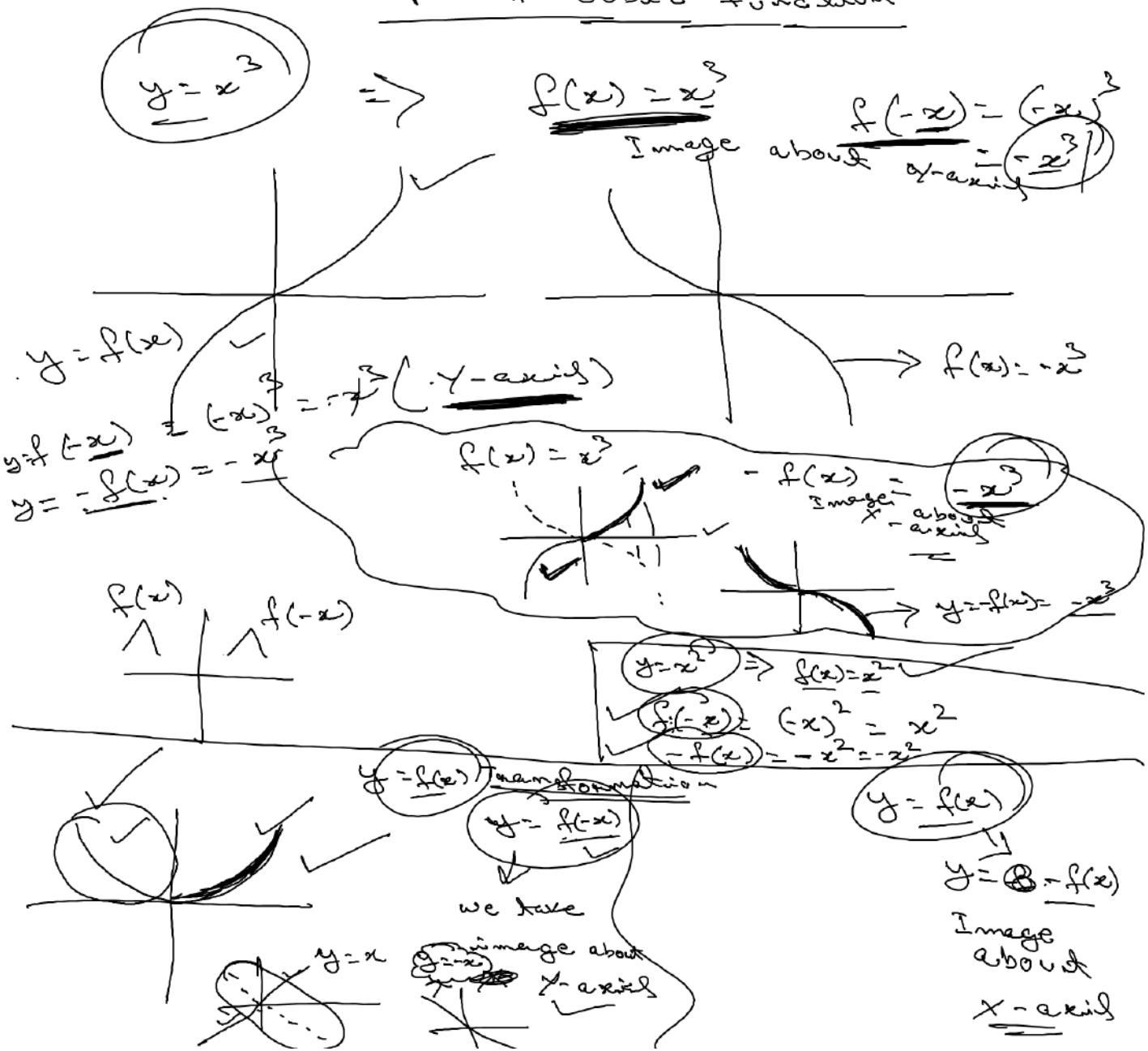
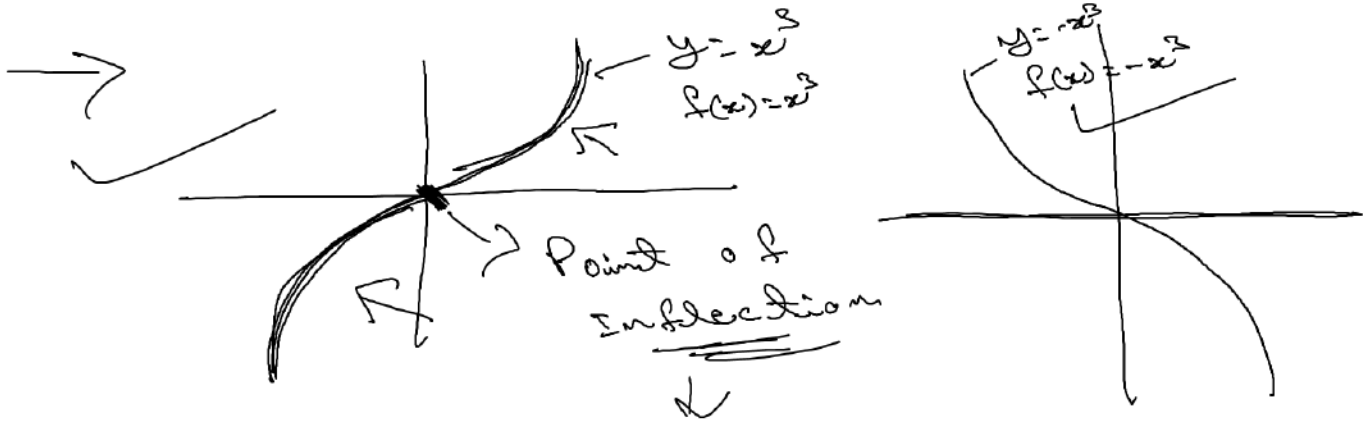
Graph of cubic function



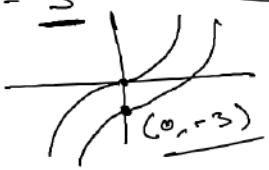


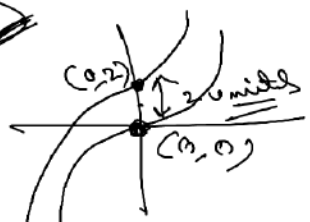
The point where func^n/graph changing its curvature

→ Vertical transformation — (up/Down shift) — $y = x^3$

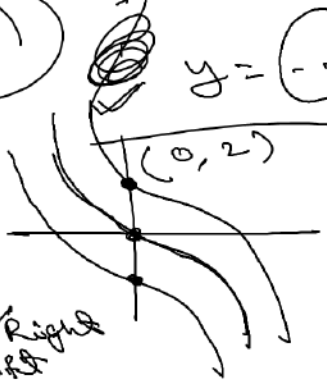
$y = x^3 + 2$

$y = x^3 - 3$





$y = -x^3 + 2$

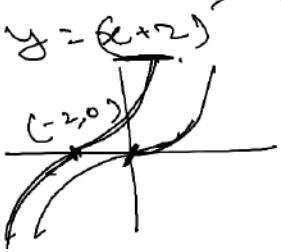


$y = -x^3 - 3$

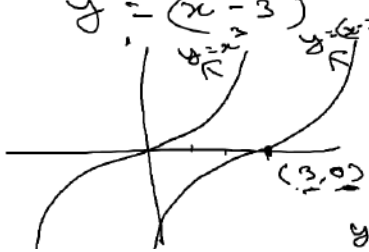
→ Horizontal transformations — Left/Right shift

$y = x^3$

$y = (x+2)^3$

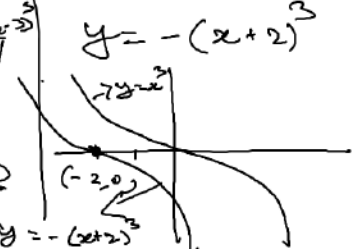


$y = (x-3)^3$

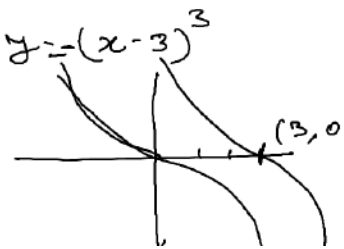


$y = -x^3$

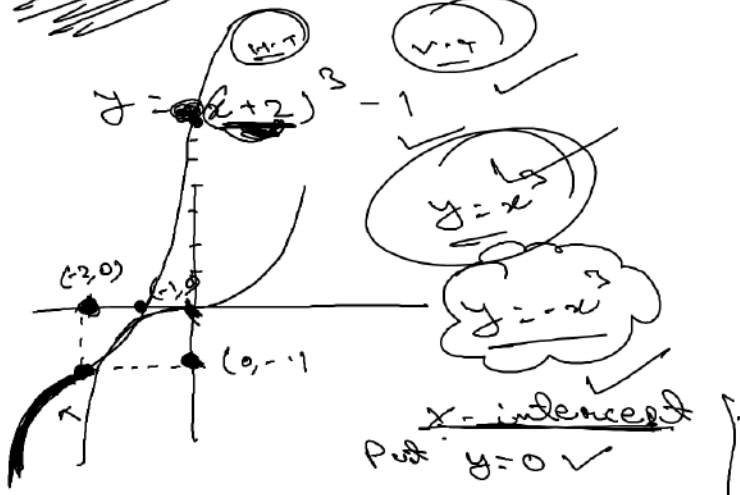
$y = -(x+2)^3$

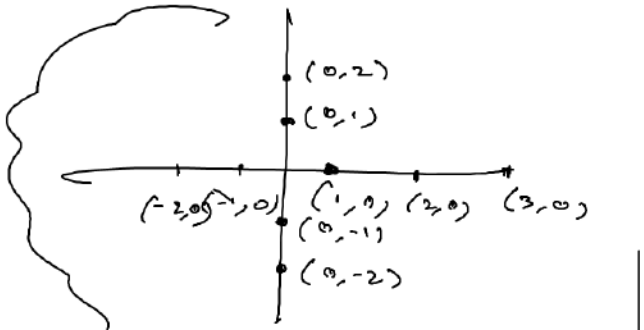


$y = -(x-3)^3$



8E.1(a)





Any pt. on x-axis
(Y-coordinate is 0)

Any pt. on y-axis
(x-coordinate is 0)

x-intercept
Put $y=0$ ✓

$y = (x+2)^3 - 1$

$0 = (x+2)^3 - 1$

$(x+2)^3 = 1$

$x = -1$

(-1,0)

x-intercept
Put $x=0$ ✓

$y = (0+2)^3 - 1$

$y = 8 - 1$

$y = 7$

(0,7)

$(x+2)^2 = 1$

$x = -1$ ✓

$x = -3$ ✓

$(x+2)^3 = 1$

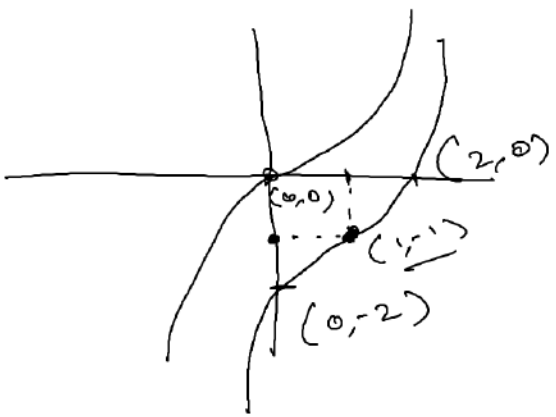
$x+2 = 1$

$x = -1$

8.E

1.b

$y = (x-1)^3 - 1$



(0, 0)
(1, -1)

Inflection pt. is (1, -1)

x-intercept

$y = 0$

$0 = (x-1)^3 - 1$

$(x-1)^3 = 1$

$(x-1)^3 = 1^3$

$x - 1 = 1$

$x = 2$

(2, 0)

y-intercept

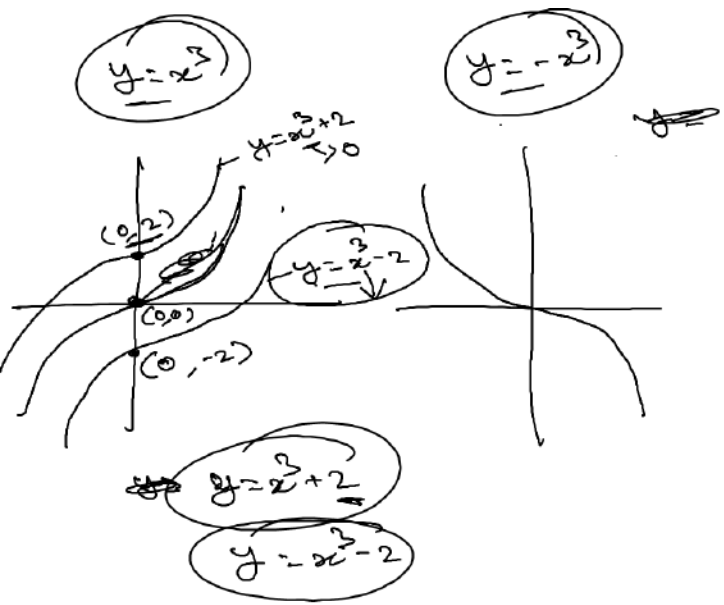
$x = 0$

$y = (0-1)^3 - 1$

$= -1 - 1$

$y = -2$

(0, -2)



Transl. 1

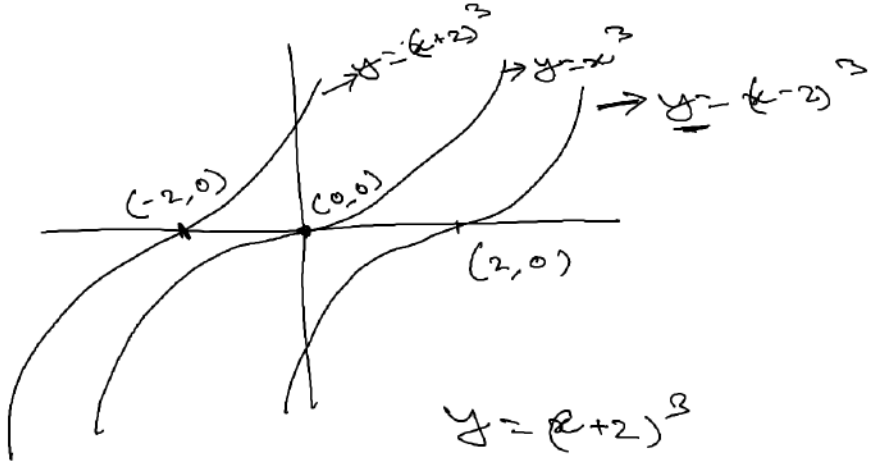
$y = f(x) \pm k$

Shift like upward (+) or downward (−)

Transl. 2

$y = f(x \pm k)$

Shift like left (+) & right (−)



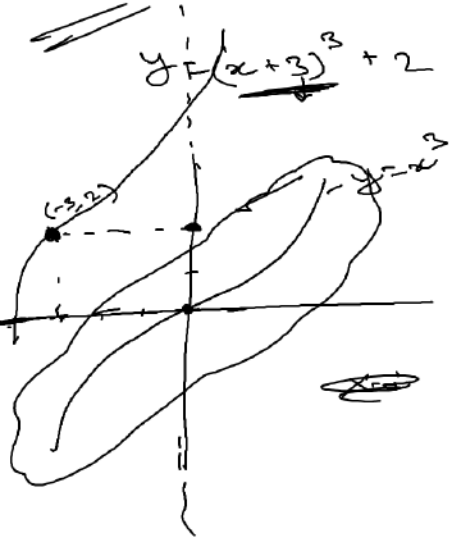
$y=(x+2)^3$

1(c)

$y = f(x) \pm c$

$y = (x+3)^3$

$y = (x+3)^3 + 2$



Inflect^n x-intercept

Pt. $P(-3,2)$

Put $y = 0$

$y = (x+3)^3 + 2$

$0 = (x+3)^3 + 2$

$(x+3)^3 = -2$

$x + 3 = \sqrt[3]{-2}$

$x = \sqrt[3]{-2} - 3$

$= -1.26 - 3$

$x = -4.26$

y-intercept

Put $x = 0$

$y = (x+3)^3 + 2$

$y = (0+3)^3 + 2$

$y = 27 + 2$

$y = 29$

$y = x^3$

$y = -x^3$

$\Rightarrow$ convex

$\Rightarrow$ concave

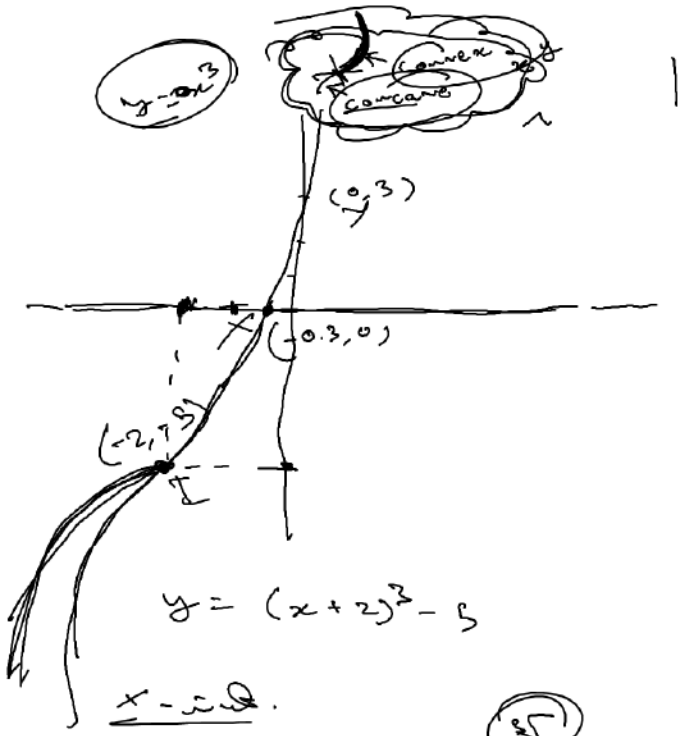
Curve with $(0,29)$

$P(-3,2)$

-3

-4

$(-4.26, 0)$



$y = (x+2)^3 - 5$

x-axis.

$y = 0$

$0 = (x+2)^3 - 5$

$(x+2)^3 = 5$

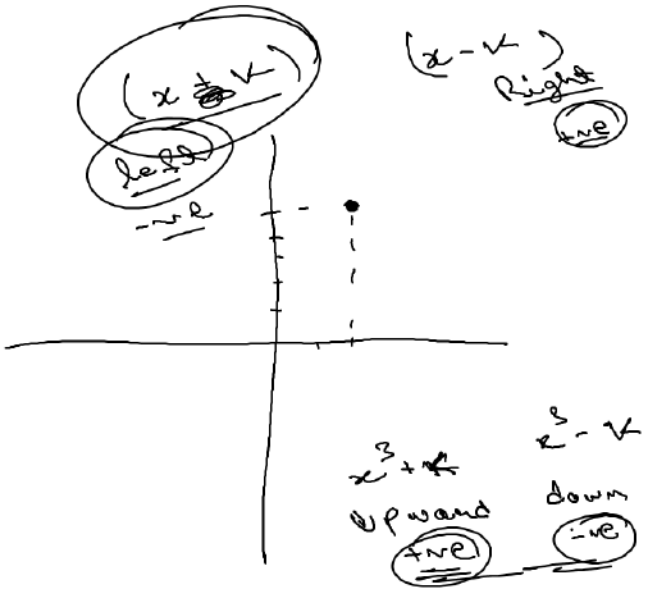
$x+2 = 5^{1/3}$

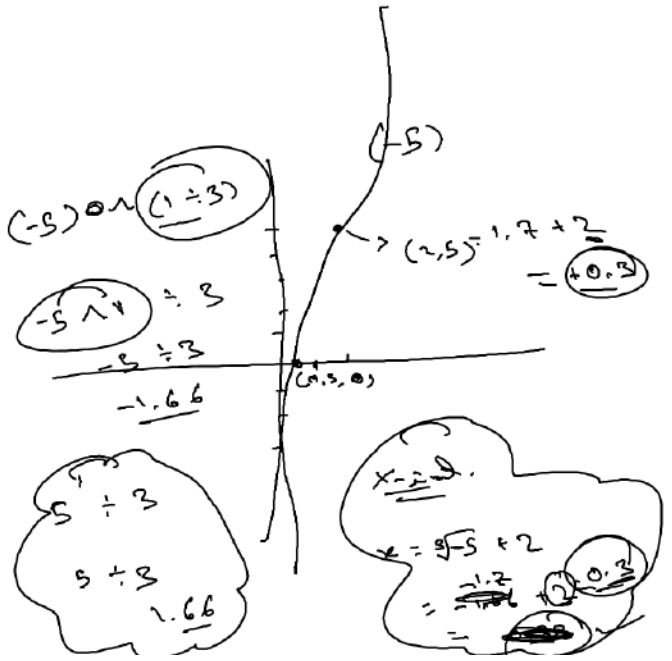
$x = 5^{1/3} - 2$

$= 1.7 - 2$

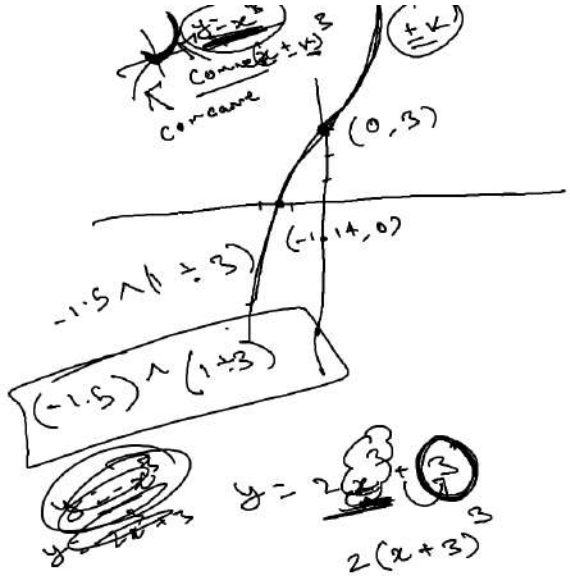
$= -0.3$

5 ∧ (1 ÷ 3)





y-axis.



Imbed

X-int.

$y = 0$

$0 = 2x^3 + 3$

$2x^3 = -3$

$x^3 = -1.5$

$x = \sqrt[3]{-1.5}$

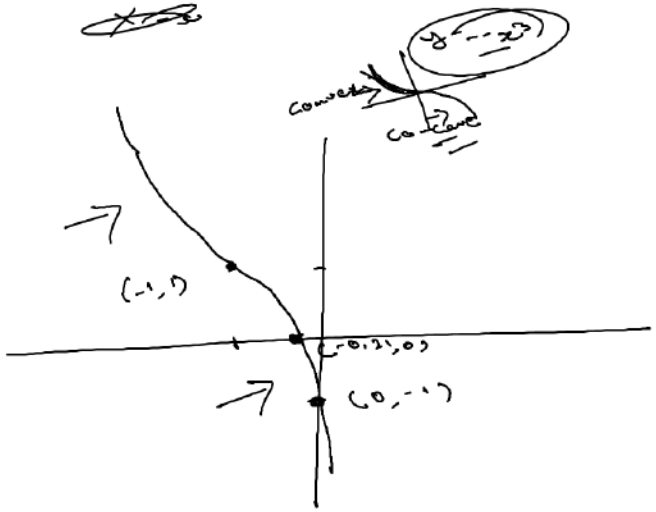
$x = -1.144$

Y-int.

$y = x^3$, $y = -x^3$

$y = -2(x+1)^3 + 1$

$(x+1) \quad +1$



x-int.

$y = -2(x+1)^3 + 1$

$y = 0$

$0 = -2(x+1)^3 + 1$

$2(x+1)^3 = 1$

$(x+1)^3 = \frac{1}{2}$

$x+1 = \sqrt[3]{0.5}$

$x = \sqrt[3]{0.5} - 1$

$= 0.79 - 1$

$x = -0.21$

$y = x^3$

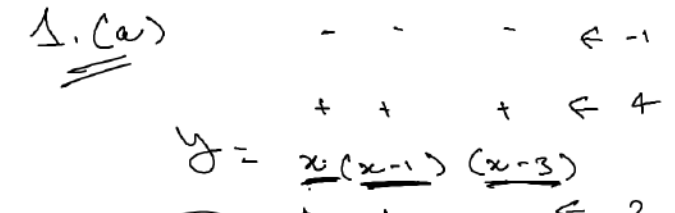
$y = (x+2)^3 + 5$

$y = -x^3$

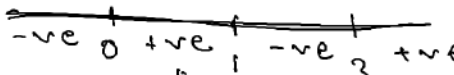
$y = -(x+2)^3 + 5$

1. (a)

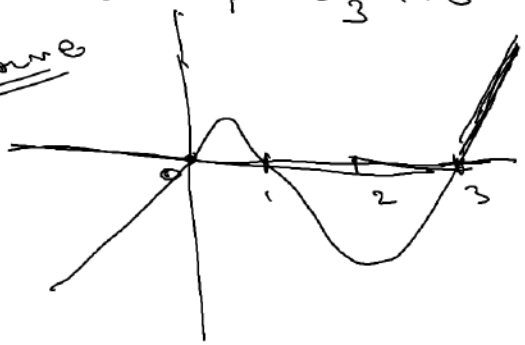
$y = x(x-1)(x-3)$



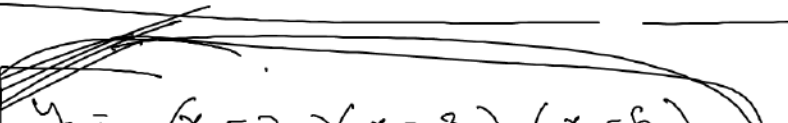
Sign Scheme



Curve



$y = (x-2)(x-3)(x-5)$

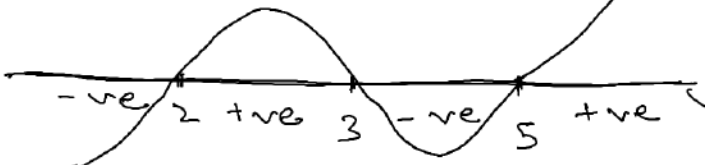


$x-2=0$
$x=2$

$x-3=0$
$x=3$

$x-5=0$
$x=5$

Sign Scheme



Real line

(i) If $x > 5$

$y = (x-2)(x-3)(x-5)$

$= (+)(+)(+)$

$= +ve$

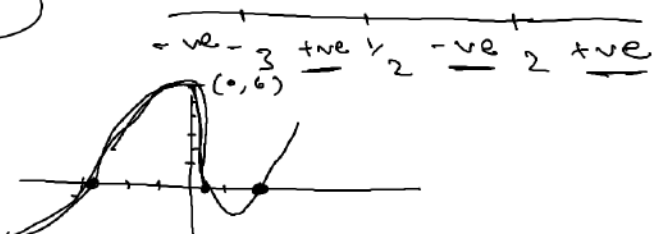
$y = -(2x-1)(x-2)(x+3)$

$0 = -(2x-1)(x-2)(x+3)$

$2x-1=0 \Rightarrow x = \frac{1}{2}$

$x-2=0 \Rightarrow x=2$

$x+3=0 \Rightarrow x=-3$



x-int. put $y=0$

$y = 6$

Repeated factor

$(x-5)^2$

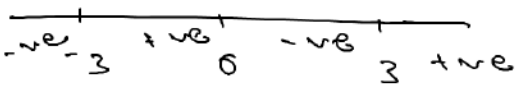
+ve + ve

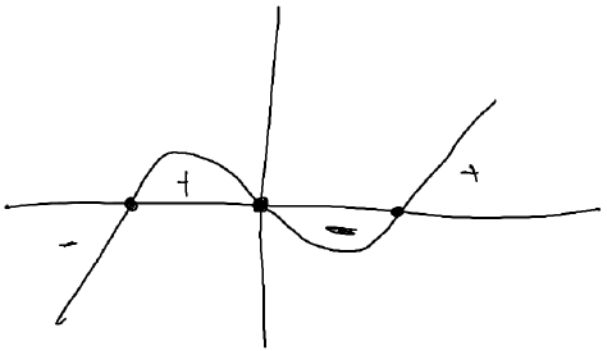
2. (a) $a^2 - b^2 = (a-b)(a+b)$ x-int. y-int.

$y = x^3 - 9x$

$= x(x^2 - 3^2)$

$y = x(x-3)(x+3)$





$y = (x-1)(x-2)(x-3)$

$(x-1)(x-2)(x-3) = 0$

$x-1=0,\ x-2=0,\ x-3=0$

$x=1,\ x=2,\ x=3$

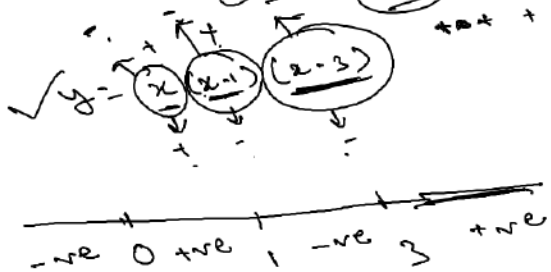
$y = x(x-1)(x-3)$

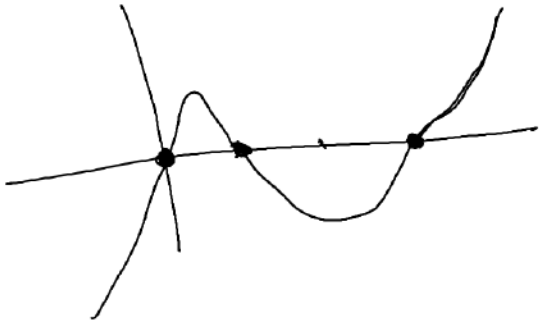
$x(x-1)(x-3) = 0$

$x=0,\ x-1=0,\ x-3=0$

$x=1,\ x=3$

$y = x(x-1)(x-3)$





y-int.

Put $x=0$

$y = 0(0-1)(0-3)$

$y = 0$

1.c

$y = (2x-1)(x-2)(x+3)$

$(2\times0-1)(0-2)(0+3)$

$(-1)(-2)(3)$

x-int. → $x = \frac{1}{2},\ x=2,\ x=-3$

y-int. → $y=6$

Sign Scheme →

1.(b)

$y = (x-1)(x+1)(x+2)$

x-int. | y-int.

$x=0$

$y = (0-1)(0+1)(0+2)$

$= -1\times1\times2$

$= -2$

$y = (x-1)(x+1)(x+2)$

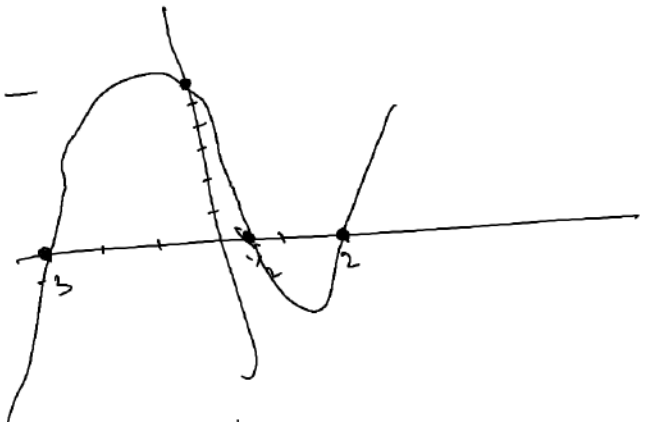
Sign Scheme

from $x=2$

$(2-1)(2+1)(2+3) = 1\times3\times4 > 0$

from $x=0$

$(0-1)(0+1)(0+2) =$



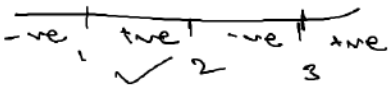
Q.

$y = (x-1)(x-2)(x-3)$

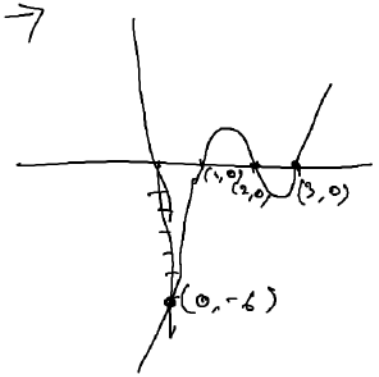
x-int. → $x=1, \quad x=2, \quad x=3$

Y-int. → $y = -6$

Sign Scheme →



Graph →



2 1 -2

$(x-3) \quad (x-1) \quad (x+2)$

$y = x^3 - 9x$

$= x(x^2 - 9)$

$= x(x^2 - 3^2)$

$y = x(x-3)(x+3)$

Repeat same process as in 1.

Q. 2. (e)

$y = 6x^3 - 5x^2 - 2x + 1$

$x = 1$ $\quad$ $y = 0$

$(x-1)$ is a factor

$$x-1 \;\big)\; 6x^3 - 5x^2 - 2x + 1 \;\big(\; 6x^2 + x - 1$$

$6x^3 - 6x^2$ (+)

$x^2 - 2x + 1$

$x^2 - x$ (−)

$-x + 1$

$-x + 1$ (−)

0

$y = (x-1)(6x^2 + x - 1)$ $\quad$ 26 | 3 | 3 | 1

2. (f) $y = 2x^3 - 9x^2 + 7x + 6$

$(x+2)$

$2 \times (-2)^3 - 9 \times (-2)^2 + 7(-2) + 6$

$= 2 \times (-8) - 9 \times 4 - 14 + 6$

$= -16 - 36 + 14 + 6$

$= -52 + 20$

$= -32$

Q. $y = x^3 + x^2$

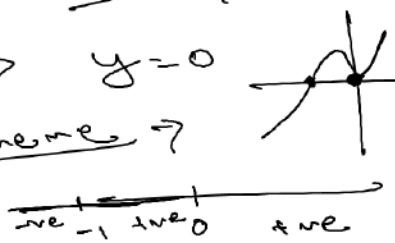
$= x^2 (x+1)$

$= x \cdot x (x+1)$

x-int. $\Rightarrow$ $x = 0, \; x = -1$

y-int. $\Rightarrow$ $y = 0$

Sign Scheme $\Rightarrow$



-ve $\quad -1 \quad$ +ve $\quad 0 \quad$ +ve

3. (a)

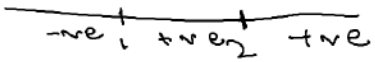
$y = (x-1)(x-2)^2$

$= (x-1)(x-2)(x-2)$

x-int. $\Rightarrow$ $1, 2$

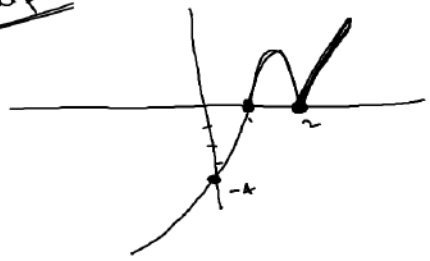
y-int. $\Rightarrow$ -4

Sign Scheme $\Rightarrow$



-ve $\quad 1 \quad$ +ve $\quad 2 \quad$ +ve

Graph



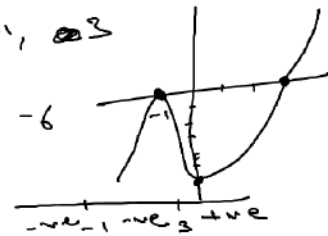
3. (c) $y = 2(x+1)^2 (x-3)$

$= 2(x+1)(x+1)(x-3)$

x-int. $\Rightarrow$ $x = -1, \; 3$

y-int. $\Rightarrow$ $y = -6$

Sign Scheme $\Rightarrow$



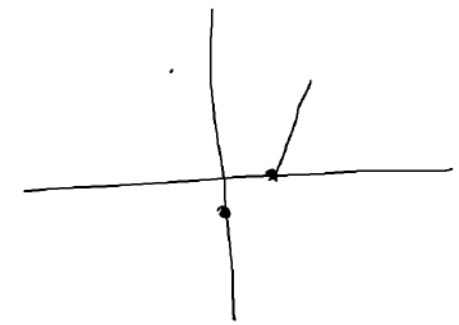
-ve $\quad -1 \quad$ -ve $\quad 3 \quad$ +ve

$x(x+1)$
x^2+x

$D = b^2 - 4ac$
$= (0)^2 - 4 \times 1 \times 1 = -4 < 0$

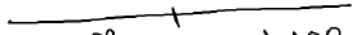
$y = (x-1)(x^2+1)$

Sign Scheme



x-int. $\Rightarrow$ $x = 1$

y-int. $\Rightarrow$ $y = -1$



$x(x-4)$

$ax^2 + bx + c = 0$

Discriminant $= b^2 - 4ac$

- $D = 0$ → factorise
- $D > 0$ (with D as perfect sq.) → factorise
- $D > 0$ (D is not perfect sq.) → can't factorise
- $D < 0$ (doesn't matter with D) → can't factorise

e.g. $x^2 - 5x + 6$

$a = 1, b = -5, c = 6$

$D = b^2 - 4ac = (-5)^2 - 4 \times 1 \times 6$

$= 25 - 24$

$= 1$ (P.S)

e.g. $x^2 - 4x + 0$

$a = 1, b = -4, c = 0$

$D = b^2 - 4ac$
$= (-4)^2 - 4 \times 1 \times 0$
$= 16$ (P.S)

Q.

$y = f(x)$

$f(x) = x^3 - x^2 - 5x - 3$

x-intercept

Put $y = 0$

$y = x^3 - x^2 - 5x - 3$

$0 = x^3 - x^2 - 5x - 3$

$x^3 - x^2 - 5x - 3 = 0$

$0, \pm 1, \pm 2, \pm 3$

-1 → $(x+1)$ is a factor

$(-1)^3 - (-1)^2 - 5 \times (-1) - 3$

$-1 - 1 + 5 - 3$

$(x+1)(x^2 - 2x - 3) = 0$

$(x+1)(x^2 - 3x + x - 3) = 0$

$(x+1)[x(x-3) + 1(x-3)] = 0$

$(x+1)(x-3)(x+1) = 0$

$(x+1)(x+1)(x-3) = 0$

touches ← $x = -1$ | $x = 3$ → cuts

$x+1 \overline{)\, x^3 - x^2 - 5x - 3 \,(x^2 - 2x - 3}$

$x^3 + x^2$

$-2x^2 - 5x - 3$

$-2x^2 - 2x$

$-3x - 3$

$-3x - 3$

$a + b = 16$ — (i)

$4a + 2b = 40$

$4a + b = 20$ — (ii)

(ii) − (i)

$3a = 4$

$a = 4/3$

Put $a = \frac{4}{3}$ in eq^n (i)

$\frac{4}{3} + b = 16$

$b = 16 - \frac{4}{3}$

$= \frac{44}{3}$

Q6

x-coord. (2, 3) y-coord.

$y = a(x-3)^3 + 1$

$(4, 12)$ lies on func^n

⇓

it will satisfy the func^n

$y = a(x-3)^3 + 1$

$12 = a(4-3)^3 + 1$

$12 = a \times 1^3 + 1$

$12 = a + 1$

$a = 11$

$y = a(x-2)(x+3)(x-1)$

$24 = a(3-2)(3+3)(3-1)$

$24 = a \times 1 \times 6 \times 2$

$a = 2$