

Linear Inequalities

degree = 1 \rightarrow highest power

variables

- > 0
- < 0
- ≥ 0
- ≤ 0

\rightarrow Lin. eqⁿ one var. ($ax + b = 0$)

$x + 5 = 0 \rightarrow x = -5$

\rightarrow Lin. eqⁿ two var. ($ax + by + c = 0$)

$2x + 5y + 7 = 0$

\rightarrow Lin. Ineqⁿ one var. ($ax + b \geq 0, ax + b > 0$
 $ax + b \leq 0, ax + b < 0$)

\rightarrow Lin. Ineqⁿ (two var.)

$ax + by + c \geq 0$
 $ax + by + c \leq 0$
 $ax + by + c > 0$
 $ax + by + c < 0$

→ Lin. Ineq. (one var.)

$x + 5 = 0$

$x = -5$



Solve

$x + 5 > 0$

$x > -5$

$x \in (-5, \infty)$



2 = y
3 = y

Solve

Q. $6x \leq 25$
 $x \leq \frac{25}{6}$

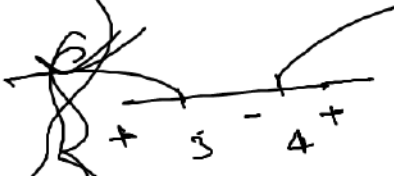
(i) $x \in \mathbb{N}$
 $x = \{1, 2, 3, 4\}$

(ii) $x \in \mathbb{Z}$
 $x = \{\dots, -3, -2, -1, 1, 2, 3, 4\}$

$(x-3)(x-4) \geq 0$

Q. $3 - 2x \geq 4x - 9$
 $12 \geq 6x$

$x \in \mathbb{R}$
 $x \in [-2, 2]$



Q. $\frac{5x-8}{3} \geq \frac{4x-7}{2}$, $x \in \mathbb{R}$

$x \in \mathbb{R}$
 $x \in (-\infty, \frac{5}{2}]$

Q. $\frac{5x+8}{4-x} \geq 2$, $x \in \mathbb{R} \rightarrow (-\infty, 0) \cup (4, \infty)$

$x \in (-\infty, 3) \cup (4, \infty)$
 $\frac{x-3}{x-4} \geq 0$

* Modulus Concept

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

$$\begin{aligned} |2| &= 2 \\ |(-2)| &= 2 \\ &= (x) \\ -(-2) &= \underline{2} \end{aligned}$$

Case I - $|x| < a$


$$-a < x < a$$

$$x \in (-a, a)$$

$x \geq 0$

$$|x| < a$$

$$x < a$$

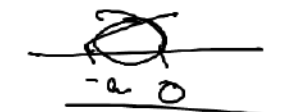
$$x \in [0, a)$$


Case II - $|x| = -x, x < 0$

$$|x| < a$$

$$-x < a$$

$$x > -a$$

$$x \in (-a, 0)$$


$$x \in (-a, a)$$

$$-a < x < a$$



$$\rightarrow |x| < a \Rightarrow -a < x < a$$

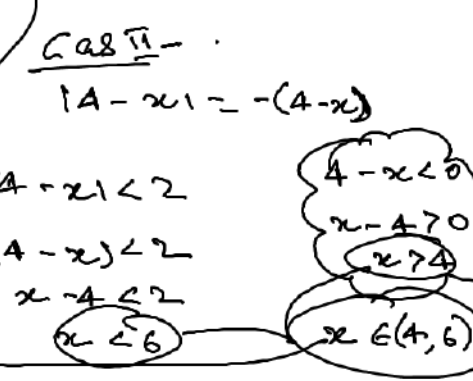
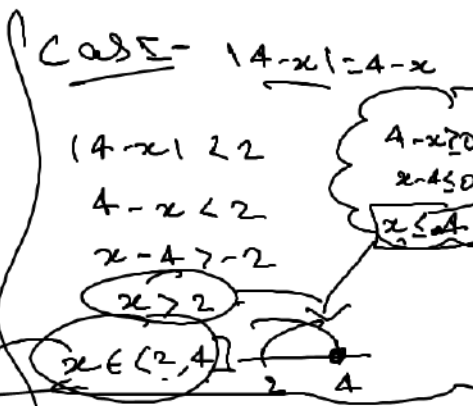
$$\rightarrow |x| \leq a \Rightarrow -a \leq x \leq a$$

$$\rightarrow |x| > a \Rightarrow x < -a, x > a$$

$$x \in (-\infty, -a) \cup (a, \infty)$$

$$\rightarrow |x| \geq a \Rightarrow x \leq -a, x \geq a$$

$$x \in (-\infty, -a] \cup [a, \infty)$$



Eg:
 $|4-x| < 2$
 $-2 < 4-x < 2$
 $-2-4 < 4-x-4 < 2-4$
 $-6 < -x < -2$

$$-6 < -x < -2$$

$$6 > x > 2$$

$$x \in (2, 6)$$



Q.

$$|5 - 2x| \leq 3$$

$$x \in [1, 4]$$

$$x \in \mathbb{R}$$

✓

~~Q.~~

$$|3x - 7| > 4$$

✓

25. ~~26.~~ 31. 30. Case II
 $x \in (\frac{3}{4}, 1) \cup (1, 0)$

$|x+2| = -(x+2)$ $x+2 < 0$
 $x < -2$
 $-\frac{(x+2)-3x}{x} < 0$
 $-\frac{-2x-2}{x} < 0$
 $\frac{2x+2}{x} < 0$
 $-2x-1$
 $-2 -ve -1 +ve 0 -ve$
 $x \in (-2, 2)$

36. ~~22.~~ $x \in \mathbb{R}$

Case I - $|x+2| = x+2$, $x+2 \geq 0$
 $x \geq -2$

25. $|x+2| - x < 2$
 $|x+2| - x - 2 < 0$

$\frac{|x+2| - 3x}{x} < 0$

$\frac{|x+2| - 2 - 2x}{x} < 0$

$\frac{x+2-3x}{x} < 0$

$\frac{|x+2| - 3x}{x} < 0$

$\frac{x-2x}{x} < 0$

$x \in \underline{[-2, 0)} \cup (1, 0)$

$x < -2$
 $x > 0$
 $x < 0$

$x \in (-2, 0) \cup (1, 0)$

110.

$$\left| \frac{2}{x-4} \right| > 1$$

$$|x| > a$$

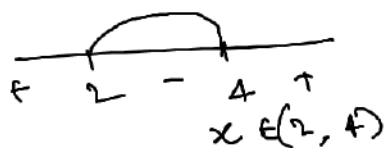
$$x < -a, \quad x > a$$

$$\frac{2}{x-4} < -1$$

$$\frac{2}{x-4} + 1 < 0$$

$$\frac{2+x-4}{x-4} < 0$$

$$\frac{x-2}{x-4} < 0$$

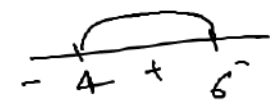


$$\frac{2}{x-4} > 1$$

$$\frac{2}{x-4} - 1 > 0$$

$$\frac{2-x+4}{x-4} > 0$$

$$\frac{6-x}{x-4} > 0$$



$$x \in (4, 6)$$

$$x \in (2, 4) \cup (4, 6)$$

$$|x+1| + |x+2| + |x+3| > 8$$



30.

$$|x+1| = x+1, x \geq -1$$

$$= -(x+1), x < -1$$

$$|x+1| + |x| > 3$$



Case I - ~~x in (0, 2)~~ $x \geq 0$

$$|x+1| + |x| > 3$$

$$x+1+x > 3$$

$$2x+1 > 3$$

$$2x > 2$$

$$x > 1$$



$$x \in (1, \infty)$$

Case III $x < -1$

$$|x+1| + |x| > 3$$

$$-(x+1) - x > 3$$

$$-x-1-x > 3$$

$$-2x-1 > 3$$

$$2x+1 < -3$$

$$2x < -4$$

$$x < -2$$

$$x \in (-\infty, -2)$$

Case II

$$-1 \leq x < 0$$

$$|x+1| + |x| > 3$$

$$x+1-x > 3$$

$$1 > 3$$

(No solution)

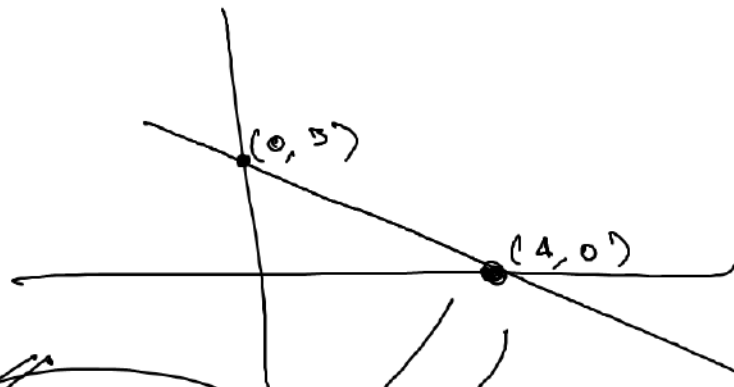
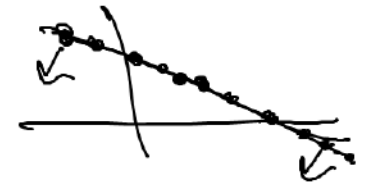
$$x \in (-\infty, -2) \cup (1, \infty)$$

Lin. Ineqⁿ (Two var.)

~~2x~~ $3x + 4y = 12$

x	0	4
y	3	0

$3x + 4y \leq 12$



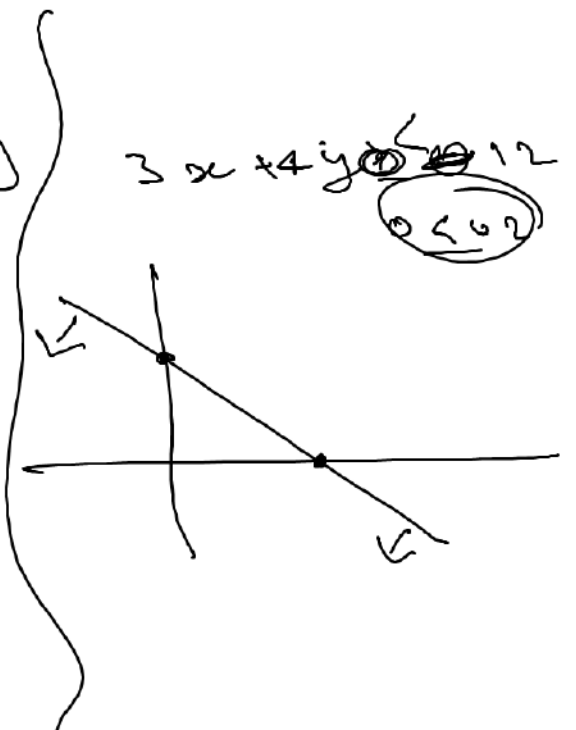
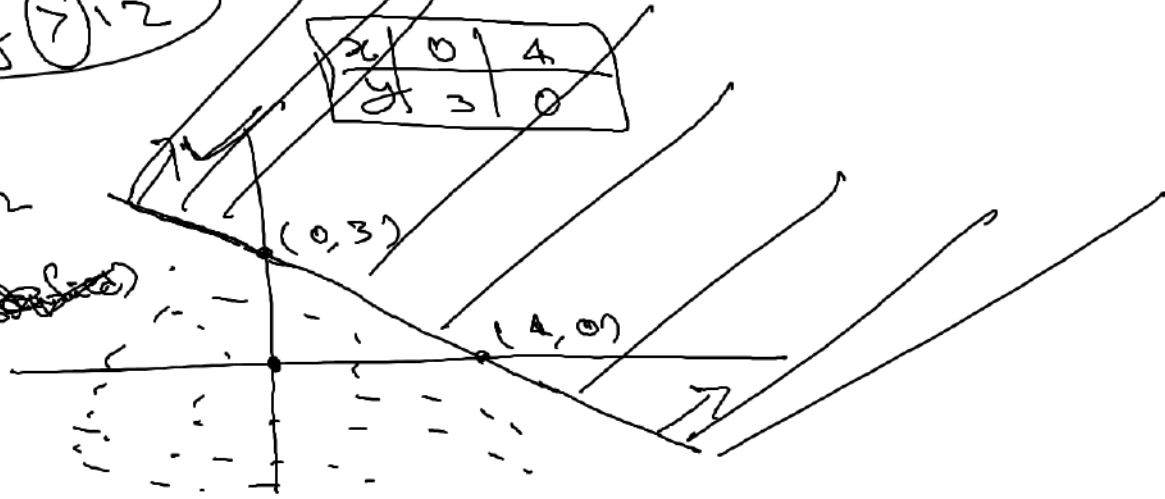
$3x + 4y = 12$

$3x + 4y < 12$
 $0 < 12$

~~$3x + 4y > 12$~~

x	0	4
y	3	0

• (0, 0)
 $0 + 0 > 12$
 $0 > 12$
 (not satisfied)



10.

x	20	0
y	0	15

$$3x + 4y \leq 60$$

x	30	0
y	0	10

$$x + 3y \leq 30$$

$$x \geq 0$$

$$y \geq 0$$

$x=0$ $y=0$
y-axis
x-axis

(1)

