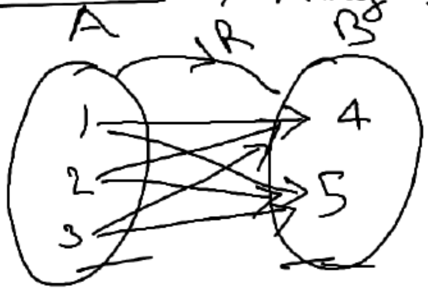
Set, Relation & function

* Relation → Any subset of $A \times B$



$\{R : A \to B$

$\emptyset$

$$\boxed{R \subseteq A \times B}$$

$A = \{1, 2, 3\}$, $B = \{4, 5\}$

$A \times B = \{(1,4), (1,5), (2,4), (2,5), (3,4), (3,5)\}$

$R = \{\emptyset \;(1,4)\}$ $R = \{(2,5)\}$

$R = \{(1,5), (2,4)\}$

cardinal no.

↑

No. of elements

$x \in (-5, 5)$

$-5 < x < 5$

$x \in [-5, 5)$ [

$-5 \le x < 5$)

$x \in [-5, 5[$

$x \in [-5, 5]$

$-5 \le x \le 5$

* Subset

$A = \{1, 2, 3\} \to$ $\{1\}, \{2\}, \{3\}$, $\{1,2\}, \{2,3\}, \{1,3\}$, $\{1,2,3\}$, $\emptyset$

Power set $= \{\emptyset, \ldots\}$

$n(P) = 8$

* Cartesian Prod. of two sets —

$A = \{1, 2, 3\}$, $B = \{4, 5\}$

$B \times A = \{(4,1), (4,2), (4,3), (5,1), (5,2), (5,3)\}$

$A \times B = \{(1,4), (1,5), (2,4), (2,5), (3,4), (3,5)\}$

$A \to \{1, 2\}$ $B \to \{1, 2\}$

$A \times B$, $B \times A$

$\{1, 4\}$ $\{a, b, c\}$

Q. $P = \{1, 2\}$, $Q = \{0, 1, 2\}$

A $P \cup Q = \{0, 1, 2\}$

B $P \cap Q = \{1, 2\}$

Q. $(a, b) = (c, d)$

$\left\{ \begin{array}{l} a = c, \quad b = d \end{array} \right.$

Q. $A = \{1, 2, 3, 4, 5\}$, $S = \{(x, y) : x \in A, y \in A\}$

(i) $x + y = 5$ $\quad \{(1,4), (2,3), (3,2), (4,1)\}$

(ii) $x + y < 5$

* Ordered triplet — (a, b, c)

$A = \{3, 5\}$, $A \times A \times A$

$A \times A = \{(3,3), (3,5), (5,3), (5,5)\} \times \{3, 5\}$

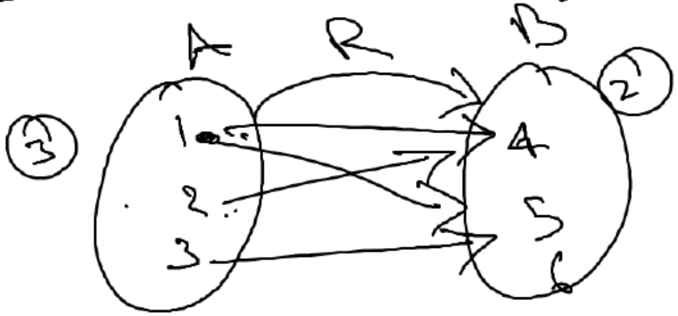
$(A \times A) \times A = \{(3,3,3), (3,5,3), (5,3,3), (5,5,3),$
$(3,3,5), (3,5,5), (5,3,5), (5,5,5)\}$

eg: $A = \{-3, -1\}$, $B = \{1, 3\}$, $C = \{3, 5\}$

$(A \times B) \times C$ $\quad$ $A \times (B \times C)$

$(2 \times 3) \times 7 = 2 \times (3 \times 7)$

* Domain & Range of a Relation



$A \times B$

5×7

$A \times B = \{\_\_\_\}$

$R = \{(1,4), (1,5), (2,4), (3,5)\}$

Dom $= \{1, 2, 3\}$

Range $= \{4, 5\}$

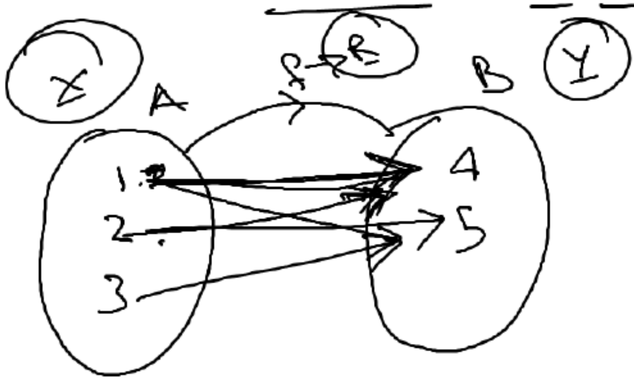
codom... $= \{4, 5, 6\}$

Domain → 1st co-ordinates are Domain

Range → Set of all 2nd co-ordinates

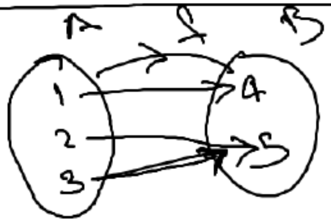
Co-Domain → Set B is called codomain

* Func^n → A certain type of Relation in which each element of X (Set A) should be associated with unique element of Y (Set B)

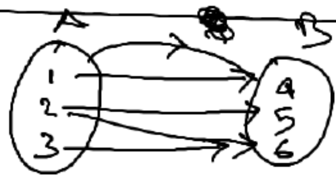


→ All element of set A ~~~~ must be associated element of set A (X)

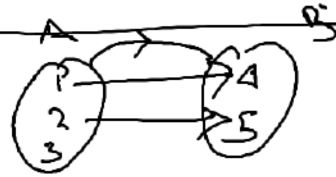
→ Each ~~~~ should ~~~~be associated with unique (element of set B) Y



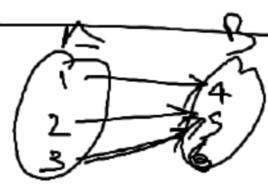
Func^n → ✓
Rel^n → ✓



Rel^n → ✓
Func^n → ✗



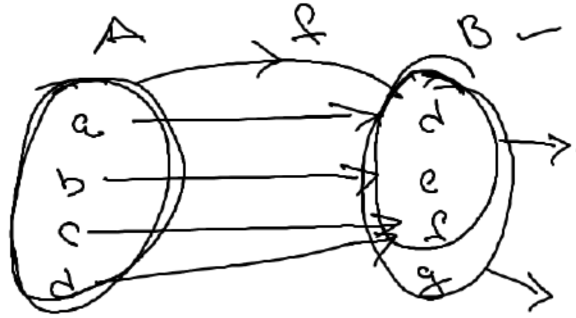
Rel^n → ✓
Func^n → ✗



Rel^n → ✓
Func^n → ✓

* Domain & Range of a func^n —

$f : A \to B$



→ Range

→ Co-domain

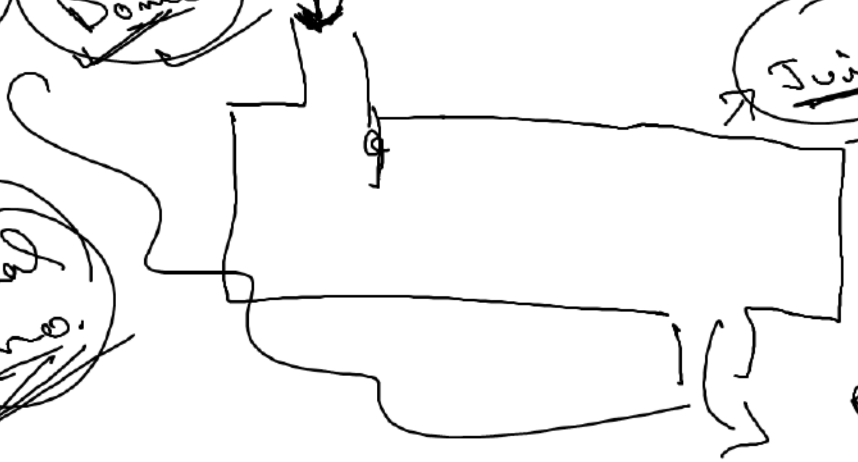
Domain

Range $\subseteq$ Co-domain

X

Domain

Fruits, vege



Juice

Machine → func^n

$y = f(x)$

output

Real no.

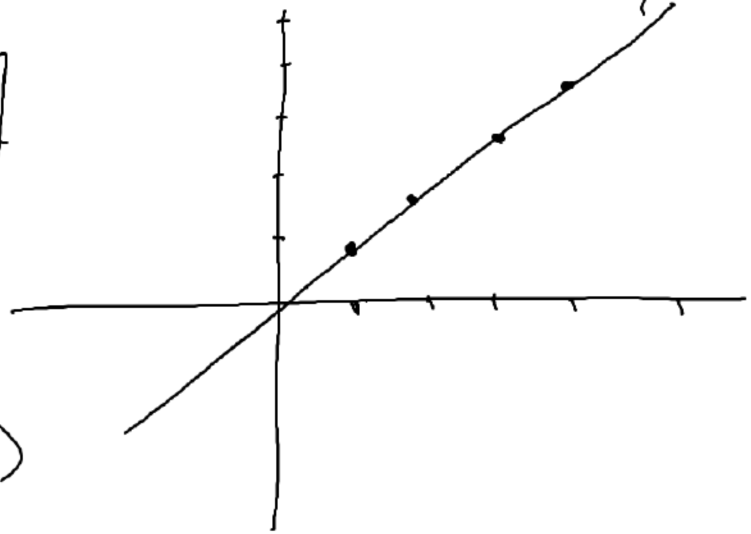
Range → Result

* Identity func^n → $f(x) = x$

$y = x$

$y = x$ Identity func^n

x	1	2	3	4	5
y	1	2	3	4	5



Dom → R → $(-\infty, \infty)$

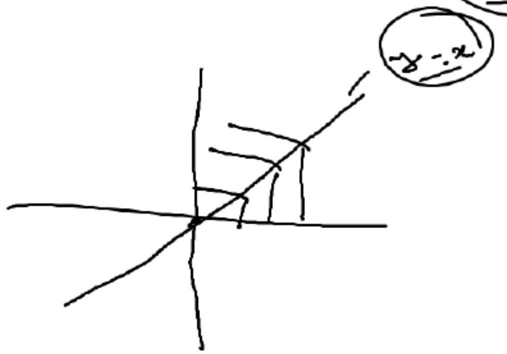
Range → R → $(-\infty, \infty)$

* Constant func^n —

Diff. func^ns

1. Identity func^n

$y = x$ → $f(x) = x$



Dom → R

Range → R

func^n → ✓
Dom → ✓
Range → ✓
Co-domain → ✓

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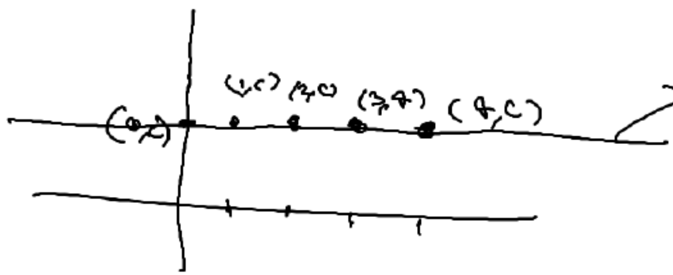
2. Constant func^n — $f(x) = c$

x	1	2	3	4
y	c	c	c	c



→ Const func^n

Dom → R

Range → c {c}

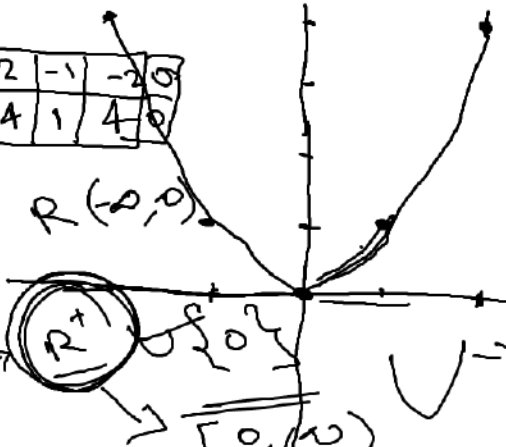
* Polynomial func^n — $y = x^2$

degree → even: x^{2n}, x^4, x^6, x^8 → upward / downward Parabola

degree → odd: x^{2n-1}, x^3, x^5, x^7, x^9

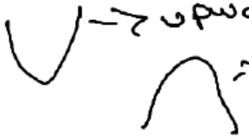
quad. func: $f(x) = x^2$

x	1	2	-1	-2	0
y	1	4	1	4	0



Dom → R $(-\infty, \infty)$

Range → $R^+ \cup \{0\}$ → $[0, \infty)$



x^2 → upward Para.

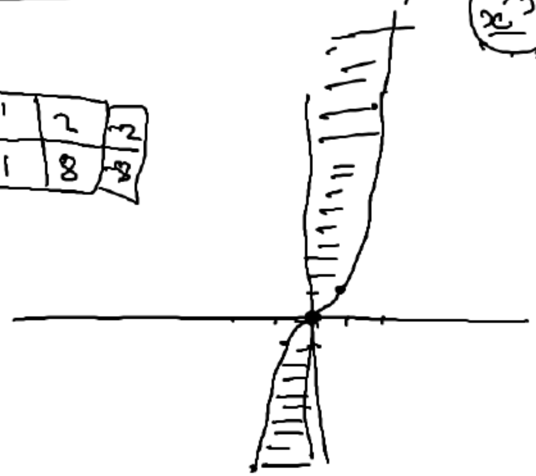
$-x^2$ → Downward Para.

$f: R \to R$

Range → $R^- \cup \{0\}$ → $(-\infty, 0]$

cubic func^n: $f(x) = x^3$

x	0	-1	1	2	-2
y	0	-1	1	8	-8



Dom → R

Range → R

* Rational funcⁿ $\rightarrow$

$\dfrac{f(x)}{g(x)}$, $g(x) \neq 0$

$f(x) = 1$

$f(x) = x$

$p \rightarrow$ Rational No.

$q \rightarrow q \neq 0$

$\text{H.C.F.}(p, q) = 1$

eg: $1 \rightarrow$ const.

$x \rightarrow$ Ident.,

$g(x) \neq 0$

$\boxed{f(x) = \dfrac{1}{x}} \rightarrow$ Reciprocal.

for $f(x)$ to be defined

$x \neq 0$

Dom $\rightarrow R - \{0\}$

x				
y				

Dom $\rightarrow R - \{0\}$

Range $\rightarrow$

$f(x) = \dfrac{1}{\sin x}$ { Dom $\rightarrow R - n\pi$

$\sin x \neq 0 \Rightarrow x \neq 0, \pi, 2\pi, 3\pi, -\pi, -2\pi$

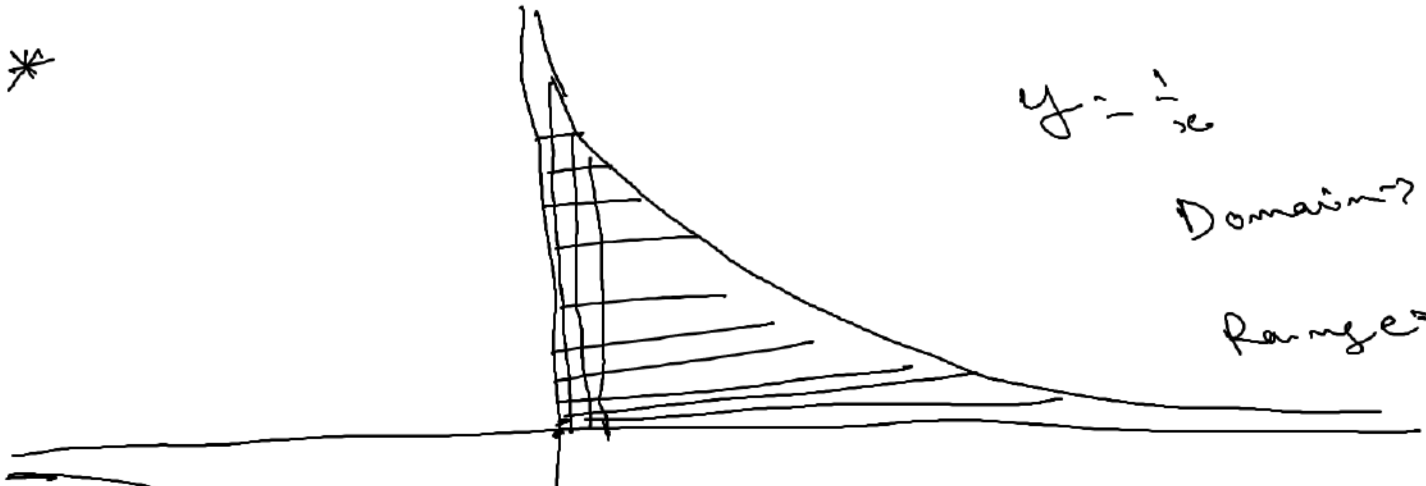
$x \neq n\pi$

eg: $f(x) = \dfrac{1}{x^2}$

$x^2 \neq 0 \Rightarrow x \neq 0$

Dom $\rightarrow R - \{0\}$

*



$y = \frac{1}{x}$

Domain $\rightarrow R - \{0\}$

Range $\rightarrow R - \{0\}$

$y = \frac{1}{x-3}$

$x - 3 = \frac{1}{y}$

$x = \frac{1}{y} + 3$

$x = \frac{1+3y}{y}$

$y \neq 0$

Rage $\rightarrow R - \{0\}$

Q. $y = \frac{1}{x-3} = \frac{p(x)}{q(x)}$,

$q(x) \neq 0$

Dom $\rightarrow$ $x - 3 \neq 0$

$x \neq 3$

$R - \{3\}$

$y = \frac{1}{x} = \frac{p(x)}{q(x)}$, $q(x) \neq 0$

Dom $\rightarrow$ $x \neq 0$

$\rightarrow R - \{0\}$

$y = \frac{1}{x}$

$x = \frac{1}{y}$

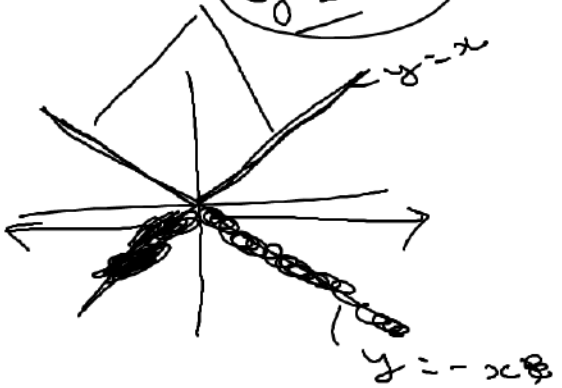
Rage $\rightarrow R - \{0\}$

* Modulus → Magnitude func.

$f(x) = |x| \rightarrow |x| = x, \; x \geq 0$

$\qquad\qquad\qquad\;\; = -x, \; x < 0$

$y = |x|$



$|2| = 2$ | $|2.5| = 2.5$

$|-2| = 2$ | $|-2.5| = 2.5$

$-(-2) = 2$

$y = |x|$

Dom → R

Range → $[0, \infty)$

$|x| = x, \; x \geq 0$

$\quad\; = -x, \; x < 0$

$|x-2| = x-2, \; x-2 \geq 0$, $x \geq 2$

$\qquad\quad\; = -(x-2), \; x-2 < 0$, $x < 2$

$x - 2 \geq 0$

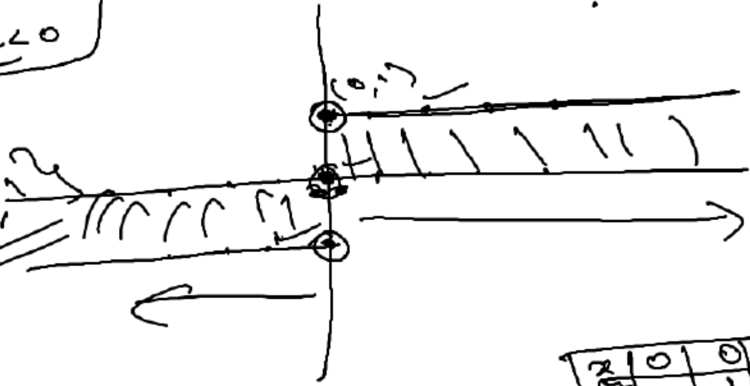
$x - \cancel{2} + \cancel{2} \geq 0 + 2$

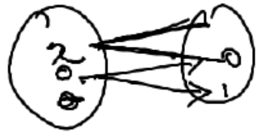
$x \geq 2$

* Signum func^n →

$$\mathrm{Sgn}(x) = \frac{|x|}{x} = \begin{cases} \frac{x}{x} = 1 & x > 0 \\ 0 & x = 0 \\ \frac{-x}{x} = -1 & x < 0 \end{cases}$$

$$\boxed{\begin{aligned} |x| &= x, \quad x \geq 0 \\ &= -x, \quad x < 0 \end{aligned}}$$





Dom → R

Range → {−1, 0, 1}

x	0	0
y	0	1

* Greatest Integer func^n

$\boxed{f(x) = [x]}$ → Dom → R

Range → Integer

[0.5] = 0

[0] = 0

[−0.99] = −1

[−1] = −1

[2.1] = 2

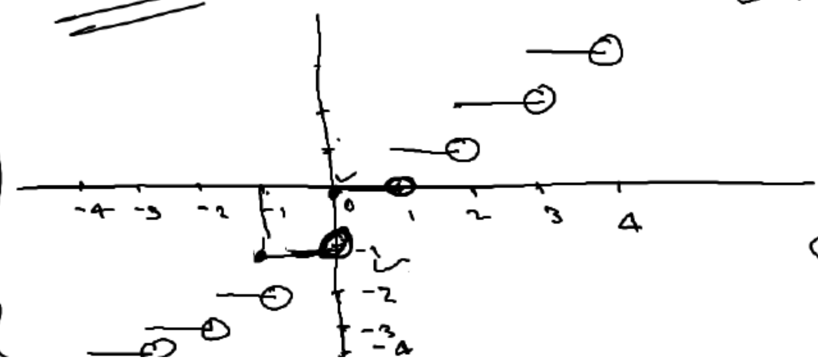
[−1.99] = −2

[2.5] = 2

[2.99] = 2

[3.01] = 3

Graph



[−1] = −1

* Domain & Range =

(i) $y = x$ $x = y$

Dom → R

Range → R

(ii) $y = x + 3$ $x = y - 3$

Dom → R

Range → R

(iii) $y = \frac{1}{x}$

Dom → $R - \{0\}$

Range → $R - \{0\}$

$x = \frac{1}{y}$, $y \neq 0$

Real No

(iv) $y = \frac{1}{x+2}$

$x + 2 \neq 0$

$x \neq -2$

Dom → $R - \{-2\}$

Range → $R - \{0\}$

$x + 2 = \frac{1}{y}$

$x = \frac{1}{y} - 2$

$x = \frac{1 - 2y}{y}$

(v) $y = \sqrt{x}$

Dom → $x \geq 0$

→ $[0, \infty)$

$y^2 = x \Rightarrow x = y^2$

Range → $[0, \infty)$

$y = \sqrt{x - 2}$ → Dom → $[2, \infty)$

$x - 2 \geq 0$

$x \geq 2$

(vi) $y = \frac{1}{\sqrt{x-2}}$

Dom → $(2, \infty)$ $x - 2 > 0$

$x > 2$

$y = \frac{+}{\sqrt{x-2}}$, $y^2 = \frac{1}{x-2}$, $x - 2 = \frac{1}{y^2}$

$+ve$

$x = 2 + \frac{1}{y^2}$

$\frac{1}{\sqrt{x-2}} \; +$; $+$; $+$

$\boxed{x = \frac{2y^2 + 1}{y^2}}$

$y \neq 0$

Range $\rightarrow R^+$

$(0, \infty)$

Q.4. $\boxed{x = \frac{2y+3}{y+1}}$ $f(x) = \frac{x-3}{2-x}$

$2 - x \neq 0$, $x \neq 2$

$y = \frac{x-3}{2-x}$ Dom $\rightarrow$ $R - \{2\}$ $x = \frac{2y+3}{y+1}$ $y+1 \neq 0$

$2y - xy = x - 3$ $y \neq -1$

$2y + 3 = xy + x$ Range $\rightarrow$ $R - \{-1\}$

$2y + 3 = x(y+1)$

Q. 6.

$$y = \frac{x^2 - 16}{x - 4} \rightarrow \cancel{x+4}$$

$\left(\frac{0}{0}\right)$

$x - 4 \neq 0$, $x \neq 4$

Dom $\rightarrow \mathbb{R} - \{4\}$

Range $\rightarrow \mathbb{R} - \{8\}$

$$y = \frac{\cancel{(x-4)}(x+4)}{\cancel{x-4}}, \quad x \neq 4$$

$y = x + 4$, $x \neq 4$

$y \neq 8$

$A \subset B$

$\Longleftrightarrow A \cup B = B$

A $A \cup B = B$

$\{1, 2\}$

$B\ \{1, 2, 3\}$

$A \cup B\ \{1, 2, 3\}$

Q. $A_1 \cup A_2 \cup A_3 \cup \ldots A_{30} = B_1 \cup B_2 \cup \ldots B_n = S$

Q. 6. (i) $A \subset (A \cup B)$

$A \subseteq B$

$x \in A, x \in B$

$A \subset B$

$y \in B, y \notin A$

$A \subset (A \cup B)$

$A = \{x\}$

$B = A \cup B = \{x, y, z\}$

$x \in A$,

$x \in A \cup B$

$x \in A$ or $x \in B$