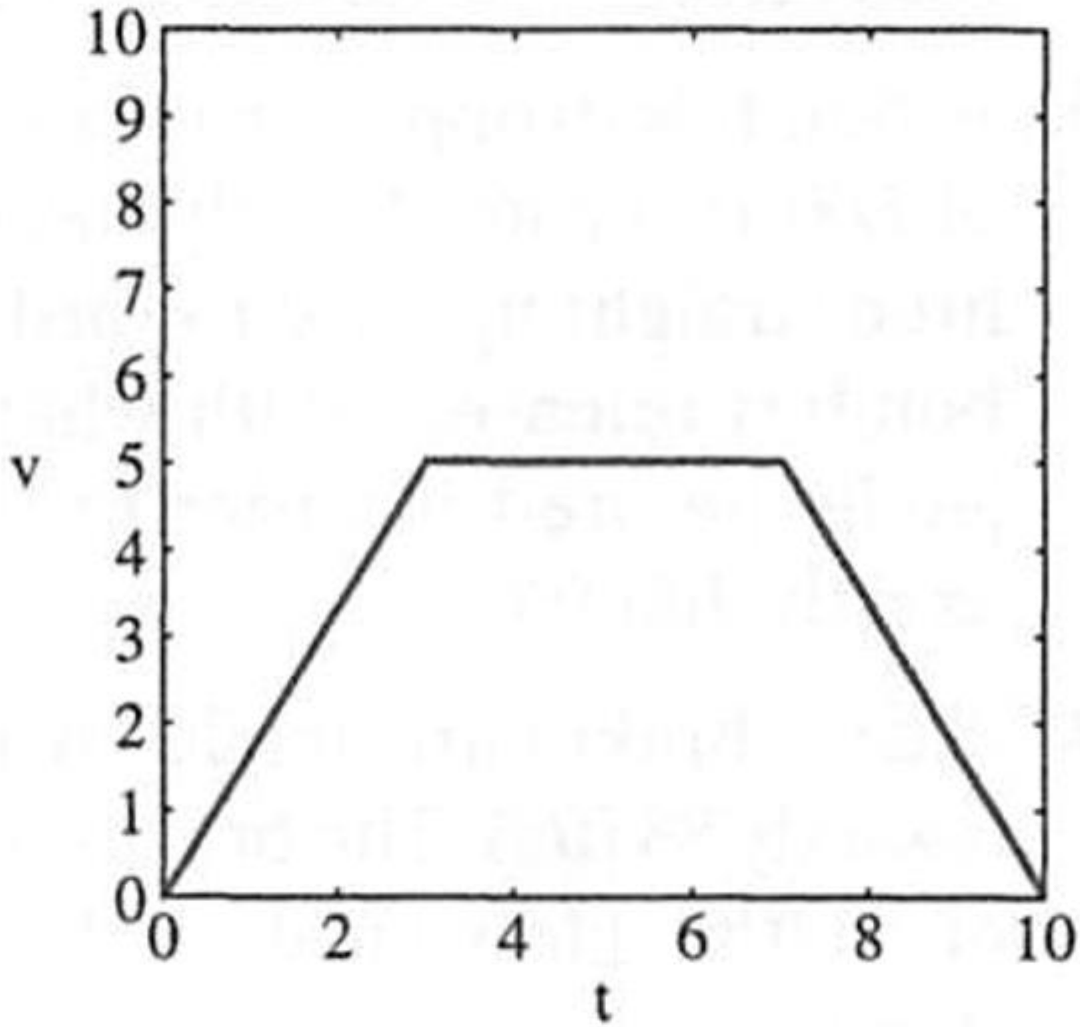
QUESTION :- A particle start from origin and travels along the x-axis with the velocity function v(t) whose graph is shown below. Sketch the graph of the resulting position function x(t) for 0<=t<=10.



** ANSWER **

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A particle starts at the origin and moves along the x-axis with the velocity function $v(t)$ whose graph is shown below.

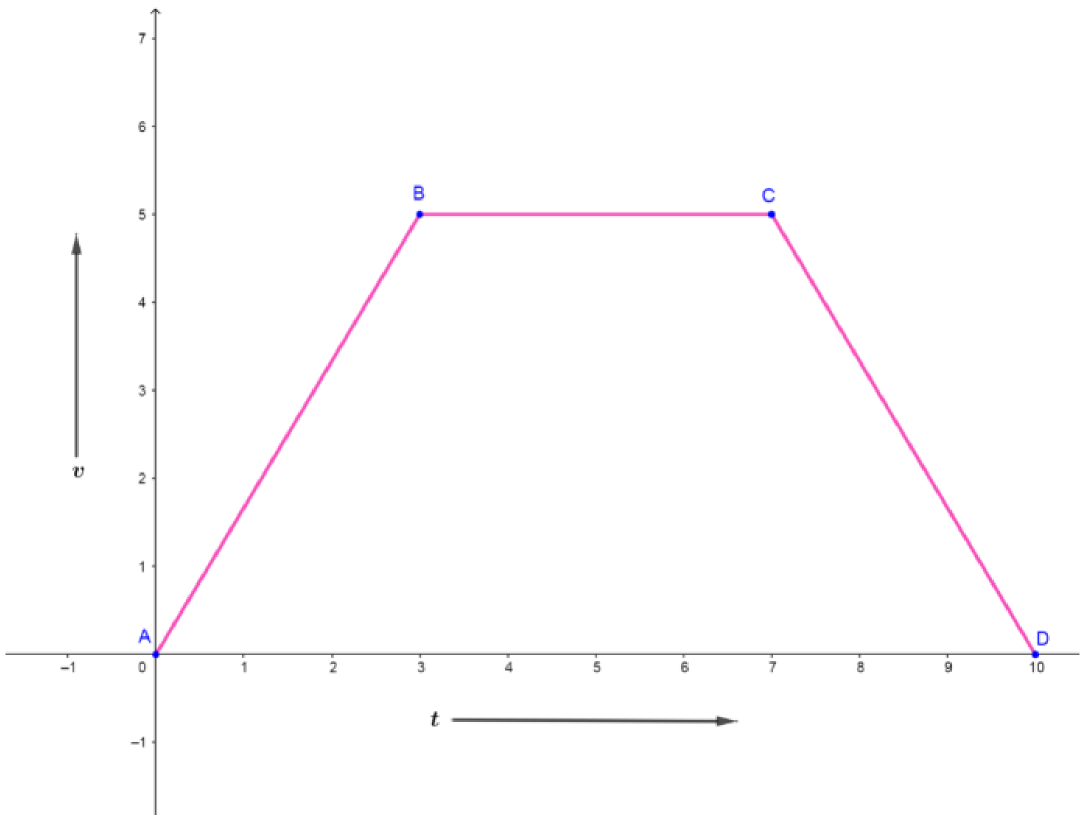

Figure 1.

We need to sketch the graph of the position function $x(t)$ in the time interval $0 \leq t \leq 10$.

To sketch the graph of the position function first of all we need to find the equation of position $x(t)$ in the time interval $0 \leq t \leq 10$.

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We know that the formula for the equation of a straight line which passes through (x_1, y_1) and (x_2, y_2) is:

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1). \tag{1}$$

First, we are going to find the equation of the velocity function $v(t)$ between A to B. Here, we can observe that it is a straight line.

The straight line passes through the points $A(0, 0)$ and $B(3, 5)$.

So, the equation of the velocity function $v(t)$ in the interval $0 \leq t \leq 3$ will be:

$$\begin{aligned} (v(t) - 0) &\overset{(1)}{=} \frac{5-0}{3-0}(t-0) \\ v(t) - 0 &= \frac{5}{3}t \\ v(t) &= \frac{5}{3}t. \end{aligned}$$

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We are going to find the equation of the velocity function $v(t)$ between B to C. Here, we can observe that it is a straight line.

The straight line passes through the points $B(3, 5)$ and $C(7, 5)$.

So, the equation of the velocity function $v(t)$ in the interval $3 \leq t \leq 7$ will be:

$$\begin{aligned} (v(t) - 5) &\overset{(1)}{=} \frac{5-5}{7-3}(t-3) \\ v(t) - 5 &= \frac{0}{5}(t-3) \\ v(t) - 5 &= 0 \\ v(t) &= 5. \end{aligned}$$

Now, we are going to find the equation of the velocity function $v(t)$ between C to D. Here, we can observe that it is a straight line.

The straight line passes through the points $C(7, 5)$ and $D(10, 0)$.

So, the equation of the velocity function $v(t)$ in the interval $7 \leq t \leq 10$ will be:

$$\begin{aligned}(v(t) - 5) &\overset{(1)}{=} \frac{0-5}{10-7}(t-7) \\ v(t) - 5 &= -\frac{5}{3}(t-7) \\ v(t) - 5 &= -\frac{5}{3}t + \frac{35}{3} \\ v(t) &= -\frac{5}{3}t + \frac{35}{3} + 5 \\ v(t) &= -\frac{5}{3}t + \frac{50}{3}.\end{aligned}$$

Finally, we found that the equation of the velocity function in the time interval $0 \leq t \leq 10$ is:

$$v(t) = \begin{cases} \frac{5}{3}t & \text{if } 0 \leq t \leq 3 \\ 5 & \text{if } 3 \leq t \leq 7 \\ -\frac{5}{3}t + \frac{50}{3} & \text{if } 7 \leq t \leq 10. \end{cases}$$

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To find the equation of the position function we will use the following formulas and properties:

$$\int x^k dx = \frac{x^{k+1}}{k+1} + C \qquad (\text{if } k \neq -1), \qquad (2)$$

$$\int cf(x)dx = c\int f(x)dx \qquad (\text{where } c \text{ is constant}), \qquad (3)$$

$$\int [f(x) \pm g(x)]\, dx = \int f(x)dx \pm \int g(x)dx. \qquad (4)$$

First, we are going to find the equation of the position function $x(t)$ in the time interval $0 \leq t \leq 3$, we know that $v(t) = dx/dt$:

$$\begin{aligned} v(t) &= \frac{5}{3}t \\ \frac{dx}{dt} &= \frac{5}{3}t \\ dx &= \frac{5}{3}tdt \end{aligned}$$

Integrating both sides with respect to t, then we get:

$$\begin{aligned} \int dx &= \int \frac{5}{3}tdt \\ x(t) &\overset{(3)}{=} \frac{5}{3}\int tdt \\ &\overset{(2)}{=} \frac{5}{3} \times \frac{t^{1+1}}{1+1} + C \\ &= \frac{5}{3} \times \frac{t^2}{2} + C \\ &= \frac{5}{6}t^2 + C. \end{aligned}$$

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Since the particle starts at the origin, we have $x(0) = 0$.

To find the value of C substitute $t = 0$ into the above equation.

$$\begin{aligned} x(t) &= \frac{5}{6}t^2 + C \\ x(0) &= \frac{5}{6}(0)^2 + C \\ 0 &= 0 + C \\ C &= 0. \end{aligned}$$

We found that the equation of the position function $x(t)$ in the time interval $0 \leq t \leq 3$ is:

$$x(t) = \frac{5}{6}t^2.$$

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We are going to find the equation of the position function $x(t)$ in the time interval $3 \leq t \leq 7$, we know that $v(t) = dx/dt$:

$$\begin{aligned} v(t) &= 5 \\ \frac{dx}{dt} &= 5 \\ dx &= 5dt \end{aligned}$$

Again integrating both sides with respect to t, then we get:

$$\begin{aligned} \int dx &= \int 5dt \\ x(t) &\stackrel{(3)}{=} 5\int t^0 dt \\ &\stackrel{(2)}{=} 5 \times \frac{t^{0+1}}{0+1} + C_1 \\ &= 5 \times \frac{t^1}{1} + C_1 \\ &= 5t + C_1. \end{aligned}$$

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Since the particle at $t=3$ will reach $\frac{15}{2}$ units, we have $x(3)=\frac{15}{2}$.

To find the value of C_1 substitute $t=3$ into the above equation.

$$\begin{aligned} x(t) &= 5t + C_1 \\ x(3) &= 5 \cdot 3 + C_1 \\ \frac{15}{2} &= 15 + C_1 \\ C_1 &= -\frac{15}{2}. \end{aligned}$$

We found that the equation of the position function $x(t)$ in the time interval $3 \leq t \leq 7$ is:

$$x(t) = 5t - \frac{15}{2}.$$

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Now, we are going to find the equation of the position function $x(t)$ in the time interval $7 \leq t \leq 10$, we know that $v(t) = dx/dt$:

$$\begin{aligned} v(t) &= -\frac{5}{3}t + \frac{50}{3} \\ \frac{dx}{dt} &= -\frac{5}{3}t + \frac{50}{3} \\ dx &= \left(-\frac{5}{3}t + \frac{50}{3}\right) dt \end{aligned}$$

Again integrating both sides with respect to t, then we get:

$$\begin{aligned} \int dx &= \int \left(-\frac{5}{3}t + \frac{50}{3}\right) dt \\ x(t) &\overset{(4)}{=} \int -\frac{5}{3}tdt + \int \frac{50}{3}dt \\ &\overset{(3)}{=} \left(-\frac{5}{3}\right) \int tdt + \frac{50}{3} \int t^0 dt \\ &\overset{(2)}{=} \left(-\frac{5}{3}\right) \times \frac{t^{1+1}}{1+1} + \frac{50}{3} \times \frac{t^{0+1}}{0+1} + C_2 \\ &= -\frac{5}{3} \times \frac{t^2}{2} + \frac{50}{3} \times \frac{t^1}{1} + C_2 \\ &= -\frac{5}{6}t^2 + \frac{50}{3}t + C_2. \end{aligned}$$

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Since the particle at $t = 7$ will reach $\frac{55}{2}$ units, we have $x(7) = \frac{55}{2}$.

To find the value of C_2 substitute $t = 7$ into the above equation.

$$\begin{aligned} x(t) &= -\frac{5}{6}t^2 + \frac{50}{3}t + C_2 \\ x(7) &= -\frac{5}{6}(7)^2 + \frac{50}{3} \cdot 7 + C_2 \\ \frac{55}{2} &= -\frac{5}{6} \cdot 49 + \frac{350}{3} + C_2 \\ \frac{55}{2} &= -\frac{245}{6} + \frac{350}{3} + C_2 \\ \frac{55}{2} &= \frac{455}{6} + C_2 \\ C_2 &= -\frac{145}{3}. \end{aligned}$$

We found that the equation of the position function $x(t)$ in the time interval $7 \leq t \leq 10$ is:

$$x(t) = -\frac{5}{6}t^2 + \frac{50}{3}t - \frac{145}{3}.$$

Finally, we found that the equation of position function $x(t)$ in the time interval $0 \leq t \leq 10$ is:

$$x(t) = \begin{cases} \frac{5}{6}t^2 & \text{if } 0 \leq t \leq 3 \\ 5t - \frac{15}{2} & \text{if } 3 \leq t \leq 7 \\ -\frac{5}{6}t^2 + \frac{50}{3}t - \frac{145}{3} & \text{if } 7 \leq t \leq 10. \end{cases}$$

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The equation of the position function $x(t)$ from A to B is:

$$x(t) = \frac{5}{6}t^2.$$

For $0 \le t \le 3$, we get the table of values as:

t	$x(t)$
0	0
1	$\frac{5}{6}$
2	$\frac{10}{3}$
3	$\frac{15}{2}$

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The equation of the position function $x(t)$ from B to C is:

$$x(t) = 5t - \frac{15}{2}.$$

For $3 \le t \le 7$, we get the table of values as:

t	$x(t)$
3	$\frac{15}{2}$
4	$\frac{25}{2}$
5	$\frac{35}{2}$
6	$\frac{45}{2}$
7	$\frac{55}{2}$

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The equation of the position function $x(t)$ from C to D is:

$$x(t) = -\frac{5}{6}t^2 + \frac{50}{3}t - \frac{145}{3}.$$

For $7 \leq t \leq 10$, we get the table of values as:

t	$x(t)$
7	$\frac{55}{2}$
8	$\frac{95}{3}$
9	$\frac{205}{6}$
10	35

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First, we are going to plot the all points of both tables on the same cartesian plane. We can see this in the below image.

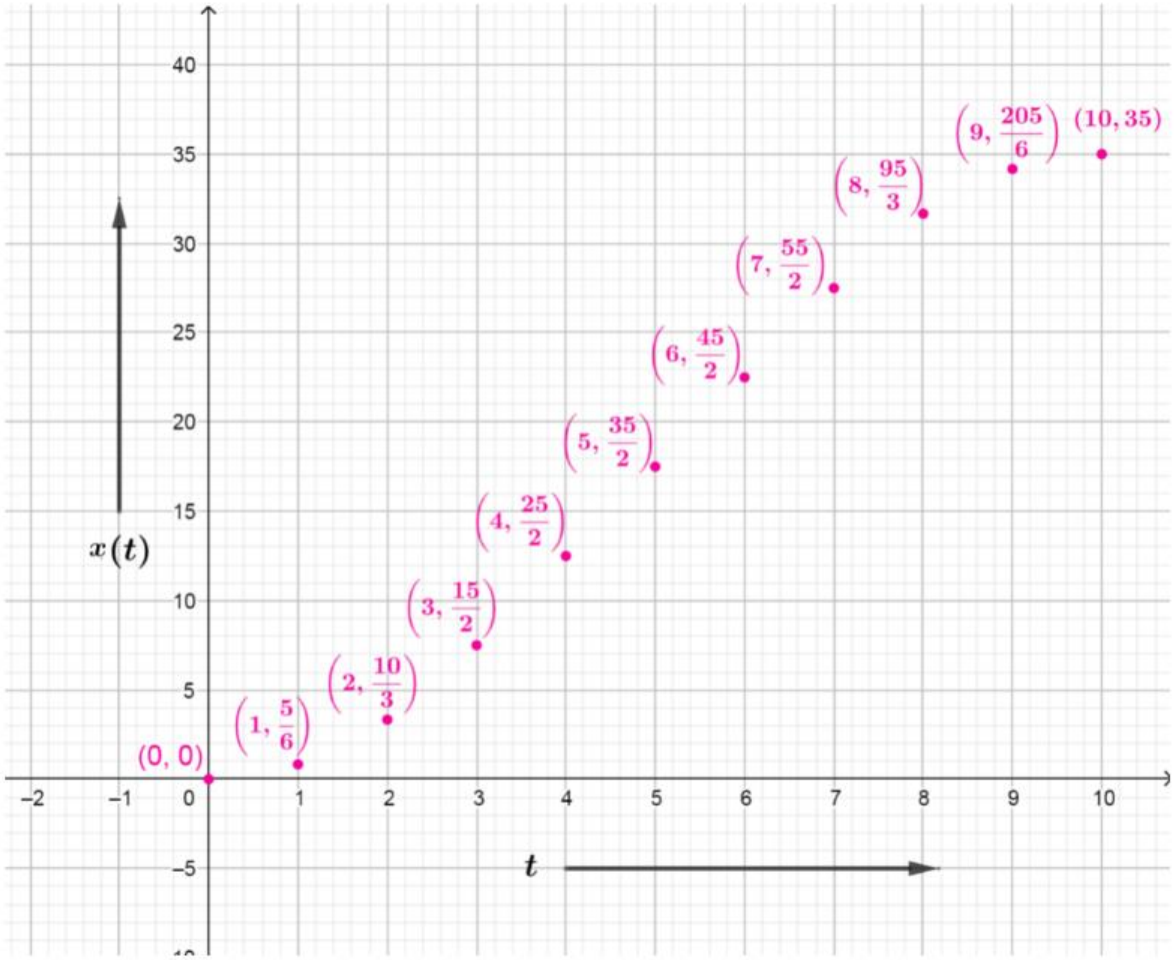

Figure 2.

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Draw a smooth line over all plotted points then we will get the graph of the position function $x(t)$ versus time t.

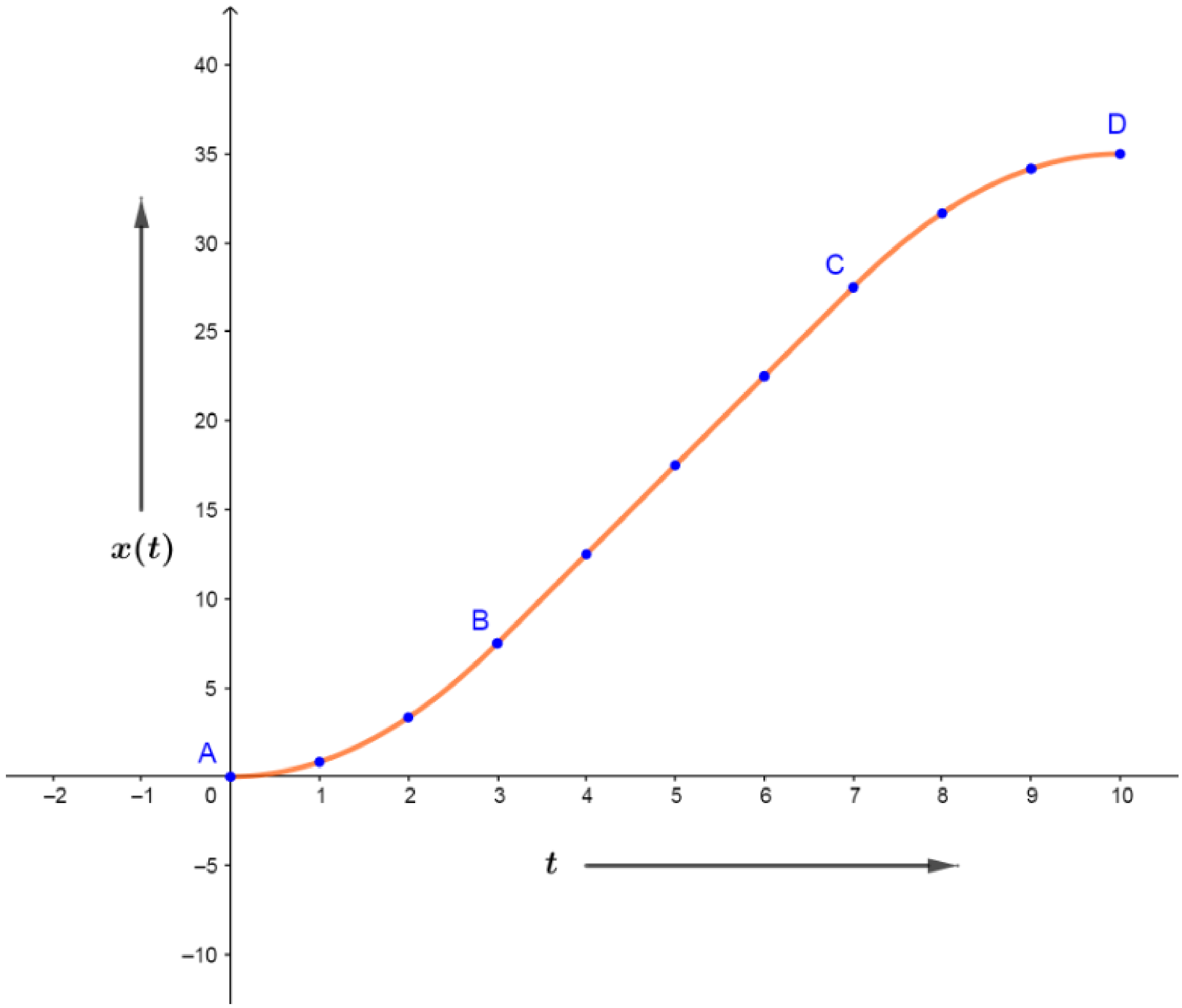

Figure 3.