. The closer end *A* acquires a negative charge while the remote end *B* acquires an equal positive charge. As soon as the glass rod is taken away, the charges at the ends *A* and *B* disappear.

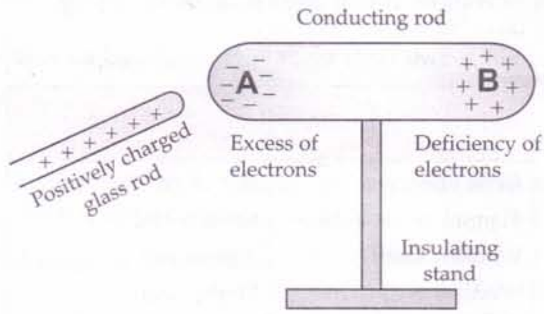


Fig. 1.4 Electrostatic induction.

Electrostatic induction *is the phenomenon of temporary electrification of a conductor in which opposite charges appear at its closer end and similar charges appear at its farther end in the presence of a nearby charged body.*

The positive and negative charges produced at the ends of the conducting rod are called *induced charges* and the charge on the glass rod which induces these charges on conducting rod is called *inducing charge*.

12. *Describe how two metal spheres can be oppositely charged by induction.*

Charging of two spheres by induction. Figure 1.5 shows the various steps involved in inducing opposite charges on two metal spheres.

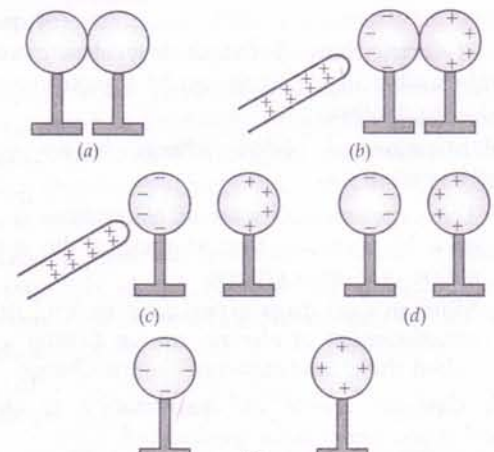


Fig. 1.5 Two metal spheres get oppositely charged by induction.

(*a*) Hold the two metal spheres on insulating stands and place them in contact, as shown in Fig. 1.5(*a*).

(*b*) Bring a positively charged glass rod near the left sphere. The free electrons of the spheres get attracted towards the glass rod. The left surface of the left sphere develops an excess of negative charge while the right side of the right sphere develops an excess of positive charge. However, all of the electrons of the spheres do not collect at the left face. As the negative charge begins to build up at the left face, it starts repelling the new incoming electrons. Soon an equilibrium is established under the action of force of attrac- tion of the rod and the force of repulsion due to the accumulated electrons. The equilibrium situation is shown in Fig. 1.5(*b*).

(*c*) Holding the glass rod near the left sphere, separate the two spheres by a small distance, as shown in Fig. 1.5(*c*). The two spheres now have opposite charges.

(*d*) Remove the glass rod. The charges on the spheres get redistributed. Their positive and negative charges face each other, as shown in Fig. 1.5(*d*). The two spheres attract each other.

(*e*) When the two spheres are separated quite apart, the charges on them get uniformly distributed, as shown in Fig. 1.5(*e*).

Thus the two metal spheres get charged by a process called ***charging by induction***. In contrast to the process of charging by contact, here the glass rod does not lose any of its charge.

13. *How can you charge a metal sphere positively without touching it ?*

Charging of a sphere by induction. Fig. 1.6 shows the various steps involved in inducing a positive charge on a metal sphere.

(*a*) Hold the metal sphere on an insulating stand. Bring a negatively charged plastic rod near it. The free electrons of the sphere are repelled to the farther end. The near end becomes positively charged due to deficit of electrons.

(*b*) When the far end of the sphere is connected to the ground by a connecting wire, its free electrons flow to the ground.

(*c*) When the sphere is disconnected from the ground, its positive charge at the near end remains held there due to the attractive force of the external charge.

(*d*) When the plastic rod is removed, the positive charge spreads uniformly on the sphere.

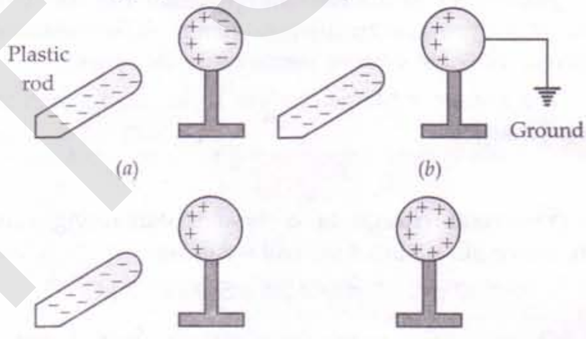
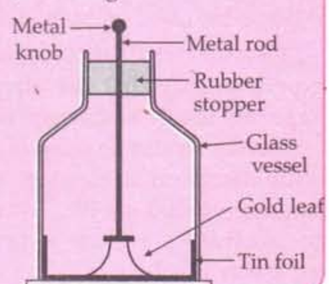


Fig. 1.6 Charging by induction.

Similarly, the metal sphere can be negatively charged by bringing a positively charged glass rod near it.

For Your Knowledge

➢ **Gold-leaf electroscope.** It is a device used for detecting an electric charge and identifying its polarity. It consists of a vertical conducting rod passing through a rubber stopper fitted in the mouth of a glass vessel. Two thin gold leaves are attached to lower end of the rod. When a charged object touches the metal knob at the outer end of the rod, the charge flows down to the leaves. The leaves diverge due to repulsion of the like charges they have received. The degree of divergence of the leaves gives a measure of the amount of charge.



1.8 BASIC PROPERTIES OF ELECTRIC CHARGE

It is observed from experiments that electric charge has following *three* basic properties :

1. Additivity **2.** Quantization **3.** Conservation.

We shall discuss these properties in detail in the next few sections.

1.9 ADDITIVITY OF ELECTRIC CHARGE

14. *What do you mean by additive nature of electric charges ?*

Additive nature of electric charges. Like mass, electric charge is a scalar quantity. Just as the mass of an extended body is the sum of the masses of its individual particles, the total charge of an extended body is the algebraic sum (*i.e.,* the sum taking into account the positive and negative signs) of all the charges located at different points inside it. Thus, *the electric charge is additive in nature.*

Additivity of electric charge *means that the total charge of a system is the algebraic sum of all the individual charges located at different points inside the system.*

If a system contains charges $q_1, q_2, ..., q_n$, then its total charge is

$$q = q_1 + q_2 + ... + q_n$$

The total charge of a system containing four charges 2 μC, – 3 μC, 4 μC and – 5 μC is

$$q = 2\ \mu C - 3\ \mu C + 4\ \mu C - 5\ \mu C = -2\ \mu C$$

1.10 QUANTIZATION OF ELECTRIC CHARGE

15. *What is meant by quantization of a physical quantity ?*

Quantization of a physical quantity. *The quantization of a physical quantity means that it cannot vary continuously to have any arbitrary value but it can change discontinuously to take any one of only a discrete set of values.* For example, a building can have different floors (ground, first, second, etc.) from the ground floor upwards but it cannot have a floor of the value in-between. Thus the energy of an electron in atom or the electric charge of a system is quantized. *The minimum amount by which a physical quantity can change is called its* ***quantum.***

16. *What is meant by quantization of electric charge ? What is the cause of quantization of electric charge ?*

Quantization of electric charge. It is found experimentally that the electric charge of any body, large or small, is always an integral multiple of a certain minimum amount of charge. This basic charge is the charge on an electron, which is denoted by e and has magnitude 1.6×10^{-19} coulomb. Thus the charge on an electron is $-e$, on a proton is $+e$ and that on α-particle is $+2e$.

The experimental fact that electric charges occur in discrete amounts instead of continuous amounts is called quantization of electric charge. The quantization of electric charge means that the total charge (q) of a body is always an integral multiple of a basic quantum of charge (e), i.e.,

$$q = ne \quad , \text{ where } n = 0, \pm 1, \pm 2, \pm 3, ...$$

Cause of quantization. The basic cause of quantization of electric charge is that during rubbing only an integral number of electrons can be transferred from one body to another.

Quantization of electric charge is an experimentally verified law :

1. The experimental laws of electrolysis discovered by Faraday first suggested the quantization of electric charge.
2. Millikan's oil drop experiment in 1912 on the measurement of electric charge further established the quantization of electric charge.

17. *Can we ignore the quantization of electric charge ? If yes, under what conditions ?*

When can we ignore the quantization of electric charge. While dealing with macroscopic charges ($q = ne$), we can ignore the quantization of electric charge. This is because the basic charge e is very small and n is very large in most practical situations, so q behaves as if it were continuous *i.e.,* as if a large amount of charge were flowing. For example, when we switch on a 60 W bulb, nearly 2×10^{18} electrons pass through its filament per second. Here the graininess or structure of charge does not show up *i.e.,* the bulb does not flicker with the entry of each electron. Quantization of charge becomes important at the microscopic level, where the charges involved are of the order of a few tens or hundreds of e.

For Your Knowledge

- The smallest amount of charge or basic quantum of charge is the charge on an electron or a proton. Its exact magnitude is $e = 1.602192 \times 10^{-19}$ C.
- Quantization of electric charge cannot be explained on the basis of classical electrodynamics or even modern physics. However, the physical and chemical properties of atoms, molecules and bulk matter cannot be explained without considering the quantization of electric charge.
- Recent discoveries in high energy physics have indicated that the elementary particles like protons and neutrons are themselves built out of more elementary units, called ***quarks***, which have charges (2/3) e and (– 1/3) e. Even if quark-model is established in future, the quantization of charge will still hold. Only the quantum of charge will reduce from e to $e/3$.
- Quantization is a universal law of nature. Like charge, energy and angular momentum of an electron are also quantized. However, quantization of mass is yet to be established.

Examples based on Quantisation of Electric Charge

Formulae Used

1. $q = ne$
2. Mass transferred during charging $= m_e \times n$

Units Used

q and e are in coulomb, n is pure integer.

Constants Used

$e = 1.6 \times 10^{-19}$ C, $m_e = 9.1 \times 10^{-31}$ kg

Example 1. *Which is bigger – a coulomb or a charge on an electron ? How many electronic charges form one coulomb of charge ?* [Haryana 01]

Solution. One coulomb of charge is bigger than the charge on an electron.

Charge on one electron, $e = 1.6 \times 10^{-19}$ C

$\therefore$ Number of electronic charges in 1 coulomb,

$$n = \frac{q}{e} = \frac{1\text{ C}}{1.6 \times 10^{-19}\text{ C}} = \mathbf{6.25 \times 10^{18}}.$$

Example 2. *A comb drawn through person's hair on a dry day causes 10^{22} electrons to leave the person's hair and stick to the comb. Calculate the charge carried by the comb.*

Solution. Here $n = 10^{22}$, $e = 1.6 \times 10^{-19}$ C

$\therefore \quad q = ne = 10^{22} \times 1.6 \times 10^{-19} = 1.6 \times 10^{3}$ C

As the comb has excess of electrons,

$\therefore$ Charge on comb $= \mathbf{-1.6 \times 10^{3}}$ **C.**

Example 3. *If a body gives out 10^9 electrons every second, how much time is required to get a total charge of 1 C from it ?* [NCERT]

Solution. Number of electrons given out by the body in one second $= 10^9$

Charge given out by the body in one second

$$= ne = 10^9 \times 1.6 \times 10^{-19}\text{ C}$$
$$= 1.6 \times 10^{-10}\text{ C}$$

Time required to get a charge of 1.6×10^{-10} C

$$= 1\text{ s}$$

Time required to get a charge of 1 C

$$= \frac{1}{1.6 \times 10^{-10}}\text{ s} = \mathbf{6.25 \times 10^{9}\text{ s}}$$

$$= \frac{6.25 \times 10^9}{365 \times 24 \times 3600}\text{ years} = \mathbf{198.18\ years.}$$

Thus from a body emitting 10^9 electrons per second, it will take nearly 200 years to get a charge of 1 C from that body. This shows how large is one coulomb as the unit of charge.

Example 4. *How much positive and negative charge is there in a cup of water ?* [NCERT]

Solution. Suppose the mass of water contained in a cup is 250 g. The molecular mass of water is 18 g.

Number of molecules present in 18 g of water

$$= \text{Avogadro's number} = 6.02 \times 10^{23}$$

$\therefore$ Number of molecules present in a cup (or 250 g) of water

$$n = \frac{6.02 \times 10^{23} \times 250}{18} = 8.36 \times 10^{24}$$

Each molecule of water (H_2O) contains $2 + 8 = 10$ electrons as well as 10 protons.

Total number of electrons or protons present in a cup of water,

$$n' = n \times 10 = 8.36 \times 10^{25}$$

Total negative charge carried by electrons or total positive charge carried by protons in a cup of water,

$$q = n'e$$
$$= 8.36 \times 10^{25} \times 1.6 \times 10^{-19}\text{ C} = \mathbf{1.33 \times 10^{7}\text{ C}}$$

Problems For Practice

1. Calculate the charge carried by 12.5×10^8 electrons. **[CBSE D 92]**
   (Ans. 2×10^{-10} C)
2. How many electrons would have to be removed from a copper penny to leave it with a positive charge of 10^{-7} C ? (Ans. 6.25×10^{11} electrons)
3. Calculate the charge on an alpha particle. Given charge on a proton $= 1.6 \times 10^{-19}$ C.
   (Ans. $+3.2 \times 10^{-19}$ C)
4. Calculate the charge on $^{56}_{26}Fe$ nucleus. Given charge on a proton $= 1.6 \times 10^{-19}$ C. (Ans. $+4.16 \times 10^{-18}$ C)
5. Determine the total charge on 75.0 kg of electrons.
   (Ans. -1.33×10^{13} C)
6. How many mega coulombs of positive (or negative) charge are present in 2.0 mole of neutral hydrogen (H_2) gas ?
7. Estimate the total number of electrons present in 100 g of water. How much is the total negative charge carried by these electrons ? Avogadro's number $= 6.02 \times 10^{23}$ and molecular mass of water $= 18$. (Ans. 5.35×10^6 C)

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3. An alpha particle contains 2 protons and 2 neutrons.
   $\therefore \quad q = +2e.$

4. $^{56}_{26}Fe$ nucleus contains 26 protons and 30 neutrons.

$\therefore \quad q = +26e$

5. $n = \dfrac{\text{Total mass}}{\text{Mass of an electron}} = \dfrac{75.0}{9 \times 10^{-31}} = \dfrac{25}{3} \times 10^{31}$

$q = -ne = -\dfrac{25}{3} \times 10^{31} \times 1.6 \times 10^{-19} = \mathbf{-1.33 \times 10^{13}}$ **C.**

6. Number of molecules in 2.0 mole of H_2 gas

$= 2.0 \times 6.02 \times 10^{23}$

As each H_2 molecule contains 2 electrons/protons, so

$n = 2 \times 2.0 \times 6.02 \times 10^{23} = 24.08 \times 10^{23}$

$\therefore \quad q = ne = 24.08 \times 10^{23} \times 1.6 \times 10^{-19}$

$= 0.3853 \times 10^{6}$ C $=$ **0.3853 MC.**

[1 MC = 10^6 C]

7. Proceed as in Example 4.

1.11 CONSERVATION OF CHARGE

18. *State the law of conservation of charge. Give some examples to illustrate this law.*

Law of conservation of charge. If some amount of matter is isolated in a certain region of space and no matter either enters or leaves this region by moving across its boundary, then whatever other changes may occur in the matter inside, its total charge will not change with time. This is the ***law of conservation of charge*** which states :

1. *The total charge of an isolated system remains constant.*
2. *The electric charges can neither be created nor destroyed, they can only be transferred from one body to another.*

The law of conservation of charge is obeyed both in large scale and microscopic processes. In fact, charge conservation is a global phenomenon *i.e., total charge of the entire universe remains constant.*

Examples :

1. When a glass rod is rubbed with a silk cloth, it develops a positive charge. But at the same time, the silk cloth develops an equal negative charge. Thus the net charge of the glass rod and the silk cloth is zero, as it was before rubbing.
2. The rocksalt ionises in aqueous solution as follows :

$$NaCl \rightleftharpoons Na^+ + Cl^-$$

As the total charge is zero before and after the ionisation, so charge is conserved.

3. Charge is conserved during the fission of a $^{235}_{92}U$ nucleus by a neutron.

$$^1_0n + ^{235}_{92}U \rightarrow ^{141}_{56}Ba + ^{92}_{36}Kr + 3\,^1_0n + \text{Energy}$$

Total charge before fission $(0 + 92)$

$=$ Total charge after fission $(56 + 36 + 3 \times 0)$

4. Electric charge is conserved during the phenomenon of ***pair production*** *in which a γ-ray photon materialises into an electron-positron pair.*

$$\underset{\text{zero charge}}{\gamma\text{ - ray}} \longrightarrow \underset{(-e)}{\text{electron}} + \underset{(+e)}{\text{positron}}$$

5. In ***annihilation of matter,*** an electron and a positron on coming in contact destroy each other, producing two γ-ray photons, each of energy 0.51 MeV.

$$\underset{(-e)}{\text{electron}} + \underset{(+e)}{\text{positron}} \longrightarrow \underset{\text{zero charge}}{2\,\gamma\text{ - rays}}$$

For Your Knowledge

- Conservation of charge implies that electric charges can be created or destroyed always in the form of equal and opposite pairs but never in isolation. For example, in the beta decay of a neutron (zero charge), a proton (charge $+e$) and an electron (charge $-e$) are produced. Total charge remains zero before and after the decay.
- The law of conservation of charge is an exact law of nature. It is valid in all domains of nature. Even in the domains of high energy physics, where mass changes into energy and vice-versa, the law of conservation of charge strictly holds good.

1.12 ELECTRIC CHARGE VS MASS

19. *Compare the properties of electric charge with those of mass of a body.*

Table 1.2 Comparison of the properties of electric charge and mass

	Electric charge	Mass
1.	Electric charge may be positive, negative or zero.	Mass of a body is always positive.
2.	Electric charge is always quantized : $q = ne$	Quantization of mass is not yet established.
3.	Charge on a body does not depend on its speed.	Mass of a body increases with its speed.
4.	Charge is strictly conserved.	Mass is not conserved by itself as some of the mass may get changed into energy or vice versa.
5.	Electrostatic forces between two charges may be attractive or repulsive.	Gravitational forces between two masses are always attractive.
6.	Electrostatic forces between different charges may cancel out.	Gravitational forces between different bodies never cancel out.
7.	A charged body always possesses some mass.	A body possessing mass may not have any net charge.

20. *How does the speed of an electrically charged particle affect its (i) mass and (ii) charge ?*

Effect of speed on mass and electric charge. According to the *special theory of relativity*, the mass of a body increases with its speed in accordance with the relation :

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

where, m_0 = rest mass of the body, c = speed of light,

and m = mass of the body when moving with speed v.

As $v < c$, therefore, $m > m_0$.

In contrast to mass, the charge on a body remains constant and does not change as the speed of the body changes.

1.13 COULOMB'S LAW OF ELECTRIC FORCE

21. *State Coulomb's law in electrostatics. Express the same in SI units. Name and define the units of electric charge.*

Coulomb's law. In 1785, the French physicist *Charles Augustin Coulomb* (1736-1806) experimentally measured the electric forces between small charged spheres by using a torsion balance. He formulated his observations in the form of Coulomb's law which is electrical analogue of Newton's law of Universal Gravitation in mechanics.

Coulomb's law *states that the force of attraction or repulsion between two stationary point charges is (i) directly proportional to the product of the magnitudes of the two charges and (ii) inversely proportional to the square of the distance between them. This force acts along the line joining the two charges.*

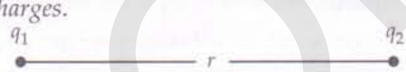


Fig. 1.7 Coulomb's law.

If two point charges q_1 and q_2 are separated by distance r, then the force F of attraction or repulsion between them is such that

$$F \propto q_1 q_2 \quad \text{and} \quad F \propto \frac{1}{r^2}$$

$$\therefore \quad F \propto \frac{q_1 q_2}{r^2} \quad \text{or} \quad F = k\,\frac{q_1 q_2}{r^2}$$

where k is a constant of proportionality, called ***electrostatic force constant***. The value of k depends on the nature of the medium between the two charges and the system of units chosen to measure F, q_1, q_2 and r.

For the two charges located in free space and in SI units, we have

$$k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{ Nm}^2\text{ C}^{-2}$$

where ε_0 is called *permittivity* of free space. So we can express Coulomb's law in SI units as

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}$$

Units of charge. (*i*) *The SI unit of charge is coulomb.* In the above equation, if $q_1 = q_2 = 1$ C and $r = 1$ m, then

$$F = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{ N}$$

So one ***coulomb*** *is that amount of charge that repels an equal and similar charge with a force of 9×10^9 N when placed in vacuum at a distance of one metre from it.*

(*ii*) In electrostatic *cgs* system, the unit of charge is known as *electrostatic unit of charge* (e.s.u. of charge) or *statcoulomb* (stat C).

One ***e.s.u. of charge*** *or one* ***statcoulomb*** *is that charge which repels an identical charge in vacuum at a distance of one centimetre from it with a force of 1 dyne.*

$$1 \text{ coulomb} = 3 \times 10^9 \text{ statcoulomb}$$
$$= 3 \times 10^9 \text{ e.s.u. of charge}$$

(*iii*) In electromagnetic *cgs* system, the unit of charge is **abcoulomb** or **electromagnetic unit of charge** (e.m.u. of charge).

$$1 \text{ coulomb} = \frac{1}{10} \text{ abcoulomb} = \frac{1}{10} \text{ e.m.u. of charge}$$

For Your Knowledge

- A torsion balance is a sensitive device to measure force.
- When the linear sizes of charged bodies are much smaller than the distance between them, their sizes may be ignored and the charged bodies are called *point charges*.
- Coulomb's law is valid only for point charges.
- In SI units, the *exact* value of the combination $4\pi\varepsilon_0$ is
$$4\pi\varepsilon_0 = \frac{10^7}{c^2}\text{ C}^2\text{N}^{-1}\text{m}^{-2}$$
where c is the speed of light in vacuum having the *exact* value $2.99792458 \times 10^8 \text{ ms}^{-1}$.
- *Electrostatic force constant,*
$$k = 8.98755 \times 10^9 \text{ Nm}^2\text{C}^{-2} \simeq 9 \times 10^9 \text{ Nm}^2\text{C}^{-2}.$$
- *Permittivity of free space,*
$$\varepsilon_0 = 8.8551485 \times 10^{-2} \text{ C}^2\text{N}^{-1}\text{m}^{-2}$$
$$\simeq 9 \times 10^{-2}\text{C}^2\text{N}^{-1}\text{ m}^{-2}.$$
- SI unit of permittivity
$$= \frac{\text{coulomb} \times \text{coulomb}}{\text{newton} \times \text{metre}^2} = \text{C}^2\text{N}^{-1}\text{m}^{-2}$$
The unit $\text{C}^2\text{N}^{-1}\text{m}^{-2}$ is usually expressed as *farad per metre* (Fm^{-1}).
- More strictly, the SI unit of charge 1 coulomb is equal to 1 ampere-second, where 1 ampere is defined in terms of the magnetic force between two current carrying wires.

1.14 COULOMB'S LAW IN VECTOR FORM

22. *Write Coulomb's law in vector form. What is the importance of expressing it in vector form ?*

Coulomb's law in vector form. As shown in Fig. 1.8, consider two positive point charges q_1 and q_2 placed in vacuum at distance r from each other. They repel each other.

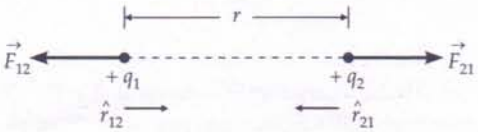


Fig. 1.8 Repulsive coulombian forces for $q_1 q_2 > 0$.

In vector form, Coulomb's law may be expressed as

$$\vec{F}_{21} = \text{Force on charge } q_2 \text{ due to } q_1$$
$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2} \hat{r}_{12}$$

where $\hat{r}_{12} = \frac{\vec{r}_{12}}{r}$, is a unit vector in the direction from q_1 to q_2.

Similarly, $\vec{F}_{12} = \text{Force on charge } q_1 \text{ due to } q_2$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2} \hat{r}_{21}$$

where $\hat{r}_{21} = \frac{\vec{r}_{21}}{r}$, is a unit vector in the direction from q_2 to q_1.

The coulombian forces between unlike charges ($q_1 q_2 < 0$) are attractive, as shown in Fig. 1.9.

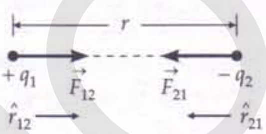


Fig. 1.9 Attractive coulombian forces for $q_1 q_2 < 0$.

Importance of vector form. The vector form of coulomb's law gives the following additional information :

1. As $\hat{r}_{21} = -\hat{r}_{12}$, therefore $\vec{F}_{21} = -\vec{F}_{12}$.

   This means that the two charges exert equal and opposite forces on each other. So *Coulombian forces obey Newton's third law of motion.*

2. As the Coulombian forces act along $\vec{F}_{12}$ or $\vec{F}_{21}$, *i.e.*, along the line joining the centres of two charges, so they are *central forces*.

23. *What is the range over which Coulombian forces can act ? State the limitations of Coulomb's law in electrostatics.*

Range of coulombian forces. Coulombian forces act over an enormous range of separations (r), from nuclear dimensions ($r = 10^{-15}$ m) to macroscopic distances as large as 10^{18} m. Inverse square is valid over this range of separation to a high degree of accuracy.

Limitations of Coulomb's law. Coulomb's law is not applicable in all situations. It is valid only under the following conditions :

1. The electric charges must be at rest.
2. The electric charges must be point charges *i.e.*, the extension of charges must be much smaller than the separation between the charges.
3. The separation between the charges must be greater than the nuclear size (10^{-15}m), because for distances $< 10^{-15}$m, the strong nuclear force dominates over the electrostatic force.

1.15 DIELECTRIC CONSTANT : RELATIVE PERMITTIVITY

24. *What do you mean by permittivity of a medium ? Define dielectric constant in terms of forces between two charges.*

Permittivity : An introduction. When two charges are placed in any medium other than air, the force between them is greatly affected. *Permittivity is a property of the medium which determines the electric force between two charges situated in that medium.* For example, the force between two charges located some distance apart in water is about 1/80th of the force between them when they are separated by same distance in air. This is because the absolute permittivity of water is about 80 times greater than the absolute permittivity of air or free space.

Dielectric constant or relative permittivity. According to Coulomb's law, the force between two point charges q_1 and q_2, placed in vacuum at distance r from each other, is given by

$$F_{vac} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2} \quad ...(1)$$

When the same two charges are placed same distance apart in any medium other than vacuum, the force between them becomes

$$F_{med} = \frac{1}{4\pi\varepsilon} \cdot \frac{q_1 q_2}{r^2} \quad ...(2)$$

The quantity ε is called *absolute permittivity* or just *permittivity* of the intervening medium. Dividing equation (1) by equation (2), we get

$$\frac{F_{vac}}{F_{med}} = \frac{\frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}}{\frac{1}{4\pi\varepsilon} \cdot \frac{q_1 q_2}{r^2}} = \frac{\varepsilon}{\varepsilon_0}$$

The ratio $(\varepsilon/\varepsilon_0)$ of the permittivity (ε) of the medium to the permittivity (ε_0) of free space is called relative permittivity (ε_r) or dielectric constant (κ) of the given medium. Thus

$$\varepsilon_r \text{ or } \kappa = \frac{\varepsilon}{\varepsilon_0} = \frac{F_{vac}}{F_{med}}$$

So one can define dielectric constant in terms of forces between charges as follows :

The ***dielectric constant*** or ***relative permittivity*** *of a medium may be defined as the ratio of the force between two charges placed some distance apart in free space to the force between the same two charges when they are placed the same distance apart in the given medium.*

Clearly, when a material medium of dielectric constant κ is placed between the charges, the force between them becomes $1/\kappa$ times the original force in vacuum. That is,

$$F_{med} = \frac{F_{vac}}{\kappa}$$

Hence the Coulomb's law for any material medium may be written as

$$F_{med} = \frac{1}{4\pi\,\varepsilon_0\kappa} \cdot \frac{q_1 q_2}{r^2}$$

$\kappa\,(\text{vacuum}) = 1$

$\kappa\,(\text{air}) = 1.00054$

$\kappa\,(\text{water}) = 80.$

Examples based on Coulomb's Law

Formulae Used

1. $F_{vac} = \dfrac{1}{4\pi\,\varepsilon_0} \cdot \dfrac{q_1\,q_2}{r^2}$
2. $F_{med} = \dfrac{1}{4\pi\,\varepsilon_0\,\kappa} \cdot \dfrac{q_1\,q_2}{r^2}$

Units Used

q_1, q_2 are in coulomb, F in newton and r in metre.

Constant Used

$$k = \frac{1}{4\pi\,\varepsilon_0} = 9 \times 10^9 \text{ Nm}^2\text{C}^{-2}$$

Example 5. *The electrostatic force of repulsion between two positively charged ions carrying equal charges is 3.7×10^{-9} N, when they are separated by a distance of 5 Å. How many electrons are missing from each ion ?*

Solution. Here $F = 3.7 \times 10^{-9}$ N,

$r = 5\text{ Å} = 5 \times 10^{-10}$ m, $q_1 = q_2 = q$ (say)

As $$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}$$

$$\therefore\quad 3.7 \times 10^{-9} = \frac{9 \times 10^9 \times q \times q}{(5 \times 10^{-10})^2}$$

or $$q^2 = \frac{3.7 \times 10^{-9} \times 25 \times 10^{-20}}{9 \times 10^9} = 10.28 \times 10^{-38}$$

or $$q = 3.2 \times 10^{-19}\text{ C}$$

Number of electrons missing from each ion is

$$n = \frac{q}{e} = \frac{3.2 \times 10^{-19}}{1.6 \times 10^{-19}} = \mathbf{2.}$$

Example 6. *A free pith-ball A of 8 g carries a positive charge of 5×10^{-8} C. What must be the nature and magnitude of charge that should be given to a second pith-ball B fixed 5 cm below the former ball so that the upper ball is stationary ?* [Haryana 01]

Solution. The pith-ball B must be of positive charge *i.e.*, of same nature as that of A, so that the upward force of repulsion balances the weight of pith-ball A.

When the pith-ball A remains stationary,

$$F = m_1 g$$

or $$\frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} = m_1 g$$

But $m_1 = 8\text{ g} = 8 \times 10^{-3}$ kg

$q_1 = 5 \times 10^{-8}$ C

$r = 5\text{ cm} = 0.05$ m

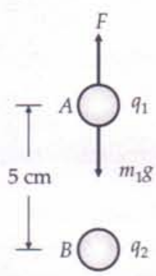


Fig. 1.10

$$\therefore\quad \frac{9 \times 10^9 \times 5 \times 10^{-8} \times q_2}{(0.05)^2} = 8 \times 10^{-3} \times 9.8$$

or $$q_2 = \frac{8 \times 9.8 \times (0.05)^2 \times 10^{-4}}{9 \times 5}$$

$$= \mathbf{4.36 \times 10^{-7}\,C\ (positive).}$$

Example 7. *A particle of mass m and carrying charge $-q_1$ is moving around a charge $+q_2$ along a circular path of radius r. Prove that the period of revolution of the charge $-q_1$ about $+q_2$ is given by*

$$T = \sqrt{\frac{16\pi^3\,\varepsilon_0\,mr^3}{q_1 q_2}}.$$

Solution. Suppose charge $-q_1$ moves around the charge $+q_2$ with speed v along the circular path of radius r. Then

Force of attraction between the two charges

= Centripetal force

or $$\frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} = \frac{mv^2}{r} \quad \text{or} \quad v = \sqrt{\frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{mr}}$$

The period of revolution of charge $-q_1$ around $+q_2$ will be

$$T = \frac{2\pi r}{v} = 2\pi r\sqrt{\frac{4\pi\,\varepsilon_0\,mr}{q_1 q_2}} \quad \text{or} \quad T = \sqrt{\frac{16\pi^3\varepsilon_0\,mr^3}{q_1 q_2}}.$$

Example 8. *Two particles, each having a mass of 5 g and charge 1.0×10^{-7} C, stay in limiting equilibrium on a horizontal table with a separation of 10 cm between them. The coefficient of friction between each particle and the table is the same. Find μ.*

Solution. Here $q_1 = q_2 = 1.0 \times 10^{-7}$ C,

$$r = 10 \text{ cm} = 0.10 \text{ m}, \quad m = 5 \text{ g} = 5 \times 10^{-3} \text{ kg}$$

The mutual electrostatic force between the two particles is

$$F = k\frac{q_1 q_2}{r^2} = \frac{9 \times 10^9 \times (1.0 \times 10^{-7})^2}{(0.10)^2} = 0.009 \text{ N}$$

The limiting force of friction between a particle and the table is

$$f = \mu \times mg = \mu \times 5 \times 10^{-3} \times 9.8 = 0.049\,\mu \text{ N}$$

As the two forces balance each other, therefore

$$0.049\,\mu = 0.009$$

or $$\mu = \frac{0.009}{0.049} = \mathbf{0.18}.$$

Example 9. (*a*) *Two insulated charged copper spheres A and B have their centres separated by a distance of 50 cm. What is the mutual force of electrostatic repulsion if the charge on each is 6.5×10^{-7} C ? The radii of A and B are negligible compared to the distance of separation. Also compare this force with their mutual gravitational attraction if each weighs 0.5 kg.*

(*b*) *What is the force of repulsion if (i) each sphere is charged double the above amount, and the distance between them is halved ; (ii) the two spheres are placed in water ? (Dielectric constant of water = 80).* [NCERT]

Solution. (*a*) Here $q_1 = q_2 = 6.5 \times 10^{-7}$ C,

$$r = 50 \text{ cm} = 0.50 \text{ m}$$

Using Coulomb's law,

$$F_{air} = k.\frac{q_1 q_2}{r^2}$$

$$= 9 \times 10^9.\frac{6.5 \times 10^{-7} \times 6.5 \times 10^{-7}}{(0.50)^2} \text{ N}$$

$$= \mathbf{1.5 \times 10^{-2}\ N}.$$

The mutual gravitational attraction,

$$F_G = G\frac{m_1 m_2}{R^2}$$

$$= \frac{6.67 \times 10^{-11} \times 0.5 \times 0.5}{(0.5)^2} = \mathbf{6.67 \times 10^{-11}\ N}$$

Clearly, $F_G << F_{air}$.

(*b*) (*i*) When charge on each sphere is doubled, and the distance between them is halved, the force of repulsion becomes

$$F'_{air} = k.\frac{2q_1.2q_2}{(r/2)^2} = 16\,k.\frac{q_1 q_2}{r^2}$$

$$= 16 \times 1.5 \times 10^{-2} = \mathbf{0.24\ N}.$$

(*ii*) The force between two charges placed in a medium of dielectric constant κ is given by

$$F = \frac{1}{4\pi\varepsilon_0}.\frac{1}{\kappa}.\frac{q_1 q_2}{r^2}$$

For water, $\kappa = 80$

$$\therefore \quad F_{water} = \frac{F_{air}}{\kappa} = \frac{1.5 \times 10^{-2}}{80}$$

$$= 1.875 \times 10^{-4} \text{ N} \simeq \mathbf{1.9 \times 10^{-4}\ N}.$$

Example 10. *Suppose the spheres A and B in Example 9 have identical sizes. A third sphere of the same size but uncharged is brought in contact with the first, then brought in contact with the second, and finally removed from both. What is the new force of repulsion between A and B ?* [NCERT]

Solution. Charge on each of the spheres *A* and *B* is

$$q = 6.5 \times 10^{-7} \text{ C}$$

When a similar but uncharged sphere *C* is placed in contact with sphere *A*, each sphere shares a charge $q/2$, equally.

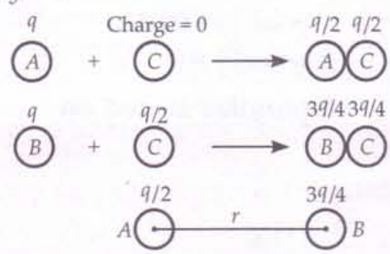


Fig. 1.11

Now when the sphere *C* (with charge $q/2$) is placed in contact with sphere *B* (with charge *q*), the charge is redistributed equally, so that

$$\text{Charge on sphere } B \text{ or } C = \frac{1}{2}\left(q + \frac{q}{2}\right) = \frac{3q}{4}$$

∴ New force of repulsion between *A* and *B* is

$$F = \frac{1}{4\pi\varepsilon_0}.\frac{\frac{3q}{4}.\frac{q}{2}}{r^2}$$

$$= \frac{3}{8} \times 1.5 \times 10^{-2} \text{ N} = 0.5625 \times 10^{-2} \text{ N}$$

$$\simeq \mathbf{5.7 \times 10^{-3}\ N}.$$

Example 11. *Two similarly equally charged identical metal spheres A and B repel each other with a force of 2.0×10^{-5} N. A third identical uncharged sphere C is touched to A, then placed at the midpoint between A and B. Calculate the net electrostatic force on C.* [CBSE OD 03]

Solution. Let the charge on each of the spheres A and B be q. If the separation between A and B is r, then electrostatic force between spheres A and B will be

$$F = k \cdot \frac{q^2}{r^2} = 2.0 \times 10^{-5}\ \text{N}$$

When sphere C is touched to A, the spheres share charge $q/2$ each, because both are identical.

$\therefore$ Force on C due to A

$$= k \cdot \frac{(q/2)^2}{(r/2)^2} = k\frac{q^2}{r^2}, \quad \text{along } AC$$

Force on C due to B

$$= k \cdot \frac{q \cdot q/2}{(r/2)^2} = k \cdot \frac{2q^2}{r^2}, \quad \text{along } BC$$

Since these forces act in opposite directions, therefore net force on C is

$$F' = k \cdot \frac{2q^2}{r^2} - k \cdot \frac{q^2}{r^2} = k\frac{q^2}{r^2} = \mathbf{2.0 \times 10^{-5}\ N, along\ \mathit{BC}.}$$

Example 12. *Two identical charges, Q each, are kept at a distance r from each other. A third charge q is placed on the line joining the above two charges such that all the three charges are in equilibrium. What is the magnitude, sign and position of the charge q ?* [CBSE OD 94, 98]

Solution. Suppose the three charges be placed in the manner, as shown in Fig. 1.12.

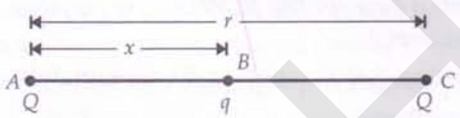


Fig. 1.12

The charge q will be in equilibrium if the forces exerted on it by the charges at A and C are equal and opposite.

$$k \cdot \frac{Qq}{x^2} = k \cdot \frac{Qq}{(r-x)^2} \quad \text{or} \quad x^2 = (r-x)^2$$

$$\text{or} \quad x = r - x \quad \text{or} \quad x = \frac{r}{2}$$

Since the charge at A is repelled by the similar charge at C, so it will be in equilibrium if it is attracted by the charge q at B, *i.e.*, the sign of charge q should be opposite to that of charge Q.

$\therefore$ Force of repulsion between charges at A and C

= Force of attraction between charges at A and B

$$\text{or} \quad k\frac{Q \cdot q}{(r/2)^2} = k\frac{Q \cdot Q}{r^2} \quad \text{or} \quad q = \frac{Q}{4}.$$

Example 13. *Two point charges + 4e and + e are 'fixed' a distance 'a' apart. Where should a third point charge q be placed on the line joining the two charges so that it may be in equilibrium ? In which case the equilibrium will be stable and in which unstable ?*

Solution. Suppose the three charges are placed as shown in Fig. 1.13. Let the charge q be positive.

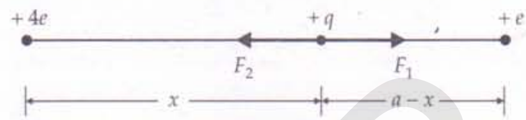


Fig. 1.13

For the equilibrium of charge $+q$, we must have

Force of repulsion F_1 between $+4e$ and $+q$

= Force of repulsion F_2 between $+e$ and $+q$

$$\text{or} \quad \frac{1}{4\pi\varepsilon_0}\frac{4e \times q}{x^2} = \frac{1}{4\pi\varepsilon_0}\frac{e \times q}{(a-x)^2}$$

$$\text{or} \quad 4(a-x)^2 = x^2$$

$$\text{or} \quad 2(a-x) = \pm x$$

$$\therefore \quad x = \frac{2a}{3} \quad \text{or} \quad 2a$$

As the charge q is placed between $+4e$ and $+e$, so only $x = 2a/3$ is possible. Hence for equilibrium, the charge q must be placed at a distance $2a/3$ from the charge $+4e$.

We have considered the charge q to be positive. If we displace it slightly towards charge e, from the equilibrium position, then F_1 will decrease and F_2 will increase and a net force $(F_2 - F_1)$ will act on q towards left *i.e.*, towards the equilibrium position. Hence the *equilibrium of positive q is stable.*

Now if we take charge q to be negative, the forces F_1 and F_2 will be attractive, as shown in Fig. 1.14.

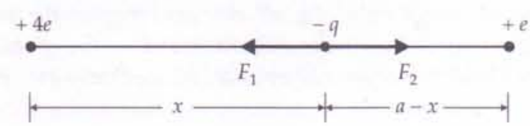


Fig. 1.14

The charge $-q$ will still be in equilibrium at $x = 2a/3$. However, if we displace charge $-q$ slightly towards right, then F_1 will decrease and F_2 will increase. A net force $(F_2 - F_1)$ will act on $-q$ towards right *i.e.*, away from the equilibrium position. So the *equilibrium of the negative q will be unstable.*

Example 14. *Two 'free' point charges + 4e and + e are placed a distance 'a' apart. Where should a third point charge q be placed between them such that the entire system may be in equilibrium ? What should be the magnitude and sign of q ? What type of a equilibrium will it be ?*

Solution. Suppose the charges are placed as shown in Fig. 1.15.

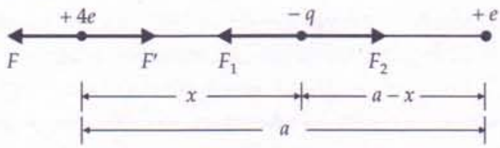


Fig. 1.15

As the charge $+e$ exerts repulsion F on charge $+4e$, so for the equilibrium of charge $+4e$, the charge $-q$ must exert attraction F' on $+4e$. This requires the charge q to be *negative*.

For equilibrium of charge $+4e$,

$$F = F'$$

$$\frac{1}{4\pi\varepsilon_0}\frac{4e \times e}{a^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{4e \times q}{x^2}$$

or $$q = \frac{ex^2}{a^2}$$

For equilibrium of charge $-q$,

Attraction F_1 between $+4e$ and $-q$

= Attraction F_2 between $+e$ and $-q$

$$\therefore \frac{1}{4\pi\varepsilon_0}\frac{4e \times q}{x^2} = \frac{1}{4\pi\varepsilon_0}\frac{e \times q}{(a-x)^2}$$

or $$x^2 = 4(a-x)^2$$

$\therefore$ $$x = 2a/3$$

Hence $$q = \frac{ex^2}{a^2} = \frac{e}{a^2}\cdot\frac{4a^2}{9} = \mathbf{\frac{4e}{9}}.$$

The equilibrium of the negative charge q will be *unstable*.

Example 15. *Two point charges of charge values Q and q are placed at distances x and x / 2 respectively from a third charge of charge value 4q, all charges being in the same straight line. Calculate the magnitude and nature of charge Q, such that the net force experienced by the charge q is zero.* [CBSE D 98]

Solution. Suppose the three charges are placed as shown in Fig. 1.16.

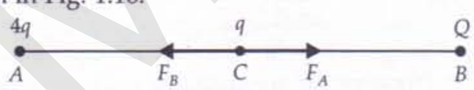


Fig. 1.16

For the equilibrium of charge q, the charge Q must have the same sign as that of q or $4q$, so that the forces F_A and F_B are equal and opposite.

As $$F_A = F_B$$

$$\therefore \frac{1}{4\pi\varepsilon_0}\cdot\frac{4q \times q}{(x/2)^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q \times Q}{(x/2)^2}$$

or $$Q = \mathbf{4q}.$$

Example 16. *A charge Q is to be divided on two objects. What should be the values of the charges on the two objects so that the force between the objects can be maximum ?*

Solution. Let q and $Q - q$ be the charges on the two objects. Then force between the two objects is

$$F = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q(Q-q)}{r^2}$$

where r is the distance between the two objects.

For F to be maximum,

$$\frac{dF}{dq} = 0$$

or $$\frac{1}{4\pi\varepsilon_0}\cdot\frac{1}{r^2}\cdot\frac{d}{dq}(qQ - q^2) = 0$$

or $$\frac{d}{dq}(qQ - q^2) = 0$$

or $$Q - 2q = 0$$

or $$q = \frac{Q}{2}$$

i.e., the charge should be divided equally on the two objects.

Example 17. *Two identical spheres, having charges of opposite sign attract each other with a force of 0.108 N when separated by 0.5 m. The spheres are connected by a conducting wire, which then removed, and thereafter they repel each other with a force of 0.036 N. What were the initial charges on the spheres ?*

Solution. Let $+q_1$ and $-q_2$ be the initial charges on the two spheres.

(*a*) When the two spheres attract each other,

$$F = k\frac{q_1 q_2}{r^2} \quad i.e., \quad 0.108 = 9 \times 10^9 \cdot \frac{q_1 q_2}{(0.5)^2}$$

$$\therefore \quad q_1 q_2 = \frac{0.108 \times (0.5)^2}{9 \times 10^9} = 3 \times 10^{-12}$$

(*b*) When the two spheres are connected by the wire, they share the charges equally.

$\therefore$ Charge on each sphere $= \frac{q_1 + (-q_2)}{2} = \frac{q_1 - q_2}{2}$

Force of repulsion between them is

$$F = \frac{k\left(\frac{q_1 - q_2}{2}\right)\left(\frac{q_1 - q_2}{2}\right)}{r^2}$$

i.e., $$0.036 = \frac{9 \times 10^9}{(0.5)^2}\cdot\left(\frac{q_1 - q_2}{2}\right)^2$$

$$\therefore (q_1 - q_2)^2 = \frac{0.036 \times (0.5)^2 \times 4}{9 \times 10^9} = 4 \times 10^{-12}$$

or $$q_1 - q_2 = 2 \times 10^{-6} \qquad ...(i)$$

Now $(q_1 + q_2)^2 = (q_1 - q_2)^2 + 4q_1 q_2$
$= (2 \times 10^{-6})^2 + 4 \times 3 \times 10^{-12}$
$= 16 \times 10^{-12}$

$\therefore \quad q_1 + q_2 = 4 \times 10^{-6}$ ...(*ii*)

On solving equations (*i*) and (*ii*), we get

$q_1 = \mathbf{3 \times 10^{-6}\ C}$ and $q_2 = \mathbf{10^{-6}\ C}$

which are the initial charges on the two spheres.

Example 18. *Two small spheres each having mass m kg and charge q coulomb are suspended from a point by insulating threads each l metre long but of negligible mass. If θ is the angle, each thread makes with the vertical when equilibrium has been attained, show that*

$$q^2 = (4\, mgl^2 \sin^2\theta \tan\theta)\, 4\pi\varepsilon_0$$ [Punjab 95]

Solution. The given situation is shown in Fig. 1.17. Each of the spheres A and B is acted upon by the following forces :

(*i*) its weight mg, (*ii*) tension T in the string

(*iii*) the force of repulsion F given by

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q^2}{AB^2} \qquad ...(i)$$

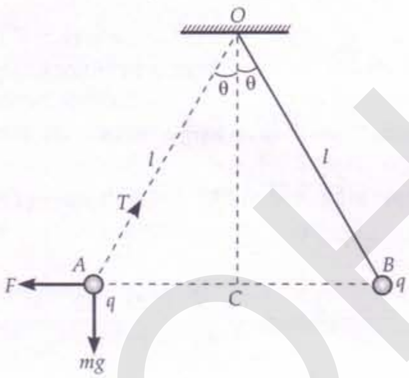


Fig. 1.17

As the forces are in equilibrium, the three forces on sphere A can be represented by the three sides of $\Delta\, AOC$ taken in the same order. Hence

$$\frac{F}{AC} = \frac{mg}{OC} = \frac{T}{AO}$$

or $$F = mg \times \frac{AC}{OC} \qquad ...(ii)$$

From (*i*) and (*ii*), we have

$$\frac{1}{4\pi\varepsilon_0} \cdot \frac{q^2}{AB^2} = mg \times \frac{AC}{OC}$$

But $AC = l \sin\theta$, $OC = l\cos\theta$, $AB = 2\ AC = 2l \sin\theta$

$$\therefore \quad \frac{1}{4\pi\varepsilon_0} \cdot \frac{q^2}{4l^2 \sin^2\theta} = mg \times \frac{l \sin\theta}{l\cos\theta}$$

or $$q^2 = (4\, mg\, l^2 \sin^2\theta \tan\theta)\, 4\pi\varepsilon_0.$$

Problems For Practice

1. Obtain the dimensional formula of ε_0. (Ans. $M^{-1}L^{-3}T^4A^2$)
2. Calculate coulomb force between two α-particles separated by a distance of 3.2×10^{-15} m in air. **[CBSE OD 92]** (Ans. 90 N)
3. Calculate the distance between two protons such that the electrical repulsive force between them is equal to the weight of either. **[CBSE D 94]** (Ans. 1.18 cm)
4. How far apart should the two electrons be, if the force each exerts on the other is equal to the weight of the electron ? Given that $e = 1.6 \times 10^{-19}$ C and $m_e = 9.1 \times 10^{-31}$ kg. **[Haryana 02]** (Ans. 5.08 m)
5. A pith-ball A of mass 9×10^{-5} kg carries a charge of 5 μC. What must be the magnitude and sign of the charge on a pith-ball B held 2 cm directly above the pith-ball A, such that the pith-ball A remains stationary ? (Ans. 7.84 pC, sign opposite to that of A)
6. Two identical metal spheres having equal and similar charges repel each other with a force of 103 N when they are placed 10 cm apart in a medium of dielectric constant 5. Determine the charge on each sphere. (Ans. 23.9×10^{-6} C)
7. The distance between the electron and proton in hydrogen atom is 5.3×10^{-11} m. Determine the magnitude of the ratio of electrostatic and gravitational force between them.
   Given $m_e = 9.1 \times 10^{-31}$ kg, $m_p = 1.67 \times 10^{-27}$ kg, $e = 1.6 \times 10^{-19}$ C and $G = 6.67 \times 10^{-11}$ Nm^2 kg^{-2}. (Ans. $F_e / F_G = 2.27 \times 10^{39}$)
8. Two identical metallic spheres, having unequal, opposite charges are placed at a distance 0.90 m apart in air. After bringing them in contact with each other, they are again placed at the same distance apart. Now the force of repulsion between them is 0.025 N. Calculate the final charge on each of them. **[CBSE D 02C]** (Ans. 1.5×10^{-6}C)
9. A small brass sphere having a positive charge of 1.7×10^{-8} C is made to touch another sphere of the same radius having a negative charge of 3.0×10^{-9} C. Find the force between them when they are separated by a distance of 20 cm. What will be the force between them when they are immersed in an oil of dielectric constant 3 ? (Ans. 1.1×10^{-5} N ; 0.367×10^{-5} N)

10. The sum of two point charges is $7\,\mu C$. They repel each other with a force of 1 N when kept 30 cm apart in free space. Calculate the value of each charge. **[CBSE F 09]**

(Ans. $5\,\mu C$, $2\,\mu C$)

11. Two point charges $q_1 = 5 \times 10^{-6}$C and $q_2 = 3 \times 10^{-6}$C are located at positions (1 m, 3 m, 2 m) and (3 m, 5 m, 1 m) respectively. Find the forces $\vec{F}_{12}$ and $\vec{F}_{21}$ using vector form of Coulomb's law.

[Ans. $\vec{F}_{12} = -5 \times 10^{-3}(2\hat{i} + 2\hat{j} - \hat{k})$N, $\vec{F}_{21} = 5 \times 10^{-3}(2\hat{i} + 2\hat{j} - \hat{k})$N]

12. Three equally charged small objects are placed as shown in Fig. 1.18. The object A exerts an electric force on object B equal to 3.0×10^{-6}N.

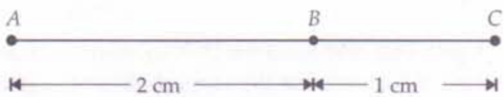


Fig. 1.18

(*i*) What electric force does C exert on B?

(*ii*) What is the net electric force on B?

[Ans. (*i*) 12.0×10^{-6} N, along BA
(*ii*) 9.0×10^{-6} N, along BA]

13. Two identical metallic spheres A and B, each carrying a charge q, repel each other with a force F. A third metallic sphere C of the same size, but uncharged, is successively made to touch the spheres A and B, and then removed away. What is the force of repulsion between A and B? (Ans. $3F/8$)

14. Two point charges $+9e$ and $+e$ are kept at a distance a from each-other. Where should we place a third charge q on the line joining the two charges so that it may be in equilibrium?

$\left(\text{Ans. } \frac{3a}{4} \text{ from } +9e \text{ charge}\right)$

15. Two point electric charges of values q and $2q$ are kept at a distance d apart from each other in air. A third charge Q is to be kept along the same line in such a way that the net force acting on q and $2q$ is zero. Calculate the position of charge Q in terms of q and d. **[CBSE D 98]**

(Ans. At a distance of $(\sqrt{2} - 1)\,d$ from charge q)

16. A charge q is placed at the centre of the line joining two equal charges Q. Show that the system of three charges will be in equilibrium if $q = -Q/4$. **[CBSE OD 05]**

17. Two pith-balls each weighing 10^{-3}kg are suspended from the same point by means of silk threads 0.5 m long. On charging the balls equally, they are found to repel each other to a distance of 0.2 m. Calculate the charge on each ball. **[Haryana 2002]**

(Ans. 2.357×10^{-6} C)

HINTS

1. $F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}$ or $\varepsilon_0 = \frac{q_1 q_2}{4\pi F r^2}$

$\therefore \quad [\varepsilon_0] = \frac{\text{AT . AT}}{\text{MLT}^{-2}\,.\,\text{L}^2} = \mathbf{[M^{-1}L^{-3}T^4\,A^2]}.$

2. Here $q_1 = q_2 = 2e = 3.2 \times 10^{-19}$ C, $r = 3.2 \times 10^{-15}$ m

$\therefore \quad F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}$

$= \frac{9 \times 10^9 \times 3.2 \times 10^{-19} \times 3.2 \times 10^{-19}}{(3.2 \times 10^{-15})^2} = \mathbf{90\ N.}$

3. For a proton, $m = 1.67 \times 10^{-27}$ kg,

$q = +e = 1.6 \times 10^{-19}$ C.

Weight of proton = Electrical repulsive force

or $\quad mg = k \cdot \frac{q \times q}{r^2}$

$\therefore \quad r^2 = \frac{kq^2}{mg} = \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{1.67 \times 10^{-27} \times 9.8}$

$= \frac{23.04}{16.36} \times 10^{-2} = 0.014$

or $\quad r = \mathbf{0.0118\ m = 1.18\ cm.}$

4. $\quad m_e\, g = k \cdot \frac{e \times e}{r^2}$

or $\quad r^2 = \frac{ke^2}{m_e\, g} = \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{9.1 \times 10^{-31} \times 9.8} = 25.84$

$r = \mathbf{5.08\ m.}$

5. The pith-ball B must have charge opposite to that of A so that the upward force of attraction balances the weight of pith-ball A.

When the pith-ball A remains stationary,

$F = m_1 g$

or $\quad \frac{1}{4\pi\,\varepsilon_0} \frac{q_1\, q_2}{r^2} = m_1\, g$

But $\quad m_1 = 9 \times 10^{-5}$ kg,

$q_1 = 5\,\mu C = 5 \times 10^{-6}$ C,

$r = 2$ cm $= 0.02$ m

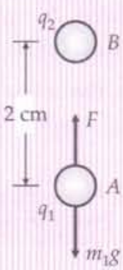


Fig. 1.19

$\therefore \quad \frac{9 \times 10^9 \times 5 \times 10^{-6} \times q_2}{(0.02)^2} = 9 \times 10^{-5} \times 9.8$

or $\quad q_2 = 7.84 \times 10^{-12}$ C $= \mathbf{7.84\ pC.}$

6. $F = \frac{1}{4\pi\,\varepsilon_0\,\kappa}\,\frac{q_1\,q_2}{r^2}$ $\therefore$ $103 = \frac{9\times10^9\times q^2}{5\times(0.10)^2}$

or $q = \mathbf{23.9\times10^{-6}}$ **C.**

7. Proceed as in illustrative problem on page 1.18.

8. The two spheres will share the final charge equally. Let q be the charge on each sphere.

$$\therefore \quad F = \frac{1}{4\pi\,\varepsilon_0}\cdot\frac{q_1q_2}{r^2} = 0.025\text{ N}$$

$$\text{or} \quad \frac{9\times10^9\times q\times q}{(0.90)^2} = 0.025$$

$$\text{or} \quad q^2 = \frac{0.025\times(0.90)^2}{9\times10^9} = 225\times10^{-14}$$

$$\text{or} \quad q = \mathbf{1.5\times10^{-6}\ C.}$$

9. Charge shared by each sphere

$$= \frac{(17-3)\times10^{-9}}{2} = 7\times10^{-9}\text{ C}$$

$$F_{\text{air}} = \frac{9\times10^9\times(7\times10^{-9})^2}{(0.20)^2} = \mathbf{1.1\times10^{-5}\,N}$$

$$F_{\text{oil}} = \frac{9\times10^9\times(7\times10^{-9})^2}{3\times(0.20)^2} = \mathbf{0.367\times10^{-5}\ N.}$$

10. Here $F = 1\text{ N}$, $r = 30\text{ m}$

As $F = k\,\frac{q_1q_2}{r^2}$

$$\therefore \quad 1 = \frac{9\times10^9\times q_1q_2}{(0.30)^2}$$

or $q_1q_2 = 10^{-11}$

But $q_1 + q_2 = 7\,\mu\text{C} = 7\times10^{-6}\text{ C}$ ...(*i*)

Now $(q_1-q_2)^2 = (q_1+q_2)^2 - 4q_1q_2$

$$= 49\times10^{-12} - 4\times10^{-11}$$

$$= 9\times10^{-12}$$

or $q_1 - q_2 = 3\times10^{-6} = 3\,\mu\text{C}$ ...(*ii*)

On solving (*i*) and (*ii*), we get

$q_1 = \mathbf{5\,\mu C}$ and $q_2 = \mathbf{2\,\mu C.}$

11. Here $\vec{r}_1 = (\hat{i}+3\hat{j}+2\hat{k})\text{m}$, $\vec{r}_2 = (3\hat{i}+5\hat{j}+\hat{k})\text{m}$

$$\therefore \quad \vec{r}_{12} = \vec{r}_2 - \vec{r}_1 = (3\hat{i}+5\hat{j}+\hat{k}) - (\hat{i}+3\hat{j}+2\hat{k})$$

$$= (2\hat{i}+2\hat{j}-\hat{k})\text{ m}$$

$$|\vec{r}_{12}| = \sqrt{2^2+2^2+(-1)^2} = 3\text{ m}$$

$$\hat{r}_{12} = \frac{\vec{r}_{12}}{|\vec{r}_{12}|} = \frac{2\hat{i}+2\hat{j}-\hat{k}}{3}$$

$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\,\frac{q_1q_2}{r_{12}^2}\,\hat{r}_{12}$$

$$= \frac{9\times10^9\times5\times10^{-6}\times3\times10^{-6}}{3^2}\cdot\frac{(2\hat{i}+2\hat{j}-\hat{k})}{3}$$

$$= 5\times10^{-3}(2\hat{i}+2\hat{j}-\hat{k})\text{ N}$$

Also, $\vec{F}_{12} = -\vec{F}_{21} = -5\times10^{-3}(2\hat{i}+2\hat{j}-\hat{k})$ N.

12. Here $AB = 2\text{ cm} = 0.02\text{ m}$, $BC = 1\text{ cm} = 0.01\text{ m}$

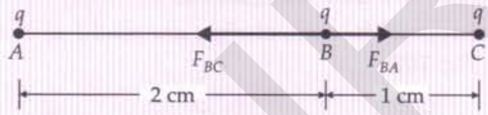


Fig. 1.20

Let q be the charge on each object.

$$F_{BA} = \frac{1}{4\pi\varepsilon_0}\,\frac{q\times q}{(AB)^2}$$

$$\text{or} \quad 3.0\times10^{-6} = \frac{9\times10^9\times q^2}{(0.02)^2}$$

$$\text{or} \quad q^2 = \frac{4}{3}\times10^{-19}\text{C.}$$

(*i*) $F_{BC} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q\times q}{(BC)^2} = 9\times10^9\times\frac{4\times10^{-19}}{3\times(0.01)^2}$

$$= \mathbf{12.0\times10^{-6}\ N,\ along\ \mathit{BA}.}$$

(*ii*) Net force on charge at B,

$$F = F_{BC} - F_{BA} = (12.0-3.0)\times10^{-6}$$

$$= \mathbf{9.0\times10^{-6}\ N,\ along\ \mathit{BA}.}$$

13. Proceed as in Example 10 on page 1.12.

14. Force between $+9e$ and q = Force between $+e$ and q

$$\therefore \quad k\,.\,\frac{9e\times q}{x^2} = k\,.\,\frac{e\times q}{(a-x)^2}$$

or $\frac{3}{x} = \frac{1}{a-x}$ or $x = \mathbf{3a/4.}$

15. For equilibrium of charges q and $2q$, the charge Q must have sign opposite to that of q or $2q$. Suppose it is placed at distance x from charge q.

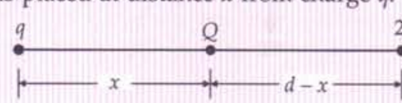


Fig. 1.21

For equilibrium of charge q,

$$k\,\frac{qQ}{x^2} = k\,\frac{q\times2q}{d^2} \quad \text{...(i)}$$

For equilibrium of charge $2q$,

$$k\,\frac{q\times2q}{d^2} = k\,\frac{Q\times2q}{(d-x)^2} \quad \text{...(ii)}$$

From (*i*) and (*ii*), we get,

$$k\frac{qQ}{x^2} = k\frac{Q \times 2q}{(d-x)^2}$$

or $$2x^2 = (d-x)^2$$

or $$\sqrt{2}x = d - x$$

or $$x = \frac{1}{\sqrt{2}+1}d = (\sqrt{2}-1)\,d$$

i.e., the charge Q must be placed at a distance of $(\sqrt{2}-1)\,d$ from the charge q.

16. Suppose the three charges are placed as shown in Fig. 1.22.

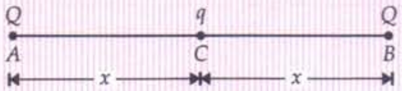


Fig. 1.22

Clearly, the net force on charge q is zero. So it is in equilibrium, the net force on other two charges should also be zero.

Total force on charge Q at point B is

$$\frac{1}{4\pi\varepsilon_0}\cdot\frac{QQ}{(2x)^2} + \frac{1}{4\pi\varepsilon_0}\frac{q\,Q}{x^2} = 0$$

or $$\frac{1}{4\pi\varepsilon_0}\frac{qQ}{x^2} = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{QQ}{(2x)^2}$$

or $$q = -\mathbf{Q/4}.$$

17. In $\Delta\,OCA$ of forces, we have

$$\frac{F}{AC} = \frac{mg}{OC} = \frac{T}{OA}$$

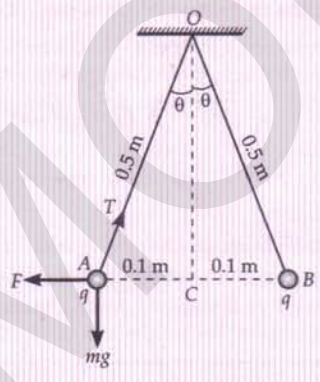


Fig. 1.23

$$\therefore \quad F = mg \times \frac{AC}{OC}$$

$$\frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{AB^2} = mg \times \frac{AC}{OC}$$

$$\frac{9\times10^9 \times q^2}{(0.2)^2} = \frac{10^{-3}\times 9.8 \times 0.1}{\sqrt{(0.5)^2-(0.1)^2}}$$

$$q = \mathbf{2.357 \times 10^{-6}\ C}.$$

1.16 COMPARING ELECTROSTATIC AND GRAVITATIONAL FORCES

25. *Give a comparison of the electrostatic and gravitational forces.*

Electrostatic force *vs* gravitational force. *Electrostatic force is the force of attraction or repulsion between two charges at rest while the gravitational force is the force of attraction between two bodies by virtue of their masses.*

Similarities :

1. Both forces obey inverse square law *i.e.*, $F \propto \frac{1}{r^2}$.
2. Both forces are proportional to product of masses or charges.
3. Both are *central forces i.e.*, they act along the line joining the centres of the two bodies.
4. Both are *conservative forces i.e.*, the work done against these forces does not depend upon the path followed.
5. Both forces can operate in vacuum.

Dissimilarities :

1. Gravitational force is attractive while electrostatic force may be attractive or repulsive.
2. Gravitational force does not depend on the nature of the medium while electrostatic force depends on the nature of the medium between the two charges.
3. Electrostatic forces are much stronger than gravitational forces.

Illustrative Problem. *Coulomb's law for electrical force between two charges and Newton's law for gravitational force between two masses, both have inverse-square dependence on the distance between charges/masses.*

(*a*) *Compare the strength of these forces by determining the ratio of their magnitude (i) for an electron and a proton and (ii) for two protons.*

(*b*) *Estimate the accelerations for electron and proton due to the electrical force of their mutual attraction when they are 1 Å (= 10^{-10} m) apart.*

How much is the electrostatic force stronger than the gravitational force ?

(*a*) (*i*) From Coulomb's law, the electrostatic force between an electron and a proton separated by distance r is

$$F_e = k\frac{q_1 q_2}{r^2} = \frac{k(-e)(e)}{r^2} = -\frac{ke^2}{r^2}$$

Negative sign indicates that the force is attractive. From Newton's law of gravitation, the corresponding gravitational attraction is

$$F_G = -G\frac{m_p\, m_e}{r^2}$$

where m_p and m_e are the masses of the proton and electron.

Hence

$$\left|\frac{F_e}{F_G}\right| = \frac{ke^2}{Gm_p\, m_e}$$

But $k = 9 \times 10^9\ \text{Nm}^2\ \text{C}^{-2},\ e = 1.6 \times 10^{-19}\ \text{C},$

$m_p = 1.67 \times 10^{-27}\ \text{kg},\ m_e = 9.1 \times 10^{-31}\ \text{kg},$

$G = 6.67 \times 10^{-11}\ \text{Nm}^2\ \text{kg}^{-2}$

$$\therefore \quad \left|\frac{F_e}{F_G}\right| = \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{6.67 \times 10^{-11} \times 1.67 \times 10^{-27} \times 9.1 \times 10^{-31}}$$

$$= \mathbf{2.27 \times 10^{39}}$$

(*a*) (*ii*) Similar to that in part (*i*), the ratio of the magnitudes of electric force to the gravitational force between two protons at a distance r is given by

$$\left|\frac{F_e}{F_G}\right| = \frac{ke^2}{Gm_p m_p} = \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{6.67 \times 10^{-11} \times (1.67 \times 10^{-27})^2}$$

$$= \mathbf{1.24 \times 10^{36}}$$

Thus the large value of the (dimensionless) ratio of the two forces indicates that the electrostatic forces are enormously stronger than the gravitational forces.

(*b*) The magnitude of the electric force exerted by a proton on an electron is equal to the magnitude of the force exerted by an electron on a proton. The magnitude of this force is

$$F = \frac{ke^2}{r^2} = \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{(10^{-10})^2}$$

$$[\because\ r = 1\ \text{Å} = 10^{-10}\ \text{m}]$$

$$= 2.3 \times 10^{-8}\ \text{N}$$

Acceleration of the electron due to the mutual attraction with the proton,

$$a_e = \frac{F}{m_e} = \frac{2.3 \times 10^{-8}\ \text{N}}{9.1 \times 10^{-31}\ \text{kg}} = \mathbf{2.5 \times 10^{22}\ ms^{-2}}$$

Acceleration of the proton due to the mutual attraction with the electron,

$$a_p = \frac{F}{m_p} = \frac{2.3 \times 10^{-8}\ \text{N}}{1.67 \times 10^{-27}\ \text{kg}} = \mathbf{1.3 \times 10^{19}\ ms^{-2}}$$

Clearly, the acceleration of an electron or a proton due to the electric force is much larger than the acceleration due to gravity. So, we can neglect the effect of gravitational field on the motion of the electron or the proton.

26. *Give two examples which illustrate that the electrical forces are enormously stronger than the gravitational forces.*

Examples : (*i*) A plastic comb passed through hair can easily lift a piece of paper upwards. The electrostatic attraction between the comb and the piece of paper overcomes the force of gravity exerted by the entire earth on the paper.

(*ii*) When we hold a book in our hand, the electric (frictional) forces between the palm of our hand and the book easily overcome the gravitational force on the book due to the entire earth.

In the words of *Feynman*, if you stand at arm's length from your friend and instead of being electrically neutral each of you had an excess of electrons over protons by just *one per cent*, then the force of repulsion between you would be enough to lift the entire earth.

1.17 FORCES BETWEEN MULTIPLE CHARGES : THE SUPERPOSITION PRINCIPLE

27. *State the principle of superposition of electrostatic forces. Hence write an expression for the force on a point charge due to a distribution of $N-1$ point charges in terms of their position vectors.*

Principle of superposition of electrostatic forces. Coulomb's law gives force between two point charges. The principle of superposition enables us to find the force on a point charge due to a group of point charges. This principle is based on the property that the forces with which two charges attract or repel each other are not affected by the presence of other charges.

*The **principle of superposition** states that when a number of charges are interacting, the total force on a given charge is the vector sum of the forces exerted on it due to all other charges. The force between two charges is not affected by the presence of other charges.*

As shown in Fig. 1.24, consider N point charges $q_1, q_2, q_3, \ldots, q_N$ placed in vacuum at points whose position vectors w.r.t. origin O are $\vec{r}_1, \vec{r}_2, \vec{r}_3, \ldots, \vec{r}_N$ respectively.

According to the principle of superposition, the total force on charge q_1 is given by

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \ldots\ldots + \vec{F}_{1N}$$

where $\vec{F}_{12}, \vec{F}_{13}, \ldots, \vec{F}_{1N}$ are the forces exerted on charge q_1 by the individual charges $q_2, q_3, \ldots\ldots, q_N$ respectively.

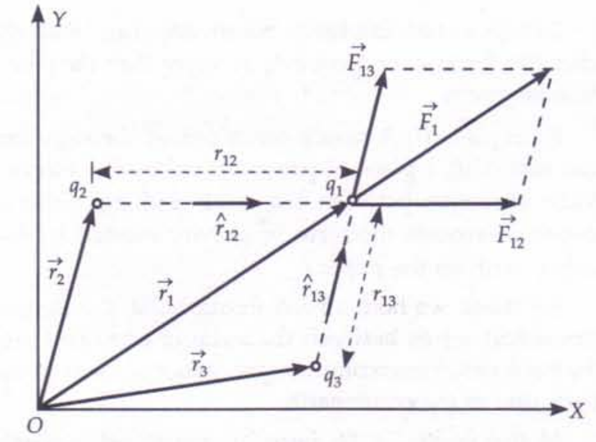


Fig. 1.24 Superposition principle : Force on charge q_1 exerted by q_2 and q_3.

According to Coulomb's law, the force exerted on charge q_1 due to q_2 is

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\,\frac{q_1 q_2}{r_{12}^2}\,\hat{r}_{12}$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^2}\cdot\frac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_1 - \vec{r}_2|}$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot q_1 q_2\,\frac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_1 - \vec{r}_2|^3}$$

where $\hat{r}_{12} = \dfrac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_1 - \vec{r}_2|}$ = a unit vector pointing from q_2 to q_1 and $r_{12} = |\vec{r}_1 - \vec{r}_2|$ = distance of q_2 from q_1.

Hence the total force on charge q_1 is

$$\vec{F}_1 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_2}{r_{12}^2}\,\hat{r}_{12} + \frac{q_1 q_3}{r_{13}^2}\,\hat{r}_{13} + \ldots\ldots + \frac{q_1 q_N}{r_{1N}^2}\,\hat{r}_{1N}\right]$$

or $$\vec{F}_1 = \frac{q_1}{4\pi\varepsilon_0}\sum_{i=2}^{N}\frac{q_i}{r_{1i}^2}\,\hat{r}_{1i}$$

In terms of position vectors,

$$\vec{F}_1 = \frac{1}{4\pi\varepsilon_0}\left[q_1 q_2\,\frac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_1 - \vec{r}_2|^3} + q_1 q_3\,\frac{\vec{r}_1 - \vec{r}_3}{|\vec{r}_1 - \vec{r}_3|^3} + \ldots\ldots + q_1 q_N\,\frac{\vec{r}_1 - \vec{r}_N}{|\vec{r}_1 - \vec{r}_N|^3}\right]$$

or $$\vec{F}_1 = \frac{q_1}{4\pi\varepsilon_0}\sum_{i=2}^{N} q_i\,\frac{\vec{r}_1 - \vec{r}_i}{|\vec{r}_1 - \vec{r}_i|^3}$$

In general, force $\vec{F}_a$ on ath charge q_a located at $\vec{r}_a$ due to all other $(N-1)$ charges may be written as

$\vec{F}_a$ = Total force on ath charge

$$\vec{F} = \frac{q_a}{4\pi\varepsilon_0}\sum_{\substack{b=1\\ b\neq a}}^{N}\frac{q_b}{r_{ab}^2}\,\hat{r}_{ab} = \frac{q_a}{4\pi\varepsilon_0}\sum_{\substack{b=1\\ b\neq a}}^{N} q_b\,\frac{\vec{r}_a - \vec{r}_b}{|\vec{r}_a - \vec{r}_b|^3}$$

where $a = 1, 2, 3, \ldots, N$.

It may be noticed that for each choice of a, the summation on b *omits* the value a. This is because summation must be taken only over *other* charges. The above expression can be written in a simpler way as follows :

$\vec{F}$ = Total force on charge q due to many point charges q'

$$\vec{F} = \frac{q}{4\pi\varepsilon_0}\sum_{\text{all point charges}} q'\,\frac{\vec{r} - \vec{r}'}{|\vec{r} - \vec{r}'|^3}.$$

Examples based on
Principle of Superposition of Electric Forces

Formulae Used

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \ldots + \vec{F}_{1N}$$

$$F = \sqrt{F_1^2 + F_2^2 + 2\,F_1\,F_2\cos\theta}$$

Units Used

Forces are in newton, charges in coulomb and distances in metre.

Example 19. *An infinite number of charges each equal to* $4\,\mu C$ *are placed along x-axis at* $x = 1\text{ m}$, $x = 2\text{ m}$, $x = 4\text{ m}$, $x = 8\text{ m}$ *and so on. Find the total force on a charge of 1 C placed at the origin.* [IIT 95]

Solution. Here $q = 4\,\mu\text{C} = 4\times10^{-6}\text{ C}$, $q_0 = 1\text{ C}$

By the principle of superposition, the total force acting on a charge of 1 C placed at the origin is

$$F = \frac{q q_0}{4\pi\varepsilon_0}\left[\frac{1}{r_1^2} + \frac{1}{r_2^2} + \frac{1}{r_3^2} + \ldots\right]$$

$$= 9\times10^9\times4\times10^{-6}\times1\left[\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{4^2} + \ldots\right]$$

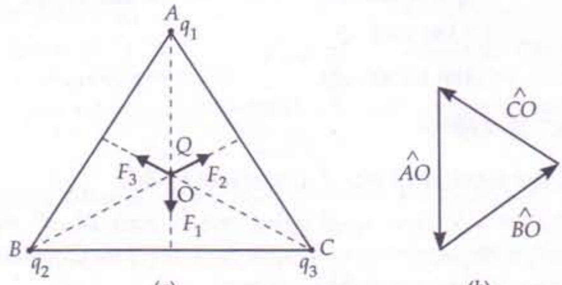
Sum of the infinite geometric progression

$$= \frac{a}{1-r} = \frac{1}{1-\frac{1}{4}} = \frac{4}{3}$$

$$\therefore\quad F = 9\times10^9\times4\times10^{-6}\times\frac{4}{3} = \mathbf{4.8\times10^4\ N}.$$

Example 20. *Consider three charges q_1, q_2, q_3 each equal to q at the vertices of an equilateral triangle of side l. What is the force on a charge Q (with the same sign as q) placed at the centroid of the triangle ?* [NCERT]

Solution. Suppose the given charges are placed as shown in Fig. 1.25(*a*).



(*a*) (*b*)

Fig. 1.25

Let $AO = BO = CO = r$

Force on charge Q due to q_1,

$$\vec{F}_1 = \frac{1}{4\pi\varepsilon_0}\frac{Qq_1}{AO^2}\hat{AO}$$

Force on charge Q due to q_2,

$$\vec{F}_2 = \frac{1}{4\pi\varepsilon_0}\frac{Qq_2}{BO^2}\hat{BO}$$

Force on charge Q due to q_3,

$$\vec{F}_3 = \frac{1}{4\pi\varepsilon_0}\frac{Qq_3}{CO^2}\hat{CO}$$

By the principle of superposition, the total force on charge Q is

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$$

$$= \frac{Qq}{4\pi\varepsilon_0 r^2}[\hat{AO} + \hat{BO} + \hat{CO}] \quad [\because q_1 = q_2 = q_3 = q]$$

As shown in Fig. 1.25(*b*), the angle between each pair of the unit vectors $\hat{AO}$, $\hat{BO}$ and $\hat{CO}$ is 120°, so they form a triangle of cyclic vectors. Consequently,

$$\hat{AO} + \hat{BO} + \hat{CO} = 0$$

Hence $\vec{F} = 0$ *i.e.*, the total force on charge Q is zero.

Example 21. *Three point charges +q each are kept at the vertices of an equilateral triangle of side 'l'. Determine the magnitude and sign of the charge to be kept at its centroid so that the charges at the vertices remain in equilibrium.* [CBSE F 2015]

Solution. At any vertex, the charge will be in equilibrium if the net electric force due to the remaining three charges is zero.

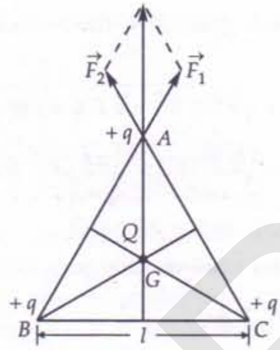


Fig. 1.26

Let Q be the charge required to be kept at the centroid G. Then,

$\vec{F}_1$ = Force at A due to the charge at B

$$= \frac{1}{4\pi\varepsilon_0}\frac{q^2}{l^2}, \text{ along } \vec{BA}$$

$$\vec{F}_2 = \text{Force at } A \text{ due to charge at } C = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{l^2}, \text{ along } \vec{CA}$$

$$\vec{F}_1 + \vec{F}_2 = 2F_1\cos 30°, \text{ along } \vec{GA} = \sqrt{3}\cdot\frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{l^2}, \text{ along } \vec{GA}$$

Force at A due to charge at G

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{Qq}{AG^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Qq}{(l/\sqrt{3})^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{3Qq}{l^2}$$

This must be equal and opposite to $(\vec{F}_1 + \vec{F}_2)$.

$$\therefore \quad 3Qq = -\sqrt{3}\,q^2 \quad \text{or} \quad Q = -\frac{q}{\sqrt{3}}.$$

Example 22. *Consider the charges q, q and – q placed at the vertices of an equilateral triangle, as shown in Fig. 1.27. What is the force on each charge ?* [NCERT]

Solution. The forces of attraction or repulsion between different pairs of charges are shown in Fig. 1.27. Each such force has magnitude,

$$F = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{l^2}$$

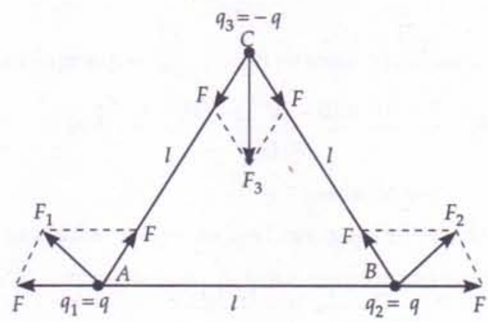


Fig. 1.27

By the parallelogram law, the net force on charge q_1 is

$$\vec{F_1} = \sqrt{F^2 + F^2 + 2\,F \times F \cos 120^\circ}\ \hat{BC}$$
$$= \sqrt{2\,F^2 + 2\,F^2(-1/2)}\ \hat{BC} = \boldsymbol{F}\,\hat{\boldsymbol{BC}}$$

where $\hat{BC}$ is a unit vector along BC.

Similarly, total force on charge q_2 is

$$\vec{F_2} = \boldsymbol{F}\,\hat{\boldsymbol{AC}}$$

where $\hat{AC}$ is a unit vector along AC.

Total force on charge q_3 is

$$\vec{F_3} = \sqrt{F^2 + F^2 + 2\,F \times F \cos 60^\circ}\ \hat{n} = \sqrt{3}\,\boldsymbol{F}\,\hat{\boldsymbol{n}}$$

where $\hat{n}$ is a unit vector along the direction bisecting $\angle ACB$.

Example 23. *Charges of $+5\ \mu C$, $+10\ \mu C$ and $-10\ \mu C$ are placed in air at the corners A, B and C of an equilateral triangle ABC, having each side equal to 5 cm. Determine the resultant force on the charge at A.*

Solution. The charge at B repels the charge at A with a force,

$$F_1 = k\,\frac{q_1 q_2}{r^2} = \frac{9 \times 10^9 \times (5 \times 10^{-6}) \times (10 \times 10^{-6})}{(0.05)^2}\ \text{N}$$
$$= 180\ \text{N, along } BA$$

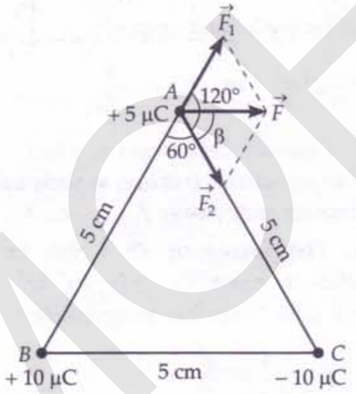


Fig. 1.28

The charge at C attracts the charge at A with a force

$$F_2 = \frac{9 \times 10^9 \times (5 \times 10^{-6}) \times (10 \times 10^{-6})}{(0.05)^2}\ \text{N}$$
$$= 180\ \text{N, along } AC.$$

By the parallelogram law of vector addition, the magnitude of resultant force $\vec{F}$ on charge at A is

$$F = \sqrt{F_1^{\,2} + F_2^{\,2} + 2\,F_1 F_2 \cos\theta}$$
$$= \sqrt{(180)^2 + (180)^2 + 2 \times 180 \times 180 \times \cos 120^\circ}\ \text{N}$$
$$= 180\sqrt{1 + 1 + 2 \times (-1/2)}\ \text{N} = \mathbf{180\ N}$$

Let the resultant force F make an angle β with the force F_2. Then

$$\tan\beta = \frac{F_2 \sin 120^\circ}{F_1 + F_2 \cos 120^\circ} = \frac{180 \times \sin 120^\circ}{180 + 180 \cos 120^\circ}$$
$$= \frac{180 \times \sqrt{3}/2}{180 + 180(-\frac{1}{2})} = \sqrt{3}$$
$$\therefore\quad \beta = 60^\circ$$

i.e., the resultant force $\vec{F}$ is parallel to BC.

Example 24. *Four equal point charges each 16 μC are placed on the four corners of a square of side 0.2 m. Calculate the force on any one of the charges.*

Solution. As shown in Fig. 1.29, suppose the four charges are placed at the corners of the square $ABCD$. Let us calculate the total force on q_4.

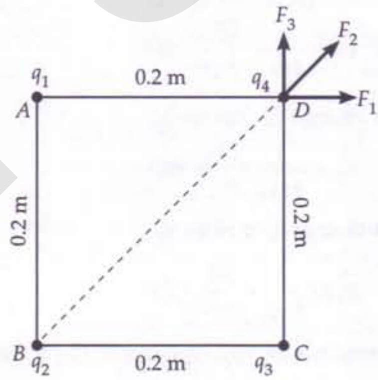


Fig. 1.29

Here $AB = BC = CD = AD = 0.2$ m

$$q_1 = q_2 = q_3 = q_4 = 16\ \mu C = 16 \times 10^{-6}\ \text{C}$$

Force exerted on q_4 by q_1 is

$$F_1 = \frac{9 \times 10^9 \times 16 \times 10^{-6} \times 16 \times 10^{-6}}{(0.2)^2}$$
$$= 57.6\ \text{N, along } AD \text{ produced}$$

Force exerted on q_4 by q_2 is

$$F_2 = \frac{9 \times 10^9 \times 16 \times 10^{-6} \times 16 \times 10^{-6}}{(0.2)^2 + (0.2)^2}$$
$$= 28.8\ \text{N, along } BD \text{ produced}$$

Force exerted on q_4 by q_3 is

$$F_3 = \frac{9 \times 10^9 \times 16 \times 10^{-6} \times 16 \times 10^{-6}}{(0.2)^2}$$
$$= 57.6\ \text{N, along } CD \text{ produced}$$

As F_1 and F_3 are perpendicular to each other, so their resultant force is

$$F' = \sqrt{F_1^2 + F_3^2} = \sqrt{57.6^2 + 57.6^2}$$
$$= 57.6\sqrt{2} = 81.5 \text{ N, in the direction of } F_2.$$

Hence total force on q_4 is

$$F = F_2 + F' = 28.8 + 81.5$$
$$= \mathbf{110.3\ N,\ along\ } \boldsymbol{BD} \mathbf{\ produced.}$$

Example 25. *Three point charges of +2 μC, −3 μC and −3 μC are kept at the vertices A, B and C respectively of an equilateral triangle of side 20 cm as shown in Fig. 1.30(a). What should be the sign and magnitude of the charge to be placed at the midpoint (M) of side BC so that the charge at A remains in equilibrium ?* [CBSE D 05]

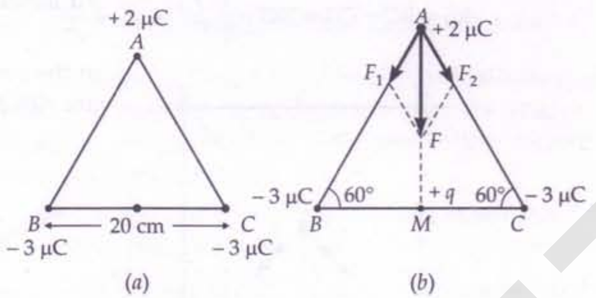


Fig. 1.30

Solution. As shown in Fig. 1.30(*b*), the force exerted on charge $+2\,\mu\text{C}$ by charge at B,

$$F_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$
$$= \frac{9\times10^9 \times 2\times10^{-6} \times 3\times10^{-6}}{(0.20)^2}$$
$$= 1.35 \text{ N, along } AB$$

Force exerted on charge $+2\,\mu\text{C}$ by charge at C,

$$F_2 = \frac{9\times10^9 \times 2\times10^{-6} \times 3\times10^{-6}}{(0.20)^2}$$
$$= 1.35 \text{ N, along } AC$$

Resultant force of F_1 and F_2

$$F = \sqrt{F_1^2 + F_2^2 + 2\,F_1F_1\cos 60^\circ}$$
$$= \sqrt{1.35^2 + 1.35^2 + 2\times1.35\times1.35\times0.5}$$
$$= 1.35\times\sqrt{3} = 2.34 \text{ N, along } AM$$

For the charge at A to be equilibrium, the charge q to be placed at point M must be a *positive charge* so that it exerts a force on $+2\,\mu\text{C}$ charge along MA.

Now, $AM = \sqrt{20^2 - 10^2}$
$$= \sqrt{300} = 10\sqrt{3} \text{ cm}$$
$$= 0.1\times\sqrt{3} \text{ m}$$

Net force on charge at A will be zero if

$$\frac{9\times10^9 \times q \times 2\times10^{-6}}{(0.1\times\sqrt{3})^2} = 2.34$$

or
$$q = \frac{2.34\times0.01\times3}{18\times10^3} = 3.9\times10^{-6}\text{ C} = \mathbf{3.9\ \mu C.}$$

Problems for Practice

1. Ten positively charged particles are kept fixed on the x-axis at points $x = 10$ cm, 20 cm, 40 cm, ..., 100 cm. The first particle has a charge 1.0×10^{-8} C, the second 8×10^{-8} C, third 27×10^{-8} C, and so on. The tenth particle has a charge 1000×10^{-8} C. Find the magnitude of the electric force acting on a 1 C charge placed at the origin. (Ans. 4.95×10^5 N)
2. Charges $q_1 = 1.5$ mC, $q_2 = 0.2$ mC and $q_3 = -0.5$ mC are placed at the points A, B and C respectively, as shown in Fig. 1.31. If $r_1 = 1.2$ m and $r_2 = 0.6$ m, calculate the magnitude of resultant force on q_2. (Ans. 3.125×10^3 N)

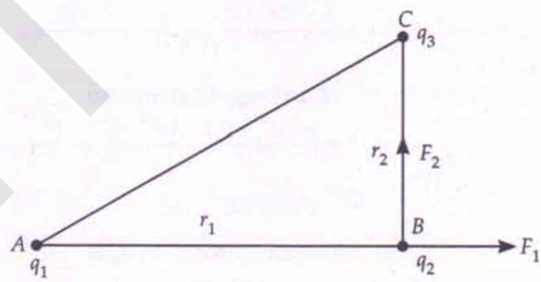


Fig. 1.31

3. Two equal positive charges, each of $2\,\mu\text{C}$ interact with a third positive charge of $3\,\mu\text{C}$ situated as shown in Fig. 1.32. Find the magnitude and direction of the force experienced by the charge of $3\,\mu\text{C}$. (Ans. 3.456×10^{-3} N, along OC produced)

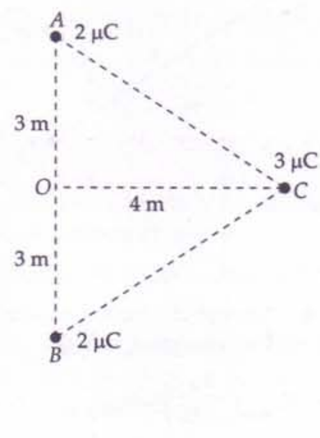


Fig. 1.32

4. Four charges $+q$, $+q$, $-q$ and $-q$ are placed respectively at the four corners A, B, C and D of a square of side a. Calculate the force on a charge Q placed at the centre of the square.

$$\left(\text{Ans. } \frac{1}{4\pi\varepsilon_0}\frac{4\sqrt{2}\,Qq}{a^2}\text{, parallel to } AD \text{ or } BC\right)$$

HINTS

1. By the principle of superposition, the total force on the 1 C charge placed at the origin is

$$F_0 = F_{01} + F_{02} + F_{03} + ... + F_{10}$$
$$= \frac{q_0}{4\pi\varepsilon_0}\left[\frac{q_1}{r_1^2} + \frac{q_2}{r_2^2} + \frac{q_3}{r_3^2} + ... + \frac{q_{10}}{r_{10}^2}\right]$$
$$= 1 \times 9 \times 10^9 \left[\frac{1.0 \times 10^{-8}}{(0.10)^2} + \frac{8 \times 10^{-8}}{(0.20)^2} + \frac{27 \times 10^{-8}}{(0.30)^2} + ... + \frac{1000 \times 10^{-8}}{(1.00)^2}\right]$$
$$= 9 \times 10^9 \times 10^{-6}\,[1 + 2 + 3 + ... + 10]$$
$$= 9 \times 55 \times 10^3 = \mathbf{4.95 \times 10^5\ N.}$$

2. $$F_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_1^2} = \frac{9 \times 10^9 \times 0.2 \times 10^{-3} \times 9 \times 10^9}{(1.2)^2}$$
$$= 1.875 \times 10^3 \text{ N, along } AB \text{ produced}$$
$$F_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_2 q_3}{r_2^2} = \frac{9 \times 10^9 \times 0.2 \times 10^{-3} \times 0.5 \times 10^{-3}}{(0.6)^2}$$
$$= 2.5 \times 10^3 \text{ N, along } BC \perp AB$$

As $F_1 \perp F_2$, so the resultant force on q_2 is

$$F = \sqrt{F_1^2 + F_2^2} = \mathbf{3.125 \times 10^3\ N.}$$

3. Here $q_A = q_B = 2\,\mu\text{C} = 2 \times 10^{-6}$ C,
$q_C = 3\,\mu\text{C} = 3 \times 10^{-6}$ C
$AC = BC = \sqrt{3^2 + 4^2} = 5$ m

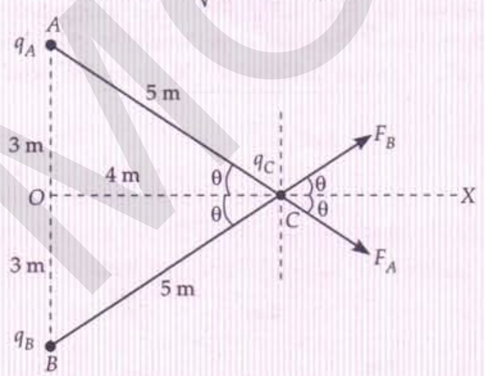


Fig. 1.33

Force exerted by charge q_A on q_C,

$$F_A = \frac{1}{4\pi\varepsilon_0}\frac{q_A\, q_C}{(AC)^2}$$
$$= \frac{9 \times 10^9 \times 2 \times 10^{-6} \times 3 \times 10^{-6}}{5^2}$$
$$= 2.16 \times 10^{-3} \text{ N, along } AC \text{ produced}$$

Similarly, force exerted by charge q_B on q_C,

$$F_B = 2.16 \times 10^{-3} \text{ N, along } BC \text{ produced}$$

Clearly, $F_A = F_B$ (in magnitude)

The components of F_A and F_B along Y-axis will cancel out and get added along X-axis.

$\therefore$ Total force on $3\,\mu$C charge,

$$F = 2F_1 \cos\theta = 2 \times 2.16 \times 10^{-3} \times \frac{4}{5}$$
$$= \mathbf{3.456 \times 10^{-3}\ N,\ \ along}\ CX.$$

4. Here $AB = BC = CD = DA = a$

$$AO = BO = CO = DO = \frac{1}{2}\sqrt{a^2 + a^2} = \frac{a}{\sqrt{2}}$$

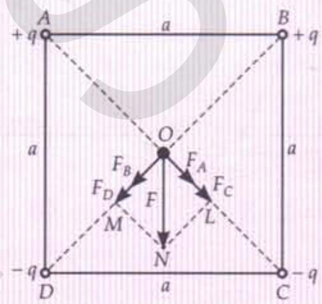


Fig. 1.34

Let F_A, F_B, F_C and F_D be the forces exerted by charges at points A, B, C and D on charge Q at point O. Then

$$F_A = F_B = F_C = F_D$$
$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{q \times Q}{(a/\sqrt{2})^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2qQ}{a^2}$$

The resultant of the forces F_A and F_C,

$$F_1 = F_A + F_C = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2qQ}{a^2} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{2qQ}{a^2}$$

or $$F_1 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{4qQ}{a^2}, \text{ along } OL$$

Similarly, resultant of the forces F_B and F_D,

$$F_2 = F_B + F_D = \frac{1}{4\pi\varepsilon_0}\cdot\frac{4qQ}{a^2}, \text{ along } OM$$

Hence the resultant force on charge Q is

$$F = \sqrt{F_1^2 + F_2^2} = \frac{1}{4\pi\varepsilon_0}\frac{4\sqrt{2}\,qQ}{a^2}, \text{ along } ON$$

As the forces F_1 and F_2 are equal in magnitude, so their resultant F will act along the bisector of $\angle COD$ *i.e.*, parallel to AD or BC.

1.18 ELECTRIC FIELD

28. *Briefly develop the concept of electric field.*

Concept of electric field. The electrostatic force acts between two charged bodies even without any direct contact between them. The nature of this *action-at-distance* force can be understood by introducing the concept of electric field.

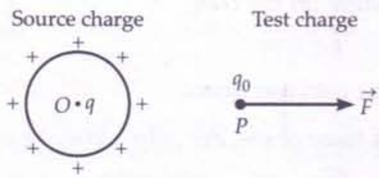


Fig. 1.35 A charged body produces an electric field around it.

Consider a charged body carrying a positive charge q placed at point O. It is assumed that the charge q produces an electrical environment in the surrounding space, called *electric field*.

To test the existence of electric field at any point P, we simply place a small positive charge q_0, called the *test charge* at the point P. If a force $\vec{F}$ is exerted on the test charge, then we say that an electric field $\vec{E}$ exists at the point P. The charge q is called the *source charge* as it produces the field $\vec{E}$.

29. *Define electric field at a point. Give its units and dimensions.*

Electric field. *An electric field is said to exist at a point if a force of electrical origin is exerted on a stationary charged body placed at that point. Quantitatively, the electric field or the electric intensity or the electric field strength $\vec{E}$ at a point is defined as the force experienced by a unit positive test charge placed at that point, without disturbing the position of source charge.*

As shown in Fig. 1.35, suppose a test charge q_0 experiences a force $\vec{F}$ at the point P. Then the electric field at that point will be

$$\vec{E} = \frac{\vec{F}}{q_0}$$

There is a difficulty in defining the electric field by the above equation. The test charge q_0 may disturb the charge distribution of the source charge and hence change the electric field $\vec{E}$ which we want to measure. The test charge q_0 must be *small enough* so that it does not change the value of $\vec{E}$. It is better to define electric field as follows :

The electric field at a point is defined as the electrostatic force per unit test charge acting on a vanishingly small positive test charge placed at that point. Hence

$$\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}}{q_0}$$

The electric field $\vec{E}$ is a *vector quantity* whose direction is same as that of the force $\vec{F}$ exerted on a positive test charge.

Units and dimensions of electric field. As the electric field is force per unit charge, so its SI unit is *newton per coulomb* (NC^{-1}). It is equivalent to *volt per metre* (Vm^{-1}).

The dimensions for $\vec{E}$ can be determined as follows :

$$[E] = \frac{\text{Force}}{\text{Charge}} = \frac{MLT^{-2}}{C}$$

$$= \frac{MLT^{-2}}{A\,.\,T} = [\mathbf{MLT^{-3}A^{-1}}] \qquad \left[\because 1\,A = \frac{1\,C}{1\,s}\right]$$

30. *Give the physical significance of electric field.*

Physical significance of electric field. The force experienced by the test charge q_0 is different at different points. So $\vec{E}$ also varies from point to point. In general, $\vec{E}$ is not a single vector but a set of infinite vectors. Each point $\vec{r}$ is associated with a unique vector $\vec{E}(r)$. So electric field is an example of *vector field*.

By knowing electric field at any point, we can determine the force on a charge placed at that point. The Coulomb force on a charge q_0 due to a source charge q may be treated as two stage process :

(*i*) The source charge q produces a definite field $\vec{E}(r)$ at every point $\vec{r}$.

(*ii*) The value of $\vec{E}(r)$ at any point $\vec{r}$ determines the force on charge q_0 at that point. This force is

$$\vec{F} = q_0\,\vec{E}(r)$$

Electrostatic force = Charge × Electric field.

Thus an electric field plays an intermediary role in the forces between two charges :

Charge ⇌ Electric field ⇌ Charge.

It is in this sense that the concept of electric field is useful. Electric field is a characteristic of the system of charges and is independent of the test charge that we place at a point to determine the field.

Examples based on
Relation between Electric Field Strength and Force

Formulae Used

$$\vec{E} = \frac{\vec{F}}{q_0} \quad \text{or} \quad \vec{F} = q_0 \vec{E}$$

Units Used

When force is in newton, charge in coulomb and distance in metre, electric field strength is in newton per coulomb (NC^{-1}) or equivalently in volt per metre (Vm^{-1}).

Example 26. *Calculate the electric field strength required to just support a water drop of mass 10^{-3} kg and having a charge 1.6×10^{-19} C.* [CBSE OD 99]

Solution. Here $m = 10^{-3}$ kg, $q = 1.6 \times 10^{-19}$ C

Let E be the strength of the electric field required to just support the water drop. Then

Force on water drop due to electric field

= Weight of water drop

or $\quad qE = mg$

$$\therefore \quad E = \frac{mg}{q} = \frac{10^{-3} \times 9.8}{1.6 \times 10^{-19}} = \mathbf{6.125 \times 10^{16}\ NC^{-1}}.$$

Example 27. *Calculate the voltage needed to balance an oil drop carrying 10 electrons when located between the plates of a capacitor which are 5 mm apart. The mass of oil drop is 3×10^{-16} kg. Take $g = 10\ ms^{-2}$.* [CBSE OD 95C]

Solution. Here $q = 10\,e = 10 \times 1.6 \times 10^{-19}$ C,
$d = 5$ mm $= 5 \times 10^{-3}$ m, $m = 3 \times 10^{-16}$ kg, $g = 10\ ms^{-2}$

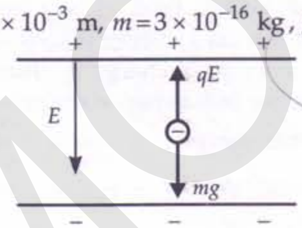


Fig. 1.36

When the drop is held stationary,

Upward force on oil drop due to electric field

= Weight of oil drop

$$qE = mg$$

or $\quad q \cdot \frac{V}{d} = mg \qquad \left[E = \frac{V}{d}\right]$

$$\therefore \quad V = \frac{mg\,d}{q} = \frac{3 \times 10^{-16} \times 10 \times 5 \times 10^{-3}}{10 \times 1.6 \times 10^{-19}}$$

$$= \mathbf{9.375\ V}.$$

Example 28. *How many electrons should be removed from a coin of mass 1.6 g, so that it may just float in an electric field of intensity $10^9\ NC^{-1}$, directed upward ?.* [Pb. 98C]

Solution. Here $m = 1.6$ g $= 1.6 \times 10^{-3}$ kg,

$$E = 10^9\ NC^{-1}$$

Let n be the number of electrons removed from the coin.

Then charge on the coin,

$$q = +\,ne$$

Fig. 1.37

When the coin just floats,

Upward force of electric field = Weight of coin

$$qE \quad \text{or} \quad neE = mg$$

$$\therefore \quad n = \frac{mg}{eE} = \frac{1.6 \times 10^{-3} \times 9.8}{1.6 \times 10^{-19} \times 10^9} = \mathbf{9.8 \times 10^7}.$$

Example 29. *A pendulum of mass 80 milligram carrying a charge of 2×10^{-8} C is at rest in a horizontal uniform electric field of $2 \times 10^4\ Vm^{-1}$. Find the tension in the thread of the pendulum and the angle it makes with the vertical.*

Solution. Here $m = 80$ mg $= 80 \times 10^{-6}$ kg,

$$q = 2 \times 10^{-8}\ C,\ E = 2 \times 10^4\ Vm^{-1}.$$

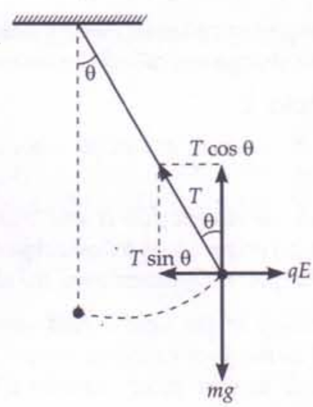


Fig. 1.38

Let T be the tension in the thread and θ be the angle it makes with vertical, as shown in Fig. 1.38. When the *bob* is in equilibrium,

$$T \sin\theta = qE\,; \quad T\cos\theta = mg$$

$$\therefore \quad \tan\theta = \frac{T\sin\theta}{T\cos\theta} = \frac{qE}{mg}$$

$$= \frac{2 \times 10^{-8} \times 2 \times 10^4}{80 \times 10^{-6} \times 9.8} = 0.51$$

or $\quad \theta = 27°$

Also, $\quad T = \frac{qE}{\sin\theta} = \frac{2 \times 10^{-8} \times 2 \times 10^4}{\sin 27°}$

$$= \mathbf{8.81 \times 10^{-4}\ N}.$$

Example 30. *An electron moves a distance of 6 cm when accelerated from rest by an electric field of strength 2×10^4 NC^{-1}. Calculate the time of travel. The mass and charge of electron are 9×10^{-31} kg and 1.6×10^{-19} C respectively.* [CBSE D 91]

Solution. Force exerted on the electron by the electric field,

$$F = eE$$

∴ Acceleration,

$$a = \frac{F}{m} = \frac{eE}{m} = \frac{1.6 \times 10^{-19} \times 2 \times 10^4}{9 \times 10^{-31}} = 0.35 \times 10^{16} \text{ ms}^{-2}$$

Now $u = 0$, $s = 6.0$ cm $= 0.06$ m, $a = 0.35 \times 10^{16}\text{ms}^{-2}$

As $\quad s = ut + \frac{1}{2}at^2$

$\therefore \quad 0.06 = 0 + \frac{1}{2} \times 0.35 \times 10^{16} \times t^2$

or $\quad t = \sqrt{\dfrac{0.06 \times 2}{0.35 \times 10^{16}}} = \mathbf{0.585 \times 10^{-8}}$ **s.**

Example 31. *An electron falls through a distance of 1.5 cm in a uniform electric field of magnitude 2.0×10^4 NC^{-1} [Fig. 1.39(a)]. The direction of the field is reversed keeping its magnitude unchanged and a proton falls through the same distance [Fig. 1.39(b)]. Compute the time of fall in each case. Contrast the situation (a) with that of 'free fall under gravity'.* [NCERT]

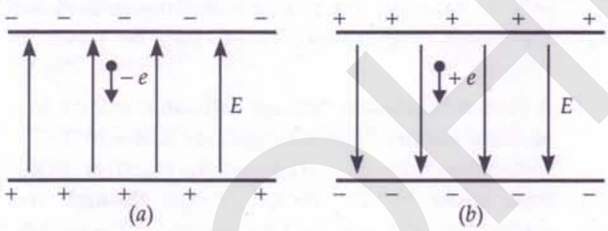


Fig. 1.39

Solution. (*a*) The upward field exerts a downward force eE on the electron.

∴ Acceleration of the electron, $a_e = \dfrac{eE}{m_e}$

As $u = 0$, $\quad s = ut + \frac{1}{2}at^2 = \frac{1}{2}at^2$

∴ Time of fall of the electron is

$$t_e = \sqrt{\frac{2s}{a_e}} = \sqrt{\frac{2sm_e}{eE}} = \sqrt{\frac{2 \times 1.5 \times 10^{-2} \times 9.1 \times 10^{-31}}{1.6 \times 10^{-19} \times 2.0 \times 10^4}}$$

$$= \mathbf{2.9 \times 10^{-9}\ s.}$$

(*b*) The downward field exerts a downward force eE on the proton.

$$\therefore \quad a_p = \sqrt{\frac{eE}{m_p}}$$

Time of fall of the proton is

$$t_p = \sqrt{\frac{2s}{a_p}} = \sqrt{\frac{2sm_p}{eE}}$$

$$= \sqrt{\frac{2 \times 1.5 \times 10^{-2} \times 1.67 \times 10^{-27}}{1.6 \times 10^{-19} \times 2.0 \times 10^4}} = \mathbf{1.25 \times 10^{-7}\ s.}$$

Thus the heavier particle takes a greater time to fall through the same distance. This is in contrast to the situation of 'free fall under gravity' where the time of fall is independent of the mass of the body. Here the acceleration due to gravity 'g', being negligibly small, has been ignored.

Example 32. *An electron is liberated from the lower of the two large parallel metal plates separated by a distance of 20 mm. The upper plate has a potential of + 2400 V relative to the lower plate. How long does the electron take to reach the upper plate ? Take $\frac{e}{m}$ of electrons 1.8×10^{11} C kg^{-1}.*

Solution. Here $V = 2400$ V, $d = 20$ mm $= 0.02$ m, $\dfrac{e}{m} = 1.8 \times 10^{11}$ C kg^{-1}

Upward force on the electron exerted by electric field is

$$F = eE = \frac{eV}{d}$$

∴ Acceleration,

$$a = \frac{F}{m} = \frac{eV}{md} = \frac{1.8 \times 10^{11} \times 2400}{0.02} \text{ ms}^{-2}$$

$$= 2.16 \times 10^{16} \text{ ms}^{-2}$$

Using, $s = \frac{1}{2}at^2$, we get

$$t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2d}{a}} = \sqrt{\frac{2 \times 0.02}{2.16 \times 10^{16}}} \text{ s} = \mathbf{1.4 \times 10^{-9}\ s.}$$

Example 33. *A stream of electrons moving with a velocity of 3×10^7 ms^{-1} is deflected by 2 mm in traversing a distance of 0.1 m in a uniform electric field of strength 18 V cm^{-1}. Determine e/m of electrons.*

Solution. Here $v_0 = 3 \times 10^7$ ms^{-1},

$$y = 2 \text{ mm} = 2 \times 10^{-3}\text{m}, \ x = 0.1 \text{ m},$$

$$E = 18 \text{ V cm}^{-1} = 1800 \text{ V m}^{-1}$$

$$ma = eE \quad \text{or} \quad a = \frac{eE}{m} \text{ and } t = \frac{x}{v_0}$$

$$\therefore \quad y = \frac{1}{2}at^2 = \frac{1}{2}\frac{eE}{m} \cdot \frac{x^2}{v_0^2}$$

or $$\frac{e}{m} = \frac{2y\,v_0^2}{Ex^2} = \frac{2 \times 2 \times 10^{-3} \times 9 \times 10^{14}}{1800 \times (0.1)^2}$$

$$= \mathbf{2 \times 10^{11} C\,kg^{-1}.}$$

Example 34. *An electric field E is set up between the two parallel plates of a capacitor, as shown in Fig. 1.40. An electron enters the field symmetrically between the plates with a speed v_0. The length of each plate is l. Find the angle of deviation of the path of the electron as it comes out of the field.*

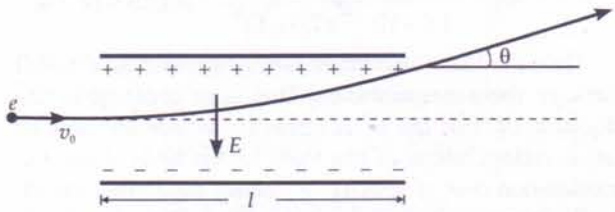


Fig. 1.40

Solution. Acceleration of the electron in the upward direction,

$$a = \frac{eE}{m}$$

Time taken to cross the field, $t = \frac{l}{v_0}$

Upward component of electron velocity on emerging from field region,

$$v_y = at = \frac{eEl}{mv_0}$$

Horizontal component remains same, $v_x = v_0$

If θ is the angle of deviation of the path of the electron, then

$$\tan\theta = \frac{v_y}{v_x} = \frac{eEl}{mv_0^2} \quad \text{or} \quad \theta = \tan^{-1}\frac{eEl}{mv_0^2}.$$

Example 35. *A charged particle, of charge $2\,\mu C$ and mass 10 milligram, moving with a velocity of 1000 m/s entres a uniform electric field of strength $10^3\, NC^{-1}$ directed perpendicular to its direction of motion. Find the velocity and displacement, of the particle after 10 s.*

[CBSE Sample Paper 11]

Solution. The velocity of the particle, normal to the direction of field.

$v_x = 1000\,\text{ms}^{-1}$, is constant

The velocity of the particle, along the direction of field, after 10 s, is given by

$$v_y = u_y + a_y t$$

$$= 0 + \frac{qE_y}{m}t = \frac{2\times10^{-6}\times10^3\times10}{10\times10^{-6}} = 2000\,\text{ms}^{-1}$$

The net velocity after 10 s,

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(1000)^2 + (2000)^2} = 1000\sqrt{5}\,\text{ms}^{-1}$$

Displacement, along the x-axis, after 10 s,

$$x = 1000\times10\,\text{m} = 10000\text{ m}$$

Displacement along y-axis (in the direction of field) after 10 s,

$$y = u_y t + \frac{1}{2}a_y t^2 = (0)t + \frac{1}{2}\frac{qE_y}{m}t^2 = \frac{1}{2}\times\frac{2\times10^{-6}\times10^3}{10\times10^{-6}}\times(10)^2$$

$$= 10000\text{ m}$$

Net displacement,

$$r = \sqrt{x^2+y^2} = \sqrt{(10000)^2+(10000)^2} = \mathbf{10000\sqrt{2}\,m}.$$

Problems For Practice

1. If an oil drop of weight 3.2×10^{-13} N is balanced in an electric field of $5\times10^5\,\text{Vm}^{-1}$, find the charge on the oil drop. **[CBSE D 93]** (Ans. 0.64×10^{-18} C)
2. Calculate the magnitude of the electric field, which can just balance a deutron of mass 3.2×10^{-27} kg. Take $g = 10\,\text{ms}^{-2}$. **[Punjab 99]** (Ans. $2.0\times10^{-7}\,\text{NC}^{-1}$)
3. A charged oil drop remains stationary when situated between two parallel plates 20 mm apart and a p.d. of 500 V is applied to the plates. Find the charge on the drop if it has a mass of 2×10^{-4} kg. Take $g = 10\,\text{ms}^{-2}$. (Ans. 8×10^{-13} C)
4. In Millikan's experiment, an oil drop of radius 10^{-4} cm remains suspended between the plates which are 1 cm apart. If the drop has charge of 5*e* over it, calculate the potential difference between the plates. The density of oil may be taken as $1.5\,\text{gcm}^{-3}$. (Ans. 770 V)
5. A proton falls down through a distance of 2 cm in a uniform electric field of magnitude $3.34\times10^3\,\text{NC}^{-1}$. Determine (*i*) the acceleration of the electron (*ii*) the time taken by the proton to fall through the distance of 2 cm, and (*iii*) the direction of the electric field. Mass of a proton is 1.67×10^{-27} kg. (Ans. $3.2\times10^{11}\,\text{ms}^{-2}$, 3.54×10^{-7}s, vertically downwards)
6. A particle of mass 10^{-3} kg and charge $5\,\mu C$ is thrown at a speed of $20\,\text{ms}^{-1}$ against a uniform electric field of strength $2\times10^5\,\text{NC}^{-1}$. How much distance will it travel before coming to rest momentarily ? (Ans. 0.2 m)

HINTS

1. Use $W = qE$.
2. $E = \frac{mg}{e} = \frac{3.2\times10^{-27}\times10}{1.6\times10^{-19}} = \mathbf{2.0\times10^{-7}\,NC^{-1}}.$
3. $mg = qE$ or $mg = q\frac{V}{d}$

$$\therefore q = \frac{mgd}{V} = \frac{2\times10^{-4}\times10\times20\times10^{-3}}{500} = \mathbf{8\times10^{-8}\,C.}$$

4. Use $\frac{4}{3}\pi r^3 \rho g = ne\frac{V}{d}$.

5. (*i*) $a = \frac{F}{m} = \frac{eE}{m} = \frac{1.6\times10^{-19}\times3.34\times10^{3}}{1.67\times10^{-27}}$

$= \mathbf{3.2\times10^{11}\ ms^{-2}}$.

(*ii*) $s = 0 + \frac{1}{2}at^2$

$\therefore \quad t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2\times0.02}{3.2\times10^{11}}} = \mathbf{3.54\times10^{-7}\,s}$

(*iii*) The field must act vertically downwards so that the positively charged proton falls downward.

6. $F = qE = 5\times10^{-6}\times2\times10^{5} = 1\,\text{N}$

As the particle is thrown against the field, so

$$a = -\frac{F}{m} = -\frac{1}{10^{-3}} = -10^3\ \text{ms}^{-2}$$

As $v^2 - u^2 = 2as \quad \therefore \quad 0^2 - 20^2 = 2\times(-10^3)\times s$

or $\qquad s = \mathbf{0.2\ m}$.

1.19 ELECTRIC FIELD DUE TO A POINT CHARGE

31. *Obtain an expression for the electric field intensity at a point at a distance r from a charge q. What is the nature of this field ?*

Electric field due to a point charge. A single point charge has the simplest electric field. As shown in Fig. 1.41, consider a point charge q placed at the origin O. We wish to determine its electric field at a point P at

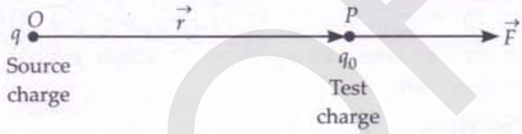


Fig. 1.41 Electric field of a point charge.

a distance r from it. For this, imagine a test charge q_0 placed at point P. According to Coulomb's law, the force on charge q_0 is

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{qq_0}{r^2}\hat{r}$$

where $\hat{r}$ is a unit vector in the direction from q to q_0.

Electric field at point P is

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$

The magnitude of the field $\vec{E}$ is

$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}}$$

Clearly, $E \propto 1/r^2$. This means that at all points on the spherical surface drawn around the point charge, the magnitude of $\vec{E}$ is same and does not depend on the direction of $\vec{r}$. Such a field is called ***spherically symmetric*** or ***radial field***, *i.e.*, a field which *looks the same* in all directions when seen from the point charge.

1.20 ELECTRIC FIELD DUE TO A SYSTEM OF POINT CHARGES

32. *Deduce an expression for the electric field at a point due to a system of N point charges.*

Electric field due to a system of point charges. Consider a system of N point charges $q_1, q_2, \ldots, q_N$ having position vectors $\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_N$ with respect to the

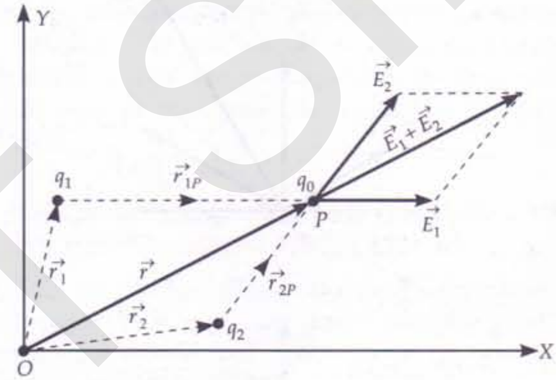


Fig. 1.42 Notations used in the determination of electric field at a point due to two point charges.

origin O. We wish to determine the electric field at point P whose position vector is $\vec{r}$. According to Coulomb's law, the force on charge test q_0 due to charge q_1 is

$$\vec{F}_1 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_0}{r_{1P}^2}\hat{r}_{1P}$$

where $\hat{r}_{1P}$ is a unit vector in the direction from q_1 to P and r_{1P} is the distance between q_1 and P. Hence the electric field at point P due to charge q_1 is

$$\vec{E}_1 = \frac{\vec{F}_1}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_{1P}^2}\hat{r}_{1P}$$

Similarly, electric field at P due to charge q_2 is

$$\vec{E}_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_2}{r_{2P}^2}\hat{r}_{2P}$$

According to ***principle of superposition of electric fields****, the electric field at any point due to a group of charges is equal to the vector sum of the electric fields produced by each charge individually at that point, when all other charges are assumed to be absent.*

Hence, the electric field at point P due to the system of N charges is

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + ... + \vec{E}_N$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r_{1P}^2}\hat{r}_{1P} + \frac{q_2}{r_{2P}^2}\hat{r}_{2P} + ... + \frac{q_N}{r_{NP}^2}\hat{r}_{NP}\right]$$

or $$\vec{E} = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N}\frac{q_i}{r_{iP}^2}\hat{r}_{iP}$$

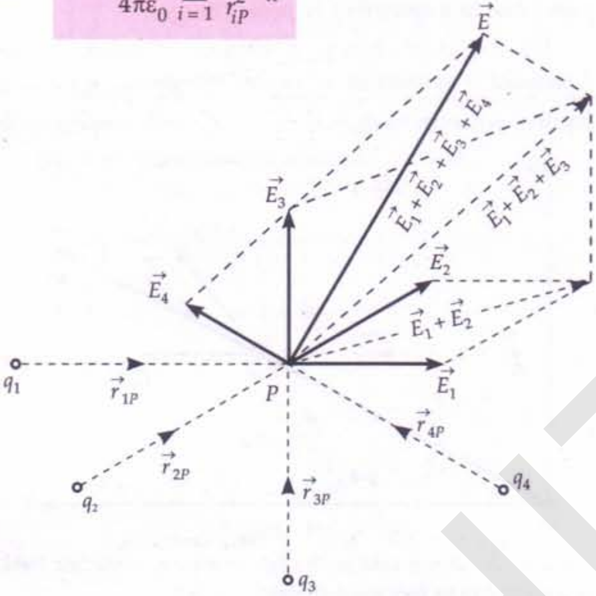


Fig. 1.43 Electric field at a point due to a system of charges is the vector sum of the electric fields at the point due to individual charges.

In terms of position vectors, we can write

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N}\frac{q_i}{|\vec{r}-\vec{r}_i|^2}\cdot\frac{\vec{r}-\vec{r}_i}{|\vec{r}-\vec{r}_i|}$$

or $$\vec{E} = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N}\frac{q_i}{|\vec{r}-\vec{r}_i|^3}(\vec{r}-\vec{r}_i).$$

Examples based on Electric Fields of Point Charges

Formulae Used

1. $E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}$
2. By the principle of superposition, electric field due to a number of point charges,
$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + ...$$

Units Used

When q is in coulomb and r in metre ; E is in NC^{-1} or Vm^{-1}.

Example 36. *Assuming that the charge on an atom is distributed uniformly in a sphere of radius 10^{-10} m, what will be the electric field at the surface of the gold atom ? For gold, Z = 79.*

Solution. The charge may be assumed to be concentrated at the centre of the sphere of radius 10^{-10} m.

$\therefore \quad r = 10^{-10}$ m, $q = Ze = 79 \times 1.6 \times 10^{-19}$ C

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2} = \frac{9\times10^9\times79\times1.6\times10^{-19}}{(10^{-10})^2}$$

$$= \mathbf{1.138\times10^{13}\ NC^{-1}}.$$

Example 37. *Two point charges of 2.0×10^{-7} C and 1.0×10^{-7} C are 1.0 cm apart. What is the magnitude of the field produced by either charge at the site of the other ? Use standard value of $1/4\pi\varepsilon_0$.* [Punjab 98]

Solution. Here $q_1 = 2.0\times10^{-7}$ C,

$$q_2 = 1.0\times10^{-7}\ C,\quad r = 1.0\ cm = 0.01\ m$$

Electric field due to q_1 at the site of q_2,

$$E_1 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1}{r^2} = \frac{9\times10^9\times2.0\times10^{-7}}{(0.10)^2}$$

$$= \mathbf{1.8\times10^7\ NC^{-1}}.$$

Electric field due to q_2 at the site of q_1,

$$E_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_2}{r^2} = \frac{9\times10^9\times1.0\times10^{-7}}{(0.10)^2}$$

$$= \mathbf{9\times10^6\ NC^{-1}}.$$

Example 38. *Two point charges of $+5\times10^{-19}$ C and $+20\times10^{-19}$ C are separated by a distance of 2 m. Find the point on the line joining them at which electric field intensity is zero.* [CBSE OD 01C]

Solution.

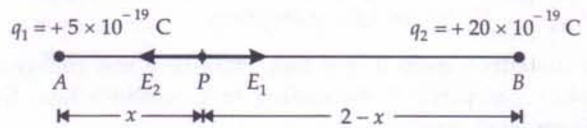


Fig. 1.44

The electric field at point P will be zero if

$$E_1 = E_2$$

$$\frac{1}{4\pi\varepsilon_0}\cdot\frac{5\times10^{-19}}{x^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{20\times10^{-19}}{(2-x)^2}$$

or $$4x^2 = (2-x)^2 \quad\text{or}\quad 2x = \pm(2-x)$$

or $$x = 2/3\ m \quad\text{or}\quad -2\ m$$

At $x = -2$ m *i.e.*, at 2 m left of q_1, electric fields due to both charges will be in same direction. So $x = -2$ m is not a possible solution.

Hence electric field will be zero at 2 / 3 m to the right of q_1.

Example 39. *Two point charges of* $+16\ \mu C$ *and* $-9\ \mu C$ *are placed 8 cm apart in air. Determine the position of the point at which the resultant field is zero.* [Punjab 94]

Solution. Let P be the point at distance x cm from A, where the net field is zero.

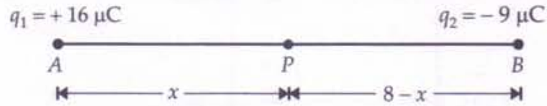


Fig. 1.45

At point P, $E_1 + E_2 = 0$

$$\frac{k \times 16 \times 10^{-6}}{(x \times 10^{-2})^2} + \frac{k \times (-9) \times 10^{-6}}{[(8-x) \times 10^{-2}]^2} = 0$$

or $$\frac{16}{x^2} = \frac{9}{(8-x)^2}$$

or $$\frac{4}{x} = \pm \frac{3}{8-x}$$

or $$x = \frac{32}{7} \text{ cm}, 32 \text{ cm}$$

At $x = \frac{32}{7}$ cm, both E_1 and E_2 will be in the same direction, therefore, net electric field cannot be zero.

Hence $x = 32$ cm

i.e., electric field is zero at a point 24 cm to the right of $-9\ \mu C$ charge.

Example 40. *Two point charges* $q_1 = +0.2$ *C and* $q_2 = +0.4$ *C are placed 0.1 m apart. Calculate the electric field at*

(*a*) *the midpoint between the charges.*

(*b*) *a point on the line joining* q_1 *and* q_2 *such that it is 0.05 m away from* q_2 *and 0.15 m away from* q_1. [CBSE D 93C]

Solution. (*a*) Let O be the midpoint between the two charges.

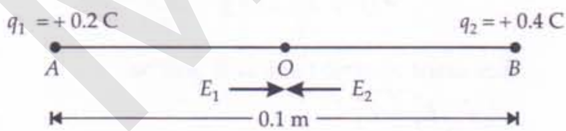


Fig. 1.46

Electric field at O due to q_1,

$$E_1 = \frac{kq_1}{r_1^2} = \frac{9 \times 10^9 \times 0.2}{(0.05)^2} = 7.2 \times 10^{11} \text{ NC}^{-1},$$

acting along AO

Electric field at O due to q_2,

$$E_2 = \frac{kq_2}{r_2^2} = \frac{9 \times 10^9 \times 0.4}{(0.05)^2}$$

$$= 14.4 \times 10^{11} \text{ NC}^{-1}, \text{ acting along } BO$$

Net field at $O = E_2 - E_1$

$$= \mathbf{7.2 \times 10^{11}\ NC^{-1}}, \textbf{ acting along } \boldsymbol{BO}.$$

(*b*) Electric field at P due to q_1,

$$E_1 = \frac{kq_1}{r_1^2} = \frac{9 \times 10^9 \times 0.2}{(0.15)^2}, \text{ acting along } AP$$

Electric field at P due to q_2,

$$E_2 = \frac{kq_2}{r_2^2} = \frac{9 \times 10^9 \times 0.4}{(0.05)^2}, \text{ acting along } BP$$

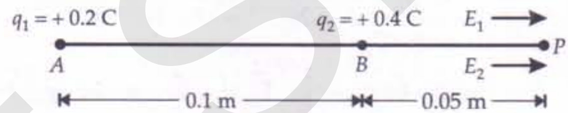


Fig. 1.47

Net electric field at point P is

$$E = E_1 + E_2 = 9 \times 10^9 \left[\frac{0.2}{(0.15)^2} + \frac{0.4}{(0.05)^2}\right]$$

$$= \mathbf{1.52 \times 10^{12}\ NC^{-1}}, \textbf{ acting along } \boldsymbol{AP}.$$

Example 41. *Two point charges* q_1 and q_2 *of* 10^{-8} *C and* -10^{-8} *C respectively are placed 0.1 m apart. Calculate the electric fields at points A, B and C shown in Fig. 1.48.* [NCERT]

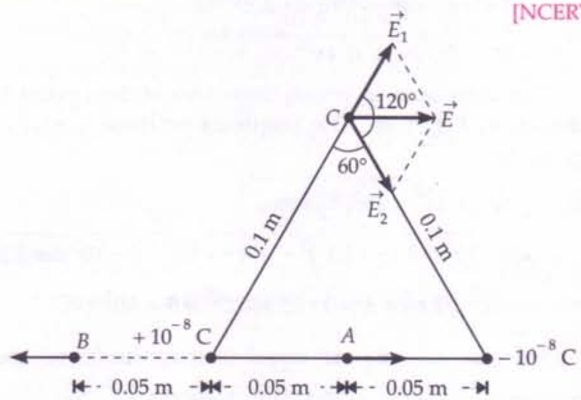


Fig. 1.48

Solution. The electric field vector $\vec{E_1}$ at A due to the positive charge q_1 points towards the right and it has a magnitude,

$$E_1 = \frac{kq_1}{r_1^2} = \frac{9 \times 10^9 \times 10^{-8}}{(0.05)^2} \text{ NC}^{-1}$$

$$= 3.6 \times 10^4 \text{ NC}^{-1}$$

The electric field vector $\vec{E_2}$ at A due to the negative charge q_2 points towards the right and it has a magnitude,

$$E_2 = \frac{9 \times 10^9 \times 10^{-8}}{(0.05)^2} \text{ NC}^{-1} = 3.6 \times 10^4 \text{ NC}^{-1}$$

Magnitude of the total electric field at A

$$E_a = E_1 + E_2$$
$$= 3.6 \times 10^4 + 3.6 \times 10^4 = \mathbf{7.2 \times 10^4 \ NC^{-1}}$$

$\vec{E_a}$ is directed towards the right.

The electric field vector $\vec{E_1}$ at B due to the positive charge q_1 points towards the left and it has a magnitude,

$$E_1 = \frac{9 \times 10^9 \times 10^{-8}}{(0.05)^2} \text{ NC}^{-1} = 3.6 \times 10^4 \text{ NC}^{-1}$$

The electric field vector $\vec{E_2}$ at B due to the negative charge q_2 points towards the right and it has a magnitude,

$$E_2 = \frac{9 \times 10^9 \times 10^{-8}}{(0.15)^2} \text{ NC}^{-1} = 4 \times 10^3 \text{ NC}^{-1}$$

Magnitude of the total electric field at B

$$E_b = E_1 - E_2 = \mathbf{3.2 \times 10^4 \ NC^{-1}}$$

$\vec{E_b}$ is directed towards the left.

Magnitude of each electric field vector, at point C, of charges q_1 and q_2 is

$$E_1 = E_2 = \frac{9 \times 10^9 \times 10^{-8}}{(0.1)^2} = 9 \times 10^3 \text{ NC}^{-1}$$

The directions in which these two vectors point are shown in Fig. 1.48. The resultant of these vectors is given by

$$E_c = \sqrt{E_1^{\ 2} + E_2^{\ 2} + 2 E_1 E_2 \cos\theta}$$
$$= \sqrt{(9 \times 10^3)^2 + (9 \times 10^3)^2 + 2 \times 9 \times 10^3 \times 9 \times 10^3 \cos 120^\circ}$$
$$= 9 \times 10^3 \sqrt{1 + 1 + 2(-1/2)} \text{ NC}^{-1} = \mathbf{9 \times 10^3 \ NC^{-1}}$$

Since $\vec{E_1}$ and $\vec{E_2}$ are equal in magnitude, so their resultant $\vec{E_c}$ acts along the bisector of the angle between $\vec{E_1}$ and $\vec{E_2}$, *i.e.*, towards right.

Example 42. *ABCD is a square of side 5 m. Charges of + 50 C, − 50 C and + 50 C are placed at A, C and D respectively. Find the resultant electric field at B.*

Solution. Electric field at B due to + 50 C charge at A is

$$E_1 = k \cdot \frac{q}{r^2} = k \cdot \frac{50}{5^2} = 2k, \text{ along } AB$$

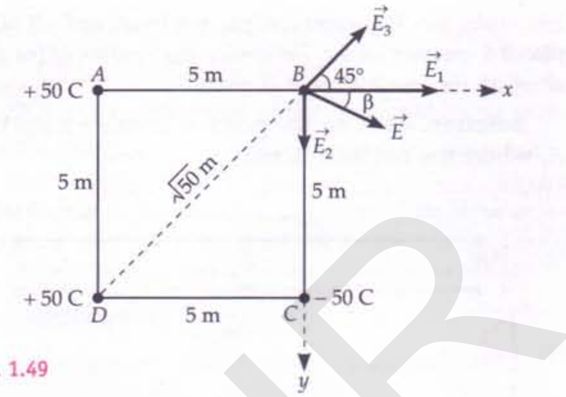


Fig. 1.49

Electric field at B due to −50 C charge at C is

$$E_2 = k \cdot \frac{50}{5^2} = 2k, \text{ along } BC$$

Electric field at B due to + 50 C charge at D is

$$E_3 = k \cdot \frac{50}{(\sqrt{5^2 + 5^2})^2} = k, \text{ along } DB$$

Component of E_1 along x-axis $= 2k$

(as it acts along x-axis)

Component of E_2 along x-axis $= 0$

(as it acts along y-axis)

Component of E_3 along x-axis

$$= E_3 \cos 45^\circ = k \cdot \frac{1}{\sqrt{2}} = \frac{k}{\sqrt{2}}.$$

∴ Total electric field at B along x-axis

$$E_x = 2k + 0 + \frac{k}{\sqrt{2}} = k\left(2 + \frac{1}{\sqrt{2}}\right)$$

Now,

Component of E_1 along y-axis $= 0$

Component of E_2 along y-axis $= 2k$

Component of E_3 along y-axis

$$E_y = E_3 \sin 45^\circ = k \cdot \frac{1}{\sqrt{2}} = \frac{k}{\sqrt{2}}$$

But the components of E_2 and E_3 act in opposite directions, therefore, total electric field at B along y-axis

$$= 2k - \frac{k}{\sqrt{2}} = k\left(2 - \frac{1}{\sqrt{2}}\right)$$

∴ Resultant electric field at B will be

$$E = \sqrt{E_x^2 + E_y^2}$$
$$= \sqrt{\left[k\left(2 + \frac{1}{\sqrt{2}}\right)\right]^2 + \left[k\left(2 - \frac{1}{\sqrt{2}}\right)\right]^2} = \sqrt{9k^2}$$
$$= 3k = 3 \times 9 \times 10^9 \text{ NC}^{-1} = \mathbf{2.7 \times 10^{10} \ NC^{-1}}$$

If the resultant field E makes angle β with x-axis, then

$$\tan\beta = \frac{E_y}{E_x} = \frac{(2 - 1/\sqrt{2})\,k}{(2 + 1/\sqrt{2})\,k} = 0.4776 \quad \text{or} \quad \beta = \mathbf{25.5^\circ}.$$

Example 43. *Four charges $+q, +q, -q, -q$ are placed respectively at the four corners A, B, C and D of a square of side 'a'. Calculate the electric field at the centre of the square.* [Punjab 96C]

Solution. Let E_A, E_B, E_C and E_D be the electric fields at the centre O of the square due to the charges at A, B, C and D respectively. Their directions are as shown in Fig. 1.50(a).

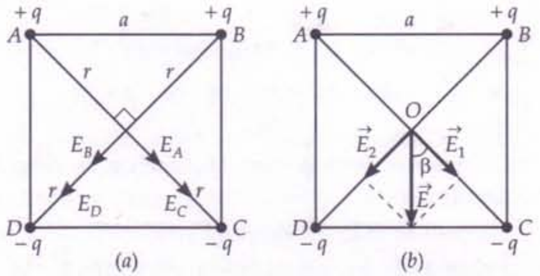


Fig. 1.50

Since all the charges are of equal magnitude and at the same distance r from the centre O, so

$$E_A = E_B = E_C = E_D = k \cdot \frac{q}{r^2} = \frac{q}{\left(\frac{a}{\sqrt{2}}\right)^2} = 2\,\frac{kq}{a^2} \qquad [\because\ r^2 + r^2 = a^2]$$

Because E_A and E_C act in the same direction, so their resultant is

$$E_1 = E_A + E_C = \frac{2kq}{a^2} + \frac{2kq}{a^2} = \frac{4kq}{a^2}$$

Similarly, resultant of E_B and E_D is

$$E_2 = E_B + E_D = \frac{4kq}{a^2}$$

Now, the resultant of E_1 and E_2 will be

$$E = \sqrt{E_1^{\,2} + E_2^{\,2}} = \sqrt{\left(\frac{4kq}{a^2}\right)^2 + \left(\frac{4kq}{a^2}\right)^2}$$

$$= 4\sqrt{2}\, k\, \frac{q}{a^2},$$

directed parallel to AD or BC, as shown in Fig. 1.50(b).

$$\cos\beta = \frac{E_1}{E} = \frac{1}{\sqrt{2}} \qquad \therefore \quad \beta = \mathbf{45^\circ}$$

i.e., the resultant field is inclined at an angle of 45° with AC.

Example 44. *Two point charges +6q and –8q are placed at the vertices 'B' and 'C' of an equilateral triangle ABC of side 'a' as shown in Fig. 1.51(a). Obtain the expression for (i) the magnitude and (ii) the direction of the resultant electric field at the vertex A due to these two charges.* [CBSE OD 14C]

Solution. (i) As shown in Fig. 1.51(b), the fields at point A due to the charges at B and C are $\vec{E}_{BA}$ and $\vec{E}_{AC}$ respectively.

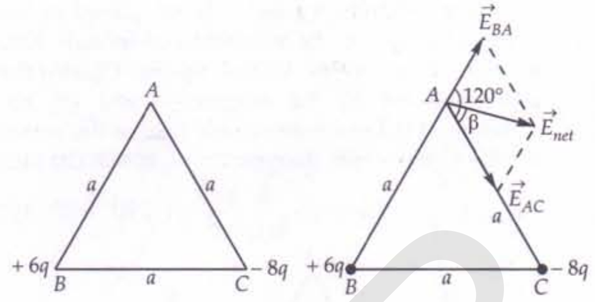


Fig. 1.51 (a) (b)

Their magnitudes are

$$E_{BA} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{6q}{a^2} = 6E, \text{ where } E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{a^2}$$

$$E_{AC} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{8q}{a^2} = 8E$$

The magnitude of the resultant field is

$$E_{net} = \sqrt{E_{BA}^2 + E_{AC}^2 + 2\,E_{BA}\,E_{AC}\cos 120^\circ}$$

$$= \sqrt{(6E)^2 + (8E)^2 + 2 \times 6E \times 8E \times \left(-\frac{1}{2}\right)}$$

$$= E\sqrt{52} = \frac{1}{4\pi\varepsilon_0}\,\frac{q\sqrt{52}}{a^2}$$

(ii) If the resultant field makes an angle β with AC, then

$$\tan\beta = \frac{E_{BA}\sin 120^\circ}{E_{AC} + E_{BA}\cos 120^\circ} = \frac{6E \times (\sqrt{3}/2)}{8E + 6E\left(-\frac{1}{2}\right)} = \frac{3\sqrt{3}}{5}$$

$$\therefore \quad \beta = \tan^{-1}\left(\frac{3\sqrt{3}}{5}\right)$$

Problems For Practice

1. An electron is separated from the proton through a distance of 0.53 Å. Calculate the electric field at the location of the electron. (Ans. 5.1×10^{11} NC^{-1})
2. Determine the electric field produced by a helium nucleus at a distance of 1 Å from it. (Ans. 2.88×10^{11} NC^{-1})
3. Two point charges $+q$ and $+4q$ are separated by a distance of $6a$. Find the point on the line joining the two charges where the electric field is zero. (Ans. At a distance $2a$ from charge $+q$)
4. Two point charges q_1 and q_2 of 2×10^{-8}C and -2×10^{-8}C respectively are placed 0.4 m apart. Calculate the electric field at the centre of the line joining the two charges. [CBSE F 94C] (Ans. 900 NC^{-1}, towards the –ve charge)

5. Two point charges $+q$ and $-2q$ are placed at the vertices 'B' and 'C' of an equilateral triangle ABC of side 'a' as given in the figure. Obtain the expression for (*i*) the magnitude and (*ii*) the direction of the resultant electric field at the vertex A due to these two charges. **[CBSE OD 14C]**

[Ans. (*i*) $\frac{1}{4\pi\varepsilon_0}\frac{q\sqrt{3}}{a^2}$ (*ii*) 30° with AC]

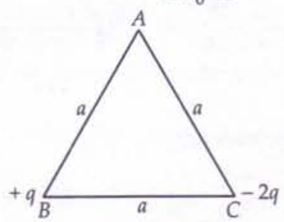


Fig. 1.52

6. Find the magnitude and direction of electric field at point P in Fig. 1.53.

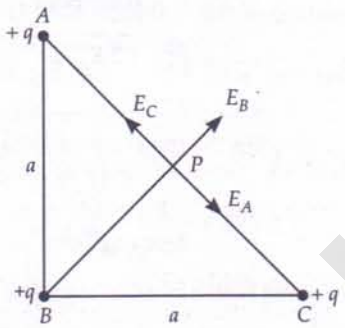


Fig. 1.53

$\left(\text{Ans. } E = \frac{1}{4\pi\varepsilon_0}\frac{2q}{a^2}, \text{ along } BP \text{ produced}\right)$

7. Three charges, each equal to q are placed at the three corners of a square of side a. Find the electric field at the fourth corner. $\left(\text{Ans. } (2\sqrt{2}+1)\frac{q}{8\pi\varepsilon_0 a^2}\right)$

8. Figure 1.54 shows four point charges at the corners of a square of side 2 cm. Find the magnitude and direction of the electric field at the centre O of the square, if $Q = 0.02\ \mu C$.

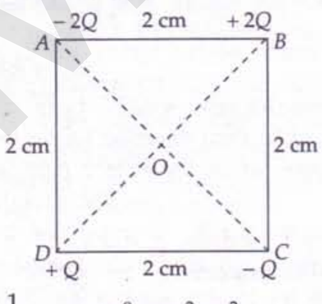


Fig. 1.54

Use $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\ \text{Nm}^2\ \text{C}^{-2}$. **[ISCE 98]**

(Ans. $9\sqrt{2} \times 10^5\ \text{NC}^{-1}$, parallel to BA)

HINTS

1. Electric field at the location of the electron,

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2} = \frac{9 \times 10^9 \times 1.6 \times 10^{-19}}{(0.53 \times 10^{-10})^2} = \mathbf{5.1 \times 10^{11}\ NC^{-1}}.$$

2. Here $q = +2e$ and $r = 1\ \text{Å} = 10^{-10}$ m.

3. Suppose the electric field is zero at distance x from the charge $+q$. Then

$$\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{x^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{4q}{(6a-x)^2}$$

or $\quad (6a-x)^2 = 4x^2$ or $6a - x = 2x$

or $\quad x = 2a$

$\therefore$ Electric field is zero at distance $2a$ from the charge $+q$.

4. Proceed as in Q. 1.8 on page 1.81.

5. Proceed as in the solution of Example 44 on page 1.33.

6. Here E_A and E_C are equal and opposite and hence cancel out.

$$BP = a \sin 45° = a/\sqrt{2}$$

Hence $\quad E = E_B = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{(a/\sqrt{2})^2}$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{2q}{a^2}, \quad \textbf{along } \boldsymbol{BP} \textbf{ produced.}$$

7. Refer to Fig. 1.55.

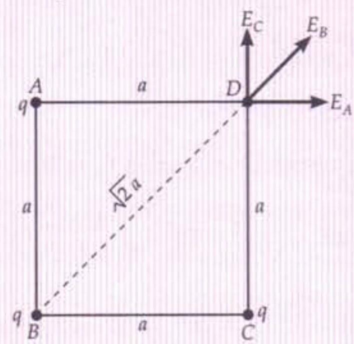


Fig. 1.55

$$E_D = \sqrt{E_A^2 + E_C^2} + E_B$$

$$= \sqrt{\left(\frac{q}{4\pi\varepsilon_0 a^2}\right)^2 + \left(\frac{q}{4\pi\varepsilon_0 a^2}\right)^2} + \frac{q}{4\pi\varepsilon_0(\sqrt{2}a)^2}$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{\sqrt{2}}{a^2} + \frac{1}{2a^2}\right] = (2\sqrt{2}+1)\frac{q}{8\pi\varepsilon_0 a^2}.$$

8. Here, $AB = BC = CD = AD = 2\,\text{cm}$

$$\therefore \quad AO = BO = CO = DO = \frac{\sqrt{2^2+2^2}}{2} = \sqrt{2}\ \text{cm}$$

$$= \sqrt{2} \times 10^{-2}\ \text{m}$$

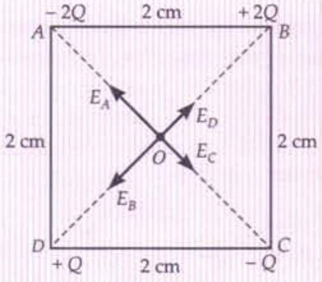


Fig. 1.56

$$\therefore \; E_A = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2Q}{(OA)^2} = \frac{1}{4\pi\varepsilon_0} \cdot Q \times 10^4, \text{ along } OA$$

$$E_B = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2Q}{(OB)^2} = \frac{1}{4\pi\varepsilon_0} \cdot Q \times 10^4, \text{ along } OD$$

$$E_C = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{(OC)^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{2} \times 10^4, \text{ along } OC$$

and $$E_D = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{(OB)^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{2} \times 10^4, \text{ along } OB$$

Net electric field along OA,

$$E_1 = E_A - E_C = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{2} \times 10^4$$

Net electric field along OD,

$$E_2 = E_B - E_D = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{2} \times 10^4$$

Hence, the resultant electric field at point O,

$$E = \sqrt{E_1^2 + E_2^2}$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{\sqrt{2}} \times 10^4, \quad \text{parallel to side } BA$$

But, $Q = 0.02\,\mu\text{C} = 0.02 \times 10^{-6}$ C

$$\therefore \quad E = 9 \times 10^9 . \frac{0.02 \times 10^{-6} \times 10^4}{\sqrt{2}}$$

$$= \mathbf{9\sqrt{2} \times 10^5 \ NC^{-1}}, \quad \textbf{parallel to side } \boldsymbol{BA}.$$

1.21 CONTINUOUS CHARGE DISTRIBUTION

33. *What is a continuous charge distribution ? How can we calculate the force on a point charge q due to a continuous charge distribution ?*

Continuous charge distribution. In practice, we deal with charges much greater in magnitude than the charge on an electron, so we can ignore the quantum nature of charges and imagine that the charge is spread in a region in a continuous manner. Such a charge distribution is known as a *continuous charge distribution.*

Calculation of the force on a charge due to a continuous charge distribution. As shown in Fig. 1.57, consider a point charge q_0 lying near a region of continuous charge distribution. This continuous charge distribution can be imagined to consist of a large number of small charges dq. According to Coulomb's law, the force on point charge q_0 due to small charge dq is

$$d\vec{F} = \frac{1}{4\pi\varepsilon_0} \frac{q_0 \, dq}{r^2} . \hat{r}$$

where $\hat{r} = \frac{\vec{r}}{r}$, is a unit vector pointing from the small charge dq towards the point charge q_0. By the principle of superposition, the total force on charge q_0 will be the vector sum of the forces exerted by all such small charges and is given by

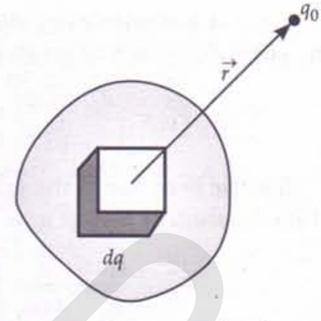


Fig. 1.57 Force on a point charge q_0 due to a continuous charge distribution.

$$\vec{F} = \int d\vec{F} = \int \frac{1}{4\pi\varepsilon_0} . \frac{q_0 \, dq}{r^2} . \hat{r}$$

or $$\boxed{\vec{F} = \frac{q_0}{4\pi\varepsilon_0} \int \frac{dq}{r^2} . \hat{r}}$$

34. *Name the different types of continuous charge distributions. Define their respective charge densities. Write expression for the electric field produced by each type of charge distribution. Hence write expression for the electric field of a general source charge distribution.*

Different types of continuous charge distributions. There are *three* types of continuous charge distributions :

(*a*) **Volume charge distribution.** *It is a charge distribution spread over a three dimensional volume or region V of space,* as shown in Fig. 1.57. We define the ***volume charge density*** *at any point in this volume as the charge contained per unit volume at that point, i.e.,*

$$\boxed{\rho = \frac{dq}{dV}}$$

The SI unit for ρ is coulomb per cubic metre (Cm^{-3}).

For example, if a charge q is distributed over the entire volume of a sphere of radius R, then its volume charge density is

$$\boxed{\rho = \frac{q}{\frac{4}{3}\pi R^3} \text{ Cm}^{-3}}$$

The charge contained in small volume dV is

$$dq = \rho \, dV$$

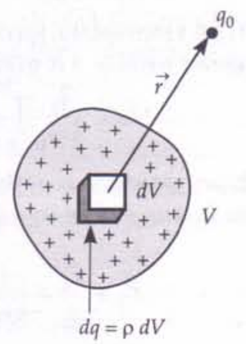


Fig. 1.58 Volume charge distribution

Total electrostatic force exerted on charge q_0 due to the entire volume V is given by

$$\vec{F}_V = \frac{q_0}{4\pi\varepsilon_0} \int_V \frac{dq}{r^2}\,\hat{r} = \frac{q_0}{4\pi\varepsilon_0} \int_V \frac{\rho}{r^2}\,dV\,\hat{r}$$

Electric field due to the volume charge distribution at the location of charge q_0 is

$$\vec{E}_V = \frac{\vec{F}_V}{q_0} = \frac{1}{4\pi\varepsilon_0} \int_V \frac{\rho}{r^2}\,dV\,\hat{r}.$$

(*b*) **Surface charge distribution.** *It is a charge distribution spread over a two-dimensional surface S in space,* as shown in Fig. 1.59. We define the ***surface charge density*** *at any point on this surface as the charge per unit area at that point, i.e.,*

$$\sigma = \frac{dq}{dS}$$

The SI unit for σ is Cm^{-2}.

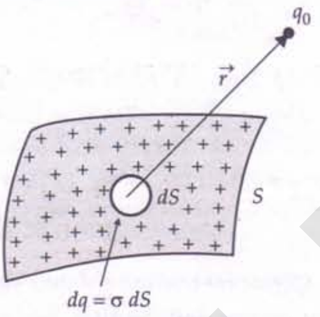


Fig. 1.59 Surface charge distribution.

For example, if a charge q is uniformly distributed over the surface of a spherical conductor of radius R, then its surface charge density is

$$\sigma = \frac{q}{4\pi R^2}\ Cm^{-2}$$

The charge contained in small area dS is

$$dq = \sigma\,dS$$

Total electrostatic force exerted on charge q_0 due to the entire surface S is given by

$$\vec{F}_S = \frac{q_0}{4\pi\varepsilon_0} \int_S \frac{\sigma}{r^2}\,dS\,\hat{r}$$

Electric field due to the surface charge distribution at the location of charge q_0 is

$$\vec{E}_S = \frac{\vec{F}_S}{q_0} = \frac{1}{4\pi\varepsilon_0} \int_S \frac{\sigma}{r^2}\,dS\,\hat{r}.$$

(*c*) **Line charge distribution.** *It is a charge distribution along a one-dimensional curve or line L in space,* as shown in Fig. 1.60. *We define the* ***line charge density*** *at any point on this line as the charge per unit length of the line at that point, i.e.,*

$$\lambda = \frac{dq}{dL}$$

The SI unit for λ is Cm^{-1}.

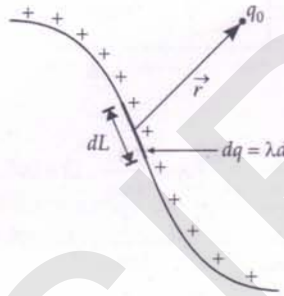


Fig. 1.60 Line charge distribution.

For example, if a charge q is uniformly distributed over a ring of radius R, then its linear charge density is

$$\lambda = \frac{q}{2\pi R}\ Cm^{-1}$$

The charge contained in small length dL is

$$dq = \lambda\,dL$$

Total electrostatic force exerted on charge q_0 due to the entire length L is given by

$$\vec{F}_L = \frac{q_0}{4\pi\varepsilon_0} \int_L \frac{\lambda}{r^2}\,dL\,\hat{r}$$

Electric field due to the line charge distribution at the location of charge q_0 is

$$\vec{E}_L = \frac{\vec{F}_L}{q_0} = \frac{1}{4\pi\varepsilon_0} \int_L \frac{\lambda}{r^2}\,dL\,\hat{r}$$

The total electric field due to a continuous charge distribution is given by

$$\vec{E}_{cont} = \vec{E}_V + \vec{E}_S + \vec{E}_L$$

or $$\vec{E}_{cont} = \frac{1}{4\pi\varepsilon_0}\left[\int_V \frac{\rho}{r^2}\,dV\,\hat{r} + \int_S \frac{\sigma}{r^2}\,dS\,\hat{r} + \int_L \frac{\lambda}{r^2}\,dL\,\hat{r}\right]$$

General charge distribution. A general charge distribution consists of continuous as well as discrete charges. Hence total electric field due to a general charge distribution at the location of charge q_0 is given by

$$\vec{E}_{total} = \vec{E}_{discrete} + \vec{E}_{cont}$$

or $$\vec{E}_{total} = \frac{1}{4\pi\varepsilon_0}\left[\sum_{i=1}^{N} \frac{q_i}{r_i^2}\,\hat{r}_i + \int_V \frac{\rho}{r^2}\,dV\,\hat{r} + \int_S \frac{\sigma}{r^2}\,dS\,\hat{r} + \int_L \frac{\lambda}{r^2}\,dL\,\hat{r}\right]$$

In all the above cases, $\hat{r} = \vec{r}/r$ is a variable unit vector directed from each point of the volume, surface or line charge distribution towards the location of the point charge q_0.

Examples based on Continuous Charge Distributions

Formulae Used

1. Volume charge density, $\rho = \dfrac{dq}{dV}$
2. Surface charge density, $\sigma = \dfrac{dq}{dS}$
3. Linear charge density, $\lambda = \dfrac{dq}{dL}$
4. Force exerted on a charge q_0 due to a continuous charge distribution,
$$\vec{F} = \frac{q_0}{4\pi\varepsilon_0}\int \frac{dq}{r^2}\,\hat{r}$$
5. Electric field due to a continuous charge distribution,
$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r^2}\,\hat{r}$$

Units Used

ρ is in Cm^{-3}, σ in Cm^{-2}, λ in Cm^{-1} and E in NC^{-1}.

Example 45. *A charged spherical conductor has a surface density of 0.7 Cm^{-2}. When its charge is increased by 0.44 C, the charge density changes by 0.14 Cm^{-2}. Find the radius of the sphere and initial charge on it.*

Solution. $\sigma = \dfrac{q}{4\pi r^2}$

In first case :
$$0.7 = \frac{q}{4\pi r^2} \qquad ...(i)$$

In second case :
$$0.7 + 0.14 = \frac{q + 0.44}{4\pi r^2}$$

or
$$0.84 = \frac{q + 0.44}{4\pi r^2} \qquad ...(ii)$$

Dividing (*ii*) by (*i*), we get,
$$\frac{0.84}{0.7} = \frac{q+0.44}{q} \quad \text{or} \quad \frac{6}{5} = 1 + \frac{0.44}{q}$$

$\therefore$ Initial charge, $q = \mathbf{2.2\ C}$.

From (*i*),
$$r = \sqrt{\frac{q}{\sigma \times 4\pi}} = \sqrt{\frac{2.2}{0.7 \times 4\pi}}$$
$$= \sqrt{\frac{2.2 \times 7}{0.7 \times 4 \times 22}} = \mathbf{0.5\ m.}$$

Example 46. *Sixty four drops of radius 0.02 m and each carrying a charge of 5 μC are combined to form a bigger drop. Find how the surface density of electrification will change if no charge is lost.*

Solution. Volume of each small drop
$$= \frac{4}{3}\pi(0.02)^3\ \text{m}^3$$

Volume of 64 small drops $= \dfrac{4}{3}\pi(0.02)^3 \times 64\ \text{m}^3$

Let R be the radius of the bigger drop formed. Then
$$\frac{4}{3}\pi R^3 = \frac{4}{3}\pi(0.02)^3 \times 64$$

or
$$R^3 = (0.02)^3 \times 4^3$$
$$\therefore \quad R = 0.02 \times 4 = 0.08\ \text{m}$$

Charge on small drop $= 5\ \mu\text{C} = 5 \times 10^{-6}$ C

Surface charge density of small drop,
$$\sigma_1 = \frac{q}{4\pi r^2} = \frac{5\times 10^{-6}}{4\pi(0.02)^2}\ \text{Cm}^{-2}$$

Surface charge density of bigger drop,
$$\sigma_2 = \frac{5\times 10^{-6} \times 64}{4\pi(0.08)^2}\ \text{Cm}^{-2}$$
$$\therefore \quad \frac{\sigma_1}{\sigma_2} = \frac{5\times 10^{-6}}{4\pi(0.02)^2} \times \frac{4\pi(0.08)^2}{5\times 10^{-6}\times 64} = \frac{1}{4} = \mathbf{1:4.}$$

Example 47. *Obtain the formula for the electric field due to a long thin wire of uniform linear charge density λ without using Gauss's law.* [NCERT 1.30]

Solution. Electric field of a line charge from Coulomb's law. Consider an infinite line of charge with uniform line charge density λ, as shown in Fig. 1.61. We wish to calculate its electric field at any point P at a distance y from it. The charge on small element dx of the line charge will be
$$dq = \lambda dx$$

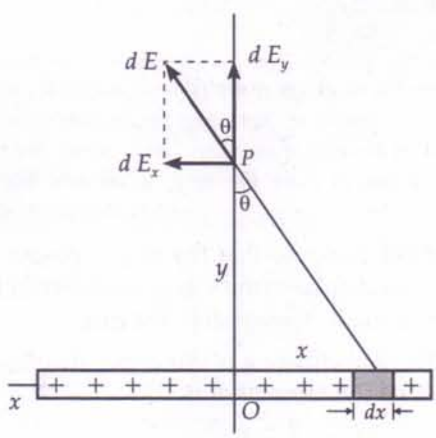


Fig. 1.61 A section of an infinite line of charge.

The electric field at the point P due to the charge element dq will be

$$dE = \frac{1}{4\pi\varepsilon_0} \cdot \frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\lambda dx}{y^2 + x^2}$$

The field dE has two components :

$$dE_x = -\,dE \sin\theta \quad \text{and} \quad dE_y = dE \cos\theta$$

The negative sign in x-component indicates that $d\vec{E}_x$ acts in the negative x-direction. Every charge element on the right has a corresponding charge element on the left. The x-components of two such charge elements will be equal and opposite and hence cancel out. The resultant field $\vec{E}$ gets contributions only from y-components and is given by

$$E = E_y = \int dE_y = \int_{x=-\infty}^{x=+\infty} \cos\theta \, dE$$

$$= 2\int_{x=0}^{x=\infty} \cos\theta \cdot \frac{1}{4\pi\varepsilon_0} \cdot \frac{\lambda\, dx}{y^2 + x^2}$$

$$= \frac{\lambda}{2\pi\varepsilon_0} \int_{x=0}^{x=\infty} \cos\theta \, \frac{dx}{y^2 + x^2}$$

Now $\quad x = y \tan\theta$

$$dx = y \sec^2\theta \, d\theta$$

$$\therefore \quad E = \frac{\lambda}{2\pi\varepsilon_0} \int_{\theta=0}^{\theta=\pi/2} \cos\theta \, \frac{y \sec^2\theta \, d\theta}{y^2(1 + \tan^2\theta)}$$

$$= \frac{\lambda}{2\pi\varepsilon_0 y} \int_{\theta=0}^{\theta=\pi/2} \cos\theta \, d\theta = \frac{\lambda}{2\pi\varepsilon_0 y} [\sin\theta]_0^{\pi/2}$$

$$= \frac{\lambda}{2\pi\varepsilon_0 y}\left(\sin\frac{\pi}{2} - \sin 0\right)$$

or $\quad \boxed{E = \dfrac{\lambda}{2\pi\varepsilon_0 y}}.$

Example 48. *A charge is distributed uniformly over a ring of radius 'a'. Obtain an expression for the electric intensity E at a point on the axis of the ring. Hence show that for points at large distances from the ring, it behaves like a point charge.* [CBSE Sample Paper 90]

Solution. Suppose that the ring is placed with its plane perpendicular to the x-axis, as shown in Fig 1.62. Consider a small element dl of the ring.

As the total charge q is uniformly distributed, the charge dq on the element dl is

$$dq = \frac{q}{2\pi a} \cdot dl$$

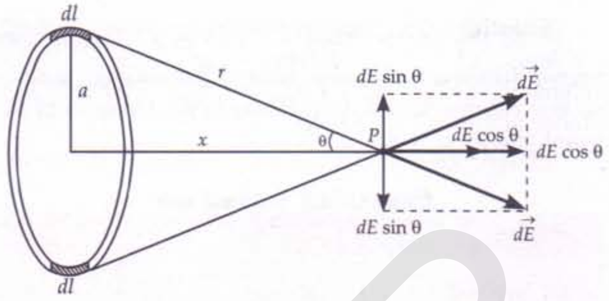


Fig. 1.62

$\therefore$ The magnitude of the field $d\vec{E}$ produced by the element dl at the field point P is

$$d\vec{E} = k \cdot \frac{dq}{r^2} = \frac{kq}{2\pi a} \cdot \frac{dl}{r^2}$$

As shown in Fig. 1.62, the field $d\vec{E}$ has two components :

1. the *axial component* $dE \cos\theta$, and

2. *the perpendicular component* $dE \sin\theta$.

Since the perpendicular components of any two diametrically opposite elements are equal and opposite, they all cancel out in pairs. Only the axial components will add up to produce the resultant field $\vec{E}$ at point P, which is given by

$$E = \int_0^{2\pi a} dE \cos\theta$$

[$\because$ Only the axial components contribute towards E]

$$= \int_0^{2\pi a} \frac{kq}{2\pi a} \cdot \frac{dl}{r^2} \cdot \frac{x}{r} = \frac{kqx}{2\pi a} \cdot \frac{1}{r^3} \int_0^{2\pi a} dl$$

$$\left[\because \cos\theta = \frac{x}{r}\right]$$

$$= \frac{kqx}{2\pi a} \cdot \frac{1}{r^3}[l]_0^{2\pi a} = \frac{kqx}{2\pi a} \cdot \frac{1}{(x^2 + a^2)^{3/2}} \cdot 2\pi a$$

$$[\because r^2 = x^2 + a^2]$$

or $\quad \boxed{E = \dfrac{kqx}{(x^2 + a^2)^{3/2}} = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{qx}{(x^2 + a^2)^{3/2}}}.$

Special case

For points at large distances from the ring, $x >> a$

$$\therefore \quad E = \frac{kq}{x^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{x^2}$$

This is the same as the field due to a point charge, indicating that for far off axial points, the charged ring behaves as a point charge.

Example 49. *A thin semicircular ring of radius a is charged uniformly and the charge per unit length is λ. Find the electric field at its centre.* [CBSE PMT 2000, AIEEE 2010]

Solution. Consider two symmetric elements each of length dl at A and B. The electric fields of the two elements perpendicular to PO get cancelled while those along PO get added.

Electric field at O due to an element of length dl is

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{a^2}\cos\theta \qquad \text{[Along } PO]$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\lambda dl}{a^2}\cos\theta$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\lambda(a d\theta)}{a^2}\cos\theta \qquad [dl = a d\theta]$$

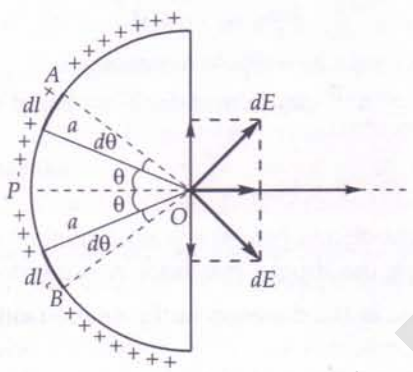


Fig. 1.63

Total electric field at the centre O is

$$E = \int_{-\pi/2}^{\pi/2} dE = 2\int_0^{\pi/2} \frac{1}{4\pi\varepsilon_0}\frac{\lambda\cos\theta\, d\theta}{a}$$

$$= \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{a}[\sin\theta]_0^{\pi/2} = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{a}\cdot 1 = \frac{\lambda}{2\pi\varepsilon_0 a}.$$

Problems for Practice

1. A uniformly charged sphere carries a total charge of $2\pi\times10^{-12}$C. Its radius is 5 cm and is placed in vacuum. Determine its surface charge density. (Ans. $2\times10^{-10}\ \text{Cm}^{-2}$)
2. What charge would be required to electrify a sphere of radius 15 cm so as to get a surface charge density of $\frac{7}{11}\mu\ \text{Cm}^{-2}$? (Ans. 1.8×10^{-7} C)
3. A metal cube of length 0.1 m is charged by $12\,\mu$C. Calculate its surface charge density. (Ans. $2\times10^{-4}\ \text{Cm}^{-2}$)
4. Two equal spheres of water having equal and similar charges coalesce to form a large sphere. If no charge is lost, how will the surface densities of electrification change ? (Ans. $\sigma_1 : \sigma_2 = 2^{2/3} : 2$)

HINTS

1. Use $\sigma = \frac{q}{4\pi r^2}$.
2. Use $q = 4\pi r^2\sigma$.
3. Surface area of cube $= 6\times l^2 = 6\times 0.01 = 0.06\ \text{m}^2$.
4. $\frac{4}{3}\pi R^3 = 2\times\frac{4}{3}\pi r^3$ or $R = 2^{1/3} r$

$$\frac{\sigma_1}{\sigma_2} = \frac{q}{4\pi r^2}\cdot\frac{4\pi R^2}{2q} = \frac{R^2}{2r^2} = \frac{2^{\frac{2}{3}} r^2}{2r^2} = 2^{2/3} : 2$$

1.22 ELECTRIC DIPOLE

35. *What is an electric dipole ? Define dipole moment and give its SI unit. Give some examples of electric dipoles. What are ideal or point dipoles ?*

Electric dipole. *A pair of equal and opposite charges separated by a small distance is called an electric dipole.*

Dipole moment. It measures the strength of an electric dipole. *The dipole moment of an electric dipole is a vector whose magnitude is either charge times the separation between the two opposite charges and the direction is along the dipole axis from the negative to the positive charge.*

As shown in Fig. 1.64, consider an electric dipole consisting of charges $+q$ and $-q$ and separated by distance $2a$. The line joining the charges is called dipole axis.

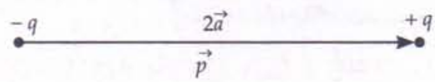


Fig. 1.64

Dipole moment = Either charge × a vector drawn from negative to positive charge

or $\qquad \vec{p} = q\times 2\,\vec{a}$

Thus the dipole moment $\vec{p}$ is a *vector* quantity. Its direction is along the dipole axis from $-q$ to $+q$ and its magnitude is

$$\boxed{p = q\times 2a}$$

The SI unit of dipole moment is *coulomb metre* (Cm). When both the charge q and separation $2a$ are finite, the dipole has a finite size (equal to $2a$), a location (midpoint between $+q$ and $-q$), a direction and a strength.

Examples of electric dipoles. Dipoles are common in nature. In molecules like H_2O, HCl, C_2H_5OH, CH_3COOH, etc., the centre of positive charges does not fall exactly over the centre of negative charges. Such molecules are electric dipoles. They have a permanent dipole moment.

Ideal or point dipole. We can think of a dipole in which size $2a \to 0$ and charge $q \to \infty$ in such a way that the dipole moment, $p = q \times 2a$ has a finite value. Such *a dipole of negligibly small size is called an ideal or point dipole.*

Dipoles associated with individual atoms or molecules may be treated as ideal dipoles. An ideal dipole is specified only by its location and a dipole moment, as it has no finite size.

1.23 DIPOLE FIELD

36. *What is a dipole field ? Why does the dipole field at large distance falls off faster than $1/r^2$?*

Dipole field. *The electric field produced by an electric dipole is called a dipole field.* This can be determined by using (*a*) the formula for the field of a point charge and (*b*) the principle of superposition.

Variation of dipole field with distance. The total charge of an electric dipole is zero. But the electric field of an electric dipole is not zero. This is because the charges $+q$ and $-q$ are separated by some distance, so the electric fields due to them when added do not exactly cancel out. However, at distances much larger than the dipole size ($r >> 2a$), the fields of $+q$ and $-q$ nearly cancel out. Hence we expect a dipole field to fall off, at larger distance, faster than $1/r^2$, typical of the field due to a single charge. In fact a dipole field at larger distances falls off as $1/r^3$.

1.24 ELECTRIC FIELD AT AN AXIAL POINT OF A DIPOLE

37. *Derive an expression for the electric field at any point on the axial line of an electric dipole.*

Electric field at an axial point of an electric dipole. As shown in Fig. 1.65, consider an electric dipole consisting of charges $+q$ and $-q$, separated by distance $2a$ and placed in vacuum. Let P be a point on the axial line at distance r from the centre O of the dipole on the side of the charge $+q$.

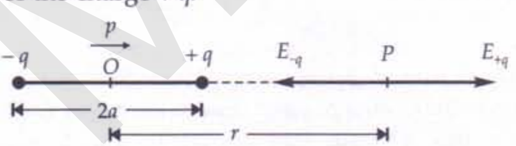


Fig. 1.65 Electric field at an axial point of dipole.

Electric field due to charge $-q$ at point P is

$$\vec{E}_{-q} = \frac{-q}{4\pi\varepsilon_0 (r+a)^2}\,\hat{p} \qquad \text{(towards left)}$$

where $\hat{p}$ is a unit vector along the dipole axis from $-q$ to $+q$.

Electric field due to charge $+q$ at point P is

$$\vec{E}_{+q} = \frac{q}{4\pi\varepsilon_0 (r-a)^2}\,\hat{p} \qquad \text{(towards right)}$$

Hence the resultant electric field at point P is

$$\vec{E}_{\text{axial}} = \vec{E}_{+q} + \vec{E}_{-q}$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right]\hat{p}$$

$$= \frac{q}{4\pi\varepsilon_0}\cdot\frac{4ar}{(r^2-a^2)^2}\,\hat{p}$$

or
$$\vec{E}_{\text{axial}} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2pr}{(r^2-a^2)^2}\,\hat{p}$$

Here $p = q \times 2a$ = dipole moment.

For $r >> a$, a^2 can be neglected compared to r^2.

$$\therefore \quad \boxed{\vec{E}_{\text{axial}} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2p}{r^3}\,\hat{p}} \qquad \text{(towards right)}$$

Clearly, electric field at any axial point of the dipole acts along the dipole axis from negative to positive charge *i.e.*, in the direction of dipole moment $\vec{p}$.

1.25 ELECTRIC FIELD AT AN EQUATORIAL POINT OF A DIPOLE

38. *Derive an expression for the electric field at any point on the equatorial line of an electric dipole.*

Electric field at an equatorial point of a dipole. As shown in Fig. 1.66, consider an electric dipole consisting of charges $-q$ and $+q$, separated by distance $2a$ and placed in vacuum. Let P be a point on the equatorial line of the dipole at distance r from it.

i.e., $\qquad OP = r$

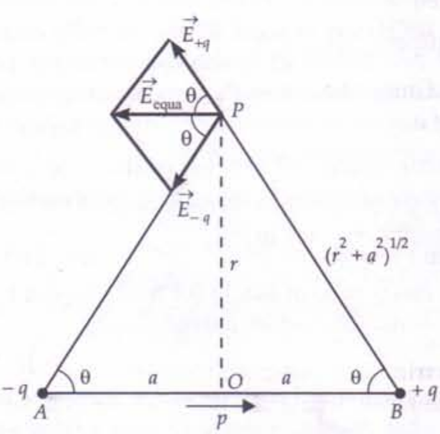


Fig. 1.66 Electric field at an equatorial point of a dipole.

Electric field at point P due to $+q$ charge is

$$\vec{E}_{+q} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2 + a^2}, \text{ directed along } BP$$

Electric field at point P due to $-q$ charge is

$$\vec{E}_{-q} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2 + a^2}, \text{ directed along } PA$$

Thus the magnitudes of $\vec{E}_{-q}$ and $\vec{E}_{+q}$ are equal *i.e.,*

$$E_{-q} = E_{+q} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2 + a^2}$$

Clearly, the components of $\vec{E}_{-q}$ and $\vec{E}_{+q}$ normal to the dipole axis will cancel out. The components parallel to the dipole axis add up. The total electric field $\vec{E}_{equa}$ is opposite to $\vec{p}$.

$$\therefore \quad \vec{E}_{equa} = -(E_{-q}\cos\theta + E_{+q}\cos\theta)\,\hat{p}$$

$$= -2\,E_{-q}\cos\theta\,\hat{p} \qquad [E_{-q} = E_{+q}]$$

$$= -2 \cdot \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2 + a^2} \cdot \frac{a}{\sqrt{r^2 + a^2}}\,\hat{p}$$

$$\left[\because \cos\theta = \frac{a}{\sqrt{r^2 + a^2}}\right]$$

or $$\vec{E}_{equa} = -\frac{1}{4\pi\varepsilon_0} \cdot \frac{p}{(r^2 + a^2)^{3/2}}\,\hat{p}$$

where $p = 2qa$, is the electric dipole moment.

If the point P is located far away from the dipole, $r >> a$, then

$$\boxed{\vec{E}_{equa} = -\frac{1}{4\pi\varepsilon_0} \cdot \frac{p}{r^3}\,\hat{p}}$$

Clearly, the direction of electric field at any point on the equatorial line of the dipole will be antiparallel to the dipole moment $\vec{p}$.

39. *Give a comparison of the magnitudes of electric fields of a short dipole at axial and equatorial points.*

Comparison of electric fields of a short dipole at axial and equatorial points. The magnitude of the electric field of a short dipole at an axial point at distance r from its centre is

$$E_{axial} = \frac{1}{4\pi\varepsilon_0} \frac{2p}{r^3}$$

Electric field at an equatorial point at the same distance r is

$$E_{equa} = \frac{1}{4\pi\varepsilon_0} \frac{p}{r^3}$$

Clearly, $\boxed{E_{axial} = 2\,E_{equa}}$

Hence the *electric field of a short dipole at a distance r along its axis is twice the electric field at the same distance along the equatorial line.*

1.26 TORQUE ON A DIPOLE IN A UNIFORM ELECTRIC FIELD

40. *Derive an expression for the torque on an electric dipole placed in a uniform electric field. Hence define dipole moment.*

Torque on a dipole in a uniform electric field. As shown in Fig. 1.67(*a*), consider an electric dipole consisting of charges $+q$ and $-q$ and of length $2a$ placed in a uniform electric field $\vec{E}$ making an angle θ with it. It has a dipole moment of magnitude,

$$p = q \times 2a$$

Force exerted on charge $+q$ by field $\vec{E} = q\vec{E}$

(along $\vec{E}$)

Force exerted on charge $-q$ by field $\vec{E} = -q\vec{E}$

(opposite to $\vec{E}$)

$$\vec{F}_{Total} = +q\vec{E} - q\vec{E} = 0.$$

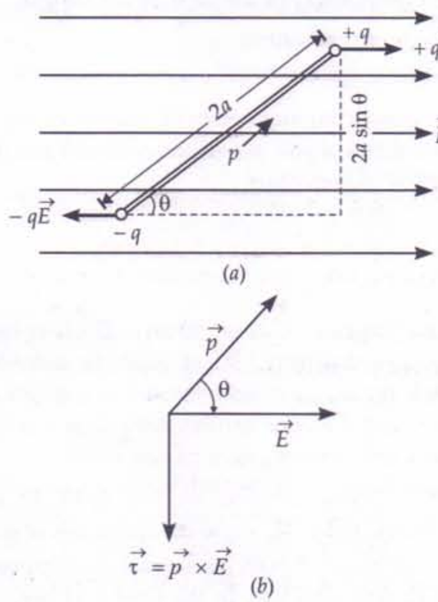


Fig. 1.67 (*a*) Torque on a dipole in a uniform electric field. (*b*) Direction of torque as given by right hand screw rule.

Hence the net translating force on a dipole in a uniform electric field is zero. But the two equal and opposite forces act at different points of the dipole. They form a couple which exerts a torque.

Torque = Either force × Perpendicular distance between the two forces

$$\tau = qE \times 2a \sin\theta = (q \times 2a)\, E \sin\theta$$

or $$\tau = pE \sin\theta \qquad (p = q \times 2a)$$

As the direction of torque $\vec{\tau}$ is perpendicular to both $\vec{p}$ and $\vec{E}$, so we can write

$$\vec{\tau} = \vec{p} \times \vec{E}$$

The direction of vector $\vec{\tau}$ is that in which a right handed screw would advance when rotated from $\vec{p}$ to $\vec{E}$. As shown in Fig. 1.67(*b*), the direction of vector $\vec{\tau}$ is perpendicular to, and points into the plane of paper.

When the dipole is released, the torque $\vec{\tau}$ tends to align the dipole with the field $\vec{E}$ *i.e.,* tends to reduce angle θ to 0. When the dipole gets aligned with $\vec{E}$, the torque $\vec{\tau}$ becomes zero.

Clearly, the torque on the dipole will be maximum when the dipole is held perpendicular to $\vec{E}$. Thus

$$\tau_{max} = pE \sin 90° = pE.$$

Dipole moment. We know that the torque,

$$\tau = pE \sin\theta$$

If $E = 1$ unit, $\theta = 90°$, then $\tau = p$

Hence ***dipole moment*** *may be defined as the torque acting on an electric dipole, placed perpendicular to a uniform electric field of unit strength.*

1.27 DIPOLE IN A NON-UNIFORM ELECTRIC FIELD

41. *What happens when an electric dipole is held in a non-uniform electric field ? What will be the force and the torque when the dipole is held parallel or anti-parallel to the electric field ? Hence explain why does a comb run through dry hair attract pieces of paper ?*

Dipole in a non-uniform electric field. In a non-uniform electric field, the + q and − q charges of a dipole experience different forces (not equal and opposite) at slightly different positions in the field and hence a net force $\vec{F}$ acts on the dipole in a non-uniform field. Also, a net torque acts on the dipole which depends on the location of the dipole in the non-uniform field.

$$\vec{\tau} = \vec{p} \times \vec{E}(\vec{r})$$

where $\vec{r}$ is the position vector of the centre of the dipole.

When the dipole is parallel or antiparallel to $\vec{E}$. In a non-uniform field, if $\vec{p}$ is parallel to $\vec{E}$ or antiparallel to $\vec{E}$, the net torque on the dipole is zero (because the forces on charges ± q become linear). However, there is a net force on the dipole. As shown in Fig. 1.68, when $\vec{p}$ is parallel to $\vec{E}$, a net force acts on the dipole in the direction of increasing $\vec{E}$. When $\vec{p}$ is antiparallel to $\vec{E}$, a net force acts in the direction of decreasing $\vec{E}$.

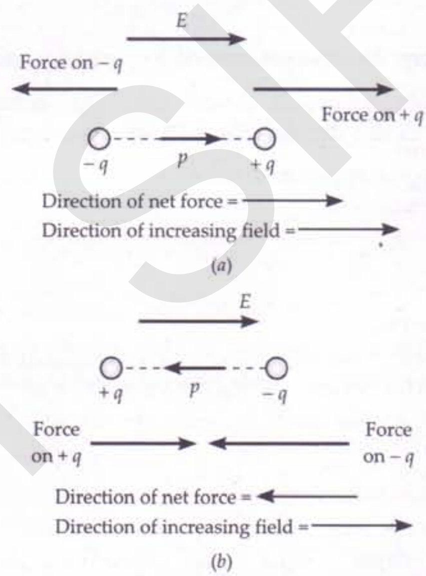


Fig. 1.68 Forces on a dipole (*a*) when $\vec{p}$ is parallel to $\vec{E}$ and (*b*) When $\vec{p}$ is antiparallel to $\vec{E}$.

A comb run through dry hair attracts small pieces of paper. As the comb runs through hair, it acquires charge due to friction. When the charged comb is brought closer to an uncharged piece of paper, it polarises the piece of paper *i.e.,* induces a net dipole moment in the direction of the field. But the electric field due to the comb on the piece of paper is not uniform. It exerts a force in the direction of increasing field *i.e.,* the piece of paper gets attracted towards the comb.

42. *Give the physical significance of electric dipoles.*

Physical significance of electric dipoles. Electric dipoles have a common occurrence in nature. A molecule consisting of positive and negative ions is an electric dipole. Moreover, a complicated array of charges can be described and analysed in terms of electric dipoles. The concept of electric dipole is used (*i*) in the study of the effect of electric field on an insulator, and (*ii*) in the study of radiation of energy from an antenna.

For Your Knowledge

- In a uniform electric field, an electric dipole experiences no net force but a non zero torque.
- As the net force on a dipole in a uniform electric field is zero, therefore, no linear acceleration is produced.
- Torque on a dipole becomes zero when it aligns itself parallel to the field.
- Torque on a dipole is maximum when it is held perpendicular to the field $\vec{E}$.
- In a non-uniform electric field, a dipole experiences a non zero force and non zero torque. In the special case when the dipole moment is parallel or antiparallel to the field, the dipole experiences a zero torque and a non zero force.
- A non-uniform or specifically an increasing E-field may be represented by field lines as shown.

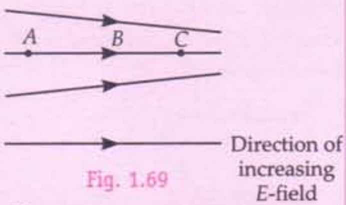


Fig. 1.69

Clearly, $E_A < E_B < E_C$.
- The direction of the electric field at an axial point of an electric dipole is same as that of its dipole moment and at an equatorial point it is opposite to that of dipole moment.
- The strength of electric field at an axial point of a short dipole is twice the strength at the same distance on the equatorial line.
- At larger distances, the dipole field $(E \propto 1/r^3)$ decreases more rapidly than the electric field of a point charge $(E \propto 1/r^2)$.

Examples based on Dipole Moment, Dipole Field and Torque on a Dipole

Formulae Used

1. Dipole moment, $p = q \times 2a$; where $2a$ is the distance between the two charges.
2. Dipole field at an axial point at distance r from the centre of the dipole is
$$E_{\text{axial}} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2pr}{(r^2 - a^2)^2}$$
When $r >> a$, $\quad E_{\text{axial}} = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{2p}{r^3}$
3. Dipole field at an equatorial point at distance r from the centre of the dipole is
$$E_{\text{equa}} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{p}{(r^2 + a^2)^{3/2}}$$
When $r >> a$, $\quad E_{\text{equa}} = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{p}{r^3}$
4. Torque, $\tau = pE \sin\theta$, where θ is the angle between $\vec{p}$ and $\vec{E}$.

Units Used

Charge q is in coulomb, distance $2a$ in metre, dipole moment p in coulomb metre (Cm), field E in NC^{-1} or Vm^{-1}.

Example 50. *Two charges, one $+5\,\mu C$ and another $-5\,\mu C$ are placed 1 mm apart. Calculate the dipole moment.* [CBSE OD 94C]

Solution. Here $\quad q = 5\,\mu C = 5 \times 10^{-6}$ C,

$$2a = 1 \text{ mm} = 10^{-3} \text{ m}$$

Dipole moment,
$$p = q \times 2a = 5 \times 10^{-6} \times 10^{-3} = \mathbf{5 \times 10^{-9}\ Cm.}$$

Example 51. *An electric dipole, when held at 30° with respect to a uniform electric field of $10^4\ NC^{-1}$ experiences a torque of 9×10^{-26} Nm. Calculate dipole moment of the dipole.* [CBSE D 96]

Solution. Here $\theta = 30°$, $E = 10^4\ NC^{-1}$,

$$\tau = 9 \times 10^{-26} \text{ Nm}$$

As $\quad \tau = pE \sin\theta$

$\therefore$ Dipole moment,
$$p = \frac{\tau}{E \sin\theta} = \frac{9 \times 10^{-26}}{10^4 \times \sin 30°} = \frac{9 \times 10^{-26}}{10^4 \times 0.5}$$
$$= \mathbf{1.8 \times 10^{-29}\ Cm.}$$

Example 52. *An electric dipole consists of two opposite charges of magnitude $1/3 \times 10^{-7}$ C, separated by 2 cm. The dipole is placed in an external field of $3 \times 10^7\ NC^{-1}$. What maximum torque does the electric field exert on the dipole?*

Solution. Here $q = \frac{1}{3} \times 10^{-7}$ C, $2a = 2$ cm $= 0.02$ m,

$$E = 3 \times 10^7\ NC^{-1}$$

$$\tau_{max} = pE \sin 90° = q \times 2a \times E \times 1$$
$$= \frac{1}{3} \times 10^{-7} \times 0.02 \times 3 \times 10^7 \times 1 = \mathbf{0.02\ Nm.}$$

Example 53. *Calculate the electric field due to an electric dipole of length 10 cm having charges of $1\,\mu C$ at an equatorial point 12 cm from the centre of the dipole.*

Solution. Here $q = 1\,\mu C = 10^{-6}$ C, $r = 12$ cm $= 0.12$ m,

$2a = 10$ cm, $a = 5$ cm $= 0.05$ m

$$E_{\text{equa}} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2qa}{(r^2 + a^2)^{3/2}}$$
$$= \frac{9 \times 10^9 \times 2 \times 10^{-6} \times 0.05}{(0.12^2 + 0.05^2)^{3/2}} = \frac{9 \times 100}{(0.13)^3}$$
$$= \mathbf{4.096 \times 10^5\ NC^{-1}.}$$

Example 54. *Two point charges, each of 5 μC but opposite in sign, are placed 4 cm apart. Calculate the electric field intensity at a point distant 4 cm from the midpoint on the axial line of the dipole.* [Punjab 02]

Solution. Here $q = 5 \times 10^{-6}$ C, $2a = 0.04$ m, $a = 0.02$ m, $r = 0.04$ m

$$E_{axial} = \frac{1}{4\pi\varepsilon_0} \frac{2pr}{(r^2 - a^2)^2} = \frac{1}{4\pi\varepsilon_0} \frac{2(q \times 2a)r}{(r^2 - a^2)^2}$$

$$= \frac{9 \times 10^9 \times 2 \times 5 \times 10^{-6} \times 0.04 \times 0.04}{[(0.04)^2 - (0.02)^2]^2}$$

$$= \frac{144}{144 \times 10^{-8}} = \mathbf{10^8\ NC^{-1}}.$$

Example 55. *Two charges ± 10 μC are placed 5.00 mm apart. Determine the electric field at (a) a point P on the axis of the dipole 15 cm away from its centre O on the side of the positive charge, (b) a point Q, 15 cm away from O on a line passing through O and normal to the axis of the dipole.* [NCERT]

Solution. Here $q = 10\,\mu C = 10^{-5}$ C,

$$2a = 5\text{ mm} = 5 \times 10^{-3}\text{ m}$$
$$r = 15\text{ cm} = 15 \times 10^{-2}\text{ m}$$

(*a*) Field at the axial point *P* of the dipole is

$$\vec{E}_P = \frac{2p}{4\pi\varepsilon_0 r^3} = \frac{2 \times q \times 2a}{4\pi\varepsilon_0 r^3}$$

$$= \frac{9 \times 10^9 \times 2 \times 10^{-5} \times 5 \times 10^{-3}}{(15 \times 10^{-2})^3}\ \text{NC}^{-1}$$

$$= \mathbf{2.66 \times 10^5\ NC^{-1}},\ \text{along } \vec{AB}.$$

This field is directed along the direction of dipole moment vector, *i.e.*, from $-q$ to $+q$, as shown in Fig. 1.70(*a*).

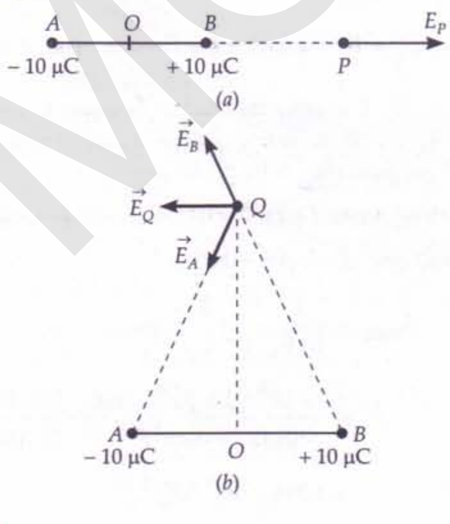


Fig. 1.70

(*b*) Field at the equatorial point *Q* of the dipole is

$$\vec{E}_Q = \frac{p}{4\pi\varepsilon_0 r^3} = \frac{q \times 2a}{4\pi\varepsilon_0 r^3}$$

$$= \frac{9 \times 10^9 \times 10^{-5} \times 5 \times 10^{-3}}{(15 \times 10^{-2})^3}\ \text{NC}^{-1}$$

$$= \mathbf{1.33 \times 10^5\ NC^{-1}},\ \text{along } \vec{BA}.$$

This field is directed opposite to the direction of the dipole moment vector, *i.e.*, from $+q$ to $-q$, as shown in Fig. 1.70(*b*).

Example 56. *The force experienced by a unit charge when placed at a distance of 0.10 m from the middle of an electric dipole on its axial line is 0.025 N and when it is placed at a distance of 0.2 m, the force is reduced to 0.002 N. Calculate the dipole length.*

Solution. $E_{axial} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2pr}{(r^2 - a^2)^2}$

In first case : $r = 0.10$ m, $E_{axial} = 0.025$ N

$$\therefore \quad 0.025 = \frac{9 \times 10^9 \times 2p \times 0.10}{[(0.10)^2 - a^2]^2} \quad ...(i)$$

In second case : $r = 0.2$ m, $E_{axial} = 0.002$ N

$$\therefore \quad 0.002 = \frac{9 \times 10^9 \times 2p \times 0.2}{[(0.2)^2 - a^2]^2} \quad ...(ii)$$

Dividing (*i*) by (*ii*), we get

$$\frac{0.025}{0.002} = \frac{0.10}{0.2} \cdot \frac{[(0.2)^2 - a^2]^2}{[(0.1)^2 - a^2]^2}$$

or $$\frac{25}{2} = \frac{1}{2} \cdot \frac{[(0.2)^2 - a^2]^2}{[(0.1)^2 - a^2]^2}$$

or $$5 = \frac{0.04 - a^2}{0.01 - a^2}$$

$$\therefore \quad a = 0.05\text{ m}$$

Dipole length $= 2a = \mathbf{0.10\ m}$.

Problems For Practice

1. An electric dipole is formed by + 4 μC and – 4 μC charges at 5 mm distance. Calculate the dipole moment and give its direction. **[Haryana 01]**

   (Ans. 2×10^{-8} Cm, from –ve to +ve charge)

2. An electric dipole of dipole moment 4×10^{-5} C m is placed in a uniform electric field of 10^{-3} NC^{-1} making an angle of 30° with the direction of the field. Determine the torque exerted by the electric field on the dipole. **[Haryana 02]**

   (Ans. 2×10^{-8} Nm)

3. A dipole consisting of an electron and a proton separated by a distance of 4×10^{-10} m is situated in an electric field of intensity 3×10^5 N C^{-1} at an angle of 30° with the field. Calculate the dipole moment and the torque acting on it. Charge on an electron = 1.602×10^{-19} C. **[Kerala 94]**
(Ans. 6.41×10^{-29} C m, 9.615×10^{-24} Nm)

4. An electric dipole is placed at an angle of 60° with an electric field of magnitude 4×10^5 NC^{-1}. It experiences a torque of $8\sqrt{3}$ Nm. If the length of the dipole is 4 cm, determine the magnitude of either charge of the dipole. (Ans. 10^{-3}C)

5. An electric dipole consists of two opposite charges of magnitude 2×10^{-6} C each and separated by a distance of 3 cm. It is placed in an electric field of 2×10^5 NC^{-1}. Determine the maximum torque on the dipole. (Ans. 1.2×10^{-2} N m)

6. Two point charges of + 0.2 μ μC and − 0.2 μ μC are separated by 10^{-8}m. Determine the electric field at an axial point at a distance of 0.1 m from their midpoint. Use the standard value of ε_0. **[Punjab 97]**
(Ans. 3.6×10^{-9} NC^{-1})

7. Calculate the field due to an electric dipole of length 10 cm and consisting of charges of ± 100 μC at a point 20 cm from each charge.
(**Ans.** 1125×10^7 N C^{-1})

HINTS

1. $p = q \times 2a = 4 \times 10^{-6} \times 5 \times 10^{-3} = \mathbf{2 \times 10^{-8}}$ **Cm.**

2. $\tau = pE \sin\theta = 4 \times 10^{-5} \times 10^{-3} \times \sin 30°$
$= \mathbf{2 \times 10^{-8}}$ **Nm.**

3. Here $q = e = 1.602 \times 10^{-19}$ C, $2a = 4 \times 10^{-10}$ m,
$E = 3 \times 10^5$ N C^{-1}, $\theta = 30°$
$p = q \times 2a = 1.602 \times 10^{-19} \times 4 \times 10^{-10}$
$\simeq \mathbf{6.41 \times 10^{-29}}$ **Cm.**
$\tau = pE\sin\theta = 6.41 \times 10^{-29} \times 3 \times 10^5 \times \sin 30°$
$= \mathbf{9.615 \times 10^{-24}}$ **Nm.**

4. $\tau = pE\sin\theta = q \times 2a \times E\sin\theta$
$$\therefore \quad q = \frac{\tau}{(2a)\,E\sin\theta} = \frac{8\sqrt{3}}{0.04 \times 4 \times 10^5 \times \sin 60°}$$
$= \mathbf{10^{-3}}$ **C.**

5. Here $q = 2 \times 10^{-6}$ C, $2a = 3\text{ cm} = 3 \times 10^{-2}$ m,
$E = 2 \times 10^5$ NC^{-1}
$\tau_{max} = p\,E\sin 90° = q \times 2a \times E \times 1$
$= 2 \times 10^{-6} \times 3 \times 10^{-2} \times 2 \times 10^5$
$= \mathbf{1.2 \times 10^{-2}}$ **Nm.**

6. Here $r >> a$
$$\therefore \quad E_{axial} = \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2(q \times 2a)}{r^3}$$
$$= \frac{9 \times 10^9 \times 2 \times 0.2 \times 10^{-12} \times 10^{-8}}{(0.1)^2}$$
$= \mathbf{3.6 \times 10^{-9}}$ **NC^{-1}.**

7. Here $q = 100\,\mu\text{C} = 10^{-4}$C, $2a = 10\text{ cm} = 0.10$ m
$p = q \times 2a = 10^{-4} \times 0.10 = 10^{-5}$ Cm

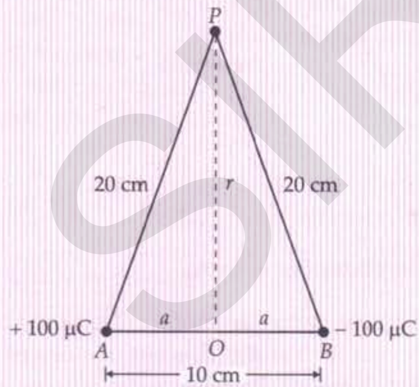


Fig. 1.71

Clearly,
$$(r^2 + a^2)^{1/2} = 20\text{ cm} = 0.20\text{ m}$$
$$E_{equa} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{p}{(r^2 + a^2)^{3/2}}$$
$$= \frac{9 \times 10^9 \times 10^{-5}}{(0.2)^3} = \frac{9}{8} \times 10^7$$
$= \mathbf{1.125 \times 10^7}$ **NC^{-1}.**

1.28 ELECTRIC FIELD LINES

43. *What are electric lines of force ? Give their important properties.*

Electric lines of force. *Michael Faraday* (1791-1867) introduced the concept of lines of force to visualize the nature of electric (and magnetic) fields. A small positive charge placed in an electric field experiences a force in a definite direction and if it is free to move, it will start moving in that direction. The path along which this charge would move will be a line of force.

An ***electric line of force*** *may be defined as the curve along which a small positive charge would tend to move when free to do so in an electric field and the tangent to which at any point gives the direction of the electric field at that point.*

In Fig. 1.72, the curve *PQR* is an electric line of force. The tangent drawn to this curve at the point *P* gives the direction of the field $\vec{E}_P$ at the point *P*. Similarly, the tangent at the point *Q* gives the direction of the field $\vec{E}_Q$ at the point *Q*, and so on.

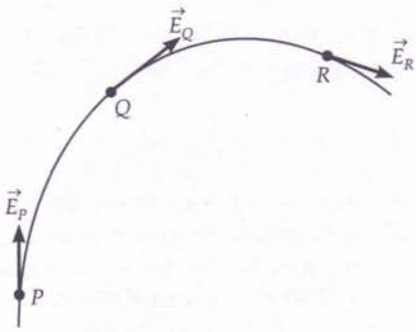


Fig. 1.72 An electric line of force.

The lines of force do not really exist, they are imaginary curves. Yet the concept of lines of force is very useful. Michael Faraday gave simple explanations for many of his discoveries (in electricity and magnetism) in terms of such lines of force.

For Your Knowledge

- The lines of force are imaginary curves, but the field which they represent is real.
- The term '***lines of force***' is misleading. It will be more appropriate to call them electric (or magnetic) '***field lines***'.
- A field line is a space curve *i.e.*, a curve in three dimensions.

Properties of Electric Lines of Force

1. The lines of force are continuous smooth curves without any breaks.
2. The lines of force start at positive charges and end at negative charges – they cannot form closed loops. If there is a single charge, then the lines of force will start or end at infinity.
3. The tangent to a line of force at any point gives the direction of the electric field at that point.
4. No two lines of force can cross each other.

   Reason. If they intersect, then there will be two tangents at the point of intersection (Fig. 1.73) and hence two directions of the electric field at the same point, which is not possible.

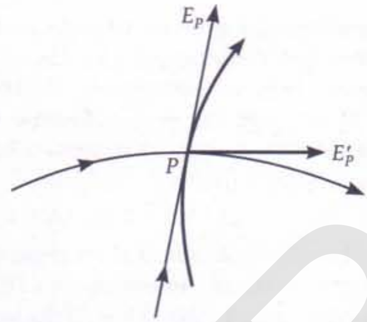


Fig. 1.73

5. The lines of force are always normal to the surface of a conductor on which the charges are in equilibrium.

   Reason. If the lines of force are not normal to the conductor, the component of the field $\vec{E}$ parallel to the surface would cause the electrons to move and would set up a current on the surface. But no current flows in the equilibrium condition.

6. The lines of force have a tendency to contract lengthwise. This explains attraction between two unlike charges.
7. The lines of force have a tendency to expand laterally so as to exert a lateral pressure on neighbouring lines of force. This explains repulsion between two similar charges.
8. The relative closeness of the lines of force gives a measure of the strength of the electric field in any region. The lines of force are
   (*i*) close together in a strong field.
   (*ii*) far apart in a weak field.
   (*iii*) parallel and equally spaced in a uniform field.
9. The lines of force do not pass through a conductor because the electric field inside a charged conductor is zero.

1.29 ELECTRIC FIELD LINES FOR DIFFERENT CHARGED CONDUCTORS

44. *Sketch and explain the field lines of (i) a positive point charge, (ii) a negative point charge, (iii) two equal and opposite charges, (iv) two equal positive charges and (v) a positively charged plane conductor.*

Electric field lines for different charge systems :

(*i*) ***Field lines of a positive point charge.*** Fig. 1.74 shows the lines of force of an isolated positive point charge. They are directed radially outwards because a small positive charge would be accelerated in the outward direction. They extend to infinity. The field is ***spherically symmetric*** *i.e.*, it looks same in all directions, as seen from the point charge.

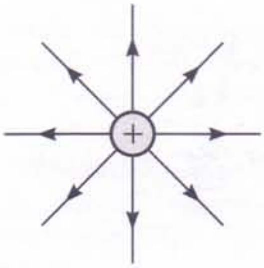


Fig. 1.74 Field lines of a positive point charge.

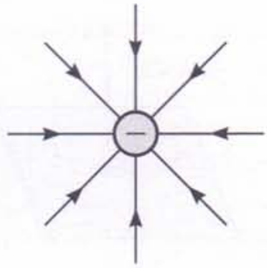


Fig. 1.75 Field lines of a negative point charge.

(*ii*) ***Field lines of a negative point charge.*** Like that of a positive point charge, the electric field of a negative point charge is also spherically symmetric but the lines of force point radially inwards as shown in Fig. 1.75. They start from infinity.

(*iii*) ***Field lines of two equal and opposite point charges.*** Fig. 1.76 shows the electric lines of force of an electric dipole *i.e.*, a system of two equal and opposite point charges ($\pm q$) separated by a small distance. They start from the positive charge and end on the negative charge. The lines of force seem to contract lengthwise as if the two charges are being pulled together. This explains attraction between two unlike charges. The field is ***cylindrically symmetric*** about the dipole axis *i.e.*, the field pattern is same in all planes passing through the dipole axis. Clearly, the electric field at all points on the equatorial line is parallel to the axis of the dipole.

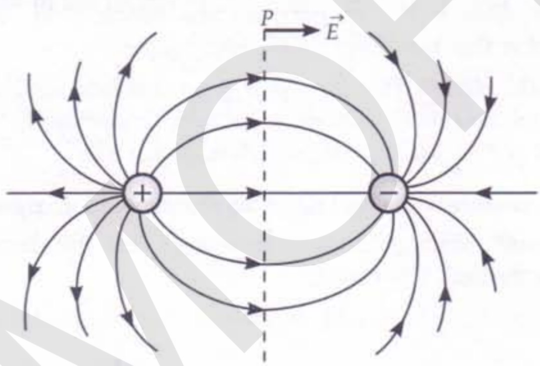


Fig. 1.76 Field lines of an electric dipole.

(*iv*) ***Field lines of two equal and positive point charges.*** Fig. 1.77 shows the lines of force of two equal and positive point charges. They seem to exert a lateral pressure as if the two charges are being pushed away from each other. This explains repulsion between two like charges. The field $\vec{E}$ is zero at the middle point N of the join of two charges. This point is called neutral point from which no line of force passes. This field also has cylindrical symmetry.

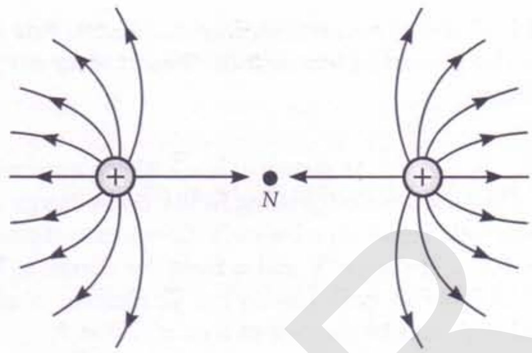


Fig. 1.77 Field lines of two equal positive charges.

(*v*) ***Field lines of a positively charged plane conductor.*** Fig. 1.78 shows the pattern of lines of force of positively charged plane conductor. A small positive charge would tend to move normally away from the plane conductor. Thus the lines of force are parallel and normal to the surface of the conductor. They are equispaced, indicating that electric field $\vec{E}$ is uniform at all points near the plane conductor.

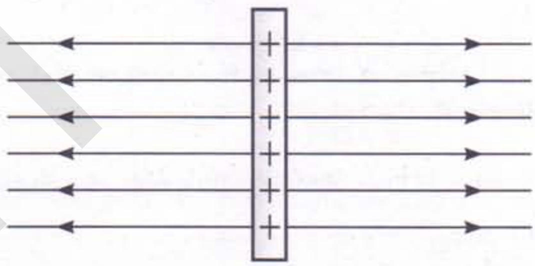


Fig. 1.78 Field pattern of a positively charged plane conductor.

45. *What is the relation between the density of lines of force and the electric field strength ? Illustrate it in a diagram.*

Relation between electric field strength and density of lines of force. Electric field strength is proportional to the density of lines of force *i.e.*, electric field strength at a point is proportional to the number of lines of force cutting a unit area element placed normal to the field at that point. As illustrated in Fig. 1.79, the electric field at P is stronger than at Q.

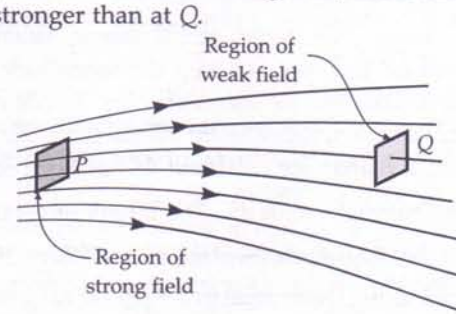


Fig. 1.79 Density of lines of force is proportional to the electric field strength.

46. *Show that the $1/r^2$ dependence of electric field of a point charge is consistent with the concept of the electric field lines.*

Consistency of the inverse square law with the electric field lines. As shown in Fig. 1.80, the number of radial lines of force originating from a point charge q in a given solid angle $\Delta\Omega$ is constant. Consider two points P_1 and P_2 at distances r_1 and r_2 from the charge q. The same number of lines (say n) cut an element of area $r_1^2\,\Delta\Omega$ at P_1 and an element of area $r_2^2\,\Delta\Omega$ at P_2.

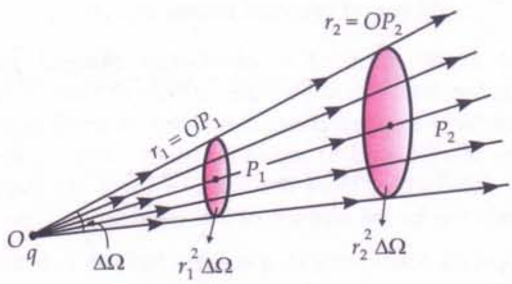


Fig. 1.80

$\therefore$ Number of lines of force cutting unit area element at $P_1 = \dfrac{n}{r_1^2\,\Delta\Omega}$

Number of lines of force cutting unit area element at $P_2 = \dfrac{n}{r_2^2\,\Delta\Omega}$

As electric field strength $\propto$ Density of lines of force

$$\therefore \qquad \frac{E_1}{E_2} = \frac{n}{r_1^2\,\Delta\Omega} \cdot \frac{r_2^2\,\Delta\Omega}{n} = \frac{r_2^2}{r_1^2}$$

i.e., $$E \propto \frac{1}{r^2}.$$

1.30 AREA VECTOR

47. *What is an area vector ? How do we specify the direction of a planar area vector ? How do we associate a vector to the area of a curved surface ?*

Area vector. We come across many situations where we need to know not only the magnitude of a surface area but also its direction. *The direction of a planar area vector is specified by the normal to the plane.* In Fig. 1.81(*a*), a planar area element dS has been represented by a normal vector $\vec{dS}$. The length of vector $\vec{dS}$ represents the magnitude dS of the area element. If $\hat{n}$ is a unit vector along the normal to the planar area, then

$$\vec{dS} = dS\,\hat{n}$$

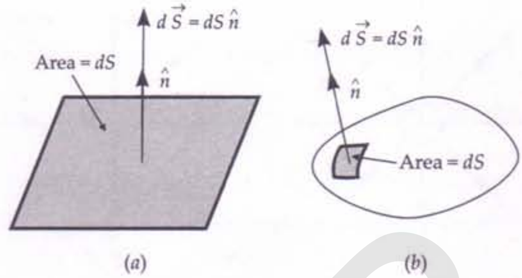


Fig. 1.81 (*a*) A planar area element. (*b*) An area element of a curved surface.

In case of a curved surface, we can imagine it to be divided into a large number of very small area elements. Each small area element of the curved surface can be treated as a planar area. By convention, the direction of the vector associated with every area element of a closed surface is along the *outward drawn normal*. As shown in Fig. 1.81(*b*), the area element $\vec{dS}$ at any point on the closed surface is equal to $dS\,\hat{n}$, where dS is the magnitude of the area element and $\hat{n}$ is a unit vector in the direction of outward normal.

1.31 ELECTRIC FLUX

48. *Define the term electric flux. How is it related to electric field intensity ? What is its SI unit ?*

Electric flux. The term flux implies some kind of flow. Flux is the property of any vector field. The electric flux is a property of electric field.

*The **electric flux** through a given area held inside an electric field is the measure of the total number of electric lines of force passing normally through that area.*

As shown in Fig. 1.82, if an electric field $\vec{E}$ passes normally through an area element ΔS, then the electric flux through this area is

$$\Delta\phi_E = E\,\Delta S$$

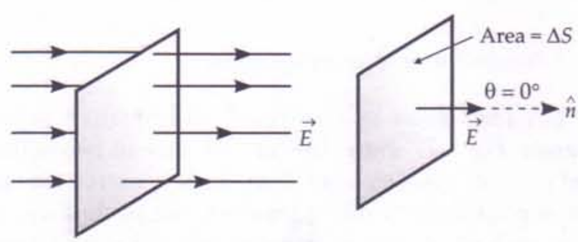


Fig. 1.82 Electric flux through normal area.

As shown in Fig. 1.83, if the normal drawn to the area element ΔS makes an angle θ with the uniform field $\vec{E}$, then the component of $\vec{E}$ normal to ΔS will be $E \cos\theta$, so that the electric flux is

$$\Delta\phi_E = \text{Normal component of } E \times \text{Surface area}$$
$$= E\cos\theta \times \Delta S$$

or $$\Delta\phi_E = E\,\Delta S\cos\theta = \vec{E}\,.\,\Delta\vec{S}$$

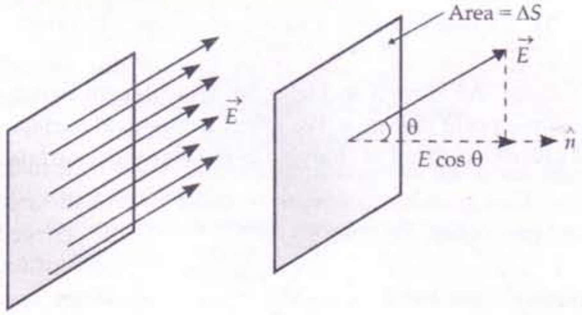


Fig. 1.83 Flux through an inclined area.

In case the field $\vec{E}$ is non-uniform, we consider a closed surface S lying inside the field, as shown in Fig. 1.84. We can divide the surface S into small area elements : $\Delta\vec{S}_1, \Delta\vec{S}_2, \Delta\vec{S}_3, ..., \Delta\vec{S}_N$. Let the corresponding electric fields at these elements be $\vec{E}_1, \vec{E}_2, ..., \vec{E}_N$.

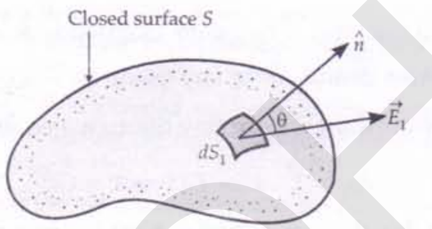


Fig. 1.84 Electric flux through a closed surface S.

Then the electric flux through the surface S will be

$$\phi_E = \vec{E}_1 \,.\, \Delta\vec{S}_1 + \vec{E}_2 \,.\, \Delta\vec{S}_2 + ... + \vec{E}_N \,.\, \Delta\vec{S}_N$$
$$= \sum_{i=1}^{N} \vec{E}_i \,.\, \Delta\vec{S}_i$$

When the number of area elements becomes infinitely large ($N \to \infty$) and $\Delta S \to 0$, the above sum approaches a surface integral taken over the closed surface. Thus

$$\phi_E = \lim_{\substack{N\to\infty \\ \Delta S \to 0}} \sum_{i=1}^{N} \vec{E}_i \,.\, \Delta\vec{S}_i = \oint_S \vec{E}\,.\,d\vec{S}$$

Thus the electric flux through any surface $\vec{S}$, open or closed, is equal to the surface integral of the electric field $\vec{E}$ taken over the surface $\vec{S}$.

Electric flux is a *scalar* quantity.

Unit of ϕ_E = Unit of $E \times$ unit of S

$\therefore$ SI unit of electric flux

$$= NC^{-1}.m^2 = \mathbf{Nm^2C^{-1}}.$$

Equivalently, SI unit of electric flux

$$= Vm^{-1}.m^2 = \mathbf{Vm}.$$

1.32 GAUSS'S THEOREM

49. *State and prove Gauss's theorem.*

Gauss's theorem. This theorem gives a relationship between the total flux passing through any *closed* surface and the net charge enclosed within the surface.

Gauss theorem *states that the total flux through a closed surface is $1/\varepsilon_0$ times the net charge enclosed by the closed surface.*

Mathematically, it can be expressed as

$$\phi_E = \oint_S \vec{E}\,.\,d\vec{S} = \frac{q}{\varepsilon_0}$$

Proof. For the sake of simplicity, we prove Gauss's theorem for an isolated positive point charge q. As shown in Fig. 1.85, suppose the surface S is a sphere of radius r centred on q. Then surface S is a *Gaussian surface*.

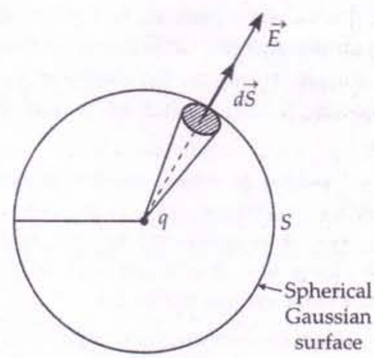


Fig. 1.85 Flux through a sphere enclosing a point charge.

Electric field at any point on S is

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$$

This field points radially outward at all points on S. Also, any area element points radially outwards, so it is parallel to $\vec{E}$, *i.e.*, $\theta = 0°$.

$\therefore$ Flux through area $d\vec{S}$ is

$$d\phi_E = \vec{E}\,.\,d\vec{S} = E\,dS\cos 0° = EdS$$

Total flux through surface S is

$$\phi_E = \oint_S d\phi_E = \oint_S E\, dS = E \oint_S dS$$

$$= E \times \text{Total area of sphere}$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2} \cdot 4\pi r^2$$

or $$\phi_E = \frac{q}{\varepsilon_0}$$

This proves Gauss's theorem.

For Your Knowledge

- Gauss's theorem is valid for a closed surface of any shape and for any general charge distribution.
- If the net charge enclosed by a closed surface is zero ($q = 0$), then flux through it is also zero.

$$\phi_E = \frac{q}{\varepsilon_0} = 0$$

- The net flux through a closed surface due to a charge lying outside the closed surface is zero.
- The charge q appearing in the Gauss's theorem includes the sum of all the charges located anywhere inside the closed surface.
- The electric field $\vec{E}$ appearing in Gauss's theorem is due to all the charges, both inside and outside the closed surface. However, the charge q appearing in the theorem is only contained within the closed surface.
- Gauss's theorem is based on the inverse square dependence on distance contained in the coulomb's law. In fact, it is applicable to any field obeying inverse square law. It will not hold in case of any departure from inverse square law.
- For a medium of absolute permittivity ε or dielectric constant κ, the Gauss's theorem can be expressed as

$$\oint \vec{E} \cdot \vec{dS} = \frac{q}{\varepsilon} = \frac{q}{\kappa\varepsilon_0}$$

1.33 GAUSSIAN SURFACE

50. *What is a Gaussian surface ? Give its importance.*

Gaussian surface. *Any hypothetical closed surface enclosing a charge is called the Gaussian surface of that charge.* It is chosen to evaluate the surface integral of the electric field produced by the charge enclosed by it, which, in turn, gives the total flux through the surface.

Importance. By a clever choice of Gaussian surface, we can easily find the electric fields produced by certain symmetric charge configurations which are otherwise quite difficult to evaluate by the direct application of Coulomb's law and the principle of superposition.

1.34 COULOMB'S LAW FROM GAUSS'S THEOREM

51. *Deduce Coulomb's law from Gauss's theorem.*

Deduction of Coulomb's law from Gauss's theorem. As shown in Fig. 1.86, consider an isolated positive point charge q. We select a spherical surface S of radius r centred at charge q as the Gaussian surface.

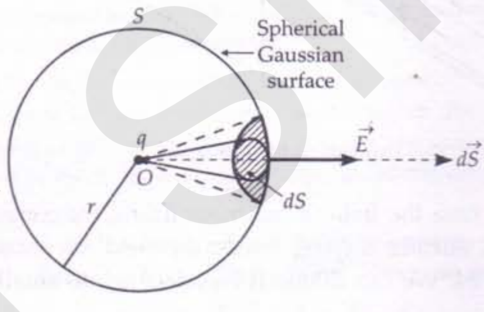


Fig. 1.86 Applying Gauss's theorem to a point charge.

By symmetry, $\vec{E}$ has same magnitude at all points on S. Also $\vec{E}$ and $\vec{dS}$ at any point on S are directed radially outward. Hence flux through area $\vec{dS}$ is

$$d\phi_E = \vec{E} \cdot \vec{dS} = EdS\cos 0° = EdS$$

Net flux through closed surface S is

$$\phi_E = \oint_S \vec{E} \cdot \vec{dS} = \oint_S E\, dS = E \oint_S dS$$

$$= E \times \text{total surface area of } S = E \times 4\pi r^2$$

Using Gauss's theorem,

$$\phi_E = \frac{q}{\varepsilon_0}$$

or $$E \times 4\pi r^2 = \frac{q}{\varepsilon_0}$$

or $$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$$

The force on the point charge q_0 if placed on surface S will be

$$F = q_0\, E = \frac{1}{4\pi\,\varepsilon_0} \frac{qq_0}{r^2}$$

This proves the Coulomb's law.

Examples based on
Electric Flux and Gauss's Theorem

Formulae Used

1. Electric flux through a plane surface area S held in a uniform electric field $\vec{E}$ is
$$\phi_E = \vec{E} \cdot \vec{S} = ES\cos\theta$$
where θ is the angle which the normal to the outward drawn normal to surface area $\vec{S}$ makes with the field $\vec{E}$.
2. According to Gauss's theorem, the total electric flux through a closed surface S enclosing charge q is
$$\phi_E = \oint_S \vec{E} \cdot \vec{dS} = \frac{q}{\varepsilon_0}$$
3. Flux density $= \frac{\text{Total flux}}{\text{Area}} = \frac{\phi_E}{S}$

Units Used

Electric flux ϕ_E is in $Nm^2 C^{-1}$ and flux density in NC^{-1}.

Constant Used

Permittivity constant of free space is
$$\varepsilon_0 = \frac{1}{4\pi \times 9 \times 10^{-9}} = 8.85 \times 10^{-12}\ C^2 N^{-1} m^{-2}$$

Example 57. *If $\vec{E} = 6\hat{i} + 3\hat{j} + 4\hat{k}$, calculate the electric flux through a surface of area 20 units in Y-Z plane.* [Haryana 97]

Solution. Electric field vector, $\vec{E} = 6\hat{i} + 3\hat{j} + 4\hat{k}$

As the area vector $\vec{S}$ in the Y-Z plane points along outward drawn normal *i.e.*, along positive X-direction, so
$$\vec{S} = 20\hat{i}$$
Flux, $\phi_E = \vec{E} \cdot \vec{S} = (6\hat{i} + 3\hat{j} + 4\hat{k}) \cdot 20\hat{i}$
$= \mathbf{120\ units.}$

Example 58. *A circular plane sheet of radius 10 cm is placed in a uniform electric field of $5 \times 10^5\ NC^{-1}$, making an angle of $60°$ with the field. Calculate electric flux through the sheet.*

Solution. Here $r = 10$ cm $= 0.1$ m, $E = 5 \times 10^5\ NC^{-1}$

As the angle between the plane sheet and the electric field is $60°$, angle made by the normal to the plane sheet and the electric field is $\theta = 90° - 60° = 30°$

Flux, $\phi_E = ES\cos\theta = E \times \pi r^2 \times \cos\theta$
$= 5 \times 10^5 \times 3.14 \times (0.1)^2 \times \cos 30°$
$= \mathbf{1.36 \times 10^4\ Nm^2\ C^{-1}}.$

Example 59. *A cylinder is placed in a uniform electric field $\vec{E}$ with its axis parallel to the field. Show that the total electric flux through the cylinder is zero.*

Solution. The situation is shown in Fig. 1.87.

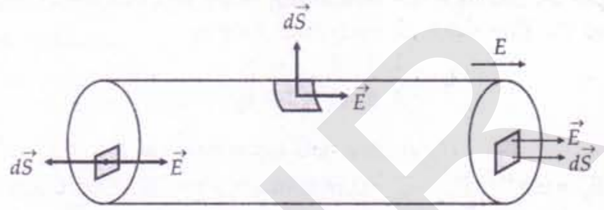


Fig. 1.87

Flux through the entire cylinder,
$$\phi_E = \underbrace{\int \vec{E} \cdot \vec{dS}}_{\text{left plane face}} + \underbrace{\int \vec{E} \cdot \vec{dS}}_{\text{right plane face}} + \underbrace{\int \vec{E} \cdot \vec{dS}}_{\text{curved surface}}$$
$= \int E\,dS\cos 180° + \int E\,dS\cos 0° + \int E\,dS\cos 90°$
$= -E\int dS + E\int dS + 0$
$= -E \times \pi r^2 + E \times \pi r^2 = \mathbf{0}.$

Example 60. *Calculate the number of electric lines of force originating from a charge of 1 C.*

Solution. The number of lines of force originating from a charge of 1 C

= Electric flux through a closed surface enclosing a charge of 1 C
$$= \frac{q}{\varepsilon_0} = \frac{1}{8.85 \times 10^{-12}} = \mathbf{1.129 \times 10^{11}}.$$

Example 61. *A positive charge of 17.7 μC is placed at the centre of a hollow sphere of radius 0.5 m. Calculate the flux density through the surface of the sphere.*

Solution. From Gauss's theorem,

Flux, $\phi_E = \frac{q}{\varepsilon_0} = \frac{17.7 \times 10^{-6}}{8.85 \times 10^{-12}} = 2 \times 10^6\ Nm^2\ C^{-1}$

Flux density $= \frac{\text{Total flux}}{\text{Area}}$
$$= \frac{2 \times 10^6}{4\pi(0.5)^2} = \mathbf{6.4 \times 10^5\ NC^{-1}}.$$

Example 62. *Calculate the electric flux through each of the six faces of a closed cube of length l, if a charge q is placed (a) at its centre and (b) at one of its vertices.*

Solution. (*a*) By symmetry, the flux through each of the six faces of the cube will be same when charge q is placed at its centre.

$$\therefore \quad \phi_E = \frac{1}{6} \cdot \frac{q}{\varepsilon_0}$$

(*b*) When charge q is placed at one vertex, the flux through each of the three faces meeting at this vertex will be zero, as $\vec{E}$ is parallel to these faces. As only one-eighth of the flux emerging from the charge q passes through the remaining three faces of the cube, so the flux through each such face is

$$\phi_E = \frac{1}{3} \cdot \frac{1}{8} \cdot \frac{q}{\varepsilon_0} = \frac{1}{24} \cdot \frac{q}{\varepsilon_0}$$

Example 63. *The electric field components in Fig. 1.88 are $E_x = \alpha x^{1/2}$, $E_y = E_z = 0$, in which $\alpha = 800\ N / Cm^2$. Calculate (i) the flux ϕ_E through the cube and (ii) the charge within the cube. Assume that $a = 0.1\ m$.* [NCERT]

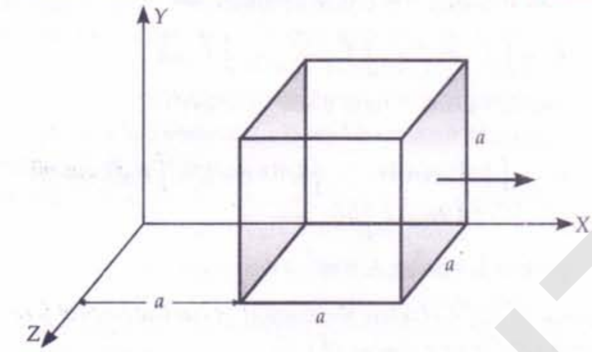


Fig. 1.88

Solution. (*i*) The electric field is acting only in *X*-direction and its *Y*-and *Z*-components are zero. For the four non-shaded faces, the angle between $\vec{E}$ and $\vec{\Delta S}$ is $+\pi/2$. So flux $\phi = \vec{E} \cdot \vec{\Delta S}$ is zero through each of these faces.

The magnitude of the electric field at the left face is

$$E_L = \alpha x^{1/2} = \alpha\, a^{1/2} \quad [x = a \text{ at the left face}]$$

Flux, $\phi_L = \vec{E}_L \cdot \vec{\Delta S} = E_L\, \Delta S \cos\theta$

$= E_L\, a^2 \cos 180° = -E_L\, a^2$

[$\theta = 180°$ for the left face]

The magnitude of the electric field at the right face is

$$E_R = \alpha\, x^{1/2} = \alpha\,(2a)^{1/2}$$

[$x = 2a$ at the right face]

Flux, $\phi_R = E_R\, \Delta S \cos 0° = E_R\, a^2$

[$\theta = 0°$ for the right face]

Net flux through the cube

$$\phi_E = \phi_L + \phi_R = E_R a^2 - E_L a^2$$
$$= a^2\,(E_R - E_L) = \alpha a^2\,[(2a)^{1/2} - a^{1/2}]$$
$$= \alpha a^{5/2}[\sqrt{2} - 1] = 800\,(0.1)^{5/2}\,(\sqrt{2} - 1)$$
$$= \mathbf{1.05\ Nm^2\ C^{-1}}.$$

(*ii*) By Gauss's theorem, the total charge inside the cube is

$$q = \varepsilon_0\, \phi_E = \frac{1}{4\pi \times 9 \times 10^9} \times 1.05 = \mathbf{9.27 \times 10^{-12}\ C}.$$

Example 64. *An electric field is uniform, and in the positive x direction for positive x and uniform with the same magnitude in the negative x direction for negative x. It is given that*

$$\vec{E} = 200\,\hat{i}\ NC^{-1} \quad \text{for } x > 0$$

and $\vec{E} = -200\,\hat{i}\ NC^{-1}$ *for* $x < 0$.

A right circular cylinder of length 20 cm and radius 5 cm has its centre at the origin and its axis along the x-axis so that one face is at $x = +10$ cm and the other is at $x = -10$ cm.

(*i*) *What is the net outward flux through each flat face ?*
(*ii*) *What is the flux through the side of the cylinder ?*
(*iii*) *What is the net outward flux through the cylinder ?*
(*iv*) *What is the net charge inside the cylinder ?* [NCERT]

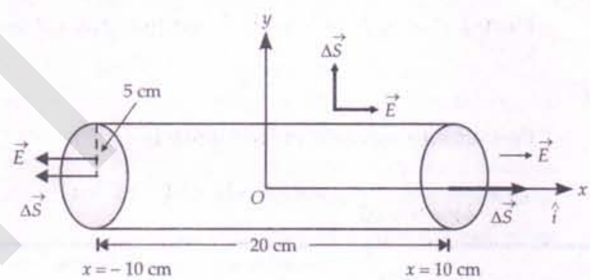


Fig. 1.89

Solution. (*i*) *On the left face* : $\vec{E} = -200\,\hat{i}\ \ NC^{-1}$,

$$\vec{\Delta S} = -\Delta S\,\hat{i} = -\pi\,(0.05)^2\,\hat{i}\ \ m^2$$

The outward flux through the left face is

$$\phi_E = \vec{E} \cdot \vec{\Delta S}$$
$$= +200 \times \pi\,(0.05)^2\,\hat{i} \cdot \hat{i}\ \ Nm^2\,C^{-1}.$$
$$= \mathbf{+1.57\ Nm^2\ C^{-1}}. \qquad [\hat{i} \cdot \hat{i} = 1]$$

On the right face :

$$\vec{E} = 200\,\hat{i}\ \ NC^{-1}$$
$$\vec{\Delta S} = \Delta S\,\hat{i} = \pi\,(0.05)^2\,\hat{i}\ \ m^2$$

The outward flux through the right face is

$$\phi_E = \vec{E} \cdot \vec{\Delta S} = \mathbf{+1.57\ Nm^2\ C^{-1}}.$$

(*ii*) For any point on the side of the cylinder $\vec{E} \perp \vec{\Delta S}$,

$\therefore$ Flux through the side of the cylinder,

$$\phi_E = \vec{E} \cdot \vec{\Delta S} = E\,\Delta S \cos 90° = 0.$$

(*iii*) Net outward flux through the cylinder,

$$\phi_E = 1.57 + 1.57 + 0 = \mathbf{3.14\ Nm^2\ C^{-1}}.$$

(*iv*) By Gauss's theorem, the net charge inside the cylinder is

$$q = \varepsilon_0\ \phi_E = 8.854 \times 10^{-12} \times 3.14 = \mathbf{2.78 \times 10^{-11}\ C.}$$

Example 65. *You are given a charge + Q at the origin O (Refer to Fig. 1.90). Consider a sphere S with centre (2, 0, 0) of radius $\sqrt{2}$ m. Consider another sphere of radius $\sqrt{2}$ m centered at the origin. Consider the spherical caps (i) PSQ (ii) PRQ (iii) PWQ, with normals outward to the respective spheres, and (iv) the flat circle PTQ with normal along the x-axis.*

(*a*) *What is the sign of electric flux through each of the surfaces (i)-(iv) ?*

(*b*) *What is the relation between the magnitudes of fluxes through surfaces (i)-(iv) ?*

(*c*) *Calculate the flux through the surface (ii) directly. Assume that the area of the cap (ii) is A.* **[NCERT]**

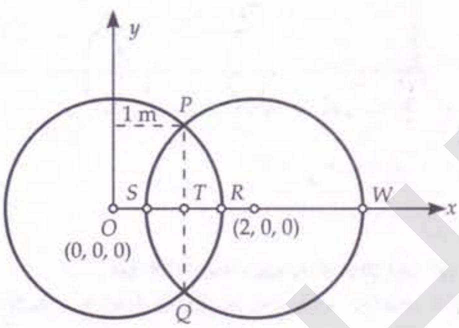


Fig. 1.90

Solution. For the charge + Q situated at origin O, the field $\vec{E}$ points along +ve x-direction *i.e.*, towards right.

(*a*) The outward drawn normal on cap PSQ points towards left while it points towards right for caps PRQ, PWQ and circle PTQ. So the flux is negative for (*i*) and positive for the rest.

(*b*) The same electric field lines crossing (*i*) also cross (*ii*), (*iii*). Also, by Gauss's law, the fluxes through (*iii*) and (*iv*) add upto zero. Hence, all magnitudes of fluxes are equal.

(*c*) Given area of the cap (*ii*) = A

Electric field through cap (*ii*) is

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r^2} = 9 \times 10^9 \times \frac{Q}{(\sqrt{2})^2}$$

$$= 4.5 \times 10^9\ Q\ \text{NC}^{-1}$$

Electric flux through the cap (*ii*) is

$$\phi_E = EA$$

$$= \mathbf{4.5 \times 10^9\ QA\ \ NC^{-1}m^2.}$$

Example 66. *Figure 1.91 shows five charged lumps of plastic and an electrically neutral coin. The cross-section of a Gaussian surface S is indicated. What is the net electric flux through the surface if*

$$q_1 = q_4 = +3.1\ nC, \qquad q_2 = q_5 = -5.9\ nC$$

and $q_3 = -3.1\ nC$?

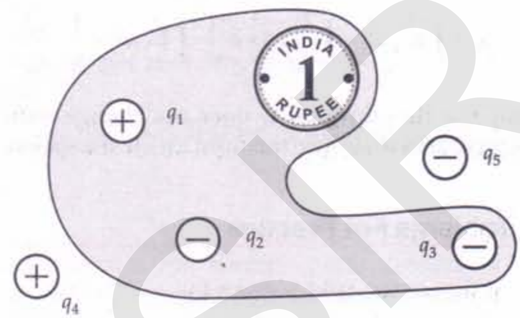


Fig. 1.91

Solution. The neutral coin and the outside charges q_4 and q_5 make no contribution towards the net charge enclosed by surface S. Applying Gauss's theorem, we get

$$\phi_E = \frac{q}{\varepsilon_0} = \frac{q_1 + q_2 + q_3}{\varepsilon_0}$$

$$= \frac{+3.1 \times 10^{-9} - 5.9 \times 10^{-9} - 3.1 \times 10^{-9}}{8.85 \times 10^{-12}}$$

$$= \mathbf{-666.67\ Nm^2\ C^{-1}.}$$

Example 67. *S_1 and S_2 are two concentric spheres enclosing charges Q and 2Q respectively as shown in Fig. 1.92.*

(*i*) *What is the ratio of the electric flux through S_1 and S_2 ?*

(*ii*) *How will the electric flux through the sphere S_1 change, if a medium of dielectric constant κ is introduced in the space inside S_1 in place of air ?*

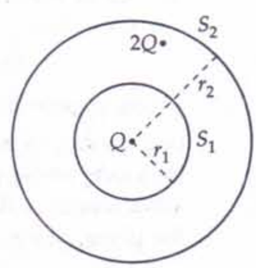


Fig. 1.92

(*iii*) *How will the electric flux through sphere S_1 change, if a medium of dielectric constant κ is introduced in the space inside S_2 in place of air ?*

[CBSE OD 02, 14, 14C]

Solution. (*i*) By Gauss's Theorem,

Flux through S_1 is $\quad \phi_1 = \dfrac{Q}{\varepsilon_0}$

Flux through S_2 is $\quad \phi_2 = \dfrac{2Q + Q}{\varepsilon_0} = \dfrac{3Q}{\varepsilon_0}$

Ratio of electric flux through S_1 and S_2 is

$$\frac{\phi_1}{\phi_2} = \frac{Q/\varepsilon_0}{3\,Q/\varepsilon_0} = \frac{1}{3} = \mathbf{1:3}$$

(*ii*) If a medium of dielectric constant κ is introduced in the space inside S_1, then flux through S_1 becomes

$$\phi_1' = \oint \vec{E}'.\,d\vec{S} = \oint \frac{\vec{E}}{\kappa}.\,d\vec{S} = \frac{1}{\kappa}\oint \vec{E}.\,d\vec{S} = \frac{1}{\kappa}.\frac{Q}{\varepsilon_0}$$

(*iii*) The flux through S_1 does not change with the introduction of dielectric medium inside the sphere S_2.

Problems For Practice

1. If the electric field is given by $\vec{E} = 8\hat{i} + 4\hat{j} + 3\hat{k}$ NC^{-1}, calculate the electric flux through a surface of area 100 m^2 lying in the *X-Y* plane. (Ans. 300 Nm^2 C^{-1})

2. The electric field in a certain region of space is $(5\hat{i} + 4\hat{j} - 4\hat{k}) \times 10^5$ NC^{-1}. Calculate electric flux due to this field over an area of $(2\hat{i} - \hat{j}) \times 10^{-2}$ m^2. (Ans. 6×10^3 Nm^2 C^{-1})

3. Consider a uniform electric field $\vec{E} = 3 \times 10^3\,\hat{i}$ NC^{-1}. Calculate the flux of this field through a square surface of area 10 cm^2 when
   (*i*) its plane is parallel to the *y-z* plane, and
   (*ii*) the normal to its plane makes a 60° angle with the *x*-axis. **[CBSE D 13C]**
   [Ans. (*i*) 30Nm^{2}C^{-1} (*ii*) 15Nm^{2}C^{-1}]

4. Given a uniform electric field $\vec{E} = 5 \times 10^3\,\hat{i}$ NC^{-1}, find the flux of this field through a square of 10 cm on a side whose plane is parallel to the *Y-Z* plane. What would be the flux through the same square if the plane makes a 30° angle with the *X*-axis? **[CBSE D 14]**
   [Ans. (*i*) 50 Nm^{2}C^{-1} (*ii*) 25 Nm^{2}C^{-1}]

5. A point charge of 17.7 μC is located at the centre of a cube of side 0.03 m. Find the electric flux through each face of the cube. **[Himachal 93]**
   (Ans. 3.3×10^5 Nm^2 C^{-1})

6. A spherical Gaussian surface encloses a charge of 8.85×10^{-8} C. (*i*) Calculate the electric flux passing through the surface. (*ii*) If the radius of the Gaussian surface is doubled, how would the flux change? **[CBSE D 01, F 07]**
   [Ans. (*i*) 10^4 Nm^2 C^{-1} (*ii*) No change]

7. A charge *q* is situated at the centre of an imaginary hemispherical surface, as shown in Fig. 1.93. Using Gauss's theorem and symmetry considerations, determine the electric flux due to this charge through the hemispherical surface. $\left(\text{Ans. } \frac{q}{2\varepsilon_0}\right)$

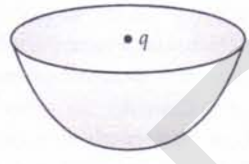


Fig. 1.93

8. A hollow cylindrical box of length 1 m and area of cross-section 25 cm^2 is placed in a three dimensional coordinate system as shown in Fig. 1.94. The electric field in the region is given by $\vec{E} = 50x\,\hat{i}$, where *E* is in NC^{-1} and *x* is in metres.

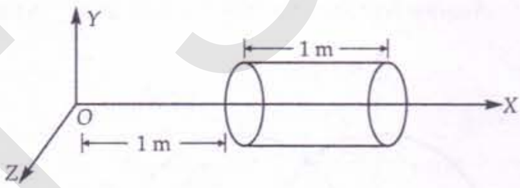


Fig. 1.94

Find
(*i*) net flux through the cylinder,
(*ii*) charge enclosed by the cylinder. **[CBSE D 13]**

9. The electric field in a region is given by $\vec{E} = \frac{E_0 x}{b}\hat{i}$. Find the charge contained in the cubical volume bounded by the surfaces $x = 0$, $x = a$, $y = 0$, $y = a$, $z = 0$ and $z = a$. Take $E_0 = 5 \times 10^3$ NC^{-1}, $a = 1$ cm and $b = 2$ cm. (Ans. 2.2×10^{-12} C)

10. The electric field components due to a charge inside the cube of side 0.1 m are as shown.

$$E_x = \alpha x, \text{ where } \alpha = 500\text{ N/C-m}$$
$$E_y = 0,\ E_z = 0.$$

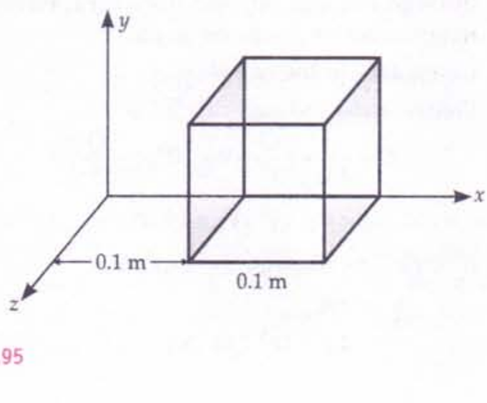


Fig. 1.95

Calculate (*i*) the flux through the cube, and (*ii*) the charge inside the cube. **[CBSE OD 08]**

[Ans. (*i*) $\phi_E = 0.656\ Nm^{-2}C^{-1}$ (*ii*) $q = 5.8 \times 10^{-12}$ C]

11. A uniform electric field $\vec{E} = E_x\ \hat{i}$ N/C for $x > 0$ and $\vec{E} = -E_x\ \hat{i}$ N/C for $x < 0$ are given. A right circular cylinder of length l cm and radius r cm has its centre at the origin and its axis along the x-axis. Find out the net outward flux. Using Gauss's law write the expression for the net charge within the cylinder. **[CBSE D 08C]**

HINTS

1. $\vec{E} = 8\hat{i} + 4\hat{j} + 3\hat{k}\ NC^{-1}, \vec{S} = 100\hat{k}\ m^2$

Flux, $\phi_E = \vec{E}.\vec{S} = (8\hat{i} + 4j + 3\hat{k}).100\hat{k}$

$= \mathbf{300\ Nm^2\ C^{-1}}$.

2. $\phi_E = \vec{E}.\vec{S} = (5\hat{i} + 4\hat{j} - 4\hat{k}) \times 10^5.(2\hat{i} - \hat{j}) \times 10^{-2}$

$= [5 \times 2 + 4 \times (-1) - 0] \times 10^3\ Nm^2\ C^{-1}$.

$= \mathbf{6 \times 10^3\ Nm^2\ C^{-1}}$.

3. (*i*) Normal to the area points in the direction of the electric field, $\theta = 0°$.

$\therefore \quad \phi_E = ES\cos\theta = 3 \times 10^3 \times (0.10)^2 \cos 0°$

$= 30\ \mathbf{Nm^2C^{-1}}$.

(*ii*) $\quad \phi_E = 3 \times 10^3 \times (0.10)^2 \times \cos 60° = \mathbf{15 Nm^2C^{-1}}$.

4. (*i*) $\phi_E = ES\cos\theta$

$= 5 \times 10^3 \times (0.10)^2 \cos 0° = \mathbf{50\ Nm^2C^{-1}}$.

(*ii*) $\phi_E = 5 \times 10^3 \times (0.10)^2 \cos(90° - 30°)$

$= 5 \times 10^3 \times (0.10)^2 \times \frac{1}{2} = \mathbf{25\ Nm^2C^{-1}}$.

5. Flux through each phase of the cube

$= \frac{1}{6}\phi_E = \frac{1}{6}\frac{q}{\varepsilon_0} = \frac{1}{6} \times \frac{17.7 \times 10^{-6}}{8.85 \times 10^{-12}}$

$= \mathbf{3.3 \times 10^5\ Nm^2\ C^{-1}}$.

6. (*i*) $\phi_E = \frac{q}{\varepsilon_0} = \frac{8.85 \times 10^{-8}\ C}{8.85 \times 10^{-12}\ C^2\ N^{-1}\ m^{-2}}$

$= \mathbf{10^4\ Nm^2\ C^{-1}}$

(*ii*) $\phi_E = 10^4 Nm^2\ C^{-1}$, because the charge enclosed is the same as in the case (*i*).

7. From Gauss's theorem, total flux through entire spherical surface is

$$\phi_E = \frac{q}{\varepsilon_0}$$

From symmetry considerations, flux through the hemispherical surface is

$$\phi_E = \frac{1}{2}.\frac{q}{\varepsilon_0}$$

8. (*i*) Flux through the curved surface of the cylinder is zero.

Magnitude of the electric field at the left face,

$E = 50 \times 1 = 50 NC^{-1}$

$\therefore$ Flux through the left face,

$\phi_L = ES\cos\theta = 50 \times 25 \times 10^{-4} \cos 180°$

$= -1250 \times 10^{-4} Nm^2C^{-1}$

Magnitude of the electric field at the right face,

$E = 50 \times 2 = 100 NC^{-1}$

$\therefore$ Flux through the right face,

$\phi_R = 100 \times 25 \times 10^{-4} \cos 0°$

$= 2500 \times 10^{-4} Nm^2C^{-1}$

Net flux through the cylinder,

$\phi_E = \phi_L + \phi_R = (2500 - 1250) \times 10^{-4} Nm^2C^{-1}$

$= 1250 \times 10^{-4} Nm^2C^{-1}$

$= \mathbf{1.250 \times 10^{-1}\ Nm^2C^{-1}}$.

(*ii*) Total charge enclosed by the cylinder,

$q = \varepsilon_0\phi_E = 8.854 \times 10^{-12} \times 1250 \times 10^{-4}C$

$= 11067.5 \times 10^{-16}C = \mathbf{1.107\ pC}$.

9. $\phi_E = \phi_L + \phi_R$

$= -E_L a^2 + E_R a^2 = -\frac{E_0 \cdot 0}{b}.a^2 + \frac{E_0 \cdot a}{b}.a^2$

$= \frac{a^3 E_0}{b} = \frac{5 \times 10^3 \times (0.01)^3}{0.02} = 0.25\ Nm^2\ C^{-1}$.

$q = \varepsilon_0\ \phi_E = 8.85 \times 10^{-12} \times 0.25 = \mathbf{2.2 \times 10^{-12}\ C}$.

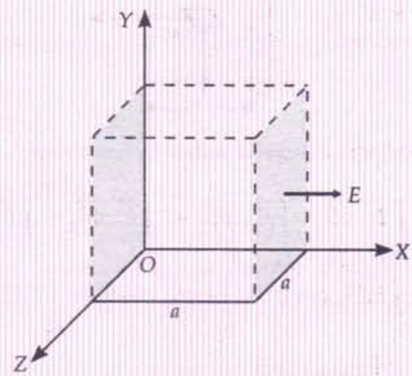


Fig. 1.96

11. Proceed as in Example 64 on page 1.52.

(*i*) $\phi_E = E_x.\pi\left(\frac{r}{100}\right)^2 + E_x.\pi\left(\frac{r}{100}\right)^2 + 0$

$= \mathbf{2\pi r^2 E_x (10)^{-4}\ Nm^2C^{-1}}$.

(*ii*) $q = \varepsilon_0 E = \mathbf{2\pi r^2 \varepsilon_0 E_x (10)^{-4}\ C}$.

1.35 FIELD DUE TO AN INFINITELY LONG CHARGED WIRE

52. *Apply Gauss's theorem to calculate the electric field of a thin infinitely long straight line of charge, with a uniform charge density of λ Cm^{-1}.*

Electric field due to an infinitely long straight charged wire. Consider a thin infinitely long straight wire having a uniform linear charge density λ Cm^{-1}. By symmetry, the field $\vec{E}$ of the line charge is directed radially outwards and its magnitude is same at all points equidistant from the line charge. To determine the field at a distance r from the line charge, we choose a cylindrical Gaussian surface of radius r, length l and with its axis along the line charge. As shown in Fig. 1.97, it has curved surface S_1 and flat circular ends S_2 and S_3. Obviously, $d\vec{S}_1 \parallel \vec{E}$, $d\vec{S}_2 \perp \vec{E}$ and $d\vec{S}_3 \perp \vec{E}$. So only the curved surface contributes towards the total flux.

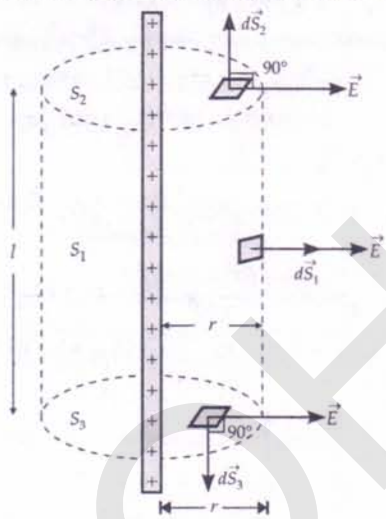


Fig. 1.97 Cylindrical Gaussian surface for line charge.

$$\phi_E = \oint_S \vec{E}\cdot d\vec{S} = \int_{S_1} \vec{E}\cdot d\vec{S}_1 + \int_{S_2} \vec{E}\cdot d\vec{S}_2 + \int_{S_3} \vec{E}\cdot d\vec{S}_3$$

$$= \int_{S_1} E dS_1 \cos 0^\circ + \int_{S_2} E dS_2 \cos 90^\circ + \int_{S_3} E dS_3 \cos 90^\circ$$

$$= E\int dS_1 + 0 + 0$$

$= E \times$ area of the curved surface

or $\quad \phi_E = E \times 2\pi r l$

Charge enclosed by the Gaussian surface, $q = \lambda l$

Using Gauss's theorem, $\phi_E = q/\varepsilon_0$, we get

or $\quad E \cdot 2\pi r l = \dfrac{\lambda l}{\varepsilon_0} \quad$ or $\quad \boxed{E = \dfrac{\lambda}{2\pi \varepsilon_0 r}}$

Thus the electric field of a line charge is inversely proportional to the distance from the line charge.

1.36 ELECTRIC FIELD DUE TO A UNIFORMLY CHARGED INFINITE PLANE SHEET

53. *Apply Gauss's theorem to calculate the electric field due to an infinite plane sheet of charge.*

Electric field due to a uniformly charged infinite plane sheet. As shown in Fig. 1.98, consider a thin, infinite plane sheet of charge with uniform surface charge density σ. We wish to calculate its electric field at a point P at distance r from it.

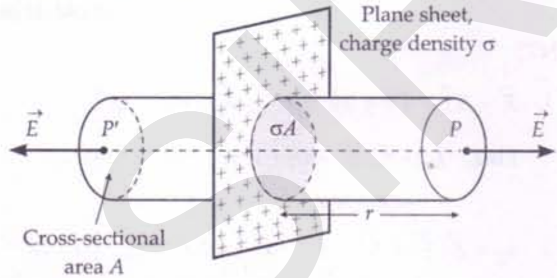


Fig. 1.98 Gaussian surface for a uniformly charged infinite plane sheet.

By symmetry, electric field E points outwards normal to the sheet. Also, it must have same magnitude and opposite direction at two points P and P' equidistant from the sheet and on opposite sides. We choose cylindrical Gaussian surface of cross- sectional area A and length $2r$ with its axis perpendicular to the sheet.

As the lines of force are parallel to the curved surface of the cylinder, the flux through the curved surface is zero. The flux through the plane-end faces of the cylinder is

$$\phi_E = EA + EA = 2\,EA$$

Charge enclosed by the Gaussian surface,

$$q = \sigma A$$

According to Gauss's theorem,

$$\phi_E = \frac{q}{\varepsilon_0}$$

$\therefore \quad 2\,EA = \dfrac{\sigma A}{\varepsilon_0} \quad$ or $\quad \boxed{E = \dfrac{\sigma}{2\varepsilon_0}}$

Clearly, E is independent of r, the distance from the plane sheet.

(*i*) If the sheet is *positively* charged ($\sigma > 0$), the field is directed *away* from it.

(*ii*) If the sheet is *negatively* charged ($\sigma < 0$), the field is directed *towards* it.

For a finite large planar sheet, the above formula will be approximately valid in the middle regions of the sheet, away from its edges.

54. *Two infinite parallel planes have uniform charge densities of σ_1 and σ_2. Determine the electric field at points (i) to the left of the sheets, (ii) between them, and (iii) to the right of the sheets.*

Electric field of two positively charged parallel plates. Fig. 1.99 shows two thin plane parallel sheets of charge having uniform charge densities σ_1 and σ_2 with $\sigma_1 > \sigma_2 > 0$. Suppose $\hat{r}$ is a unit vector pointing from left to right.

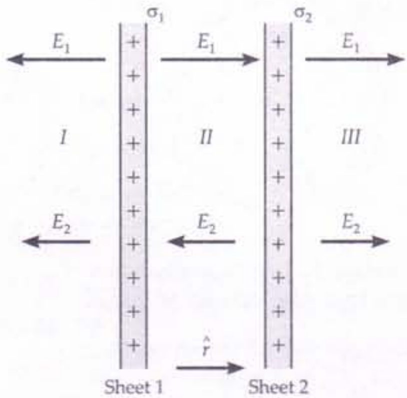


Fig. 1.99

In the region I : Fields due to the two sheets are

$$\vec{E}_1 = -\frac{\sigma_1}{2\varepsilon_0}\hat{r}, \quad \vec{E}_2 = -\frac{\sigma_2}{2\varepsilon_0}\hat{r}$$

From the principle of superposition, the total electric field at any point of region *I* is

$$\vec{E}_I = \vec{E}_1 + \vec{E}_2 = -\frac{\hat{r}}{2\varepsilon_0}(\sigma_1 + \sigma_2)$$

In the region II : Fields due to the two sheets are

$$\vec{E}_1 = \frac{\sigma_1}{2\varepsilon_0}\hat{r}, \quad \vec{E}_2 = -\frac{\sigma_2}{2\varepsilon_0}\hat{r}$$

$\therefore$ Total field, $\vec{E}_{II} = \frac{\hat{r}}{2\varepsilon_0}(\sigma_1 - \sigma_2)$

In the region III : Fields due to the two sheets are

$$\vec{E}_1 = \frac{\sigma_1}{2\varepsilon_0}\hat{r}, \quad \vec{E}_2 = \frac{\sigma_2}{2\varepsilon_0}\hat{r}$$

$\therefore$ Total field, $\vec{E}_{III} = \frac{\hat{r}}{2\varepsilon_0}(\sigma_1 + \sigma_2)$

55. *Two infinite parallel planes have uniform charge densities $\pm\sigma$. Determine the electric field in (i) the region between the planes, and (ii) outside it.*

Electric field of two oppositely charged plane parallel plates. As shown in Fig. 1.100, consider two plane parallel sheets having uniform surface charge densities of $\pm\sigma$. Suppose $\hat{r}$ be a unit vector pointing from left to right.

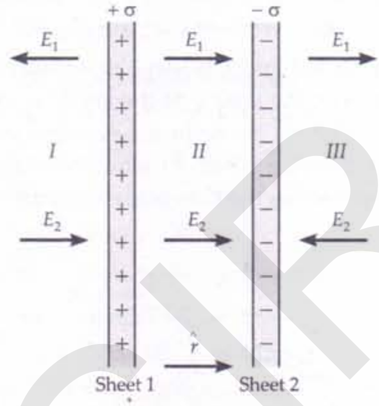


Fig. 1.100

In the region I : Fields due to the two sheets are

$$\vec{E}_1 = -\frac{\hat{r}}{2\varepsilon_0}\sigma, \quad \vec{E}_2 = \frac{\hat{r}}{2\varepsilon_0}\sigma$$

Total field, $\vec{E}_I = \vec{E}_1 + \vec{E}_2 = -\frac{\hat{r}}{2\varepsilon_0}\sigma + \frac{\hat{r}}{2\varepsilon_0}\sigma = 0$

In the region II : Fields due to the two sheets are

$$\vec{E}_1 = \frac{\hat{r}}{2\varepsilon_0}\sigma, \quad \vec{E}_2 = \frac{\hat{r}}{2\varepsilon_0}\sigma$$

Total field, $\vec{E}_{II} = \frac{\hat{r}}{2\varepsilon_0}\sigma + \frac{\hat{r}}{2\varepsilon_0}\sigma = \frac{\sigma}{\varepsilon_0}\hat{r}$

In the region III : Fields due to the two sheets are

$$E_1 = \frac{\hat{r}}{2\varepsilon_0}\sigma, \quad E_2 = -\frac{\hat{r}}{2\varepsilon_0}\sigma$$

Total field, $\vec{E}_{III} = 0$.

Thus the electric field between two oppositely charged plates of equal charge density is uniform which is equal to $\frac{\sigma}{\varepsilon_0}$ and is directed from the positive to the negative plate, while the field is zero on the outside of the two sheets. This arrangement is used for producing *uniform electric field.*

1.37 FIELD DUE TO A UNIFORMLY CHARGED THIN SPHERICAL SHELL

56. *Apply Gauss's theorem to show that for a spherical shell, the electric field inside the shell vanishes, whereas outside it, the field is as if all the charge had been concentrated at the centre.*

Electric field due to a uniformly charged thin spherical shell. Consider a thin spherical shell of charge of radius R with uniform surface charge density σ. From symmetry, we see that the electric field $\vec{E}$ at any point is radial and has same magnitude at points equidistant from the centre of the shell *i.e.*, the field is *spherically symmetric*. To determine electric field at any point P at a distance r from O, we choose a concentric sphere of radius r as the Gaussian surface.

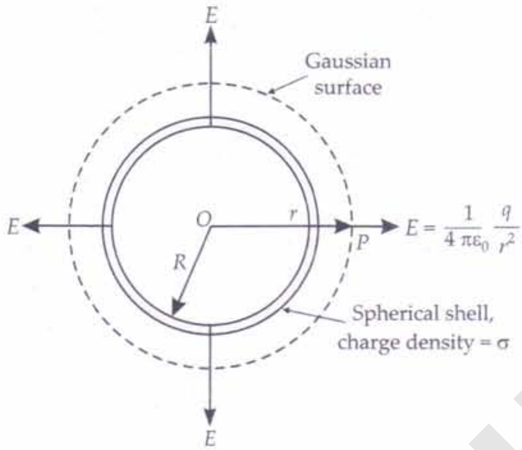


Fig. 1.101 Gaussian surface for outside points of a thin spherical shell of charge.

(*a*) *When point P lies outside the spherical shell.* The total charge q inside the Gaussian surface is the charge on the shell of radius R and area $4\pi R^2$.

$$\therefore \quad q = 4\pi R^2 \sigma$$

Flux through the Gaussian surface,

$$\phi_E = E \times 4\pi r^2$$

By Gauss's theorem,

$$\phi_E = \frac{q}{\varepsilon_0}$$

$$\therefore \quad E \times 4\pi r^2 = \frac{q}{\varepsilon_0}$$

or $$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2} \qquad [\text{For } r > R]$$

This field is the same as that produced by a charge q placed at the centre O. Hence *for points outside the shell, the field due to a uniformly charged shell is as if the entire charge of the shell is concentrated at its centre.*

(*b*) *When point P lies on the spherical shell.* The Gaussian surface just encloses the charged spherical shell.

Applying Gauss's theorem,

$$E \times 4\pi R^2 = \frac{q}{\varepsilon_0}$$

or $$E = \frac{q}{4\pi\varepsilon_0 R^2} \qquad [\text{For } r = R]$$

or $$E = \frac{\sigma}{\varepsilon_0} \qquad [\because q = 4\pi R^2 \sigma]$$

(*c*) *When point P lies inside the spherical shell.* As is clear from Fig. 1.102, the charge enclosed by the Gaussian surface is zero, *i.e.*,

$$q = 0$$

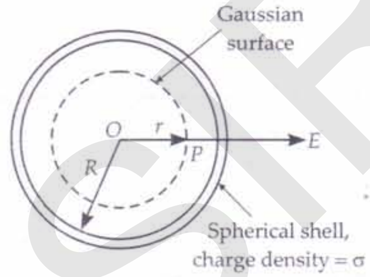


Fig. 1.102 Gaussian surface for inside points of a thin spherical shell of charge.

Flux through the Gaussian surface,

$$\phi_E = E \times 4\pi r^2$$

Applying Gauss's theorem,

$$\phi_E = \frac{q}{\varepsilon_0}$$

$$E \times 4\pi r^2 = 0$$

or $$E = 0 \qquad [\text{For } r < R]$$

Hence electric field due to a uniformly charged spherical shell is zero at all points inside the shell.

Figure 1.103 shows how E varies with distance r from the centre of the shell of radius r. E is zero from $r = 0$ to $r = R$; and beyond $r = R$, we have

$$E \propto \frac{1}{r^2}.$$

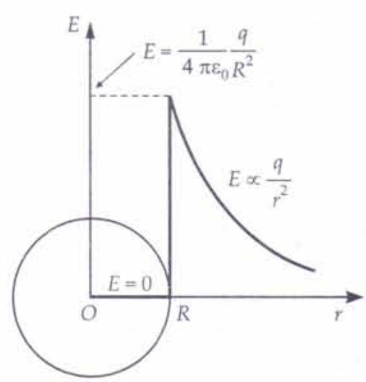


Fig. 1.103 Variation of E with r for a spherical shell of charge.

Examples based on Applications of Gauss's Theorem

Formulae Used

1. Electric field of a long straight wire of uniform linear charge density λ,
$$E = \frac{\lambda}{2\pi \varepsilon_0 r}$$
where r is the perpendicular distance of the observation point from the wire.

2. Electric field of an infinite plane sheet of uniform surface charge density σ,
$$E = \frac{\sigma}{2\varepsilon_0}$$

3. Electric field of two positively charged parallel plates with charge densities σ_1 and σ_2 such that $\sigma_1 > \sigma_2 > 0$,
$$E = \pm \frac{1}{2\varepsilon_0}(\sigma_1 + \sigma_2) \qquad \text{(Outside the plates)}$$
$$E = \frac{1}{2\varepsilon_0}(\sigma_1 - \sigma_2) \qquad \text{(Inside the plates)}$$

4. Electric field of two equally and oppositely charged parallel plates,
$$E = 0 \qquad \text{(For outside points)}$$
$$E = \frac{\sigma}{\varepsilon_0} \qquad \text{(For inside points)}$$

5. Electric field of a thin spherical shell of charge density σ and radius R,
$$E = \frac{1}{4\pi \varepsilon_0} \cdot \frac{q}{r^2} \qquad \text{For } r > R \text{ (Outside points)}$$
$$E = 0 \qquad \text{For } r < R \text{ (Inside points)}$$
$$E = \frac{1}{4\pi \varepsilon_0} \cdot \frac{q}{R^2} \qquad \text{For } r = R \text{ (At the surface)}$$
Here $q = 4\pi R^2 \sigma$.

6. Electric field of a solid sphere of uniform charge density ρ and radius R :
$$E = \frac{1}{4\pi \varepsilon_0} \cdot \frac{q}{r^2} \qquad \text{For } r > R \text{ (Outside points)}$$
$$E = \frac{1}{4\pi \varepsilon_0} \cdot \frac{qr}{R^3} \qquad \text{For } r < R \text{ (Inside points)}$$
$$E = \frac{1}{4\pi \varepsilon_0} \cdot \frac{q}{R^2} \qquad \text{For } r = R \text{ (At the surface)}$$
Here $q = \frac{4}{3}\pi R^3 \rho$

Units Used

Here charges are in coulomb, r and R in metre, λ in Cm^{-1}, σ in Cm^{-2}, ρ in Cm^{-3} and electric field E in NC^{-1} or Vm^{-1}.

Example 68. *Two long straight parallel wires carry charges λ_1 and λ_2 per unit length. The separation between their axes is d. Find the magnitude of the force exerted on unit length of one due to the charge on the other.*

Solution. Electric field at the location of wire 2 due to charge on 1 is
$$E = \frac{\lambda_1}{2\pi\varepsilon_0 d}$$
Force per unit length of wire 2 due to the above field
$$f = E \times \text{charge on unit length of wire } 2 = E\lambda_2$$
or
$$f = \frac{\lambda_1 \lambda_2}{2\pi\varepsilon_0 d}.$$

Example 69. *An electric dipole consists of charges $\pm 2 \times 10^{-8}$ C, separated by a distance of 2 mm. It is placed near a long line charge of density 4.0×10^{-4} Cm^{-1}, as shown in Fig. 1.104, such that the negative charge is at a distance of 2 cm from the line charge. Calculate the force acting on the dipole.*

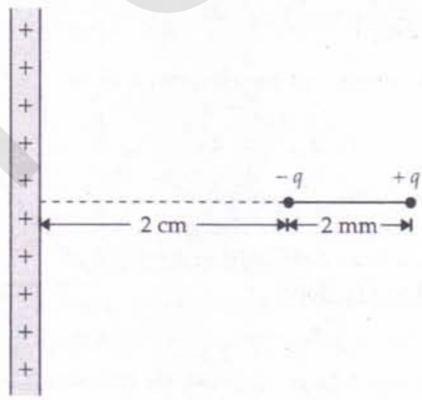


Fig. 1.104

Solution. Electric field due to a line charge at distance r from it,
$$E = \frac{1}{4\pi\varepsilon_0} \frac{2\lambda}{r}$$
Force exerted by this field on charge q,
$$F = qE = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2q\lambda}{r}$$
Force exerted on negative charge ($r = 0.02$ m),
$$F_1 = \frac{9 \times 10^9 \times 2 \times 2 \times 10^{-8} \times 4 \times 10^{-4}}{0.02} \text{ N}$$
$= 7.2$ N, acting towards the line charge

Force exerted on positive charge ($r = 2.2 \times 10^{-2}$ m),
$$F_2 = \frac{9 \times 10^9 \times 2 \times 2 \times 10^{-8} \times 4 \times 10^{-4}}{2.2 \times 10^{-2}}$$
$= 6.5$ N, acting away from the line charge

Net force on the dipole,

$$F = F_1 - F_2 = 7.2 - 6.5$$

$= \mathbf{0.7\ N}$, acting towards the line charge.

Example 70. *(a) An infinitely long positively charged wire has a linear charge density λCm^{-1}. An electron is revolving around the wire as its centre with a constant velocity in a circular plane perpendicular to the wire. Deduce the expression for its kinetic energy. (b) Plot a graph of the kinetic energy as a function of charge density λ.* [CBSE F 13]

Solution. The electrostatic force exerted by the line charge on the electron provides the centripetal force for the revolution of electron.

∴ Force exerted by electric field = Centripetal force

$$eE = \frac{mv^2}{r}$$

Here v is the orbital velocity of the electron

But $$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

$$\therefore \quad \frac{e\lambda}{2\pi\varepsilon_0 r} = \frac{mv^2}{r} \quad \text{or} \quad v^2 = \frac{e\lambda}{2\pi\varepsilon_0 m}$$

Kinetic energy of the electron will be

$$E_k = \frac{1}{2}mv^2 = \frac{e\lambda}{4\pi\varepsilon_0}.$$

(*b*) As $E_k \propto \lambda$, the graph of kinetic energy E_k *vs.* charge density λ will be a straight line as shown in Fig. 1.105.

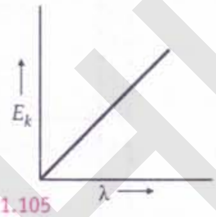


Fig. 1.105

Example 71. *A charge of 17.7×10^{-4} C is distributed uniformly over a large sheet of area 200 m^2. Calculate the electric field intensity at a distance of 20 cm from it in air.* [CBSE OD 03C]

Solution. Surface charge density of the sheet,

$$\sigma = \frac{q}{A} = \frac{17.7 \times 10^{-4}\ \text{C}}{200\ \text{m}^2} = 8.85 \times 10^{-6}\ \text{Cm}^{-2}$$

Electric field at a distance of 20 cm from it in air,

$$E = \frac{\sigma}{2\varepsilon_0} = \frac{8.85 \times 10^{-6}}{2 \times 8.85 \times 10^{-12}} = \mathbf{5 \times 10^5\ NC^{-1}}.$$

Example 72. *A charged particle having a charge of -2.0×10^{-6} C is placed close to a non-conducting plate having a surface charge density of 4.0×10^{-6} Cm^{-2}. Find the force of attraction between the particle and the plate.*

Solution. Here $q = -2.0 \times 10^{-6}$C,

$$\sigma = 4.0 \times 10^{-6}\text{Cm}^{-2}$$

Field produced by charged plate,

$$E = \frac{\sigma}{2\varepsilon_0}$$

Force of attraction between the charged particle and the plate,

$$\therefore \quad F = qE = \frac{\sigma q}{2\varepsilon_0} = \frac{4 \times 10^{-6} \times 2.0 \times 10^{-6}}{2 \times 8.85 \times 10^{-12}}$$

$= \mathbf{0.45\ N.}$

Example 73. *A particle of mass 9×10^{-5}g is kept over a large horizontal sheet of charge density 5×10^{-5} Cm^{-2}. What charge should be given to the particle, so that if released, it does not fall ?*

Solution. Here $m = 9 \times 10^{-5}\text{g} = 9 \times 10^{-8}$ kg,

$$\sigma = 5 \times 10^{-5}\ \text{Cm}^{-2}$$

The particle must be given a positive charge q. It will not fall if

Upward force exerted on the particle by electric field = Weight of the particle

or $$qE = mg$$

or $$q \cdot \frac{\sigma}{2\varepsilon_0} = mg$$

or $$q = \frac{2\varepsilon_0 mg}{\sigma}$$

$$= \frac{2 \times 8.85 \times 10^{-12} \times 9 \times 10^{-8} \times 9.8}{5 \times 10^{-5}}$$

$= \mathbf{3.12 \times 10^{-13}\ C.}$

Example 74. *A large plane sheet of charge having surface charge density 5.0×10^{-16} Cm^{-2} lies in the X-Y plane. Find the electric flux through a circular area of radius 0.1 m, if the normal to the circular area makes an angle of 60° with the Z-axis.*

Given that : $\varepsilon_0 = 8.85 \times 10^{-12}\ C^2\ N^{-1} m^{-2}$.

Solution. Here $\sigma = 5.0 \times 10^{-16}\ \text{Cm}^{-2}$, $r = 0.1$ m, $\theta = 60°$

Field due to a plane sheet of charge,

$$E = \frac{\sigma}{2\varepsilon_0}$$

Flux through circular area,

$$\phi_E = E\Delta S \cos\theta = \frac{\sigma}{2\varepsilon_0} \times \pi r^2 \cos\theta$$

$$= \frac{5.0 \times 10^{-16} \times 3.14 \times (0.1)^2 \cos 60°}{2 \times 8.85 \times 10^{-12}}$$

$= \mathbf{4.44 \times 10^{-7}\ Nm^2C^{-1}}.$

Example 75. *A spherical conductor of radius 12 cm has a charge of 1.6×10^{-7} C distributed uniformly over its surface. What is the electric field (i) inside the sphere, (ii) just outside the sphere, (iii) at a point 18 cm from the centre of the sphere ?* [NCERT]

Solution. Here $q = 1.6 \times 10^{-7}$ C,

$$R = 12 \text{ cm} = 0.12 \text{ m}$$

(*i*) *Inside the sphere,* $E = \mathbf{0}$. This is because the charge resides on the outer surface of the spherical conductor.

(*ii*) *Just outside the sphere,* $r = R = 0.12$ m. Here the charge may be assumed to be concentrated at the centre of the sphere.

$$\therefore \quad E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R^2}$$

$$= \frac{9 \times 10^9 \times 1.6 \times 10^{-7}}{(0.12)^2} = \mathbf{10^5\ NC^{-1}}.$$

(*iii*) *At a point 18 cm from the centre,*

$$r = 18 \text{ cm} = 0.18 \text{ m}.$$

$$\therefore \quad E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2} = \frac{9 \times 10^9 \times 1.6 \times 10^{-7}}{(0.18)^2}$$

$$= \mathbf{4.44 \times 10^4\ NC^{-1}}.$$

Problems For Practice

1. An infinite line charge produces a field of $9 \times 10^4 \text{NC}^{-1}$ at a distance of 4 cm. Calculate the linear charge density. **[Haryana 01]**
(Ans. $2 \times 10^{-7}\ \text{Cm}^{-1}$)

2. A cylinder of large length carries a charge of $2 \times 10^{-8} \text{Cm}^{-1}$. Find the electric field at a distance of 0.2 m from it. (Ans. $1800\ \text{Vm}^{-1}$)

3. An infinitely long wire is stretched horizontally 4 metre above the surface of the earth. It carries a charge 1μC per cm of its length. Calculate its electric field at a point on the earth's surface vertically below the wire. (Ans. $4.5 \times 10^5 \text{Vm}^{-1}$)

4. Two large metal plates each of area 1 m^2 are placed facing each other at a distance of 10 cm and carry equal and opposite charges on their faces. If the electric field between the plates is 100 NC^{-1}, find the charge on each plate. (Ans. 8.85×10^{-10} C)

5. An electron is revolving around a long line charge having charge density $2 \times 10^{-8} \text{Cm}^{-1}$. Find the kinetic energy of the electron, assuming that it is independent of the radius of electron's orbit.
(Ans. 2.88×10^{-17}J)

6. A particle of mass 5×10^{-6}g is kept over a large horizontal sheet of charge density $4 \times 10^{-6} \text{Cm}^{-2}$. What charge should be given to this particle, so that if released, it does not fall down. How many electrons should be removed to give this charge ?
(Ans. 2.16×10^{-13}C, 1.355×10^6)

7. A spherical shell of metal has a radius of 0.25 m and carries a charge of 0.2 μC. Calculate the electric field intensity at a point (*i*) inside the shell, (*ii*) just outside the shell and (*iii*) 3.0 m from the centre of the shell. [Ans. (*i*) 0 (*ii*) $2.88 \times 10^4\ \text{NC}^{-1}$ (*iii*) $200\ \text{NC}^{-1}$]

HINTS

1. $\lambda = 2\pi\varepsilon_0 Er = 4\pi\varepsilon_0 \frac{Er}{2} = \frac{9 \times 10^4 \times 0.04}{9 \times 10^9 \times 2}$
$= \mathbf{2 \times 10^{-7}\ Cm^{-1}}$.

2. Here $\lambda = 2 \times 10^{-8} \text{Cm}^{-1}$, $r = 0.2$ m
$\therefore E = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2\lambda}{r} = 9 \times 10^9 \times \frac{2 \times 2 \times 10^{-8}}{0.2}$
$= \mathbf{1800\ Vm^{-1}}$.

3. $E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2\lambda}{r} = \frac{9 \times 10^9 \times 2 \times 10^{-4}}{4} = \mathbf{4.5 \times 10^5\ Vm^{-1}}$.

4. $E = \frac{\sigma}{\varepsilon_0} = \frac{q}{\varepsilon_0\ \Delta S}$
$\therefore \quad q = \varepsilon_0\ \Delta S\ E = 8.85 \times 10^{-12} \times 1 \times 100$
$= \mathbf{8.85 \times 10^{-10}\ C}$

5. From Example 70,
$E_k = \frac{e\lambda}{4\pi\varepsilon_0} = 9 \times 10^9 \times 1.6 \times 10^{-9} \times 2.0 \times 10^{-8}$
$= \mathbf{2.88 \times 10^{-17}\ J}$.

6. Upward electric force on particle
= Weight of the particle
$$mg = qE = q \cdot \frac{\sigma}{2\varepsilon_0}$$
or $$q = \frac{2\varepsilon_0 mg}{\sigma}$$
$$= \frac{2 \times 8.85 \times 10^{-12} \times 5 \times 10^{-9} \times 9.8}{4 \times 10^{-6}}$$
$$= \mathbf{2.16 \times 10^{-13}\ C}.$$
Number of electrons required to be removed,
$$n = \frac{q}{e} = \frac{2.16 \times 10^{-13}}{1.6 \times 10^{-19}} = \mathbf{1.355 \times 10^6}.$$

7. (*i*) Electric field at any point inside the shell = **0**.
(*ii*) $E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R^2}$
$$= \frac{9 \times 10^9 \times 0.2 \times 10^{-6}}{(0.25)^2} = \mathbf{2.88 \times 10^4\ NC^{-1}}.$$
(*iii*) $E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$
$$= \frac{9 \times 10^9 \times 0.2 \times 10^{-6}}{(3.0)^2} = \mathbf{200\ NC^{-1}}.$$

VERY SHORT ANSWER CONCEPTUAL PROBLEMS

Problem 1. **The electric charge of any body is actually a surplus or deficit of electrons. Why not protons ?**

Solution. Electrons are loosely bound to atoms and can be readily exchanged during rubbing. Protons are firmly bound inside the nucleus. They cannot be easily detached. Hence electric charge of any body is just a surplus or deficit of electrons and not protons.

Problem 2. **When a glass rod is rubbed with silk, both acquire charges. What is the source of their electrification ?**

Solution. For the electrification of a body, only electrons are responsible. During rubbing electrons are transferred from glass rod to silk. The glass rod acquires a positive charge and silk acquires an equal negative charge.

Problem 3. **Is the mass of a body affected on charging ?**

Solution. Yes. Electrons have a definite mass. The mass of a body slightly increases if it gains electrons while the mass decreases if the body loses electrons.

Problem 4. **Two identical metallic spheres of exactly equal masses are taken. One is given a positive charge q coulombs and other an equal negative charge. Are their masses after charging equal ?** [IIT]

Solution. No. The positive charge of a body is due to deficit of electrons while the negative charge is due to surplus of electrons. Hence the mass of the negatively charged sphere will be slightly more than that of the positively charged spheres.

Problem 5. **A positively charged rod repels a suspended object. Can we conclude that the object is positively charged ?**

Solution. Yes, the object is positively charged. Repulsion is the surest test of electrification.

Problem 6. **A positively charged rod attracts a suspended object. Can we conclude that the object is negatively charged ?**

Solution. No. A positively charged rod can attract both a neutral object and a less positively charged object.

Problem 7. **How does a positively charged glass rod attract a neutral piece of paper ?**

Solution. The positively charged rod induces negative charge on the closer end and positive charge on the farther end of the paper. The rod exerts greater attraction than repulsion on the paper because negative charge is closer to the rod than the positive charge. Hence the rod attracts the piece of paper.

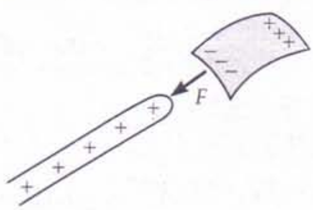


Fig. 1.106

Problem 8. **Can two like charges attract each other ? If yes, how ?**

Solution. Yes. If one charge is larger than the other, the larger charge induces equal and opposite charge on the nearer end of the body with smaller charge. The opposite induced charge is larger than the small charge initially present on it.

Problem 9. **Why do the gramophone records get covered with dust easily ?**

Solution. The gramophone records get charged due to the rubbing action of the needle. So they attract the dust particles from the air.

Problem 10. **An ebonite rod held in hand can be charged by rubbing with flannel but a copper rod cannot be charged like this. Why ?** [Himachal 97]

Solution. Ebonite rod is insulating. Whatever charge appears on it due to rubbing, stays on it. Copper is good conductor. Any charge developed on it flows to the earth through our body. So copper rod cannot be charged like this. It can be charged by providing it a plastic or rubber handle.

Problem 11. **Electrostatic experiments do not work well on humid days. Give reason.**

Solution. Electrostatic experiments require accumulation of charges. Whatever charges appear during the experimentation, they are drained away through humid air which is more conducting than dry air due to the presence of a larger number of charged particles in it.

Problem 12. **A comb run through one's dry hair attracts small bits of paper. Why ? What happens if the hair is wet or if it is a rainy day ?** [NCERT]

Solution. When the comb runs through dry hair, it gets charged by friction. The molecules in the paper get polarized by the charged comb, resulting in a net force of attraction. If the hair is wet, or if it is rainy day, friction between hair and the comb reduces. The comb does not get charged and thus it will not attract small bits of paper.

Problem 13. **Ordinary rubber is an insulator. But the special rubber tyres of aircrafts are made slightly conducting. Why is this necessary ?** [NCERT]

Solution. During landing, the tyres of aircraft may get highly charged due to friction between tyres and the air strip. If the tyres are made slightly conducting, they will lose the charge to the earth otherwise too much of static electricity accumulated may produce spark and result in fire.

Problem 14. **Vehicles carrying inflammable materials usually have metallic ropes touching the ground during motion. Why ?** [Himanchal 98 ; Punjab 99 ; NCERT]

Solution. Moving vehicle gets charged due to friction. The inflammable material may catch fire due to the spark produced by charged vehicle. When metallic rope is used, the charge developed on the vehicle is transferred to the ground and so the fire is prevented.

Problem 15. **An inflated balloon is charged by rubbing with fur. Will it stick readily to a conducting wall or to an insulating wall ? Give reason.** [Roorkee]

Solution. It will stick readily to the conducting wall. It induces an equal amount of charge on the conducting wall and much smaller charge on insulating wall. So a large force of attraction acts between the balloon and the conducting wall.

Problem 16. **A metal sphere is fixed on a smooth horizontal insulating plate. Another metal sphere is placed a small distance away. If the fixed sphere is given a charge, how will the other sphere react ?**

Solution. The charge on the fixed sphere induces unlike charge at the closer end and like charge on the far end of the free sphere. Net attraction acts on the free sphere and so it gets accelerated towards the fixed sphere.

Problem 17. **Is there some way of producing high voltage on your body without getting a shock ?**

Solution. If we stand on an insulating surface and touch the live wire of a high power supply, a high potential is developed on our body, without causing any shock.

Problem 18. **A charged rod attracts bits of dry cork which after touching the rod, often jump away from it violently. Why ?**

Solution. The charged rod attracts the bits of dry cork by inducing unlike charge at their near ends and like charge at their far ends. When the cork bits touch the rod, they share the charge of the rod of the same sign and so get strongly repelled away.

Problem 19. **What does $q_1 + q_2 = 0$ signify in electrostatics ?** [CBSE OD 01C]

Or

Two charges q_1 and q_2, separated by a small distance satisfy the equation $q_1 + q_2 = 0$. What does it tell about the charges ? [CBSE F 03]

Solution. The equation signifies that the electric charges are algebraically additive and here q_1 and q_2 are equal and opposite.

Problem 20. **Name the experiment which established the quantum nature of electric charge.** [CBSE OD 98]

Solution. Millikan's oil drop experiment for determining electronic charge.

Problem 21. **Can a body have a charge of 0.8×10^{-19} C ? Justify your answer by comment ?** [Himachal 99C]

Solution. The charge on any body is always an integral multiple of e. Here

$$n = \frac{q}{e} = \frac{0.8 \times 10^{-19}\text{ C}}{1.6 \times 10^{-19}\text{ C}} = 0.5$$

This is not an integer. So a body cannot have a charge of 0.8×10^{-19} C.

Problem 22. **If the distance between two equal point charges is doubled and their individual charges are also doubled, what would happen to the force between them ?** [ISCE 95]

Solution. The original force between the two charges is

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q \times q}{r^2}$$

When the individual charges and the distance between them are doubled, the force becomes

$$F' = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2q \times 2q}{(2r)^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q \times q}{r^2} = F$$

Hence the force will remain same.

Problem 23. **The electrostatic force between two charges is a central force. Why ?**

Solution. The electrostatic force between two charges acts along the line joining the two charges. So it is a central force.

Problem 24. **How is the Coulomb force between two charges affected by the presence of a third charge ?**

Solution. The Coulomb force between two charges does not depend on the presence of a third charge.

Problem 25. **Two equal balls having equal positive charge 'q' coulombs are suspended by two insulating strings of equal length. What would be the effect on the force when a plastic sheet is inserted between the two ?** [CBSE OD 14]

Solution. The force between the two balls decreases because $\kappa(\text{Plastic}) > 1$ and $F \propto 1/\kappa$.

Problem 26. **Force between two point charges kept at a distant d apart in air is F. If these charges are kept at the same distance in water, how does the electric force between them change ?** [CBSE OD 11]

Solution. Dielectric constant for water, $\kappa = 80$

$$\therefore \quad F_{\text{water}} = \frac{F_{\text{air}}}{\kappa} = \frac{F}{80}$$

Thus the force in water is $1/80$ times the original force in air.

Problem 27. **The dielectric constant of water is 80. What is its permittivity ?** [Haryana 97C]

Solution. Dielectric constant, $\kappa = \dfrac{\varepsilon}{\varepsilon_0}$

$\therefore$ Permittivity, $\varepsilon = \kappa\varepsilon_0 = 8.854 \times 10^{-12} \times 80$

$$= 7.083 \times 10^{-10}\,\mathrm{C^2N^{-1}m^{-2}}.$$

Problem 28. **Give an example to illustrate that electrostatic forces are much stronger than gravitational forces.**

Solution. A charged glass rod can lift a piece of paper against the gravitational pull of the earth on this piece. This shows that the electrostatic force on the piece of paper is much greater than the gravitational force on it.

Problem 29. **Two electrically charged particles, having charges of different magnitude, when placed at a distance '*d*' from each other, experience a force of attraction '*F*'. These two particles are put in contact and again placed at the same distance from each other.**

What is the nature of new force between them ?

Is the magnitude of the force of interaction between them now more or less than *F* ? [CBSE Sample Paper 11]

Solution. When the two particles are put in contact, they share the difference of charge identically. Hence the two particles *repel*, with a force *less than F*.

Problem 30. **An electron moves along a metal tube with variable cross-section, as shown in Fig. 1.107. How will its velocity change when it approaches the neck of the tube ?**

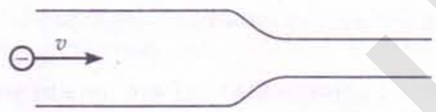


Fig. 1.107

Solution. The positive charge induced on the neck of the tube will accelerate the electron towards the neck.

Problem 31. **Why should a test charge be of negligibly small magnitude ?**

Solution. The magnitude of the test charge must be small enough so that it does not disturb the distribution of the charges whose electric field we wish to measure otherwise the measured field will be different from the actual field.

Problem 32. **In defining electric field due to a point charge, the test charge has to be vanishingly small. How this condition can be justified, when we know that charge less than that on an electron or a proton is not possible ?**

Solution. Because of charge quantisation, the test charge q_0 cannot go below *e*. However, in macroscopic situations, the source charge is much larger than the charge on an electron or proton, so the limit $q_0 \to 0$ for the test charge is justified.

Problem 33. **What is the advantage of introducing the concept of electric field ?**

Solution. By knowing the electrical field at a point, the force on a charge placed at that point can be determined.

Problem 34. **How do charges interact ?**

Solution. The electric field of one charge exerts a force on the other charge and vice versa.

$$\text{Charge} \rightleftharpoons \text{Electric field} \rightleftharpoons \text{Charge}.$$

Problem 35. **An electron and a proton are kept in the same electric field. Will they experience same force and have same acceleration ?**

Solution. Both electron and proton will experience force of same magnitude, $F = eE$. Since a proton has 1836 times more mass than an electron, so its acceleration will be 1/1836 times that of the electron.

Problem 36. **Why direction of an electric field is taken outward (away) for a positive charge and inward (towards) for a negative charge ?**

Solution. By convention, the direction of electric field is the same as that of force on a unit positive charge. As this force is outward in the field of a positive charge, and inward in the field of a negative charge, so the directions are taken accordingly.

Problem 37. **A charged particle is free to move in an electric field. Will it always move along an electric field ?** [IIT]

Solution. The tangent at any point to the line of force gives the direction of electric field and hence of force on a charge at that point. If the charged particle starts from rest, it will move along the line of force. If it is in motion and moves initially at an angle with the line of force, then resultant path is not along the line of force.

Problem 38. **A small test charge is released at rest at a point in an electrostatic field configuration. Will it travel along the line of force ?** [NCERT]

Solution. Not necessarily. The test charge will move along the line of force only if it is a straight line. This is because a line of force gives the direction of acceleration and not that of velocity.

Problem 39. **Why do charges reside on the surface of the conductor ?**

Solution. Charges lie at the ends of lines of force. These lines of force have a tendency to contract in length. The lines of force pull charges from inside a conductor to its outer surface.

Problem 40. **Why is electric field zero inside a charged conductor ?**

Solution. This is because charges reside on the surface of a conductor and not inside it.

Problem 41. **Why do the electrostatic field lines not form closed loops ?** [CBSE OD 14, 15]

Solution. Electrostatic field lines start from a positive charge and end on a negative charge or they fade out at infinity in case of isolated charges without forming any closed loop.

Alternatively, electrostatic field is a conservative field. The work done in moving a charge along a closed path must be zero. Hence, electrostatic field lines cannot form closed loops.

Problem 42. Do the electric lines of force really exist ? What is about the field they represent ?

Solution. Lines of force do not really exist. These are hypothetical curves used to represent an electric field. But the electric field which they represent is real.

Problem 43. Draw lines of force to represent a uniform electric field. [CBSE OD 95]

Solution. The lines of force of a uniform electric field are equidistant parallel lines as shown in Fig. 1.108.

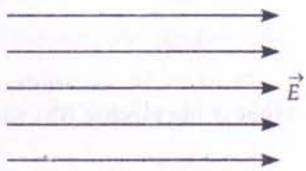


Fig. 1.108 Uniform electric field.

Problem 44. Fig. 1.109 shows electric lines of force due to point charges q_1 and q_2 placed at points A and B respectively. Write the nature of charge on them. [CBSE F 03]

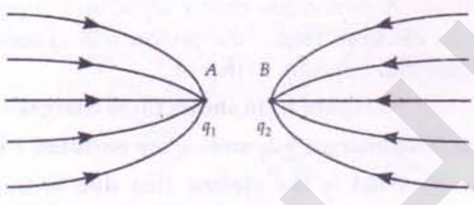


Fig. 1.109

Solution. As the lines of force are pointing towards q_1 as well as q_2, so both q_1 and q_2 must be negative charges.

Problem 45. A positive point charge (+ q) is kept in the vicinity of an uncharged conducting plate. Sketch electric field lines originating from the point charge on to the surface of the plate. [CBSE OD 09]

Solution. Starting from the charge + q, the lines of force will terminate at the metal plate, inducing negative charge on it. At all positions, the lines of force will be perpendicular to the metal surface, as shown in Fig. 1.110.

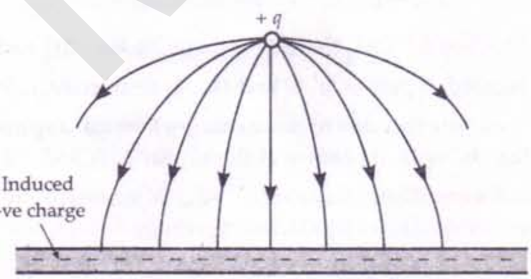


Fig. 1.110

Problem 46. Why is it necessary that the field lines from a point charge placed in the vicinity of a conductor must be normal to the conductor at every point. [CBSE F 09]

Solution. If the field lines are not normal, then the field $\vec{E}$ would have a tangential component which will make electrons move along the surface creating surface currents and the conductor will not be in equilibrium.

Problem 47. Fig. 1.111 shows two large metal plates, P_1 and P_2, tightly held against each other and placed between two equal and unlike point charges perpendicular to the line joining them.

(i) What will happen to the plates when they are released ?

(ii) Draw the pattern of the electric field lines for the system. [CBSE F 09]

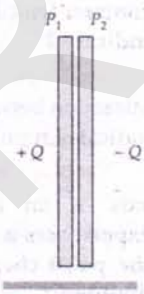


Fig. 1.111

Solution.

(i) When released, the two plates tend to move apart slightly due to the charges induced in them.

(ii) The pattern of the electric field lines for the system is shown in Fig. 1.112.

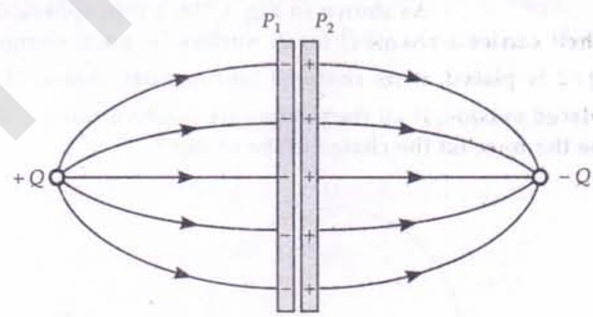


Fig. 1.112

Problem 48. In the electric field shown in Fig. 1.113, the electric field lines on the left have twice the separation as that between those on the right. If the magnitude of the field at point A is 40 NC^{-1}, calculate the force experienced by a proton placed at point A Also find the magnitude of electric field at point B.

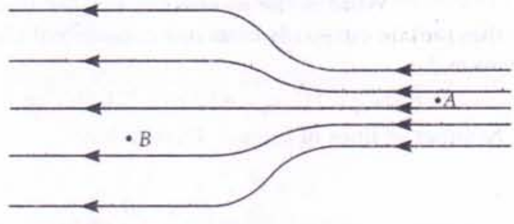


Fig. 1.113

Solution. Force on proton at point A,

$$F = eE_A = 1.6 \times 10^{-19} \times 40 = \mathbf{6.4 \times 10^{-18}\ N}$$

As the separation between the lines of force at point B is twice of that at point A, so

$$E_B = \frac{1}{2} E_A = \frac{1}{2} \times 40 = \mathbf{20\ NC^{-1}}.$$

Problem 49. The electric lines of force tend to contract lengthwise and expand laterally. What do they indicate ?

Solution. The lengthwise contraction indicates attraction between unlike charges while lateral expansion indicates repulsion between like charges.

Problem 50. A point charge placed at any point on the axis of an electric dipole at some large distance experiences a force F. What will be the force acting on the point charge when its distance from the dipole is doubled ? [CPMT 91]

Solution. At any axial point of a dipole, electric field varies as

$$E \propto \frac{1}{r^3} \quad \text{or} \quad \frac{F}{q} = \frac{1}{r^3} \quad \text{or} \quad F \propto \frac{1}{r^3}$$

$\therefore$ When the distance of the point charge is doubled, the force reduces to $F/8$.

Problem 51. As shown in Fig. 1.114, a thin spherical shell carries a charge Q on its surface. A point charge $Q/2$ is placed at its centre O and another charge $2Q$ placed outside. If all the charges are positive, what will be the force on the charge at the centre ?

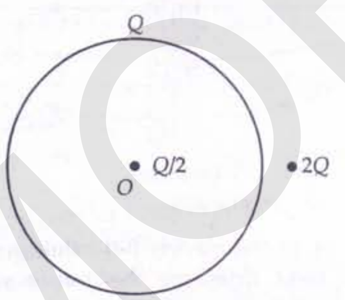


Fig. 1.114

Solution. Zero, because the electric field inside the spherical shell is zero.

Problem 52. What is the number of electric lines of force that radiate outwards from one coulomb of charge in vacuum ?

Solution. Here $q = 1\,C$, $\varepsilon_0 = 8.85 \times 10^{-12}\ C^2N^{-1}m^{-2}$

Number of lines of force = Electric flux

$$= \frac{q}{\varepsilon_0} = \frac{1}{8.85 \times 10^{-12}}$$

$$= \mathbf{1.13 \times 10^{11}}.$$

Problem 53. Consider the situation shown in Fig. 1.115. What are the signs of q_1 and q_2 ? If the lines are drawn in proportion to the charge, what is the ratio q_1 / q_2 ?

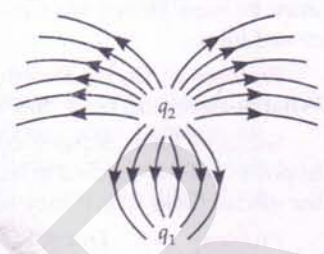


Fig. 1.115

Solution. Here q_1 is a negative charge and q_2 is a positive charge.

$$\frac{q_1}{q_2} = \frac{6}{18}$$

$$= \mathbf{1 : 3}.$$

Problem 54. An arbitrary surface encloses a dipole. What is the electric flux through this surface ? [Exemplar Problem]

Solution. As the total charge of a dipole is zero, so by Gauss's theorem, the electric flux through the closed surface is zero.

Problem 55. The force on an electron kept in an electric field in a particular direction is F. What will be the magnitude and direction of the force experienced by a proton at the same point in the field ? Mass of the proton is 1836 times the mass of the electron. [CBSE F07]

Solution. A proton has charge equal and opposite to that of an electron. Hence the proton will experience a force equal and opposite to that of F.

Problem 56. Figure 1.116 shows three charges $+2q$, $-q$ and $+3q$. Two charges $+2q$ and $-q$ are enclosed within a surface 'S'. What is the electric flux due to this configuration through the surface 'S' ? [CBSE D 10]

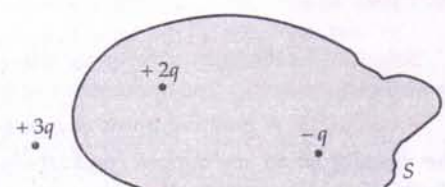


Fig. 1.116

Solution. $\phi_E = \dfrac{\text{Net charge enclosed by the surface } S}{\varepsilon_0}$

$$= \frac{+2q - q}{\varepsilon_0} = \frac{q}{\varepsilon_0}$$

Problem 57. Two charges of magnitudes $-2Q$ and $+Q$ are located at points $(a, 0)$ and $(4a, 0)$ respectively. What is the electric flux due to these charges through a sphere of radius '$3a$' with its centre at the origin ? *[CBSE OD 13]*

Solution. Only the charge $-2Q$ is enclosed by the sphere of radius $3a$. By Gauss's theorem.

$$\phi_E = -\frac{2Q}{\varepsilon_0}.$$

GUIDELINES TO NCERT EXERCISES

1.1. *What is the force between two small charged spheres having charges of 2×10^{-7} C and 3×10^{-7} C placed 30 cm apart in air ?*

Ans. Here $q_1 = 2 \times 10^{-7}$C, $q_2 = 3 \times 10^{-7}$C,

$$r = 30\text{ cm} = 0.30\text{ m}$$

According to Coulomb's law,

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2} = 9 \times 10^9 \times \frac{2 \times 10^{-7} \times 3 \times 10^{-7}}{(0.30)^2}$$

$$= \mathbf{6 \times 10^{-3}\ N\ (repulsive)}.$$

1.2. *The electrostatic force on a small sphere of charge 0.4 μC due to another small sphere of charge −0.8μC in air is 0.2 N. (i) What is the distance between two spheres ? (ii) What is the force on the second sphere due to the first ?*

Ans. (*i*) Here $q_1 = 0.4\ \mu\text{C} = 0.4 \times 10^{-6}$ C

$$q_2 = -0.8\ \mu\text{C} = -0.8 \times 10^{-6}\text{ C},\ F = 0.2\text{ N},\ r = ?$$

As $F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}$

$$\therefore\ r^2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{F}$$

$$= \frac{9 \times 10^9 \times 0.4 \times 10^{-6} \times 0.8 \times 10^{-6}}{0.2} = 144 \times 10^{-4}$$

or $r = 12 \times 10^{-2} = 0.12\text{ m} = \mathbf{12\ cm}$.

(*ii*) The two charges mutually exert equal and opposite forces.

∴ Force on the second sphere due to the first

$$= \mathbf{0.2\ N\ (attractive)}.$$

1.3. *Check that the ratio $ke^2/Gm_e m_p$ is dimensionless. Look up a table of physical constants and determine the value of this ratio. What does this ratio signify ?*

Ans. $$\left[k\frac{e^2}{Gm_e m_p}\right] = \frac{[\text{Nm}^2\text{C}^{-2}] \times [\text{C}]^2}{[\text{Nm}^2\text{kg}^{-2}] \times [\text{kg}][\text{kg}]} = \text{no unit}$$

As the ratio $k e^2 / Gm_e m_p$ has no unit, so it is dimensionless.

Now $k = 9 \times 10^9\ \text{Nm}^2\text{C}^{-2}$

$G = 6.67 \times 10^{-11}\ \text{Nm}^2\ \text{kg}^{-2}$

$e = 1.6 \times 10^{-19}$ kg

$m_e = 9.1 \times 10^{-31}$ kg

and $m_p = 1.66 \times 10^{-27}$ kg

$$\therefore\ k\frac{e^2}{Gm_e m_p} = \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{6.67 \times 10^{-11} \times 9.1 \times 10^{-31} \times 1.66 \times 10^{-27}}$$

$$= \mathbf{2.287 \times 10^{39}}.$$

The factor $k e^2 / Gm_e m_p$ represents the ratio of electrostatic force to the gravitational force between an electron and a proton. Also, the large value of the ratio signifies that the electrostatic force is much stronger than the gravitational force.

1.4. (*i*) *Explain the meaning of the statement 'electric charge of a body is quantised.'*

(*ii*) *Why can one ignore quantisation of electric charge when dealing with macroscopic i.e., large scale charges ?*

Ans. (*i*) Quantisation of electric charge means that the total charge (*q*) of a body is always an integral multiple of a basic charge (*e*) which is the charge on an electron. Thus $q = ne$, where $n = 0, \pm 1, \pm 2, \pm 3, ...$

(*ii*) While dealing with macroscopic charges ($q = ne$), we can ignore quantisation of electric charge. This is because *e* is very small and *n* is very large and so *q* behaves as if it were continuous *i.e.*, as if a large amount of charge is flowing continuously.

1.5. *When a glass rod is rubbed with a silk cloth, charges appear on both. A similar phenomenon is observed with many other pairs of bodies. Explain how this observation is consistent with the law of conservation of charge.*

Ans. It is observed that the positive charge developed on the glass rod has the same magnitude as the negative charge developed on silk cloth. So total charge after rubbing is zero as before rubbing. Hence the law of conservation of charge is being obeyed here.

1.6. *Four point charges $q_A = 2\ \mu$C, $q_B = -5\ \mu$C, $q_C = 2\ \mu$C, $q_D = -5\ \mu$C are located at the corners of a square ABCD of side 10 cm. What is the force on a charge of 1 μC placed at the centre of the square ?*

Ans. Here $OA = OB = OC = OD = \frac{\sqrt{10^2 + 10^2}}{2}$

$$= 5\sqrt{2}\text{ cm} = 5\sqrt{2} \times 10^{-2}\text{ m}$$

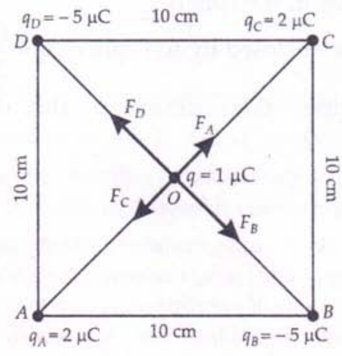


Fig. 1.149

Forces exerted on the charge of 1 μC located at the centre are

$$\vec{F}_A = \frac{9\times10^9 \times 2\times10^{-6} \times 1\times10^{-6}}{(5\sqrt{2}\times10^{-2})^2}$$

$$= 3.6 \text{ N, along } \vec{OC}$$

$$\vec{F}_B = \frac{9\times10^9 \times 5\times10^{-6} \times 1\times10^{-6}}{(5\sqrt{2}\times10^{-2})^2}$$

$$= 9 \text{ N, along } \vec{OB}$$

$$\vec{F}_C = \frac{9\times10^9 \times 2\times10^{-6} \times 1\times10^{-6}}{(5\sqrt{2}\times10^{-2})^2}$$

$$= 3.6 \text{ N, along } \vec{OA}$$

$$\vec{F}_D = \frac{9\times10^9 \times 5\times10^{-6} \times 1\times10^{-6}}{(5\sqrt{2}\times10^{-2})^2}$$

$$= 9 \text{ N, along } \vec{OD}$$

Clearly, $\vec{F}_C = -\vec{F}_A$ and $\vec{F}_D = -\vec{F}_B$

Hence total force on 1μC charge is

$$\vec{F} = \vec{F}_A + \vec{F}_B + \vec{F}_C + \vec{F}_D$$

$$= \vec{F}_A + \vec{F}_B - \vec{F}_A - \vec{F}_B = \textbf{zero N.}$$

1.7. *(a) An electrostatic field line is a continuous curve. That is, a field line cannot have sudden breaks. Why not ?*

(b) Explain why two field lines never cross each other at any point ? [Punjab 01, 02 ; CBSE D 05, 03 ; OD 14]

Ans. (*a*) Electric lines of force exist throughout the region of an electric field. The electric field of a charge decreases gradually with increasing distance from it and becomes zero at infinity *i.e.*, electric field cannot vanish abruptly. So a line of force cannot have sudden breaks, it must be a continuous curve.

(*b*) If two lines of force intersect, then there would be two tangents and hence two directions of electric field at the point of intersection, which is not possible.

1.8. *Two point charges $q_A = +3$ μC and $q_B = -3$ μC are located 20 cm apart in vacuum. (i) Find the electric field at the midpoint O of the line AB joining the two charges. (ii) If a negative test charge of magnitude 1.5×10^{-9} C is placed at the centre, find the force experienced by the test charge.* [CBSE OD 03]

Ans. The directions of the fields E_A and E_B due to the charges q_A and q_B at the midpoint P are as shown in Fig. 1.150.

Electric field at the midpoint O due to q_A,

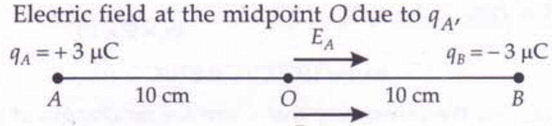


Fig. 1.150

$$E_A = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_A}{r^2} = \frac{9\times10^9 \times 3\times10^{-6}}{(0.10)^2}$$

$$= 2.7\times10^6 \text{ NC}^{-1}, \text{ along } OB$$

Electric field at the midpoint O due to q_B,

$$E_B = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_B}{r^2} = \frac{9\times10^9 \times 3\times10^{-6}}{(0.10)^2}$$

$$= 2.7\times10^6 \text{ NC}^{-1}, \text{ along } OB$$

Resultant field at the midpoint O is

$$E = E_A + E_B = (2.7 + 2.7)\times10^6$$

$$= \mathbf{5.4\times10^6 \text{ NC}^{-1}}, \text{ along } OB.$$

(*ii*) Force on a negative charge of 1.5×10^{-9}C placed at the midpoint O,

$$F = qE = 1.5\times10^{-9} \times 5.4\times10^6$$

$$= \mathbf{8.1\times10^{-3} \text{ N}}, \text{ along } OA$$

The force on a negative charge acts in a direction opposite to that of the electric field.

1.9. *A system has two charges $q_A = 2.5\times10^{-7}$ C and $q_B = -2.5\times10^{-7}$ C, located at points A (0, 0, – 15 cm) and B (0, 0, +15 cm) respectively. What is the total charge and electric dipole moment of the system ?*

Ans. Clearly, the two charges lie on Z-axis on either side of the origin and at 15 cm from it, as shown in Fig. 1.151.

$\therefore$ $2a = 30 \text{ cm} = 0.30 \text{ m}, q = 2.5\times10^{-7}$ C

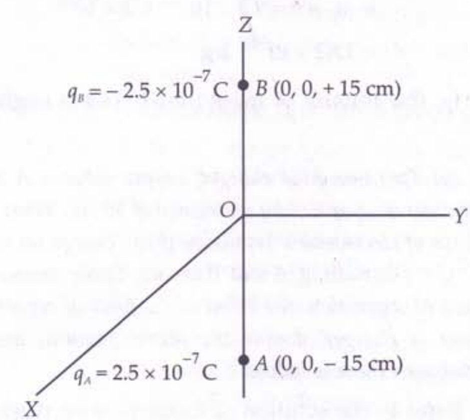


Fig. 1.151

Total charge $= q_A + q_B = 2.5\times10^{-7} - 2.5\times10^{-7} = 0$

Dipole moment,

$$p = q\times 2a = 2.5\times10^{-7} \times 0.30$$

$$= \mathbf{0.75\times10^{-7} \text{ Cm}}$$

The dipole moment acts in the direction from B to A *i.e.*, along negative Z-axis.

1.10. *An electric dipole with dipole moment 4×10^{-9} Cm is aligned at 30° with the direction of a uniform electric field of magnitude 5×10^4 NC^{-1}. Calculate the magnitude of the torque acting on the dipole.*

Ans. Here $p = 4 \times 10^{-9}$ Cm, $\theta = 30°$, $E = 5 \times 10^4$ NC^{-1}

$$\therefore \text{ Torque, } \tau = pE \sin\theta$$
$$= 4 \times 10^{-9} \times 5 \times 10^4 \times \sin 30°$$
$$= \mathbf{10^{-4}\ Nm.}$$

1.11. *A polythene piece rubbed with wool is found to have a negative charge of 3.2×10^{-7} C. (i) Estimate the number of electrons transferred. (ii) Is there a transfer of mass from wool to polythene ?*

Ans. (*i*) Here $q = 3.2 \times 10^{-7}$ C, $e = 1.6 \times 10^{-19}$ C

As $q = ne$, therefore

Number of electrons transferred,

$$n = \frac{q}{e} = \frac{3.2 \times 10^{-7}}{1.6 \times 10^{-19}} = \mathbf{2 \times 10^{12}}$$

Since polythene has negative charge, so electrons are transferred from wool to polythene during rubbing.

(*ii*) Yes, there is a transfer of mass from wool to polythene because each electron has a finite mass of 9.1×10^{-31} kg.

Mass transferred

$$= m_e \times n = 9.1 \times 10^{-31} \times 2 \times 10^{12}$$
$$= \mathbf{1.82 \times 10^{-18}\ kg}$$

Clearly, the amount of mass transferred is negligibly small.

1.12. *(a) Two insulated charged copper spheres A and B have their centres separated by a distance of 50 cm. What is the mutual force of electrostatic repulsion if the charge on each is 6.5×10^{-7} C ? The radii of A and B are negligible compared to the distance of separation. (b) What is the force of repulsion if each sphere is charged double the above amount, and the distance between them is halved ?*

Ans. Refer to the solution of Example 9 on page 1.12.

1.13. *Suppose the spheres A and B in Exercise 1.12 have identical sizes. A third sphere of the same size but uncharged is brought in contact with the first, then brought in contact with the second, and finally removed from both. What is the new force of repulsion between A and B ?*

Ans. Refer to the solution of Example 10 on page 1.12.

1.14. *Figure 1.152 shows tracks of three charged particles in a uniform electrostatic field. Give the signs of the three charges. Which particle has the highest charge to mass ratio ?*

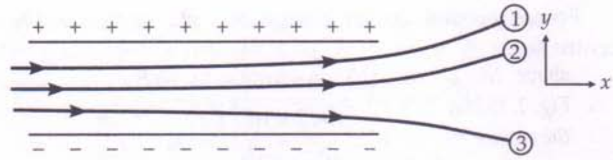


Fig. 1.152

Ans. Refer to the solution of Problem 9 on page 1.73.

1.15. *Consider a uniform electric field : $\vec{E} = 3 \times 10^3\ \hat{i}\ NC^{-1}$ (i) What is the flux of this field through a square of 10 cm on a side whose plane is parallel to the Y-Z-plane ? (ii) What is the flux through the same square if the normal to its plane makes a 60° angle with the X-axis ?*

Ans. (*i*) Normal to a plane parallel to Y-Z plane points in X-direction, so

$$\Delta\vec{S} = 0.10 \times 0.10\,\hat{i}\ \ m^2 = 0.01\,\hat{i}\ \ m^2$$

Electric flux,

$$\phi_E = \vec{E}\,.\,\Delta\vec{S} = 3 \times 10^3\ \hat{i}\ .\ 0.01\,\hat{i}$$
$$= 30\,\hat{i}\,.\,\hat{i} = \mathbf{30\ Nm^2C^{-1}}.$$

(*ii*) Here $\theta = 60°$

$$\therefore \quad \phi_E = E\Delta S \cos 60° = 3 \times 10^3 \times 0.01 \cos 60°$$
$$= 30 \times \frac{1}{2} = \mathbf{15\ Nm^2C^{-1}}.$$

1.16. *Consider a uniform electric field : $\vec{E} = 3 \times 10^3\ \hat{i}\ NC^{-1}$. What is the net flux of this field through a cube of side 20 cm oriented so that its faces are parallel to the coordinate planes ?*

Ans. The flux entering one face parallel to Y-Z plane is equal to the flux leaving other face parallel to Y-Z plane. Flux through other faces is zero. Hence net flux through the cube is zero.

1.17. *Careful measurement of the electric field at the surface of a black box indicates that the net outward flux through the surface of the box is 8.0×10^3 Nm^2C^{-1}. (i) What is the net charge inside the box ? (ii) If the net outward flux through the surface of the box were zero, could you conclude that there were no charges inside the box ? Why or why not ?*

Ans. (*i*) $\phi_E = 8.0 \times 10^3$ Nm^2C^{-2}

Using Gauss theorem,

$$\phi_E = \frac{q}{\varepsilon_0}$$

$$\text{Charge,} \quad q = \varepsilon_0 \cdot \phi_E = 8.0 \times 10^3 \times \frac{1}{4\pi \times 9 \times 10^9}\ \text{C}$$
$$= 0.07 \times 10^{-6}\ \text{C} = \mathbf{0.07\,\mu C}$$

(*ii*) No, we cannot say that there are no charges at all inside the box. We can only say that the net charge inside the box is zero.

1.18. *A point charge +10 μC is a distance 5 cm directly above the centre of a square of side 10 cm as shown in Fig. 1.153(a). What is the magnitude of the electric flux through the square ? (Hint : Think of the square as one face of a cube with edge 10 cm.)*

Ans. We can imagine the square as face of a cube with edge 10 cm and with the charge of + 10 μC placed at its centre, as shown in Fig. 1.153(*b*).

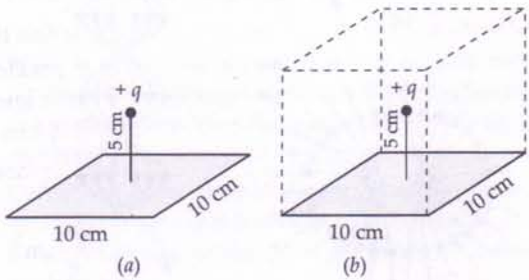


Fig. 1.153

Symmetry of six faces of a cube about its centre ensures that the flux ϕ_S through each square face is same when the charge q is placed at the centre.

∴ Total flux,

$$\phi_E = 6 \times \phi_S = \frac{q}{\varepsilon_0}$$

or

$$\phi_S = \frac{q}{6\varepsilon_0} = \frac{1}{6} \times 10 \times 10^{-6} \times 4\pi \times 9 \times 10^9$$

$$= \mathbf{1.88 \times 10^5\ Nm^2C^{-1}}.$$

1.19. *A point charge of 2.0 μC is at the centre of a cubic Gaussian surface 9.0 cm on edge. What is the net electric flux through the surface ?*

Ans. Here $q = 2.0\,\mu\text{C} = 2.0 \times 10^{-6}\text{C}$,

$$\varepsilon_0 = 8.85 \times 10^{-12} \text{C}^2\text{N}^{-1}\text{m}^{-2}$$

By Gauss's theorem, electric flux is

$$\phi_E = \frac{q}{\varepsilon_0} = \frac{2.0 \times 10^{-6}}{8.85 \times 10^{-12}} = \mathbf{2.26 \times 10^5\ Nm^2\ C^{-1}}.$$

1.20. *A point charge causes an electric flux of -1.0×10^3 $Nm^2\ C^{-1}$ to pass through a spherical Gaussian surface of 10.0 cm radius centred on the charge. (i) If the radius of the Gaussian surface were doubled, how much flux would pass through the surface ? (ii) What is the value of the point charge ?*

Ans. (*i*) $\phi_E = -10^3\ \text{Nm}^2\text{C}^{-1}$, because the charge enclosed is the same in both the cases.

(*ii*) Charge,

$$q = \varepsilon_0\,\phi_E$$

$$= \frac{1}{4\pi \times 9 \times 10^9} \times (-1.0 \times 10^3)$$

$$= -8.84 \times 10^{-9}\ \text{C} = \mathbf{-8.84\ nC.}$$

1.21. *A conducting sphere of radius 10 cm has an unknown charge. If the electric field 20 cm from the centre of the sphere is 1.5×10^3 NC^{-1} and points radially inward, what is the net charge on the sphere ?*

Ans. Electric field at the outside points of a conducting sphere is

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$$

$$\therefore \quad q = 4\pi\varepsilon_0 Er^2 = \frac{1}{9 \times 10^9} \times 1.5 \times 10^3 \times (0.20)^2\ \text{C}$$

$$= 6.67 \times 10^{-9}\ \text{C} = 6.67\ \text{nC}$$

As the field acts inwards, the charge q must be negative.

$$\therefore \quad q = -\mathbf{6.67\ nC}.$$

1.22. *A uniformly charged conducting sphere of 2.4 m diameter has a surface charge density of 80.0 μC/m². (i) Find the charge on the sphere. (ii) What is the total electric flux leaving the surface of the sphere ?* [CBSE D 09C]

Ans. Here $R = \dfrac{2.4}{2} = 1.2\,\text{m}$

$$\sigma = 80.0\mu\text{Cm}^{-2} = 80 \times 10^{-6}\ \text{Cm}^{-2}$$

(*i*) Charge on the sphere is

$$q = 4\pi\ R^2\ \sigma = 4 \times 3.14 \times (1.2)^2 \times 80 \times 10^{-6}\text{C}$$

$$= \mathbf{1.45 \times 10^{-3}\ C}.$$

(*ii*) Flux,

$$\phi_E = \frac{q}{\varepsilon_0} = 1.45 \times 10^{-3} \times 4\pi \times 9 \times 10^9$$

$$= \mathbf{1.6 \times 10^8\ Nm^2\ C^{-1}}.$$

1.23. *An infinite line charge produces a field of 9×10^4 NC^{-1} at a distance of 2 cm. Calculate the linear charge density.*

Ans. $E = 9 \times 10^4\ \text{NC}^{-1}$, $r = 2\text{cm} = 0.02\,\text{m}$

Electric field of a line charge, $E = \dfrac{\lambda}{2\pi\varepsilon_0 r}$

∴ Linear charge density,

$$\lambda = 2\pi\varepsilon_0 Er = 2\pi \times \frac{1}{4\pi \times 9 \times 10^9} \times 9 \times 10^4 \times 0.02$$

$$= 0.01 \times 10^{-5}\ \text{Cm}^{-1} = \mathbf{0.1\,\mu Cm^{-1}}.$$

1.24. *Two large, thin metal plates are parallel and close to each other. On their inner faces, the plates have surface charge densities of opposite signs and of magnitude 17.0×10^{-22} Cm^{-2}. What is E (a) to the left of the plates, (b) to the right of the plates, and (c) between the plates ?*

Ans. Here $\sigma = 17.0 \times 10^{-22}\ \text{Cm}^{-2}$

(*a*) On the left, the fields of the two plates are equal and opposite, so E = **Zero.**

(b) On the right, the fields of the two plates are equal and opposite, so E = **Zero.**

(c) Between the plates, the fields due to both plates are in same direction. So the resultant field is

$$E = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0} = 17 \times 10^{-22} \times 4\pi \times 9 \times 10^9$$

$$= \mathbf{19.2 \times 10^{-10}\ NC^{-1}}.$$

1.25. *An oil drop of 12 excess electrons is held stationary under a constant electric field of $2.55 \times 10^4\ Vm^{-1}$ in Millikan's oil drop experiment. The density of the oil is $1.26\ g\ cm^{-3}$. Estimate the radius of the drop. ($g = 9.81\ ms^{-2}$; $e = 1.60 \times 10^{-19}$ C)*

Ans. Force on the oil drop due to electric field

$$= qE = neE$$

Weight of oil drop

$$= mg = \text{volume} \times \text{density} \times g = \frac{4}{3}\pi r^3 \rho g$$

The field E must act vertically downward so that the negatively charged oil drop experiences an upward force and balances the weight of the drop.

Fig. 1.154

When the drop is held stationary,

Weight of oil drop

= Force on the oil drop due to electric field

or $\frac{4}{3}\pi r^3 \rho g = neE \quad \therefore \quad r = \left[\frac{3neE}{4\pi\rho g}\right]^{1/3}$

Now $n = 12, \quad e = 1.6 \times 10^{-19}$ C,

$E = 2.55 \times 10^4\ \text{Vm}^{-1}, g = 9.81\ \text{ms}^{-2}$

$\rho = 1.26\ \text{g cm}^{-3} = 1.26 \times 10^3\ \text{kg m}^{-3}$

$$\therefore \quad r = \left[\frac{3 \times 12 \times 1.6 \times 10^{-19} \times 2.55 \times 10^4}{4 \times 3.14 \times 1.26 \times 10^3 \times 9.81}\right]^{1/3}$$

$$= \left[\frac{9 \times 16 \times 255}{314 \times 126 \times 981} \times 10^{-15}\right]^{1/3}$$

$$= (9.46 \times 10^{-4})^{1/3} \times 10^{-5}$$

$$= 0.0981 \times 10^{-5}\ \text{m} = \mathbf{9.81 \times 10^{-4}\ mm.}$$

1.26. *Which among the curves shown in Fig. 1.155, cannot possibly represent electrostatic field lines ?*

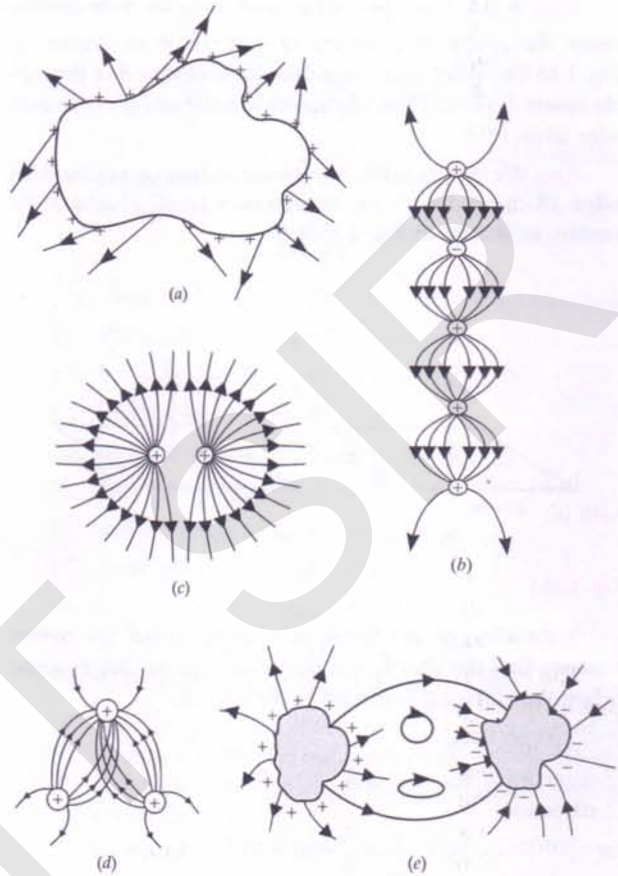


Fig. 1.155

Ans. Only Fig. 1.155(c) is right and the remaining figures cannot represent the electrostatic field lines.

Figure 1.155(a) is wrong because field lines must be normal to a conductor.

Figure 1.155(b) is wrong because lines of force cannot start from a negative charge.

Figure 1.155(c) is right because it satisfies all the properties of lines of force.

Figure 1.155(d) is wrong because lines of force cannot intersect each other.

Figure 1.155(e) is wrong because electrostatic field lines cannot form closed loops.

1.27. *In a certain region of space, electric field is along the Z-direction throughout. The magnitude of electric field is, however, not constant but increases uniformly along the positive Z-direction at the rate of $10^5\ NC^{-1}m^{-1}$. What are the force and torque experienced by a system having a total dipole moment equal to 10^{-7} C m in the negative Z-direction ?*

Ans. The situation is shown in Fig. 1.156.

As the electric field changes uniformly in the positive Z-direction, so

$$\frac{\partial E_z}{\partial z} = +\ 10^5\ \text{NC}^{-1}\text{m}^{-1}, \quad \frac{\partial E_x}{\partial x} = 0, \quad \frac{\partial E_y}{\partial y} = 0$$

As the system has a total dipole moment in the negative Z-direction, so

$$p_z = -\ 10^{-7}\ \text{Cm}, \ p_x = 0, \ p_y = 0$$

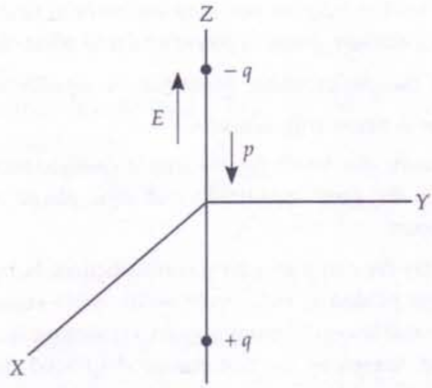


Fig. 1.156

In a non-uniform electric field, the force on the dipole will be

$$F = p_x \frac{\partial E_x}{\partial x} + p_y \frac{\partial E_y}{\partial y} + p_z \frac{\partial E_z}{\partial z}$$

$$= 0 + 0 - 10^{-7} \times 10^5 = \mathbf{-10^{-2}\ N}$$

The negative sign shows that the force on the dipole acts in the negative Z-direction.

As the dipole moment p acts in the negative Z-direction while the electric field E acts in the positive Z-direction, so $\theta = 180°$.

Torque, $\tau = pE \sin 180° = pE \times 0 = \mathbf{0}$.

1.28. *(i) A conductor A with a cavity [Fig. 1.157(a)] is given a charge Q. Show that the entire charge must appear on the outer surface of the conductor.*

(ii) Another conductor B with charge q is inserted into the cavity keeping B insulated from A. Show that the total charge on the outside surface of A is Q + q [Fig. 1.157(b)].

(iii) A sensitive instrument is to be shielded from the strong electrostatic fields in its environment. Suggest a possible way.

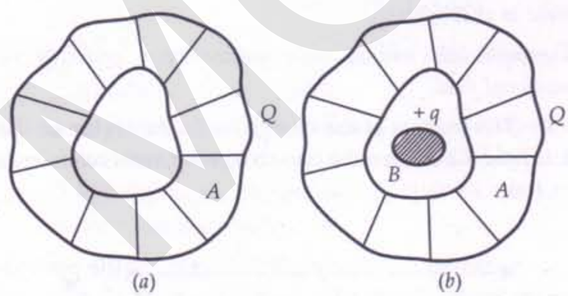


Fig. 1.157

Ans. (*i*) Refer answer to Q.25(6) on page 2.25.

(*ii*) Consider a Gaussian surface inside the conductor but quite close to the cavity.

Inside the conductor, $E = 0$.

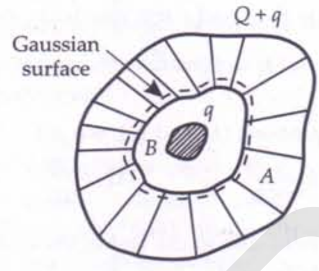


Fig. 1.158

By Gauss's theorem,

$$\phi_E = \oint \vec{E} \cdot \vec{dS} = \frac{\text{Total charge}}{\varepsilon_0} = 0$$

i.e., the total charge enclosed by the Gaussian surface must be zero. This requires a charge of $-q$ units to be induced on inner surface of conductor A. But an equal and opposite charge of $+q$ units must appear on outer surface A so that charge on the surface of A is $Q + q$.

Hence the total charge on the surface of A is $Q + q$.

(*iii*) The instrument should be enclosed in a metallic case. This will provide an electrostatic shielding to the instrument.

1.29. *A hollow charged conductor has a tiny hole cut into its surface. Show that the electric field in the hole is* $\frac{\sigma}{2\varepsilon_0}\hat{n}$, *where* $\hat{n}$ *is the unit vector in the outward normal direction, and* σ *is the surface charge density near the hole.*

Ans. Consider the charged conductor with the hole filled up, as shown by shaded portion in Fig. 1.159. Applying Gauss's theorem, we find that field just outside is $\frac{\sigma}{\varepsilon_0}\hat{n}$ and is zero inside. This field can be viewed as the superposition of the field E_2 due to the filled up hole plus

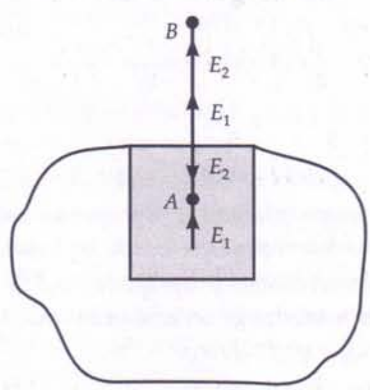


Fig. 1.159

the field E_1 due to the rest of the charged conductor. Since inside the conductor the field vanishes, the two fields must be equal and opposite, *i.e.*,

$$E_1 - E_2 = 0 \qquad ...(1)$$

And outside the conductor, the fields are added up :

$$E_1 + E_2 = \frac{\sigma}{\varepsilon_0} \quad ...(2)$$

Adding equations (1) and (2), we get

$$2E_1 = \frac{\sigma}{\varepsilon_0} \quad \text{or} \quad E_1 = \frac{\sigma}{2\varepsilon_0}$$

Hence the field due to the rest of the conductor or the field in the hole is

$$E = \frac{\sigma}{2\varepsilon_0}\hat{n}$$

where $\hat{n}$ is a unit vector in the outward normal direction.

1.30. *Obtain the formula for the electric field due to a long thin wire of uniform linear charge density λ without using Gauss's law.*

[Hint. Use Coulomb's law directly and evaluate the necessary integral.]

Ans. Refer to the solution of Example 47 on page 1.37.

1.31. *It is now believed that protons and neutrons are themselves built out of more elementary units called quarks. A proton and a neutron consists of three quarks each. Two types of quarks, the so called 'up' quark (denoted by u) of charge + (2/3) e, and the 'down' quark (denoted by d) of charge (− 1/3) e, together with electrons build up ordinary matter. Suggest a possible quark composition of a proton and neutron.*

Ans. Charge on 'up' quark $(u) = +\frac{2}{3}e$

Charge on 'down' quark $(d) = -\frac{1}{3}e$

Charge on a proton $= e$

Charge on a neutron $= 0$

Let a proton contain x 'up' quarks and $(3 - x)$ 'down' quarks. Then total charge on a proton is

$$ux + d(3-x) = e$$

or $$\frac{2}{3}ex - \frac{1}{3}e(3-x) = e$$

or $$\frac{2}{3}x - 1 + \frac{x}{3} = 1$$

or $$x = 2 \quad \text{and} \quad 3 - x = 3 - 2 = 1$$

Thus a proton contains 2 'up' quarks and 1 'down' quark. Its quark composition should be : ***uud***.

Let a neutron contain y 'up' quarks and $(3 - y)$ 'down' quarks. Then total charge on a neutron must be

$$uy + d(3-y) = 0$$

or $$\frac{2}{3}ey - \frac{1}{3}e(3-y) = 0$$

or $$\frac{2}{3}y - 1 + \frac{y}{3} = 0$$

or $$y = 1 \text{ and } 3 - y = 3 - 1 = 2$$

Thus a neutron contains 1 'up' quark and 2 'down' quarks. Its composition should be : ***udd***.

1.32. *(a) Consider an arbitrary electrostatic field configuration. A small test charge is placed at a null point (i.e., where $\vec{E} = 0$) of the configuration. Show that the equilibrium of the test charge is necessarily unstable.*

(b) Verify this result for the simple configuration of two charges of the same magnitude and sign placed a certain distance apart.

Ans. (*a*) We can prove it by contradiction. Suppose the test charge placed at null point be in stable equilibrium. Since the stable equilibrium requires restoring force in all directions, therefore, the test charge displaced slightly in any direction will experience a restoring force towards the null point. That is, all field lines near the null point should be directed towards the null point. This indicates that there is a net inward flux of electric field through a closed surface around the null point. But, by Gauss's law, the flux of electric field through a surface enclosing no charge must be zero. This contradicts our assumption. Hence the test charge placed at the centre must be necessarily in unstable equilibrium.

(*b*) The null point lies on the midpoint of the line joining the two charges. If the test charge is displaced slightly on either side of the null point along this line, it will experience a restoring force. But if it is displaced normal to this line, the net force takes it away from the null point. That is, no restoring force acts in the normal direction. But stable equilibrium demands restoring force in all directions, hence test charge placed at null point will not be in stable equilibrium.

1.33. *A particle of mass m and charge (− q) enters the region between the two charged plates initially moving along x-axis with speed v_x (like particle 1 in Fig. 1.152). The length of plate is L and a uniform electric field E is maintained between the plates. Show that the vertical deflection of the particle at the far edge of the plate is $qEL^2/(2m\,v_x^2)$.*

Compare this motion with motion of a projectile in gravitational field.

Ans. The motion of the charge $-q$ in the region of the electric field E between the two charged plates is shown in Fig. 1.160.

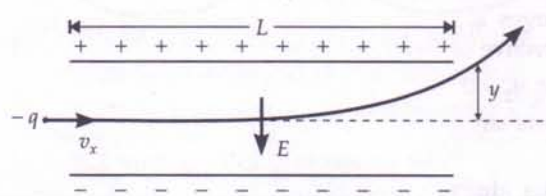


Fig. 1.160

Force on the charge $-q$ in the upward direction is

$$ma = qE$$

$\therefore$ Acceleration, $\quad a = \dfrac{qE}{m}$

Time taken to cross the field, $t = \dfrac{L}{v_x}$

Vertical deflection at the far edge of the plate will be

$$y = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \cdot \frac{qE}{m} \cdot \frac{L^2}{v_x^2} = \frac{qEL^2}{2mv_x^2}$$

Like the motion of a projectile in gravitational field, the path of a charged particle in an electric field is parabolic.

1.34. *Suppose that the particle in Exercise 1.33 is an electron projected with velocity $v_x = 2.0 \times 10^6$ ms^{-1}. If E between the plates separated by 0.5 cm is 9.1×10^2 N/C, where will the electron strike the upper plate ?*
($|e| = 1.6 \times 10^{-19}$ C, $m_e = 9.1 \times 10^{-31}$ kg).

Ans. Here $y = 0.5\text{ cm} = 0.5 \times 10^{-2}$ m,

$$v_x = 2.0 \times 10^6 \text{ ms}^{-1}, \; E = 9.1 \times 10^2 \text{ NC}^{-1}, \; L = ?$$

From the above exercise, the vertical deflection of an electron is given by

$$y = \frac{eEL^2}{2m_e v_x^2}$$

$$\therefore \quad L^2 = \frac{2ym_e v_x^2}{eE}$$

$$= \frac{2 \times 0.5 \times 10^{-2} \times 9.1 \times 10^{-31} \times 4 \times 10^{12}}{1.6 \times 10^{-19} \times 9.1 \times 10^2}$$

$$= 2.5 \times 10^{-4}$$

or $\quad L = 1.58 \times 10^{-2}$ m $\simeq$ **1.6 cm.**

Text Based Exercises

TYPE A : VERY SHORT ANSWER QUESTIONS (1 mark each)

1. What is the cause of charging a body ?
2. An ebonite rod is rubbed with wool or fur. What type of charges do they acquire ? [Haryana 93]
3. A glass rod is rubbed with silk. What type of charges do they acquire ? [CBSE OD 90]
4. Why does an ebonite rod get negatively charged on rubbing with wool ?
5. Consider three charged bodies P, Q and R. If P and Q repel each other and P attracts R, what is the nature of the force between Q and R ?
6. A positively charged glass rod is brought near an uncharged pith ball pendulum. What happens to the pith ball ?
7. When a polythene piece is rubbed with wool, it acquires negative charge. Is there transfer of mass from wool to polythene ?
8. Is the force acting between two point electric charges q_1 and q_2 kept at some distance in air, attractive or repulsive when :
   (*i*) $q_1 q_2 > 0$ (*ii*) $q_1 q_2 < 0$? [CBSE 03, 07]
9. Name any two basic properties of electric charges. [CBSE D 95C ; Punjab 05C]
10. What do you understand by quantisation of electric charges ? [Punjab 07, 10C ; CBSE OD 92]
11. What is the cause of quantisation of electric charge ? [Punjab 10C]
12. What do you mean by additivity of electric charge ?
13. What do you mean by conservation of electric charge ?
14. Is the total charge of the universe conserved ?
15. A glass rod, when rubbed with silk cloth, acquires a charge of 1.6×10^{-13} C. What is the charge on silk cloth ? [CBSE D 91 ; Himachal 99 ; Haryana 99]
16. Two insulated charged copper spheres A and B of identical size have the charges q_A and q_B respectively. A third sphere C of the same size but uncharged is brought in contact with the first and then with the second and finally removed from both. What are the new charges on A and B ? [CBSE F 11]
17. What is the least possible value of charge ? [Haryana 02]
18. State Coulomb's law of force between charges at rest. Express the same in SI units. [CBSE OD 94 ; ISCE 93 ; Haryana 02]
19. In Coulomb's law, $F = k\frac{q_1 q_2}{r^2}$, what are the factors on which the proportionality constant k depends ? [Himachal 02 ; CPMT 93]
20. Name and define the SI unit of charge. [Punjab 09C, 11]
21. In the relation $F = k\frac{q_1 q_2}{r^2}$, what is the value of k in free space ? [Haryana 02]

22. Give the SI unit of electrical permittivity of free space. [Haryana 02]

23. Write down the value of absolute permittivity of free space. [Punjab 96]

24. Deduce the dimensional formula for the proportionality constant k in Coulomb's law.

25. Write the dimensional formula for the permittivity constant ε_0 of free space.

26. What is the force of repulsion between two charges of 1 C each, kept 1 m apart in vacuum ?

27. Two point charges 'q_1' and 'q_2' are placed at a distance 'd' apart as shown in the figure. The electric field intensity is zero at a point 'P' on the line joining them as shown. Write two conclusions that you can draw from this. [CBSE D 14C]

28. Define dielectric constant of a medium in terms of force between electric charges. [CBSE D 05 11C ; F 10 ; Punjab 11]

29. In a medium the force of attraction between two point electric charges, distance d apart, is F. What distance apart should these be kept in the same medium so that the force between them becomes $3F$? [CBSE OD 98]

30. The force between two charges placed in vacuum is F. What happens to the force if the two charges are dipped in kerosene oil of dielectric constant, $\kappa = 2$?

31. State the superposition principle for electrostatic force on a charge due to a number of charges. [NCERT ; Haryana 01]

32. A force F is acting between two point charges q_1 and q_2. If a third charge q_3 is placed quite close to q_2, what happens to the force between q_1 and q_2 ?

33. How many electrons are present in 1 coulomb of charge ? [Himachal 92 ; Punjab 99]

34. Define volume charge density at a point. Write its SI unit.

35. Define surface charge density at a point. Write its SI unit.

36. Define line charge density at a point. Write its SI unit.

37. Define electric field at a point. [CBSE OD 95 ; Punjab 2000]

38. Is electric field intensity a scalar or vector quantity ? Give its SI unit. [CBSE D 99C]

39. Write the dimensional formula of electric field.

40. Name the physical quantity whose SI unit is newton coulomb^{-1}. [CBSE D 98]

41. Draw the pattern of electric field around a point charge (*i*) $q > 0$ and (*ii*) $q < 0$. [CBSE D95, 95C]

42. Sketch the lines of force due to two equal positive charges placed near each other. [CBSE D 96C, 03]

43. Sketch the lines of force of a + ve point charge placed near a –ve point charge of the same magnitude. [CBSE D 96C]

44. Draw the lines of force of an electric dipole. [CBSE OD 95C]

45. Two point charges q_1 and q_2 placed a distance d apart are such that there is no point where the field vanishes. What can be concluded from this ?

46. A proton is placed in a uniform electric field directed along the positive x-axis. In which direction will it tend to move ? [CBSE D 11C]

47. What is an electric dipole ? [CBSE OD 08, 11]

48. Define electric dipole moment. Write its SI unit. [CBSE OD 08, 11 ; F 13]

49. Is electric dipole moment a scalar or vector quantity ? [CBSE 06C ; F 13]

50. What is a point (ideal) dipole ? Give example.

51. How much is the dipole moment of non-polar molecule ?

52. An electric dipole is placed in a uniform electric field. What is the net force acting on it ? [CBSE D 92C ; F 94C]

53. When is the torque on a dipole in a field maximum ?

54. What is the effect of torque on a dipole in an electric field ?

55. When does an electric dipole placed in a non-uniform electric field experience a zero torque but non-zero force ?

56. What is the nature of symmetry of dipole field ?

57. Will an electric dipole have translational motion when placed in a non-uniform electric field ? Give reason for your answer.

58. Does the torque exerted on a dipole in a non-uniform field depend on the orientation of the dipole with respect to the field ?

59. What is the charge of a dipole ? [CBSE D 10C]

60. Under what condition will a charged circular loop behave like a point charge in respect of its electric field ?

61. Define electric flux. [Punjab 2000, 01 ; CBSE D 13C]

62. Name the principle which is mathematical equivalent of Coulomb's law and superposition principle.

63. What is the relation between electric intensity and flux ? [Punjab 97, 98, 99]

64. How is electric flux expressed in terms of surface integral of the electric field ?

65. State Gauss theorem in electrostatics. [Punjab 02 ; CBSE D 08C]

66. Is electric flux a scalar or a vector ? [CBSE Sample Paper 96]
67. Give the SI unit of electric flux ? [CBSE D 13C]
68. Give the SI unit of surface integral $[\oint_S \vec{E} \cdot d\vec{S}]$ of an electric field ?
69. What is the direction of an area vector ?
70. What is a Gaussian surface ?
71. What is the use of Gaussian surface ?
72. How much is the electric flux through a closed surface due to a charge lying outside the closed surface ?
73. Two plane sheets of charge densities $+\sigma$ and $-\sigma$ are kept in air as shown in Fig. 1.161. What are the electric field intensities at points A and B? [CBSE D 03C]

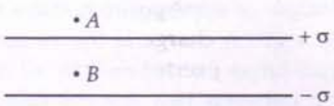


Fig. 1.161

74. Two small balls, having equal positive charge q coulomb are suspended by two insulating strings of equal length l metre from a hook fixed to a stand. The whole set up is taken in a satellite into space where there is no gravity. What is the angle between the two strings and the tension in each string ? [IIT 86]
75. An electric dipole of dipole moment 20×10^{-6} Cm is enclosed by a closed surface. What is the net flux coming out of the surface ? [CBSE D 05]
76. How does the coulomb force between two point charges depend upon the dielectric constant of the medium ? [CBSE OD 05]
77. Two fixed point charges $+4e$ and $+e$ units are separated by a distance a. Where should a third charge q be placed for it to be in equilibrium ? [CBSE OD 05]
78. What is the angle between the directions of electric field at any (*i*) axial point and (*ii*) equitorial point due to an electric dipole ? [CBSE Sample Paper 08]
79. If the radius of the Gaussian surface enclosing a charge q is halved, how does the electric flux through the Gaussian surface change ? [CBSE OD 08]
80. A dipole, of dipole moment $\vec{p}$, is present in a uniform electric field $\vec{E}$. Write the value of the angle between $\vec{p}$ and $\vec{E}$ for which the torque, experienced by the dipole, is minimum. [CBSE D 09C]
81. A charge 'q' is placed at the centre of a cube. What is the electric flux passing through the cube ? [CBSE OD 12]
82. A charge 'q' is placed at the centre of a cube of side l. What is the electric flux passing through each face of the cube ? [CBSE OD 12]
83. A charge 'q' is placed at the centre of a cube of side l. What is the electric flux passing through two opposite faces of the cube ? [CBSE OD 12]
84. A charge $Q\,\mu$C is placed at the centre of a cube. What is the flux coming out from any one surface ? [CBSE F 10]
85. Charges of magnitudes $2Q$ and $-Q$ are located at points (a, o, o) and $(4a, o, o)$. Find the ratio of the flux of electric field, due to these charges, through concentric spheres of radii $2a$ and $8a$ centered at the origin. [CBSE Sample Paper 11]
86. Two charges of magnitudes $-3Q$ and $+2Q$ are located at points $(a, 0)$ and $(4a, 0)$ respectively. What is the electric flux due to these charges through a sphere of radius $5a$ with its centre at the origin ? [CBSE OD 13]
87. Two concentric spherical shells of radii R and $2R$ are given charges Q_1 and Q_2 respectively. The surface charge densities on the outer surfaces are equal. Determine the ratio $Q_1 : Q_2$. [CBSE F13]
88. Write the expression for the torque $\vec{\tau}$ acting on a dipole of dipole moment $\vec{p}$ placed in an electric field $\vec{E}$. [CBSE F 15]
89. What is the electric flux through a cube of side 1 cm which encloses an electric dipole ? [CBSE D 15]

Answers

1. Charging occurs due to the transfer of electrons from one body to another.
2. The ebonite rod acquires a negative charge and fur or wool acquires an equal positive charge.
3. The glass rod acquires positive charge and silk acquires an equal negative charge.
4. This is because electrons in wool are less tightly bound than electrons in ebonite rod.
5. Attractive.
6. The pith ball is attracted towards the rod, touches it and then thrown away.

7. The polythene piece acquires negative charge due to transfer of material particles like electrons from wool to it, so there is a transfer of mass from wool to polythene.

8. (*i*) When $q_1 q_2 > 0$, the force is repulsive
(*ii*) When $q_1 q_2 < 0$, the force is attractive.

9. Electric charges are (*i*) quantised, (*ii*) additive and (*iii*) conserved.

10. Quantisation of electric charge means that the total charge (q) of a body is always an integral multiple of a basic charge (e) which is the charge on an electron.
Thus $\quad q = ne, \quad$ where $n = 0, \pm 1, \pm 2, \pm 3, ...$

11. The basic cause of quantisation of electric charge is that during rubbing only an integral number of electrons can be transferred from one body to another.

12. Additivity of electric charge means that the total charge on a system is the algebraic sum (taking into account proper signs) of all individual charges in the system.

13. Conservation of electric charge means that the total charge of an isolated system remains unchanged with time.

14. Yes, charge conservation is a global phenomenon.

15. To conserve charge, the silk cloth acquires negative charge of 1.6×10^{-13}C.

16. New charge on sphere A,
$$q'_A = \frac{q_A}{2}$$
New charge on sphere B,
$$q'_B = \frac{q_B + q_A/2}{2} = \frac{2q_B + q_A}{4}$$

17. The least possible value of charge is the magnitude of the charge on an electron or proton and it is $e = 1.6 \times 10^{-19}$ C.

18. Refer to point 14 of Glimpses on page 1.100.

19. The proportionality constant k depends on the nature of the medium between the two charges and the system of units chosen.

20. The SI unit of electric charge is coulomb. One coulomb is that amount of charge which repels an equal and similar charge with a force of 9×10^9N when placed in vacuum at a distance of 1 metre from it.

21. $k = 9 \times 10^9 \text{ Nm}^2\text{C}^{-2}$.

22. SI unit of $\varepsilon_0 = \text{C}^2\text{N}^{-1}\text{m}^{-2}$.

23. Permittivity of free space,
$$\varepsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}.$$

24. $[k] = \dfrac{Fr^2}{q_1 q_2} = \dfrac{\text{MLT}^{-2}\text{L}^2}{(\text{AT})^2} = [\text{ML}^3\text{T}^{-4}\text{A}^{-2}]$.

25. $[\varepsilon_0] = \dfrac{1}{4\pi F} \cdot \dfrac{q_1 q_2}{r^2} = \dfrac{(\text{AT})^2}{[\text{MLT}^{-2}\text{L}^2]} = [\text{M}^{-1}\text{L}^{-3}\text{T}^4\text{A}^2]$.

26. $F = 9 \times 10^9$ N.

27. (*i*) The point charges q_1 and q_2 must be of opposite nature or signs.
(*ii*) The magnitude of charge q_1 must be greater than that of charge q_2.

28. The dielectric constant of a medium is the ratio of the force between two charges placed some distance apart in vacuum to the force between the same two charges when they are placed the same distance apart in the given medium.

29. As $F \propto \dfrac{1}{d^2} \quad \therefore \quad \dfrac{3F}{F} = \dfrac{d^2}{d'^2} \quad$ or $\quad d' = \dfrac{1}{\sqrt{3}} d$.

30. $F_{\text{kerosene}} = \dfrac{F_{\text{air}}}{\kappa} = \dfrac{F}{2}$.

31. The principle of superposition states that the total force on a given charge is the vector sum of the individual forces exerted on it by all other charges, the force between two charges being exerted in such a manner as if all other charges were absent.
$$\vec{F} = \vec{F}_{12} + \vec{F}_{13} + ... + \vec{F}_{1N}$$

32. By the superposition principle, the force between two charges does not depend on the presence of third charge. Hence the force between q_1 and q_2 remains equal to F.

33. $n = \dfrac{q}{e} = \dfrac{1\text{C}}{1.6 \times 10^{-19}\text{ C}} = \mathbf{6.25 \times 10^{18}}$ **electrons.**

34. The volume charge density (ρ) at a point is defined as the charge contained per unit volume around that point.
$$\rho = \frac{dq}{dV}$$
The SI unit of ρ is coulomb per cubic metre (C m^{-3}).

35. The surface charge density (σ) at a point is the charge per unit area around that point
$$\sigma = \frac{dq}{dS}.$$
The SI unit for σ is Cm^{-2}.

36. The line charge density at a point on a line is the charge per unit length of the line at that point
$$\lambda = \frac{dq}{dL}$$
The SI unit for λ is Cm^{-1}.

37. The electric field at a point is defined as the electrostatic force per unit positive charge acting on a vanishingly small test charge placed at that point.
Mathematically, $\quad \vec{E} = \lim\limits_{q_0 \to 0} \dfrac{\vec{F}}{q_0}$.

38. Electric field is a vector quantity. Its direction is same as that of the force on a unit positive test charge.

SI unit of electric field = NC^{-1} or Vm^{-1}.

39. [Electric Field]

$$= \frac{\text{Force}}{\text{Charge}} = \frac{MLT^{-2}}{C}$$

$$= \frac{MLT^{-2}}{AT} = [MLT^{-3}A^{-1}].$$

40. NC^{-1} is the SI unit of electric field.

41. (*i*) See Fig. 1.74 on page 1.47 (*ii*) See Fig. 1.75 on page 1.47.

42. See Fig. 1.77 on page 1.47.

43. See Fig. 1.76 on page 1.47.

44. See Fig. 1.76 on page 1.47.

45. The point charges q_1 and q_2 are equal and opposite.

46. As the proton has a positive charge, it will tend to move along positive x-axis *i.e.*, along the direction of the electric field.

47. An electric dipole is a pair of equal and opposite charges separated by some distance.

48. Electric dipole moment is the product of either charge q and the vector $2\vec{a}$ drawn from the – ve charge to the + ve charge.

$$\vec{p} = q \times 2\vec{a}$$

Its SI unit is coulomb metre (Cm).

49. Electric dipole moment is a vector quantity.

50. A point dipole is one which has negligibly small size. In such a dipole, charge $q \to \infty$ and size $2a \to 0$ in such a way that the product $p = q \times 2a$ has a finite value. Atomic dipoles are point dipoles.

51. Zero.

52. Zero.

53. Torque is maximum when dipole is held perpendicular to the electric field.

54. Torque tends to align the dipole along the direction of the electric field.

55. When the dipole is placed parallel to the non-uniform electric field.

56. The dipole field is cylindrically symmetric.

57. Yes. In a non-uniform electric field, an electric dipole experiences unequal forces at its ends. The two forces add up to give a resultant force which gives a translatory motion to the dipole.

58. Yes. In a non-uniform electric field, the field vector $\vec{E}\ (\vec{r})$ changes from point to point, either in magnitude or in direction or both. Therefore, the torque $\vec{\tau} = \vec{p} \times \vec{E}\ (\vec{r})$ for a dipole located at $\vec{r}$ changes with the change in orientation of the dipole with respect to the field.

59. Zero.

60. When the observation point on the axis of the circular loop lies at a distance much greater than its radius, the electric field of the circular loop is similar to that of a point charge.

61. Electric flux over an area in an electric field represents the total number of electric lines of force crossing this area normally. If the normal drawn to the surface area $\Delta\vec{S}$ makes an angle θ with the field $\vec{E}$, then the electric flux through this area is

$$\phi_E = E\Delta S \cos\theta = \vec{E} \cdot \Delta\vec{S}$$

62. Gauss's theorem of electrostatics.

63. The relation between electric intensity $\vec{E}$ and flux ϕ_E is $\phi_E = E\,\Delta S \cos\theta$

64. The electric flux ϕ_E through any surface, open or closed, is equal to the surface integral of the electric field $\vec{E}$ over the surface $\vec{S}$,

$$\phi_E = \int_S \vec{E} \cdot d\vec{S}$$

65. Gauss's theorem states that the flux of electric field through any closed surface S is $1/\varepsilon_0$ times the total charge q enclosed by S.

Mathematically,

$$\phi_E = \oint_S \vec{E} \cdot d\vec{S} = \frac{q}{\varepsilon_0}.$$

66. Electric flux is a scalar.

67. SI unit of electric flux = $Nm^2\,C^{-1}$.

68. SI unit of $\oint \vec{E} \cdot d\vec{S} = Nm^2C^{-1}$.

69. The direction of an area vector is along the outward drawn normal to the surface.

70. An imaginary closed surface enclosing a charge is called the Gaussian surface of that charge.

71. By a clever choice of Gaussian surface, we can easily find the electric field produced by certain charge systems which are otherwise quite difficult to determine by the application of Coulomb's law and superposition principle.

72. Zero.

73. $E_A = 0$ and $E_B = \dfrac{\sigma}{\varepsilon_0}$.

74. As the two balls are in the state of weightlessness, the strings would become horizontal due to the force of repulsion.

$\therefore$ Angle between the two strings = 180°

Tension in each string $= \frac{1}{4\pi\varepsilon_0} \cdot \frac{q^2}{(2l)^2}$ N.

75. Zero, because the net charge on the dipole is zero.

76. $F_{med} = \frac{F_{vac}}{\kappa}$ *i.e.,* $F_{med} \propto \frac{1}{\kappa}$

77. Refer to the solution of Example 13 on page 1.13.

78. (*i*) At any axial point, $\vec{E}$ acts in the direction $\vec{p}$.

(*ii*) At any equatorial point, $\vec{E}$ acts in the opposite direction of $\vec{p}$

Hence the angle between the directions of the above two electric fields is 180°.

79. Electric flux $(\phi_E = q / \varepsilon_0)$ remains unchanged because the charge enclosed by the Gaussian surface remains same.

80. Torque experienced by the dipole is minimum when angle between $\vec{p}$ and $\vec{E}$ is 0°.

$$\tau = pE \sin 0° = 0.$$

81. $\phi_E = \frac{q}{\varepsilon_0}$

82. $\phi_E = \frac{1}{6} \cdot \frac{q}{\varepsilon_0}$

83. $\phi_E = \frac{2}{6} \cdot \frac{q}{\varepsilon_0} = \frac{1}{3} \cdot \frac{q}{\varepsilon_0}$

84. Flux through each face of the cube,

$$\phi_E = \frac{1}{6} \cdot \frac{Q}{\varepsilon_0} \ \mu\text{Nm}^2\text{C}^{-1}$$

85. $\frac{\phi(r = 2a)}{\phi(r = 8a)} = \frac{2Q}{2Q - Q} = 2:1$

86. $\phi_E = \frac{\text{Net charge enclosed}}{\varepsilon_0}$

$$= \frac{-3Q + 2Q}{\varepsilon_0} = -\frac{Q}{\varepsilon_0}$$

87. $\frac{Q_1}{Q_2} = \frac{4\pi R^2 \sigma}{4\pi (2R)^2 \sigma} = \mathbf{1:4.}$

88. $\vec{\tau} = \vec{p} \times \vec{E}$

89. Zero, because the net charge on the dipole is zero.

TYPE B : SHORT ANSWER QUESTIONS (2 or 3 marks each)

1. What is frictional electricity ? Briefly describe the electronic theory of frictional electricity.
2. What is electric charge ? Is it a scalar or vector ? Name its SI unit.
3. How will you show experimentally that there are only two kinds of electric charges ?
4. Define electrostatic induction. Briefly explain how an insulated metal sphere can be positively charged by induction.
5. What is meant by quantization of electric charge ? What is its cause ? [Haryana 2000 ; Punjab 01]
6. Give six properties of electric charges. [Punjab 99C]
7. State the law of conservation of charge. Give two examples to illustrate it. [Himachal 96 ; Haryana 98, 2000 ; Punjab 06C, 10C]
8. How does the speed of an electrically charged particle affects its (*i*) mass and (*ii*) charge ? [CBSE D 93]
9. State Coulomb's law of force between two electric charges and state its limitations. Also define the SI unit of electric charge. [Haryana 96 ; Punjab 2003]
10. Write Coulomb's law in vector form. What is the importance of expressing it in vector form ? [Haryana 91, 95 ; Punjab 98C, 2000]
11. Write the vector form of force acting between two charges q_1 and q_2 having $\vec{r_1}$ and $\vec{r_2}$ as their position vectors respectively. [Himachal 2000]
12. State Coulomb's law in vector form and prove that
$$\vec{F}_{21} = -\vec{F}_{12}$$
where letters have their usual meanings. [Haryana 97]
13. Define electric field intensity. What is its SI unit ? What is relation between electric field and force ? [CBSE OD 91]
14. Define electric field at a point. Give its physical significance.
15. Derive an expression for electric field intensity at a point at distance r from a point charge q. [CBSE OD 94 ; Haryana 95, 99]
16. Write an expression for the force exerted on a test charge by a continuous charge distribution.
17. Define the term electric dipole moment of a dipole. State its SI unit. [CBSE OD 08, 11]
18. Define electric field intensity and derive an expression for it at a point on the axial line of a dipole. Also determine its direction. [Punjab 2000, 01 ; Haryana 98, 02 ; CBSE D 92, 95]

19. Define the term 'electric dipole moment'. Is it a scalar or vector ?

Deduce an expression for the electric field at a point on the equatorial plane of an electric dipole of length $2a$. [Haryana 02 ; CBSE F 09 ; OD 13]

20. Define electric field intensity. Write its SI unit. Write the magnitude and direction of electric field intensity due to an electric dipole of length $2a$ at the midpoint of the line joining the two charges. [CBSE OD 05]

21. What is an electric dipole ? Derive an expression for the torque acting on an electric dipole, when held in a uniform electric field. Hence define the dipole moment. [Haryana 01, 02 ; CBSE D 08 ; OD 03C]

22. Define the term electric dipole moment. Give its unit. Derive an expression for the maximum torque acting on an electric dipole, when held in a uniform electric field. [CBSE D 02]

23. An electric dipole is placed in uniform external electric field $\vec{E}$. Show that the torque on the dipole is given by $\vec{\tau} = \vec{p} \times \vec{E}$

where $\vec{p}$ is the dipole moment of the dipole. What is the net force experienced by the dipole ? Identify two pairs of perpendicular vectors in the expression. [CBSE D 15C]

24. Draw a labelled diagram showing an electric dipole making an angle θ with a uniform electric field $\vec{E}$. Derive an expression for the torque experienced by the dipole. [ISCE 95 ; CBSE OD 14]

25. An electric dipole is held in a uniform electric field.
   (*i*) Using suitable diagram, show that it does not undergo any translatory motion, and
   (*ii*) Derive an expression for the torque acting on it and specify its direction. When is this torque maximum ? [CBSE D 05, 08]

26. In a non-uniform electric field, is there any torque or force acting on a dipole held parallel or antiparallel to the field. If yes, show them by suitable diagrams.

27. Briefly explain how does a comb run through dry hair attract small pieces of paper.

28. Define an electric field line. Draw the pattern of the field lines around a system of two equal positive charges separated by a small distance. [CBSE D 03 ; Sample Paper 11]

29. Define electric line of force and give its two important properties. [CBSE D 05]

30. What do electric lines of force represent ? Explain repulsion between two like charges on their basis. [Punjab 97C]

31. Define electric flux. Write its SI unit.

A charge q is enclosed by a spherical surface of radius R. If the radius is reduced to half, how would the electric flux through the surface change ? [CBSE OD 09]

32. Prove that $1/r^2$ dependence of electric field of a point charge is consistent with the concept of the electric field lines.

33. State and prove Gauss's theorem in electrostatics. [Punjab 03 ; CBSE OD 92C, 95]

34. Using Gauss's theorem, obtain an expression for the force between two point charges. [CBSE OD 91]

35. State Gauss's theorem and express it mathematically. Using it, derive an expression for the electric field intensity at a point near a thin infinite plane sheet of charge density $\sigma\,\text{Cm}^{-2}$. [Punjab 03 ; CBSE D 07, 09, 12 ; CBSE OD 01, 04, 05, 06C]

36. Using Gauss's law establish that the magnitude of electric field intenisty, at a point, due to an infinite plane sheet with uniform charge density σ, is independent of the distance of the field point. [CBSE Sample Paper 11]

37. Use Gauss's law to derive the expression for the electric field between two uniformly charged large parallel sheets with surface charge densities σ and $-\sigma$ respectively. [CBSE OD 09]

38. State Gauss's theorem in electrostatics. Using this theorem, prove that no electric field exists inside a hollow charged conducting sphere. [Punjab 03 ; CBSE D 02, 03 C ; CBSE OD 97]

39. A thin conducting spherical shell of radius R has charge Q spread uniformly over its surface. Using Gauss's law, derive an expression for an electric field at a point outside the shell. Draw a graph of electric field $E(r)$ with distance r from the centre of the shell for $0 \le r \le \infty$. [CBSE D 04, 08, 09 ; OD 06C, 07]

40. Using Gauss's law obtain the expression for the electric field due to a uniformaly charged thin spherical shell of radius R at a point outside the shell. Draw a graph showing the variation of electric field with r, for $r > R$ and $r < R$. [CBSE D 11]

41. A thin straight infinitely long conducting wire having charge density λ is enclosed by a cylindrical surface of radius r and length l, its axis coinciding with the length of wire. Find the expression for the electric flux through the surface of the cylinder. [CBSE OD 11]

42. State Gauss's theorem in electrostatics. Using this theorem, derive an expression for the electric field intensity due to an infinitely long, straight wire of linear charge density $\lambda\,\text{Cm}^{-1}$. [CBSE D 04, 08, 09 ; OD 05, 06C, 07]

Answers

1. Refer to points 2 and 6 of Glimpses on page 1.99.
2. Refer to point 3 of Glimpses on page 1.99.
3. Refer answer to Q. 5 on page 1.2.
4. Refer answer to Q. 11 on page 1.4 and Q. 13 on page 1.5.
5. Refer answer to Q. 16 on page 1.6.
6. The properties of electric charges are as follows :
   (i) Like charges repel and unlike charges attract each other.
   (ii) Electric charges are quantized.
   (iii) Electric charges are additive.
   (iv) Electric charges are conserved.
   (v) The magnitude of elementary negative charge is same as that of elementary positive charge and is equal to 1.6×10^{-19} C.
   (vi) Unlike mass, the electric charge on a body is not affected by its motion.
7. Refer answer to Q. 18 on page 1.8.
8. Refer answer to Q. 19 on page 1.8.
9. Refer to point 14 of Glimpses and the solution of Problem 3 on page 1.67.
10. Refer answer to Q. 22 on page 1.10.
11. Refer answer to Q. 22 on page 1.10.
12. Refer answer to Q. 22 on page 1.10.
13. Refer answer to Q. 29 and Q. 30 on page 1.25.
14. Refer answer to Q. 29 and Q. 30 on page 1.25.
15. Refer answer to Q. 31 on page 1.29.
16. Refer answer to Q. 33 on page 1.35.
    Add the forces $\vec{F}_V$, $\vec{F}_S$ and $\vec{F}_L$.
17. Refer answer to Q. 48 on page 1.91.
18. Refer answer to Q. 37 on page 1.40.
19. Refer answer to Q. 38 on page 1.40.
20. Refer answer to Q. 29 on page 1.25. At any equatorial point of a dipole,
$$\vec{E}_{\text{equa}} = -\frac{1}{4\pi\varepsilon_0} \cdot \frac{p}{(r^2 + a^2)^{3/2}} \hat{p}$$
    At the midpoint of the dipole ($r = 0$), the magnitude of the field is
$$E_{\text{equa}} = \frac{1}{4\pi\varepsilon_0} \frac{p}{a^3}$$
    The direction of the field is from +ve to –ve charge.
21. Refer answer to Q. 40 on page 1.41.
22. Refer answer to Q. 40 on page 1.41.
23. Refer answer to Q. 40 on page 1.41.
24. Refer answer to Q. 40 on page 1.41.
25. Refer answer to Q. 40 on page 1.41.
26. Refer answer to Q. 41 on page 1.42.
27. Refer answer to Q. 41 on page 1.42.
28. See Fig. 1.77 on page 1.47.
29. Refer answer to Q. 43 on page 1.45.
30. Refer answer to Q. 44(iv) on page 1.47.
31. Refer to point 33 of Glimpses. If the radius of the spherical surface is reduced to half, the electric flux would not change as the charge enclosed remains the same.
32. Refer answer to Q. 46 on page 1.48.
33. Refer answer to Q. 49 on page 1.49.
34. Refer answer to Q. 51 on page 1.50.
35. Refer answer to Q. 53 on page 1.56.
36. Refer answer to Q. 53 on page 1.56.
37. Refer answer to Q. 55 on page 1.57.
38. Refer answer to Q. 56(c) on page 1.58.
39. Refer answer to Q. 56(a) on page 1.58 and see Fig. 1.103.
40. Refer answer to Q. 56(a) on page 1.58 and see Fig. 1.103.
41. Refer for answer to Q. 52 on page 1.56.
42. Refer answer to Q. 52 on page 1.56.

TYPE C : LONG ANSWER QUESTIONS (5 marks each)

1. State the principle of superposition and use it to obtain the expression for the total force exerted on a point charge due to an assembly of $(N-1)$ discrete point charges. [Haryana 02]
2. Obtain an expression for the electric field at any point due to a continuous charge distribution. Hence extend it for the electric field of a general source charge distribution.
3. (a) Consider a system of n charges $q_1, q_2, \ldots, q_n$ with position vectors $\vec{r}_1, \vec{r}_2, \vec{r}_3, \ldots, \vec{r}_n$ relative to some origin 'O'. Deduce the expression for the net electric field $\vec{E}$ at a point P with position vector $\vec{r}_p$, due to this system of charges.

(b) Find the resultant electric field due to an electric dipole of dipole moment $2aq$, ($2a$ being the separation between the charges $\pm q$) at a point distant 'x' on its equator. [CBSE F 15]

4. A dipole is made up of two charges $+q$ and $-q$ separated by a distance $2a$. Derive an expression for the electric field $\vec{E}_e$ due to this dipole at a point distant r from the centre of the dipole on the equatorial plane.

Draw the shape of the graph, between $|\vec{E}_e|$ and r when $r >> a$. If this dipole were to be put in a uniform external electric field $\vec{E}$, obtain an expression for the torque acting on the dipole. [CBSE SP 15]

5. (a) An electric dipole of dipole moment $\vec{p}$ consists of point charges $+q$ and $-q$ separated by a distance $2a$ apart. Deduce the expression for the electric field $\vec{E}$ due to the dipole at a distance x from the centre of the dipole on its axial line in terms of the dipole moment $\vec{p}$. Hence show that in the limit $x >> a$, $\vec{E} \to 2\vec{p}/(4\pi\varepsilon_0 x^3)$.

(b) Given the electric field in the region $\vec{E} = 2x\hat{i}$, find the net electric flux through the cube and the charge enclosed by it. [CBSE D 15]

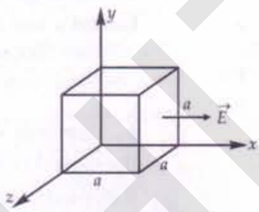


Fig. 1.162

6. (a) State the theorem which relates total charge enclosed within a closed surface and the electric flux passing through it. Prove it for a single point charge.

(b) An 'atom' was earlier assumed to be a sphere of radius a having a positively charged point nucleus of charge $+Ze$ at its centre. This nucleus was believed to be surrounded by a uniform density of negative charge that made the atom neutral as a whole. Use this theorem to find the electric field of this 'atom' at a distance r $(r < a)$ from the centre of the atom. [CBSE SP 15]

7. (a) Define electric flux. Write its SI units.

(b) Using Gauss's law, prove that the electric field at a point due to a uniformly charged infinite plane sheet is independent of the distance from it.

(c) How is the field directed if (i) the sheet is positively charged, (ii) negatively charged ? [CBSE D 12]

8. State Gauss's law in electrostatics. Using this theorem, show mathematically that for any point outside the shell, the field due to uniformly charged thin spherical shell is the same as if entire charge of the shell is concentrated at the centre. Why do you expect the electric field inside the shell to be zero according to this theorem ? [CBSE D 92 ; OD 06]

9. Using Gauss' law, deduce the expression for the electric field due to a uniformly charged spherical conducting shell of radius R at a point (i) outside and (ii) inside the shell.

Plot a graph showing variation of electric field as a function of $r > R$ and $r < R$ (r being the distance from the centre of the shell). [CBSE OD 13, 13C]

10. (a) Using Gauss' law, derive an expression for the electric field intensity at any point outside a uniformly charged thin spherical shell of radius R and the density $\sigma\,C/m^2$. Draw the field lines when the charge density of the sphere is (i) positive, (ii) negative.

(b) A uniformly charged conducting sphere of 2.5 m in diameter has a surface charge density of $100\,\mu C/m^2$. Calculate the (i) charge on the sphere (ii) total electric flux passing through the sphere. [CBSE D 08]

11. (a) Define electric flux. Write its SI unit.

(b) State and explain Gauss's law. Find out the outward flux due to a point charge $+q$ placed at the centre of a cube of side 'a'. Why is it found to be independent of the size and shape of the surface enclosing it ? Explain. [CBSE OD 15]

12. (a) Define electric flux. Write its SI unit.

"Gauss's law in electrostatics is true for any closed surface, no matter what its shape or size is". Justify this statement with the help of a suitable example.

(b) Use Gauss's law to prove that the electric field inside a uniformly charged spherical shell is zero. [CBSE OD 15]

Answers

1. Refer answer to Q. 27 on page 1.19.
2. Refer answer to Q. 34 on page 1.35.
3. (a) Refer answer to Q. 32 on page 1.29.

   (b) Refer answer to Q. 38 on page 1.40.
4. Refer answer to Q. 38 on page 1.40 and Q. 40 on page 1.41.

   For $r >> a$, $E_{equa} \propto \frac{1}{r^3}$.

So the graph between E_{equa} and r is of the type as shown in the figure given below.

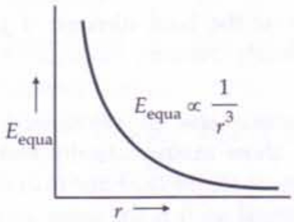


Fig. 1.163

5. (*a*) Refer answer to Q. 37 on page 1.40.

(*b*) Only the faces perpendicular to the *x*-axis contribute towards the electric flux. The contribution from the remaining faces is zero.

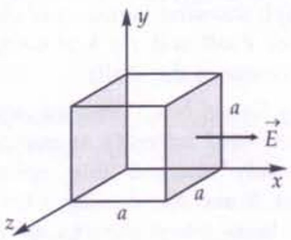


Fig. 1.164

Flux through the left face,

$$\phi_L = ES\cos 180° = 2(0)a^2(-1) = 0$$

Flux through the right face,

$$\phi_R = ES\cos 0° = 2a \times a^2 \times 1 = 2a^3$$

$\therefore$ Net flux through the cube, $\phi_E = \phi_L + \phi_R = 2a^3$

6. (*a*) Refer answer to Q. 49 on page 1.49.

(*b*) Refer to the solution of Problem 29 on page 1.79.

7. (*a*) Refer answer to Problem 18 on page 1.71.

(*b*), (*c*), Refer answer to Q. 53 on page 1.56.

8. Refer answer to Q. 56 on page 1.57. Any Gaussian surface lying inside spherical shell does not enclose any charge. So by Gauss's theorem, electric field inside the shell is zero.

9. Refer answer to Q. 56 on page 1.57.

10. (*a*) Refer answer to Q. 56 (*a*) on page 1.58. The lines of force for positively and negatively charged spherical shells are shown below :

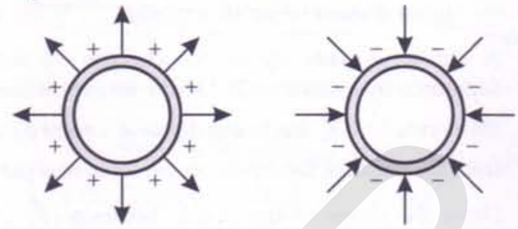


Fig. 1.165

(*b*) (*i*) $q = 4\pi R^2 \sigma = 4 \times 3.14 \times (1.25)^2 \times 10^{-4}$

$= 1.963 \times \mathbf{10^{-3}}$ **C**

(*ii*) $\phi_E = \frac{q}{\varepsilon_0} = 1.963 \times 10^{-3} \times 4\pi \times 10^9$

$= \mathbf{2.465 \times 10^7\ Nm^2C^{-1}}$

11. (*a*) Refer to the solution of Problem 17 on page 1.71.

(*b*) For Gauss's law, refer to point 35 of Glimpses on page 1.102.

Outward flux due to a point charge $+q$ placed at the centre of a cube of side a is given by Gauss's law as

$$\phi_E = \frac{\text{Total charge enclosed}}{\varepsilon_0} = +\frac{q}{\varepsilon_0}$$

ϕ_E depends only the total charge enclosed by the closed surface and not on its size and shape.

12. (*a*) Refer to the solution of Problem 17 on page 1.71.

According to Gauss's law, the electric flux through a closed surface depends on the net charge enclosed by the surface and not upon the size of the surface.

For any closed surface of arbitrary shape enclosing a charge, the outward flux is same as that due to a spherical Gaussian surface enclosing the same charge. This is because of the fact that :

(*i*) electric field is radial, and

(*ii*) the electric field, $E \propto \frac{1}{r^2}$.

(*b*) Refer answer to Q. 56(*c*) on page 1.58.

TYPE D : VALUE BASED QUESTIONS (4 marks each)

1. Aneesha has dry hair. She runs a plastic comb through her hair and finds that the comb attracts small bits of paper. But her friend Manisha has oily hair. The comb passed to Manisha hair could not attract small bits of paper. Aneesha goes to her Physics teacher and gets an explanation of this phenomenon from her. She then goes to different junior classes and demonstrates this experiment to the students. The junior students feel very happy and promise her to join her science club set up for searching such interesting phenomena of nature.

Answer the following questions based on the above information :

(*a*) What are the values displayed by Aneesha ?

(b) A comb run through one's dry hair attracts small bits of paper. But it does not attract when run through wet hair. Why ?

2. Neeta's grandmother, who was illiterate, was wrapping her satin saree. She found some sparks coming out from it. She frightened and called Neeta. Neeta calmed down her grandmother and explained to her the scientific reason behind these sparks.

Answer the following questions based on the above information :

(a) What according to you, are the values displayed by Neeta ?

(b) Why do sparks appear when a satin cloth is folded ?

Answers

1. (a) Curiosity, leadership and compassion.

   (b) When the comb runs through dry hair, it gets charged by friction and attracts small bits of paper. The comb does not get charged when run through wet hair due to less friction and so it does not attract bits of paper.

2. (a) Awareness and sensitivity.

   (b) The different portions of the cloth get charged due to friction. Then the flow of charge gives rise to sparks.

COMPETITION SECTION

Electric Charges and Field

GLIMPSES

1. Electrostatics. It is the study of electric charges at rest.
2. Frictional electricity. The property of rubbed substances due to which they attract light objects is called electricity. The electricity developed by rubbing or friction is called frictional or static electricity. The rubbed substances which show this property of attraction are said to be electrified or electrically charged substances.
3. Electric charge. It is an intrinsic property of elementary particles of matter which gives rise to electric force between various objects. It is a scalar quantity and its SI unit is coulomb (C).
4. Positive and negative charges. Benjamin Franklin introduced the present day convention that
   (*i*) The charge developed on a glass rod when rubbed with silk is called positive charge.
   (*ii*) The charge developed on a plastic/ebonite rod when rubbed with fur is called negative charge.
5. Fundamental law of electrostatics. Like charges repel and unlike charges attract each other.
6. Electronic theory of frictional electricity. During rubbing, electrons are transferred from one object to another. The object with excess of electrons develops a negative charge, while the object with deficit of electrons develops a positive charge.
7. Electrostatic induction. It is the phenomenon of temporary electrification of a conductor in which opposite charges appear at its closer end and similar charges appear at its farther end in the presence of a nearby charged body. An insulated conductor can be positively or negatively charged by induction.
8. Electroscope. A device used for detecting an electric charge and identifying its polarity is called electroscope.
9. Three basic properties of electric charges. These are : (*i*) quantization, (*ii*) additivity, and (*iii*) conservation.
10. Additivity of electric charge. This means that the total charge of a system is the algebraic sum of all the individual charges located at different points inside the system.
11. Quantization of electric charge. This means that the total charge (q) of a body is always an integral multiple of a basic quantum of charge (e) *i.e.*,

    $$q = ne, \text{ where } n = 0, \pm 1, \pm 2, \pm 3, ...$$

    Faraday's laws of electrolysis and Millikan's oil drop experiment established the quantum nature of electric charge.

    For macroscopically large charges, the quantization of charge can be ignored.
12. Basic quantum of charge. The smallest amount of charge or the basic quantum of charge is the charge on an electron or proton. Its exact magnitude is

    $$e = 1.602182 \times 10^{-19} \text{ C}$$
13. Law of conservation of charge. It states that the total charge of a system remains unchanged with time. This means that when bodies are charged through friction, there is only transfer of charge from one body to another b it no net creation or destruction of charge takes place.

14. **Coulomb's law.** The force of attraction or repulsion between two stationary point charges q_1 and q_2 is directly proportional to the product $q_1 q_2$ and inversely proportional to the square of the distance r between them. Mathematically,

$$F = k\,\frac{q_1 q_2}{r^2}$$

The proportionality constant k depends on the nature of the medium between the two charges and the system of units chosen to measure F, q_1, q_2 and r. For free space and in SI units,

$$k = \frac{1}{4\pi\varepsilon_0} = 9\times10^9\ \text{Nm}^2\text{C}^{-2},$$

ε_0 is called permittivity of free space and its value is $8.854\times10^{-12}\ \text{C}^2\text{N}^{-1}\text{m}^{-2}$.

Hence Coulomb's law in SI units may be expressed as

$$F = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r^2}.$$

15. **SI unit of charge is coulomb (C).** It is that amount of charge that repels an equal and similar charge with a force of 9×10^9 N when placed in vacuum at a distance of one metre from it.

16. **Permittivity (ε).** It is the property of a medium which determines the electric force between two charges situated in that medium.

17. **Dielectric constant or relative permittivity.** The ratio $(\varepsilon/\varepsilon_0)$ of the permittivity of the given medium to that of free space is known as relative permittivity (ε_r) or dielectic constant (κ) of the given medium,

$$\varepsilon_r \quad \text{or} \quad \kappa = \frac{\varepsilon}{\varepsilon_0} = \frac{F_{\text{vac}}}{F_{\text{med}}}$$

The dielectric constant of a medium may be defined as the ratio of the force between two charges placed some distance apart in free space to the force between the same two charges when they are placed the same distance apart in the given medium.

Coulomb's law for any medium other than vacuum can be written as

$$F_{\text{med}} = \frac{1}{4\pi\varepsilon}\,\frac{q_1 q_2}{r^2} = \frac{1}{4\pi\varepsilon_0\kappa}\,\frac{q_1 q_2}{r^2} = \frac{F_{\text{vac}}}{\kappa}.$$

18. **Electrostatic force *vs.* gravitational force.** Electrostatic forces are much stronger than gravitational forces. The ratio of the electric force and gravitational force between a proton and an electron is

$$\frac{F_e}{F_G} = \frac{ke^2}{Gm_p m_e} \simeq 2.27\times10^{39}$$

19. **Principle of superposition of electrostatic forces.** When a number of charges are interacting, the total force on a given charge is the vector sum of the forces exerted on it due to all other charges. The force between two charges is not affected by the presence of other charges. The total force on charge q_1 due to the charges $q_2, q_3, \ldots\ldots, q_N$ will be

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \ldots\ldots + \vec{F}_{1N}$$

$$= \frac{q_1}{4\pi\varepsilon_0}\sum_{i=2}^{N}\frac{q_i}{r_{1i}^2}\,\hat{r}_{1i}$$

$$= \frac{q_1}{4\pi\varepsilon_0}\sum_{i=2}^{N}\frac{q_i(\vec{r}_1 - \vec{r}_i)}{|\vec{r}_1 - \vec{r}_i|^3}$$

where $\hat{r}_{1i} = \dfrac{\vec{r}_1 - \vec{r}_i}{|\vec{r}_1 - \vec{r}_i|}$

= a unit vector pointing from q_i to q_1.

20. **Electric field.** An electric field is said to exist at a point, if a force of electrical origin is exerted on a stationary charge placed at that point. Quantitatively, it is defined as the electrostatic force per unit test charge acting on a vanishingly small positive test charge placed at the given point. Mathematically,

$$\vec{E} = \lim_{q_0\to 0}\frac{\vec{F}}{q_0}$$

Electric field is a vector quantity whose direction is same as that of the force exerted on a positive test charge.

21. **Units and dimensions of electric field.** The SI unit of electric field is newton per coulomb (NC^{-1}) or volt per metre (Vm^{-1}). The dimensions of electric field are

$$[E] = \frac{\text{Force}}{\text{Charge}} = \frac{\text{MLT}^{-2}}{\text{C}}$$

$$= \frac{\text{MLT}^{-2}}{\text{AT}} = [\text{MLT}^{-3}\text{A}^{-1}]$$

22. Electric field due to a point charge. The electric field of a point charge q at distance r from it is given by

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$$

If q is positive, E points radially outwards and if q is negative, E points radially inwards. This field is spherically symmetric.

23. Electric field due to a system of point charges : Superposition principle for electric fields. The principle states that the electric field at any point due to a group of point charges is equal to the vector sum of the electric fields produced by each charge individually at that point, when all other charges are assumed to be absent.

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \ldots\ldots + \vec{E}_N$$

$$= \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{N} \frac{q_i}{r_{iP}^2} \hat{r}_{iP}$$

$$= \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{N} \frac{q_i}{|\vec{r} - \vec{r}_i|^3} (\vec{r} - \vec{r}_i)$$

24. Continuous charge distribution. When the charge involved is much greater than the charge on an electron, we can ignore its quantum nature and assume that the charge is distributed in a continuous manner. This is known as a continuous charge distribution.

Volume charge density, $\rho = \frac{dq}{dV}$ Cm^{-3}

Surface charge density, $\sigma = \frac{dq}{dS}$ Cm^{-2}

Linear charge density, $\lambda = \frac{dq}{dL}$ Cm^{-1}

25. Electrostatic force and field due to a continuous charge distribution. The total force on a charge q_0 due to a continuous charge distribution is given by

$$\vec{F}_{\text{cont}} = \vec{F}_V + \vec{F}_S + \vec{F}_L$$

or $$\vec{F}_{\text{cont}} = \frac{q_0}{4\pi\varepsilon_0}\left[\int_V \frac{\rho}{r^2}\hat{r}\, dV + \int_S \frac{\sigma}{r^2}\hat{r}\, dS + \int_L \frac{\lambda}{r^2}\hat{r}\, dL\right]$$

$$\vec{E}_{\text{cont}} = \frac{\vec{F}_{\text{cont}}}{q_0}$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\int_V \frac{\rho}{r^2}\hat{r}\, dV + \int_S \frac{\sigma}{r^2}\hat{r}\, dS + \int_L \frac{\lambda}{r^2}\hat{r}\, dL\right]$$

26. Electric field due to a general charge distribution. It is given by

$$\vec{E}_{\text{total}} = \vec{E}_{\text{discreat}} + \vec{E}_{\text{cont}}$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\sum_{i=1}^{N} \frac{q_i}{r_i^2}\hat{r}_i + \int_V \frac{\rho}{r^2}\hat{r}\, dV + \int_S \frac{\sigma}{r^2}\hat{r}\, dS + \int_L \frac{\lambda}{r^2}\hat{r}\, dL\right]$$

27. Electric dipole and dipole moment. An electric dipole is a pair of equal and opposite charges $+q$ and $-q$ separated by some distance $2a$. Its dipole moment is given by

$$\vec{p} = \text{Either charge} \times \text{vector drawn from } -q \text{ to } +q$$

$$= q \times 2\vec{a}$$

Magnitude of dipole moment, $p = q \times 2a$

Dipole moment is a vector quantity having direction along the dipole axis from $-q$ to $+q$. Its SI unit is coulomb metre (Cm).

28. Electric field at an axial point of a dipole. The dipole field on the axis at distance r from the centre is

$$E_{\text{axial}} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2pr}{(r^2 - a^2)^2} \simeq \frac{1}{4\pi\varepsilon_0} \cdot \frac{2p}{r^3} \text{ for } r >> a.$$

At any axial point, the direction of dipole field is along the direction of dipole moment $\vec{p}$

29. Electric field at an equatorial point of a dipole. The electric field at a point on the perpendicular bisector of the dipole at distance r from its centre is

$$E_{\text{equa}} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{p}{(r^2 + a^2)^{3/2}} \simeq \frac{1}{4\pi\varepsilon_0} \cdot \frac{p}{r^3} \text{ for } r >> a.$$

At any equatorial point, the direction of dipole field is antiparallel to the direction of dipole moment $\vec{p}$.

In contrast to $1/r^2$ dependence of the electric field of a point charge, the dipole field has $1/r^3$ dependence. Moreover, the electric field due to a short dipole at a certain distance along the axis is twice the electric field at the same distance along the equatorial line.

30. Torque on a dipole in a uniform electric field. The torque on a dipole of moment p when placed in a uniform electric field at an angle θ with it is given by

$$\tau = pE \sin \theta$$

In vector rotation, $\vec{\tau} = \vec{p} \times \vec{E}$

When the dipole is released, the torque $\vec{\tau}$ tends to align the dipole along the field $\vec{E}$.

If $E = 1$ unit and $\theta = 90°$, then $\tau = p$. So dipole moment may also be defined as the torque acting on an electric dipole placed perpendicular to a uniform electric field of unit strength.

31. **Electric lines of force.** An electric line of force may be defined as the curve along which a small positive charge would tend to move when free to do so in an electric field and the tangent to which at any point gives the direction of electric field at that point.

32. **Important properties of electric lines of force.** These are :

(*i*) Lines of force are continuous curves without any breaks.

(*ii*) No two lines of force can cross each other.

(*iii*) They start at positive charges and end at negative charges—they cannot form closed loops.

(*iv*) The relative closeness of the lines of force indicates the strength of electric field at different points.

(*v*) They are always normal to the surface of a conductor.

(*vi*) They have a tendency to contract lengthwise and expand laterally.

33. **Electric flux.** The electric flux through a given area represents the total number of electric lines of force passing normally through that area. If the electric field $\vec{E}$ makes an angle θ with the normal to the area elements ΔS, then the electric flux is

$$\Delta\phi_E = E\Delta S \cos \theta = \vec{E} \cdot \Delta \vec{S}$$

The electric flux through any surface S, open or closed, is equal to the surface integral of $\vec{E}$ over the surface S.

$$\phi_E = \int_S \vec{E} \cdot \vec{dS}$$

Electric flux is a scalar quantity.

SI unit of electric flux = $\text{Nm}^2\ \text{C}^{-1}$.

34. **Gaussian surface.** Any hypothetical closed surface enclosing a charge is called the Gaussian surface of that charge.

35. **Gauss's theorem.** The total flux of electric field $\vec{E}$ through a closed surface $\vec{S}$ is equal to $1/\varepsilon_0$ times the charge q enclosed by the surface $\vec{S}$.

$$\phi_E = \oint_S \vec{E} \cdot \vec{dS} = \frac{q}{\varepsilon_0}$$

36. **Electric field of a line charge.** The electric field of a long straight wire of uniform linear charge density λ,

$$E = \frac{\lambda}{2\pi\varepsilon_0 r} \quad i.e., \quad E \propto \frac{1}{r}$$

where r is the perpendicular distance of the wire from the observation point.

37. **Electric field of an infinite plane sheet of charge.** It does not depend on the distance of the observation point from the plane sheet.

$$E = \frac{\sigma}{2\varepsilon_0}$$

where σ = uniform surface charge density.

38. **Electric field of two positively charged parallel plates.** If the two plates have surface charge densities σ_1 and σ_2 such that $\sigma_1 > \sigma_2 > 0$, then

$$E = \pm \frac{1}{2\varepsilon_0}(\sigma_1 + \sigma_2) \quad \text{(Outside the plates)}$$

$$E = \frac{1}{2\varepsilon_0}(\sigma_1 - \sigma_2) \quad \text{(Inside the plates)}$$

39. **Electric field of two equally and oppositely charged parallel plates.** If the two plates have surface charge densities $\pm\sigma$, then

$$E = 0 \quad \text{(For outside points)}$$

$$E = \frac{\sigma}{\varepsilon_0} \quad \text{(For inside points)}$$

40. **Electric field of a thin spherical shell.** If R is the radius and σ, the surface charge density of the shell, then

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2} \quad \text{For } r > R \quad \text{(Outside points)}$$

$$E = 0 \qquad \text{For } r < R \qquad \text{(Inside points)}$$

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R^2} \qquad \text{For } r = R \qquad \text{(At the surface)}$$

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2} \qquad \text{For } r > R \qquad \text{(Outside points)}$$

where $q = 4\pi R^2 \sigma$

41. Electric field of a uniformly charged solid sphere. If ρ is the uniform volume charge density and R radius of the sphere, then

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{qr}{R^3} \qquad \text{For } r < R \qquad \text{(Inside points)}$$

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R^2} \qquad \text{For } r = R \qquad \text{(At the surface)}$$

where $q = \frac{4}{3}\pi R^3 \rho$