

Ratio, Proportion & Variation

Comparison

A
70 Rs B
30 Rs

A : B = 70 : 30

= 7 : 3
10

$\frac{a}{b} = \frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12}$

$a \neq 2, b \neq 3$

$a = 2k, b = 3k$

$\frac{a}{b} = \frac{2}{3}$

$a = 2k, b = 3k$
 $8, 12$

$a + b = 20$

$2k + 3k = 20$

$5k = 20$
 $k = 4$

Properties of Ratios

(i) $\frac{a}{b} = \frac{ka}{kb} = \frac{a}{b}$

(ii) $\frac{a}{b} = \frac{a/k}{b/k} = \frac{a/m}{b/m}$

$\frac{2 \times 100}{3 \times 100}$

(iii)

$\frac{a/b}{c/d} = \frac{a \cdot d}{b \cdot c}$

(iv)

$a:b = 2:3, c:d = 5:7$
Compounded ratio
 $\frac{a}{b} \times \frac{c}{d} = \frac{2}{3} \times \frac{5}{7} = \frac{10}{21}$

(v) Compounded Specific types

(b)

(a) Duplicate ratio

$\frac{a}{b} \times \frac{a}{b} = \frac{a^2}{b^2}$
 $\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$
Duplicate ratio

(b) TriPLICATE ratio

$\frac{a}{b} \times \frac{a}{b} \times \frac{a}{b} = \frac{a^3}{b^3}$

(c) Sub-duplicate ratio

$\frac{a}{b} \rightarrow \sqrt{\frac{a}{b}} = \left(\frac{a}{b}\right)^{1/2}$

(d) Sub-triplicate ratio

$\frac{a}{b} \rightarrow \sqrt[3]{\frac{a}{b}} = \left(\frac{a}{b}\right)^{1/3}$

(vi) * $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \dots = k$

$$\frac{a+c+e+g+\dots}{b+d+f+h+\dots} = k$$

eg:

$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6}$$

$$\frac{1+2}{2+4} = \frac{3}{6}$$

* $a:b = 2:3$ $b:c = 3:5$

$$a:b:c = 2:3:5$$

→ $a:b = 2:3$ $b:c = 6:10$

$$a:b:c = 4:6:15$$

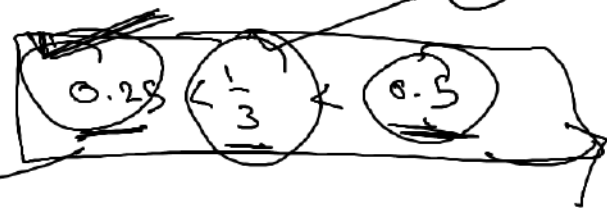
→ $a:b = 2 \times 5 : 3 \times 5$ $b:c = 5:7$

(vii) $\frac{a}{b}, \frac{c}{d}, \frac{e}{f}, \dots$
 \rightarrow Diff. ratios

Lowest ratio $\frac{a+c+e+\dots}{b+d+f+\dots}$ Highest ratio

eg: $0.5, 0.33, 0.25$
 $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$

$$\frac{1+1+1}{2+3+4} = \frac{3}{9} = \frac{1}{3}$$



$$a:b:c = 10:5:2 = 10:15:21$$

$$c:d = 5:14$$

$a:b:c:d = ?$

*

$\frac{a}{b}$

$k \rightarrow +ve \text{ number}$

$\frac{a+k}{b+k} < \frac{a}{b}$

$\frac{a-k}{b-k} > \frac{a}{b}$

eg:

$\frac{5}{2} > 1$ $\frac{a}{b} < \frac{a+k}{b+k}$
 $\frac{a}{b} = \frac{5}{2} = 2.5$

$k = 0.1$

$\frac{5+0.1}{2+0.1} = \frac{5.1}{2.1} = 2.428 < 2.5$

$\frac{5-0.1}{2-0.1} = \frac{4.9}{1.9} = 2.578 > 2.5$

*

$\frac{a}{b}$

$k \rightarrow +ve \text{ number}$

$\frac{a+k}{b+k} > \frac{a}{b}$

$\frac{a-k}{b-k} < \frac{a}{b}$

$\frac{a}{b} < 1$ $\frac{a}{b} < \frac{a}{b}$

$\frac{a}{b} = \frac{2}{5} = 0.4$

$k = 1$

$\frac{2+1}{5+1} = \frac{3}{6} = 0.5 > 0.4$

$\frac{2-1}{5-1} = \frac{1}{4} = 0.25 < 0.4$

11. Divide 14 dollars among Anshu & Anshul in the ratio 5:2.

12. 3 boys are aged 2 years, 4 years, 8 years. They want to divide 70 rupees in the ratio of their ages. How much money would each get?

$$\frac{2}{3} : 2 : 8 = 2 : 6 : 15$$

$$2 : 6 : 15 : 45 = \frac{4 : 12 : 30 : 45}{2}$$

13. $a:b = 1:3 \times \frac{2}{3}$, $b:c = 2:5$, $c:d = 2:3 \times \frac{15}{2}$. Find $a:b:c:d = ?$

14. Simplify the following ratios

(a) $\frac{1}{6} : \frac{1}{8}$ (b) $2\frac{1}{3} : 3\frac{1}{2}$ (c) $\frac{5}{6} : \frac{3}{8} : 3\frac{3}{4}$

15. $a:b = 1:2$, $b:c = 3:4$, $c:d = 5:6$, $d:e = 7:8$. Find $a:e = ?$

$$\frac{a}{5} = \frac{3}{4}, \quad \frac{7a-4b}{3a+b} = ?$$

$$\frac{1 \times 35}{2 \times 4} = \frac{35}{8}$$

16. The incomes of A, B, C are in the ratio of 12:9:7 and their spending are in the ratio 15:9:8. If A saves 25% of his income, what is the ratio of the savings of A, B & C.

inc. $\rightarrow A = 12x$
 $\rightarrow B = 9x$
 $\rightarrow C = 7x$

exp. $\rightarrow 15y$ (A) $\rightarrow 15 \times \frac{3x}{5}$
 $\rightarrow 9y$ (B) $\rightarrow 9 \times \frac{3x}{5}$
 $\rightarrow 8y$ (C) $\rightarrow 8 \times \frac{3x}{5}$

$B = 3 \text{ inc.} - \text{exp.}$

$$\frac{3}{5}y = \frac{9x}{5} - \frac{8x}{5}$$

$$y = \frac{3x}{5}$$

* Proportion - Equality of two ratios is called Proportion.

$$\frac{a}{b} = \frac{c}{d}$$

\Rightarrow we can say a, b, c, d are in proportion



$a, d \rightarrow$ extremes

$b, c \rightarrow$ means

* Concept of components related to Proportion
Dividendo

$$2:3 :: 4:6$$

Properties of Proportion

\rightarrow Invertendo -

$$\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{b}{a} = \frac{d}{c}$$

$$\frac{1}{2} = \frac{2}{4} \Rightarrow \frac{2}{1} = \frac{4}{2}$$

\rightarrow Alternendo -

$$\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a}{c} = \frac{b}{d}$$

\rightarrow Componendo -

$$\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a+b}{b} = \frac{c+d}{d}$$

\rightarrow Dividendo -

$$\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a-b}{b} = \frac{c-d}{d}$$

(i)

$$a:b :: c:d$$

$$\Rightarrow \frac{a}{b} = \frac{c}{d}$$

$$\Rightarrow a \times d = b \times c$$

Product of extremes = Product of means

Product of means =

(ii)

$$a:b = b:c$$

\rightarrow continued Proportion

$$\frac{a}{b} = \frac{b}{c} \Rightarrow b^2 = ac$$

$b \rightarrow$ Mean Proportional

$c \rightarrow$ Third Proportional

e.g.:



$$\frac{a+b}{b} = \frac{c+d}{d} \Rightarrow \frac{a+b}{b} = \frac{c+d}{c-d}$$

third proportional

$$3:9 = 9:k$$

$$9^2 = k \times 3 \Rightarrow 81 = 3k \Rightarrow k = 27$$

Q. If $81, x, x, 256$ are in proportion, find 'x'
 $x^2 = 81 \times 256 \Rightarrow x = \sqrt{81 \times 256} = 9 \times 16 = 144$

Q. What is the least number which must be subtracted from 16, 19 & 23 so that the resulting numbers are in continued proportion.

Q. $(a+b) : (a-b) = (15) : (1)$, find $\frac{a^2}{b^2}$
 $\frac{(a+b)(a-b)}{(a+b) - (a-b)} = \frac{15+1}{15-1}$

Q. Mean proportional of 8 & 98 is $\frac{a}{x} = \frac{x}{b} = \frac{16}{14} \times 8 = \frac{64}{49}$
 $\frac{a^2}{b^2} = \frac{64}{49}$

Q. The students in 3 classes are in the ratio of $2:3:4$. If 40 students are added in each class the ratio becomes $4:5:6$. Find the total no. of students in all the three classes is

$$2x + 40 = 4y$$

$$3x + 40 = 5y$$

$$4x + 40 = 6y$$

$$9x = 9 \times 20$$

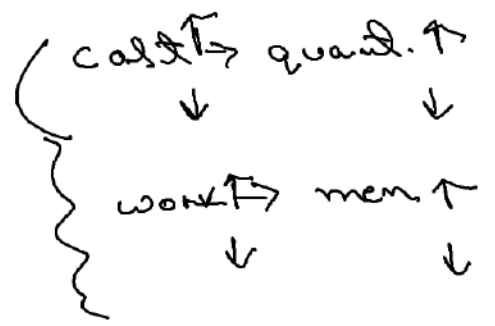
$$= 180$$

Unitary method
concept

→ Direct related quantities

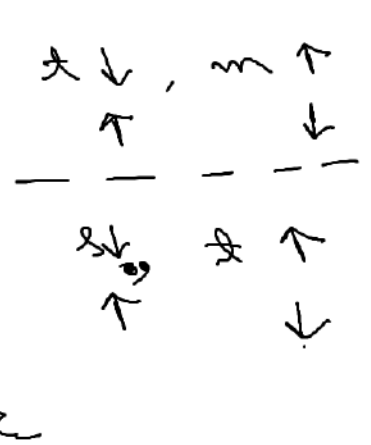
eg.: If 6 notebook cost Rs 45. How much would 8 notebook cost?

$$\begin{aligned}
 6n &\rightarrow 45 \\
 \swarrow \quad \searrow \\
 1n &\rightarrow \frac{45}{6} \\
 8n &\rightarrow \frac{45}{6} \times 8 = \boxed{60 \text{ Rs}}
 \end{aligned}$$



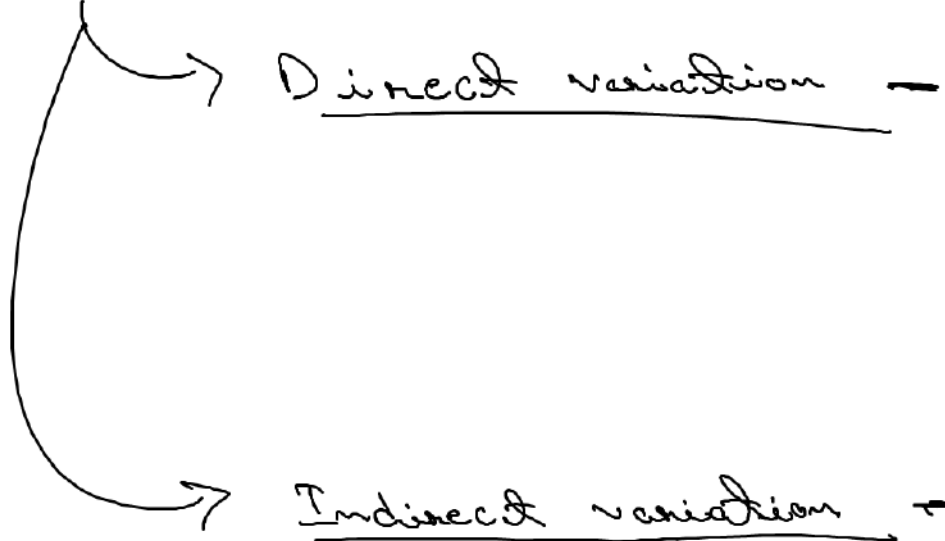
→ Inversely related quantities

eg.: If 45 students can consume a stock of food in 2 months, find how many days the same stock of food will last for 27 students?



$$\begin{aligned}
 \textcircled{45 \text{ s}} &\rightarrow 60 \text{ d} \checkmark \\
 \swarrow \quad \searrow \\
 1 \text{ s} &\rightarrow \frac{60 \times 45}{27} \\
 27 \text{ s} &\rightarrow \frac{60 \times 45}{27} = \boxed{100 \text{ days}}
 \end{aligned}$$

* Variation



Direct variation

$a \propto b$

$a = kb$

Proportionality
Const.

Indirect variation

$a \propto \frac{1}{b}$

$a = \frac{k}{b}$

~~Direct/Indirect~~

$A \propto B$ ✓ $A \propto \frac{1}{C}$ ✓

$A=12, B=6, C=2$

$A=?$, $B=12, C=3$

$A = \frac{kB}{C}$

$\frac{12}{2} = \frac{k \cdot 6}{2}$

$k=4$

$A = \frac{4B}{C}$

$= \frac{4 \times 12}{3}$

$A=16$ ✓

Q. The ratio of ages of K & B is 3:4 four years earlier the ratio was 5:7. Find the present ages of K & B.

$$\begin{array}{cc} 3K & , & 4K \\ \downarrow & & \downarrow \\ \underline{24} & & \underline{32} \end{array}$$

$$\frac{3K-4}{4K-4} = \frac{5}{7} \Rightarrow$$

$$21K - 28 = 20K - 20$$

$$\boxed{K = 8}$$

Q. The age of Sachin is 4 times that of his son. Five years ago Sachin was 9 times as old as his son was at that time. The present age of the Sachin is?