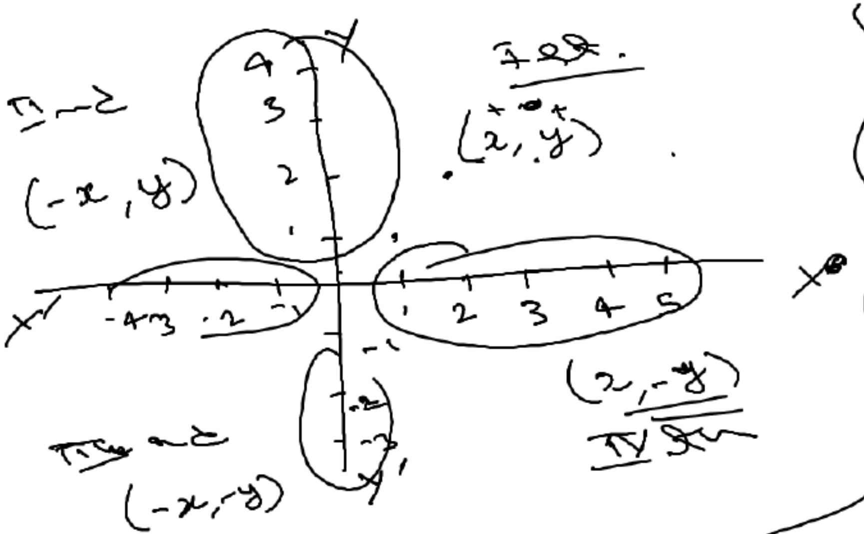
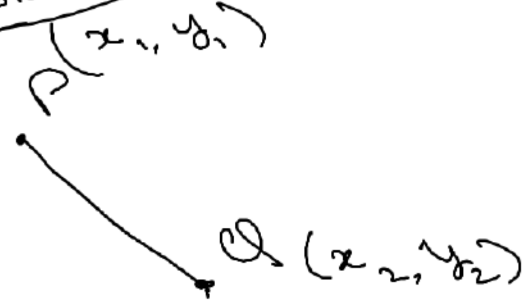
Co-ordinate Geometry



Distance formula



$$PQ = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Q. find a pt. on X-axis which is equidistant from pt. P(2, 0), Q(6, 0)

Sol^n A(x, 0) $\Rightarrow AP = AQ$

$\Rightarrow AP^2 = AQ^2$

$$(x-2)^2 = (x-6)^2$$

$$\cancel{x^2} - 4x + 4 = \cancel{x^2} - 12x + 36$$

$$\cancel{8}x = \cancel{32}^{\,4}$$

$$x = 4$$

* Collinearity —

A, B, C

A B C

→ Distance formula

A B C | B C A | C A B

$AB + BC = AC$ | $BC + CA = AB$ | $CA + AB = BC$

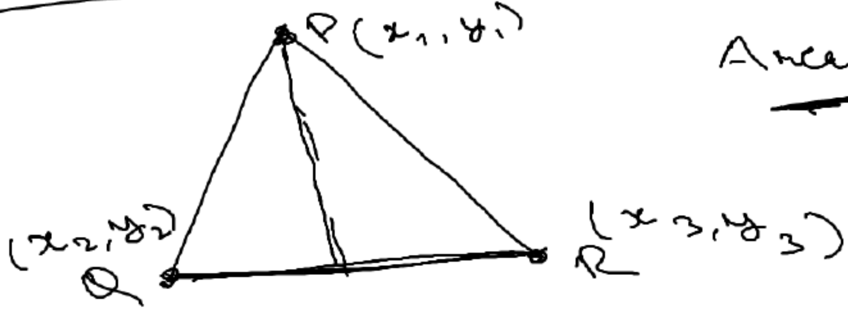
→ Area Method

Area = 0 ← collinear

Q. $A(2,1)$
$B(8,7)$
$C(4,4)$

1, 2, 3
2, 3, 1
3, 1, 2

* Area of Δ if co-ordinates are given



$$\text{Area} = \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$$

$= \frac{1}{2}(-48)$

$= \frac{1}{2} \times \overset{24}{48} = 24$

* Section Formulae —

Internal Division

$P(x_1, y_1)$ $Q(x_2, y_2)$

m n $A(x, y)$

PA, QA

$$x = \frac{m x_2 + n x_1}{m + n}$$

$$y = \frac{m y_2 + n y_1}{m + n}$$

External Division

(x_1, y_1) (x_2, y_2) (x, y)

P Q A

PA, QA

$m : n$

$$x = \frac{m x_2 - n x_1}{m - n}$$

$$y = \frac{m y_2 - n y_1}{m - n}$$

* Special Case

$P(x_1, y_1)$ $Q(x_2, y_2)$

$m : n = 1 : 1$

m A n

$$x = \frac{1 \times x_2 + 1 \times x_1}{1 + 1}$$

$$x = \frac{x_2 + x_1}{2} = \frac{x_1 + x_2}{2}$$

$$y = \frac{1 \times y_2 + 1 \times y_1}{1 + 1}$$

$$\boxed{y = \frac{y_1 + y_2}{2}}$$

Q

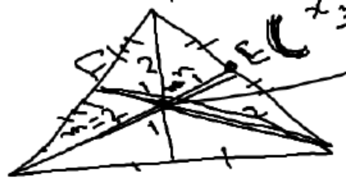
$P(3, 5)$ $Q(6, 10)$ $R(7, 7)$

$S(x, 2)$

$\left(\frac{3+7}{2}, \frac{5+7}{2}\right)$

$(5, 6)$

* Centroid —



A (x_1, y_1)

E $\left(\frac{x_3+x_1}{2}, \frac{y_1+y_3}{2}\right)$

G (x, y)

B (x_2, y_2) D C (x_3, y_3)

$$x = \frac{2 \times \left(\frac{x_3+x_1}{2}\right) + 1 \times x_2}{2+1}$$

$$= \frac{x_1+x_2+x_3}{3}$$

$$y = \frac{2 \times \left(\frac{y_3+y_1}{2}\right) + 1 \times y_2}{2+1}$$

$$\boxed{y = \frac{y_1+y_2+y_3}{3}}$$

* St. line

$\boxed{ax+by+c=0}$

Lin eq^n in two var x & y represents a st. line

↓

Eq^n of st. line

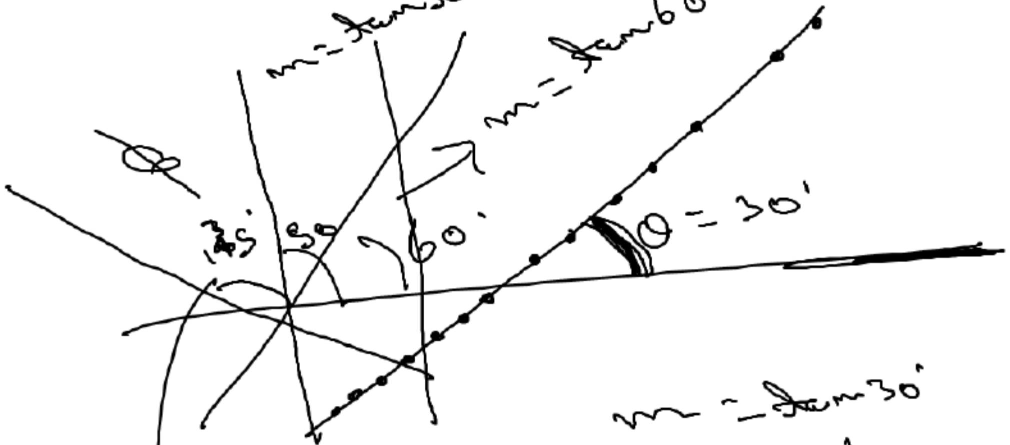
$\boxed{m=\tan\theta}$

Slope → Inclination with +ve x-axis

θ↑ , Slope↑

$m=\tan 90^\circ=\infty$

$m=\tan 60^\circ=\sqrt{3}$



$m=\tan 30^\circ$
$=\frac{1}{\sqrt{3}}$

$m=\tan 135^\circ$
$=\tan(90+45)$
$=-\cot 45^\circ$
$m=-1$

90 × odd → func^n change
90 × even → func^n same

sin → cos
sec → cosec
tan → cot

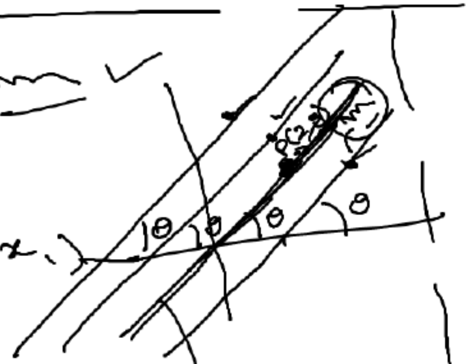
$\tan(90+\theta)$
$=-\cot\theta$

* Eqⁿ of St. line — (Parallel lines slope → same, equal)

1. Point slope form

$P(x_1, y_1)$, m ✓

$$y - y_1 = m(x - x_1)$$



eg: $P(2,3)$, & 5

$$y - 3 = 5(x - 2)$$

$$y - 3 = 5x - 10$$

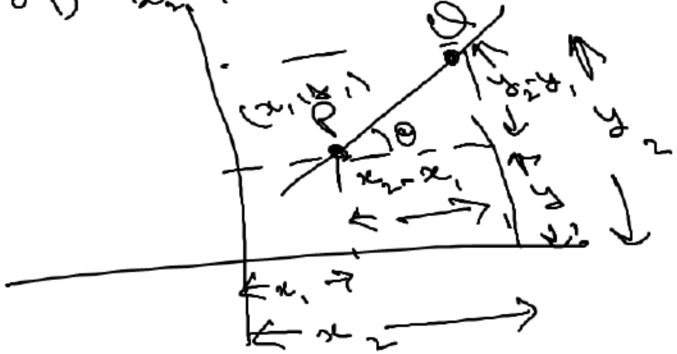
$$\boxed{5x - y - 7 = 0}$$ ✓

2. Two pt. form

$P(x_1, y_1)$, $Q(x_2, y_2)$

$$y - y_1 = m(x - x_1)$$

$$(y - y_1) = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$



$$m = \tan\theta = \frac{P}{B}$$

$$\boxed{m = \frac{y_2 - y_1}{x_2 - x_1}}$$ ✓

Q. $P(2, -3)$, $m = 3$ ✓

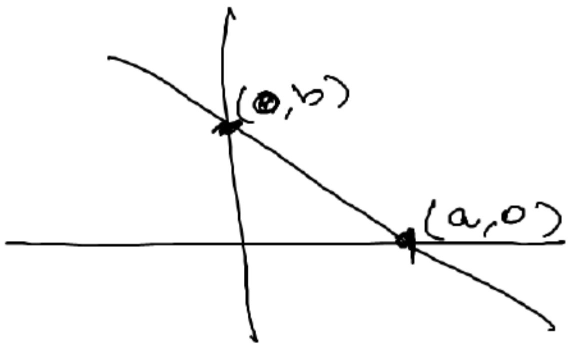
Q. $P(4, 6)$, $Q(5, 7)$

Q. $9x + 8y - 72 = 0$, X-axis, Y-axis

$3x$

* Intercept form

$$\boxed{\frac{x}{a} + \frac{y}{b} = 1}$$



$$9x + 8y - 72 = 0$$

$$\frac{\cancel{72}^{8}}{\cancel{9}} = 8 \qquad \frac{\cancel{72}^{9}}{\cancel{8}} = 9$$

$$9x + 8y = 72$$

$$\boxed{\frac{x}{8} + \frac{y}{9} = 1}$$

x-intercept

y-intercept

eg:

7, 24

$$\frac{x}{7} + \frac{y}{24} = 1$$

$$ax + by + c = 0$$

$$ax + by = -c$$

$$\frac{ax}{-c} + \frac{by}{-c} = 1$$

$$\boxed{\frac{x}{-c/a} + \frac{y}{-c/b} = 1}$$

X-intercept → $-\frac{c}{a}$

Y-intercept → $\frac{-c}{b}$

Slope → $\frac{-a}{b}$

* Slope -intercept form —

$y = mx + c$

$m \to$ slope

$c \to$ intercept on y-axis

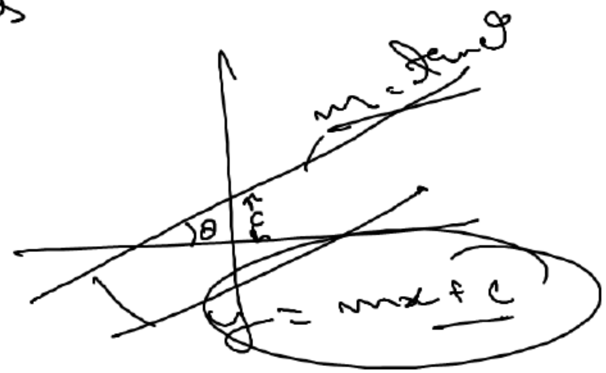
$ax + by + c = 0$

$by = -ax - c$

$y = -\frac{a}{b}x - \frac{c}{b}$

$-\frac{a}{b} \to$ slope

$-\frac{c}{b} \to$ In y-int.



$\parallel \Rightarrow m_1 = m_2$

$\perp \Rightarrow m_1 \times m_2 = -1 \Rightarrow$

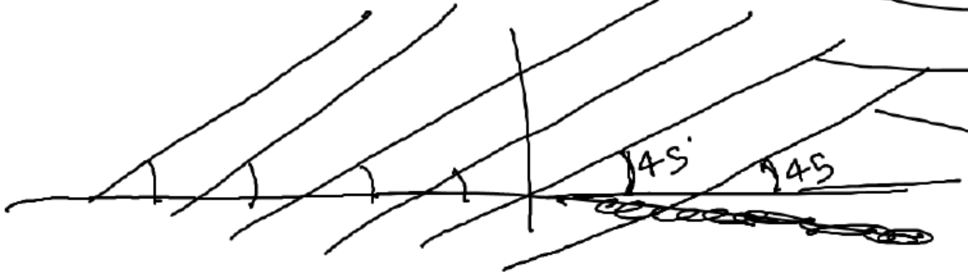
Co-ordinate Geometry

St. lines

→ ∥ lines →

$S_1 \to m_1$, $S_2 \to m_2$

$m_1 = m_2$



$m = \tan 45° = 1$

→ Slope $= \tan 45° = 1$

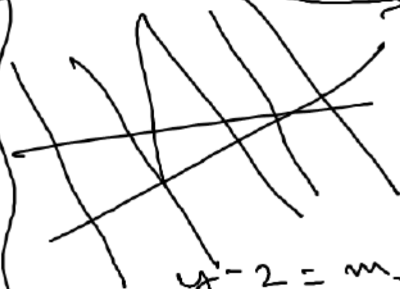
$y = mx + c$

$2 \times 1 - 2 = 0$

$\boxed{2x - y = 0}$

eg: $x + 2y = 5$, $P(1, 2)$

$\to m_1 = -\frac{a}{b} = \frac{-1}{2}$



$m_1 \times m_2 = -1$

$\frac{-1}{2} \times m_2 = -1$

$m_2 = 2$

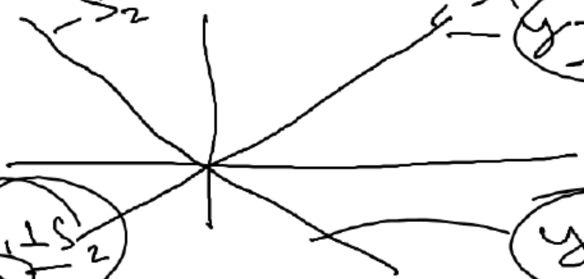
$y - 2 = m_2(x - 1)$

$y - 2 = 2(x - 1)$

$y - \cancel{2} = 2x - \cancel{2}$

$\boxed{y - 2x = 0}$

→ ⊥ lines →



S_1: $y = 2x \to m_1 = 1$

$S_1 \perp S_2$

$y = -x$ → $m_2 = -1$

$m_1 \times m_2 = -1$

eg: $m_1 = 2$, $P(3, 4)$

$2 \times m_2 = -1$

$m_2 = \frac{-1}{2}$

$(x - 3)$

$S_2 \to ?$

$y - 4 = m_2$

$y - 4 = -\frac{1}{2}(x - 3)$

$ax + by = c$

↓

⊥ line

$bx - ay = ?$

Q.

$S_1 \Rightarrow$ $\boxed{2x + 7y = 16}$ P(1, 2)

1

$S_2 \Rightarrow$

$7x - 2y = 3$ ✓

Q. $ax + by = c$

(1)

$ax + by =$

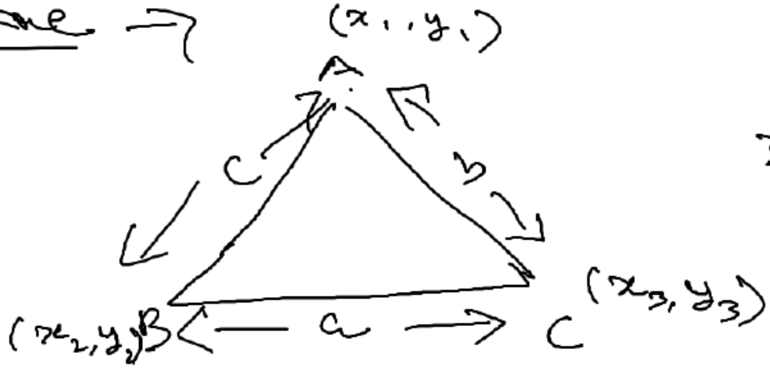
(Parallel case)

Q. $x + 2y = 5$ (5, 5)

$5 + 2 \times 5$

$\boxed{x + 2y = 15}$

Incentre → Intersection of internal angle bisectors

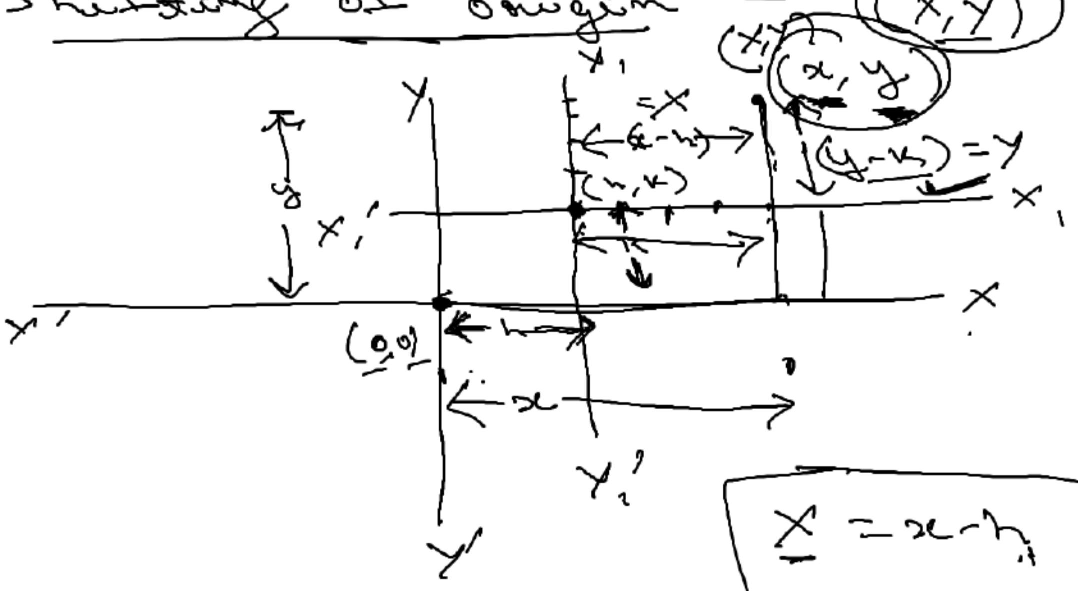


$$I\left(\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c}\right)$$

$$\frac{x_1+x_2+x_3}{3}$$

Equilateral Δ → C

Q.
* Shifting of origin (Translation of axis)



(x, y) (0, 0)
respect to
Cartesian origin

$$\boxed{\begin{aligned} X &= x-h \\ Y &= y-k \end{aligned}}$$

eg:

O (0, 0), P $\overset{x\ \ y}{(2, 3)}$

O' $\overset{h\ k}{(1, 1)}$, P' $\overset{(1, 2)}{(X, Y)}$

$X = x - h$

$= 2 - 1$

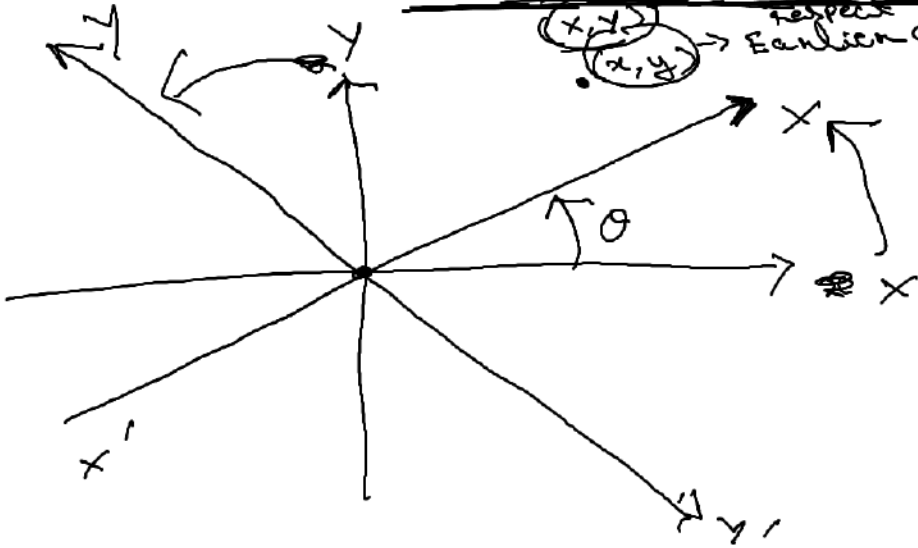
$\boxed{X = 1}$

$Y = 3 - 1$

$\boxed{Y = 2}$

Rotation of axis

(x, y) → respect Earlier axis
(X, Y)



Rotation
→ anticlockwise → +ve
→ clockwise → −ve

$x = X\cos\theta - Y\sin\theta$

$y = X\sin\theta + Y\cos\theta$

(X, Y) → Respect to new axis

eg: If axis is rotated anti. by $90°$ → θ $P(1, 2)$
$P'(2, -1)$

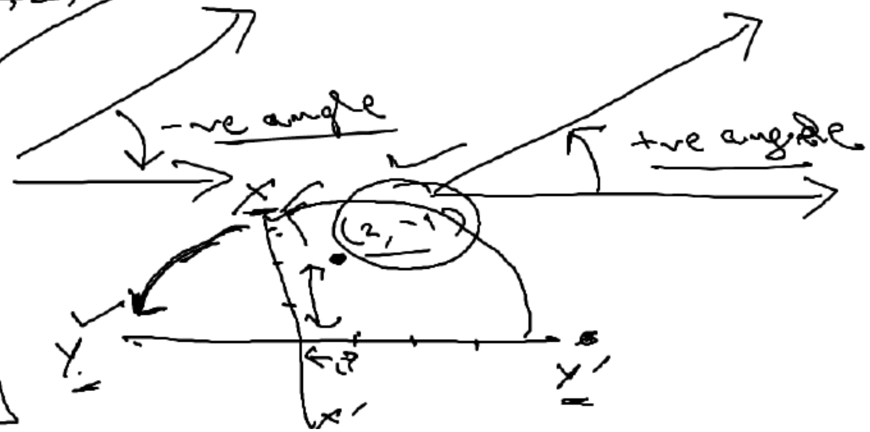
$x = X\cos\theta - Y\sin\theta$

$1 = 0 - Y \times 1$

$\boxed{Y = -1}$

$y = X\sin\theta + Y\cos\theta$

$2 = X \times 1 + Y \times 0$ $\boxed{X = 2}$



* Angle b/w two lines =

$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right| = \left|\frac{\tan\theta_2 - \tan\theta_1}{1 + \tan\theta_1 \cdot \tan\theta_2}\right|$$

Parallel

$\theta = 0$

$$0 = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right| \Rightarrow m_2 - m_1 = 0$$

$$\boxed{m_2 = m_1}$$

$\perp$arity

$$\frac{\sin 90}{\cos 90} = \frac{1}{0}$$

$$\tan 90 = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$$

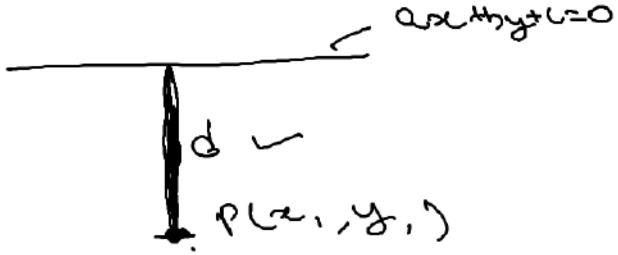
$$\frac{1}{0} = \frac{m_2 - m_1}{1 + m_1 m_2}$$

$$1 + m_1 m_2 = 0$$

$$\boxed{m_1 m_2 = -1}$$

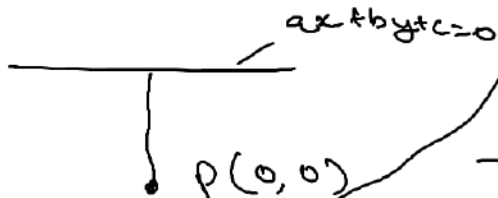
* Distance from a line —

$ax+by+c=0$, $P(x_1, y_1)$



$$\boxed{d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}}$$

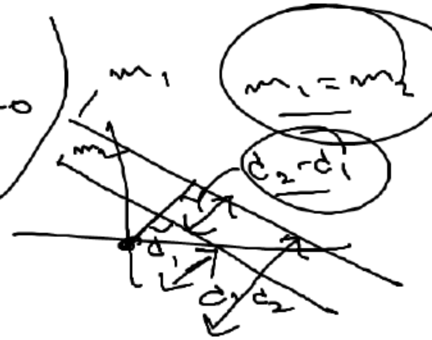
→ ⊥ar distance of a line from origin —



$$d = \frac{|a\times 0 + b\times 0 + c|}{\sqrt{a^2+b^2}}$$

$$d = \frac{|c|}{\sqrt{a^2+b^2}}$$

→ Distance b/w two parallel lines —



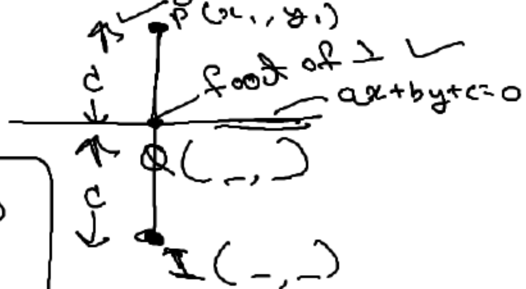
$$d_1 = \left|\frac{c_1}{\sqrt{a^2+b^2}}\right|$$

$$d_2 = \left|\frac{c_2}{\sqrt{a^2+b^2}}\right|$$

$$\boxed{d = \frac{|c_2 - c_1|}{\sqrt{a^2+b^2}}}$$

* foot of the $\perp$ (h, k) on $ax+by+c=0$

from (x_1, y_1)



foot of $\perp$

$$\frac{h-x_1}{a} = \frac{k-y_1}{b} = \frac{-(ax_1+by_1+c)}{\sqrt{a^2+b^2}}$$

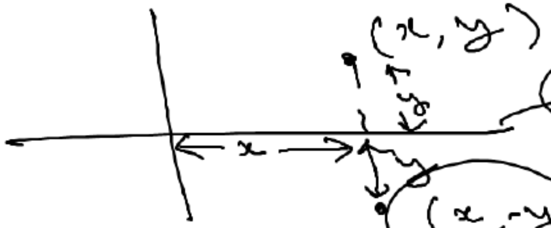
Image

$$\frac{h-x_1}{a} = \frac{k-y_1}{b} = \frac{-2(ax_1+by_1+c)}{\sqrt{a^2+b^2}}$$

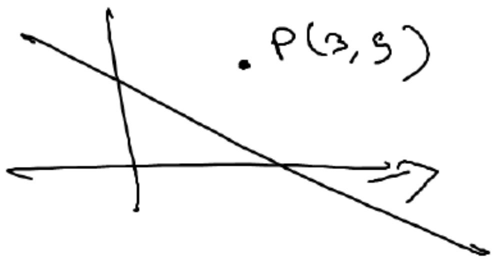
$(1, 2)$ (x_1, y_1) $(3, 4)$ (x, y) (x_2, y_2)

$$\frac{1+x_2}{2} = 3$$

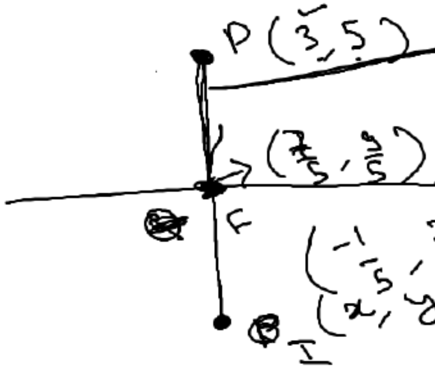
$1+x_2 = 6$ $x_2 = 5$ $x = \frac{x_1+x_2}{2}$, $y = \frac{y_1+y_2}{2}$



x-axis



Q.



$\boxed{2x - y = 1}$

$x + 2y = 5$

$\left(\frac{-1}{5}, \frac{7}{5}\right)$

(x, y)

$$\frac{3+x}{2} = \frac{7}{5}$$

$$15 + 5x = 14$$

$$5x = -1$$

$$x = \frac{-1}{5}$$

$$\frac{5+y}{2} = \frac{9}{5}$$

$$25 + 5y = 18$$

$$5y = -7$$

$$y = \frac{-7}{5}$$

$$2y = 5 - x$$

$$2y = 5 - \frac{7}{5}$$

$$= \frac{25 - 7}{5}$$

$$2y = \frac{18}{5}$$

$$y = \frac{9}{5}$$

$$(2x - y = 1) \times 2$$

$$x + 2y = 5$$

$$5x = 7$$

$$x = 7/5$$

* **Circle** →

$$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

diameter

$$r = \sqrt{(x-h)^2 + (y-k)^2}$$

Gen. eqn.

$$\boxed{r^2 = (x-h)^2 + (y-k)^2}$$

→ $h = 0,\ k = 0$

$$(x-0)^2 + (y-0)^2 = r^2$$

$$\boxed{x^2 + y^2 = r^2}$$

Eqn of circle having centre at origin

Q. angle of inclination $135'$ $\tan(135') = \tan(90+45')$

slope = ? $= -\cot 45'$

$= -1$

Q. which of the following points nearest to origin.

(a) (1, 3) (b) (2, −3) (c) (−3, 0) (d) (1, 2)

Q. find distance from the origin of the line $5x + 12y + 26 = 0$

Q. The angle made by the line $x - \sqrt{3}y + 4 = 0$ on x-axis → θ — $m = \tan\theta$

$\frac{1}{\sqrt{3}} = \tan\theta$

$\theta = 30'$

Q. The distance b/w the parallel lines $y = 7$ & $y + 7 = 0$ $\frac{A_1}{\sqrt{2}}$ = 1 unit

Q. The lines $4x + 3y - 7 = 0$ & $\cancel{2}0x + \cancel{1}5y - \cancel{3}5 = 0$ are

(i) parallel lines (ii) ten lines (iii) coincident lines (iv) Intersecting lines

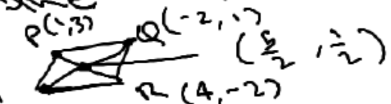
Q. The lines $2x+3y+5=0$ & $x+y+2=0$ intersect in which quadrant (3rd)

Q. The area of the triangle formed by the line $3x+4y=24$ with co-ordinate axes in sq. units

(24)

Q. If (1, 1) & (−1, −1) are the two vertices of an equilateral triangle third vertex could be.

Q. If (1, 3), (−2, 1), (4, −2) are three consecutive vertices of ||gm then the 4^{th} vertex is



→ (7, 0)

Q. If $x-2y+1=0$ and $2x-4y+6=0$ are two opposite sides of a square, then area of the square in sq. units. → $\frac{4}{5}$

Q. If the line $x-2y-k=0$ passes through the pt. of intersection of $2x+y=2$ & $x-3y=1$ then $k=?$

$k=1$

Q. If the line $(4x-5y+6)+\lambda(5x-3y+2)=0$ is parallel to y-axis then the value of λ is ?

$(4+5\lambda)x-(5+3\lambda)y \ (\ \ )=0$

$m=\frac{1}{0}=\tan 90°$

$\frac{(4+5\lambda)}{(5+3\lambda)}=\tan 90°=\frac{1}{0}$

$5+3\lambda=0$

$\lambda=-5/3$