(set of rule)

Sequences & Series

eg: 1, 3, 5, 7, 9, ... $+2$

eg: 1, 2, 4, 8, 16, ... $\times 2$

eg: $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots$ H.P

Fibonacci seq

eg: 0, 0, 1, 1, 2, 3, 5, 8, 13, 21, ...

$a, a+d, a+2d, \ldots, l-2d, l-d, l$

$a_1 = a + 0d$

$a_2 = a + 1d$

$a_3 = a + 2d$

$a_4 = a + 3d$

$$\boxed{a_n = a + (n-1)d}$$

$l_1 = l$

$l_2 = l - d$

$l_3 = l - 2d$

$l_4 = l - 3d$

$$\boxed{l_n = l - (n-1)d}$$

A.P

$+k$ $-k$

nth term from starting

$$\boxed{a_n = a + (n-1)d}$$

$$\boxed{l_n = l - (n-1)d}$$

$$\boxed{S_n = \frac{n}{2}(2a + (n-1)d) = \frac{n}{2}(a+l)}$$

G.P

H.P

$\checkmark\ S_n = a + (a+d) + (a+2d) + \ldots + \overset{l-d+\ l}{\cancel{a+(n-1)d}}$ — ①

$\checkmark\ S_n = \cancel{a+(n-1)d}\ l + (l-d) + (l-2d) + \ldots + a$ — ⑪

$1+2+3 = 6$

$3+2+1 = 6$

$2S_n = (a+l) + (a+l) + (a+l) + \ldots + (a+l)$

n times

$2S_n = n(a+l)$

$$\boxed{S_n = \frac{n}{2}(a+l)}$$

$2+2+2 \ldots 10$ times

10×2

$l = a_n = a+(n-1)d$

$= \frac{n}{2}(a + a + (n-1)d)$

$$\boxed{S_n = \frac{n}{2}(2a+(n-1)d)}$$

* Arithmetic mean (A)

a, A, b

$A - a = b - A$

$2A = a + b$

$$\boxed{A = \frac{a+b}{2}}$$

a_1, a_2, a_3

$$\boxed{a_2 - a_1 = a_3 - a_2}$$

eg.:

$2, \overset{A}{4}, 6$

$A = \frac{2+6}{2} = \frac{\overset{4}{\cancel{8}}}{\cancel{2}}$

---

* 'n' no. of A.M b/w two numbers —

c.d = d

first, last

$a, A_1, A_2, A_3, \dots A_n, b$

$\xleftrightarrow{\; n+2 \;}$

$A_1 = a + d = a + \left(\frac{b-a}{n+1}\right)$

$A_2 = a + 2d = a + 2\left(\frac{b-a}{n+1}\right)$ $b = a + (n+2-1)d$

$A_3 = a + 3d = a + 3\left(\frac{b-a}{n+1}\right)$ $b = a + (n+1)d$

$A_n = A + nd = a + n\left(\frac{b-a}{n+1}\right).$

$l = a + (n-1)d$

l → last term an

a → 1st

n → no. of terms

d → c.d

$$\boxed{d = \frac{b-a}{n+1}}$$

b → last term

a → 1st

n → No. of A.M to be inserted

6.

$\overset{a}{16}\ ,\ \overset{b}{65}$

$A_5 = 51$

$a + 5d$

$16 + 5d = 51$

$\cancel{5}d = \cancel{35}^{7}$

$\boxed{d = 7}$

$a, A_1, A_2 \ldots$

$A_1 = a + d$

$A_5 = a + 5d$

$d = \frac{b-a}{n+1}$

$n+1 = \frac{b-a}{d}$

$n+1 = \frac{\cancel{49}\ 7}{\cancel{7}}$

$\boxed{n = 6}$

G.P ✓

$a, ar, ar^2, ar^3, \ldots, ar^{n-1}$

$\rightarrow \frac{\cancel{a}r}{\cancel{a}} = \frac{\cancel{a}r^2}{\cancel{ar}} = \frac{\cancel{a}r^3}{\cancel{ar^2}} = \ldots$

$a_1, a_2, a_3, \ldots, a_n$

$\frac{a_2}{a_1} = \frac{a_3}{a_2} = \frac{a_4}{a_3} \ldots$

common ratio

$2, 4, 8, \quad \times 2$

$\div 2$

* nth term of a G.P

$\boxed{a_n = ar^{n-1}}$

from starting

$a_1 = a$

$a_2 = ar^1$

$a_3 = ar^2$

$a_4 = ar^3$

$\boxed{a_n = ar^{n-1}}$

from ending

$a, ar, ar^2, \ldots, \frac{l}{r^2}, \frac{l}{r}, l$

$l_1 = l$

$l_2 = \frac{l}{r}$

$l_3 = \frac{l}{r^2}$

$l_4 = l/r^3$

$\boxed{l_n = \frac{l}{r^{n-1}}}$

* Sum of 'n' terms of a G.P : $a, ar, ar^2, \dots, \frac{l}{r^2}, \frac{l}{r}, \underset{\boxed{ar^{n-1}}}{l}$

$$S_n = \frac{a(r^n - 1)}{r-1}, \quad r > 1 \quad \Rightarrow \quad S_n = \frac{ar^n - a}{r-1}$$

$$= \frac{(ar^{n-1}).r - a}{r-1}$$

$$\boxed{S_n = \frac{lr - a}{r-1}}$$

$$S_n = \frac{a(1 - r^n)}{1 - r}, \quad r < 1$$

$$\boxed{S_n = \frac{a - lr}{1 - r}}$$

---

*

$$S_n = a + \cancel{ar} + \cancel{ar^2} + \dots + \overset{l}{\cancel{ar^{n-1}}} \quad — ①$$

$$rS_n = \cancel{ar} + \cancel{ar^2} + \cancel{ar^3} + \dots \cancel{ar^{n-1}} + ar^n \quad — ⑪$$

$$rS_n - S_n = ar^n - a$$

$$(r-1)S_n = a(r^n - 1)$$

$$* \boxed{S_n = \frac{a(r^n - 1)}{r-1}}$$

* G.M → b/w two numbers

a , G , b

$$\frac{G}{a} = \frac{b}{G}$$

$$G^2 = ab$$

$$\boxed{G = \sqrt{ab}}$$

a , A , b

$A - a = b - A$

$2A = a + b$

$$A = \frac{a+b}{2}$$

Relⁿ b/w A.M ≥ G.M
(A) (G)

$A = \frac{a+b}{2}$, $G = \sqrt{ab}$

$R^2 \geq 0$

$(\sqrt{a} - \sqrt{b})^2 \geq 0$

$a + b - 2\sqrt{ab} \geq 0$

$a + b \geq 2\sqrt{ab}$

$\frac{a+b}{2} \geq \sqrt{ab}$

$A \geq G$

* 'n' no. of G.M b/w two numbers

$$\overset{\longleftarrow n \longrightarrow}{a, G_1, G_2, G_3, \dots, G_n, b} \rightarrow r$$

$\longleftarrow (n+2) \longrightarrow$

$b = a r^{(n+2-1)}$

$b = a r^{(n+1)}$

$\frac{b}{a} = r^{n+1}$

$$\boxed{r = \left(\frac{b}{a}\right)^{1/n+1}}$$

~~100 term~~

$l = ar^{n-1}$

n−1 → no. of terms

r → c.r

l → last term

$G_1 = ar = a\left(\frac{b}{a}\right)^{1/n+1}$

$G_2 = ar^2 = a\left(\frac{b}{a}\right)^{2/n+1}$

$G_3 = ar^3 = a\left(\frac{b}{a}\right)^{3/n+1}$

Ex - 12 A

Q.4

$\frac{1}{2}\left(-\frac{2}{3}\right)^8$

$\frac{\cancel{256}\ 128}{2 \times 3^8}$

$\frac{128}{81 \times 81}$

$\frac{128}{6561}$

$\frac{a/b}{c/d}$

$\frac{a}{b} \times \frac{d}{c}$

Q.9

$\frac{\cancel{x_n^3}}{\cancel{a_n 3}} = \frac{1/8}{-1/486}$ ✓

$\frac{1}{r^3} = -\frac{\cancel{486}\ \cancel{54}}{\cancel{18}}\ 27$ ✓

$\frac{1}{r^3} = -27 \Rightarrow r^3 = -\frac{1}{27}$

$r = -\frac{1}{3}$

$r = -\frac{1}{3}$

$\log_{3^6} 2^5 = \frac{5}{6}\log_3 2$

$\log_3 2^5 = 5\log_3 2$

$\log_{3^6} 2 = \frac{1}{6}\log_3 2$

* **Logarithm** → It is inverse funcⁿ of exponential funcⁿ

$Y = \log_a x$ → base

$x > 0$

$a \neq 1$

$x = a^y$

e → const.

→ 2.71828184

$\log_2 5$

$\log_7 10$

$\log_e 10$

Eg: $8 = \log_2 x$

$x = 2^8$

Prop. →

1. $\log_a 1 = 0$
2. $\log a = \log_e a$
3. $\log_5 2 = \frac{1}{\log_2 5}$

3. $\log_e e^m = m \log e$

4. $\log_{b^n} a^m = \frac{m}{n}\log_b a$

5. $\log_b a = \frac{\log_e a}{\log_e b} = \frac{\log_c a}{\log_c b}$

6. $\log_b a = \frac{\log_c a}{\log_c b} = \frac{1}{\frac{\log b}{\log a}}$

* $\log(ab) = \log a + \log b$

* $\log\left(\frac{a}{b}\right) = \log a - \log b$

Q. $\log(a^m b^m) = \log(ab)^m$

$= m \log ab$

$= m(\log a + \log b)$

$\frac{a^m}{a^n} = a^{m-n}$

$a^m \times a^n = a^{m+n}$

Q. 14

1st term ⇐ (A), (R) → C.R

$a_p = a \Rightarrow A(R)^{p-1} = a$

$a_q = b \Rightarrow A(R)^{q-1} = b$

$a_r = c \Rightarrow A(R)^{r-1} = c$

$(q-r)\log a + (r-p)\log b + (p-q)\log c$

$= (q-r)\log(AR^{p-1}) + (r-p)\log(AR^{q-1}) + (p-q)(AR^{r-1})$

$= (q-r)(\log A + \log R^{(p-1)}) + (r-p)(\log A + \log R^{(q-1)}) + (p-q)(\log A + \log R^{(r-1)})$

$= \cancel{(q-r)\log A} + (q-r)\log R^{(p-1)} + \cancel{(r-p)\log A} + (r-p)\log R^{(q-1)} + \cancel{(p-q)\log A} + (p-q)\log R^{(r-1)}$

Q.18

$x^2-3x+p=0$ (a,b) ✓

$x^2-12x+q=0$ (c,d)

Sum of roots $= -\frac{b}{a}$

Prod. of roots $= \frac{c}{a}$

$a+b=3$

$ab=p$

$a \quad ar \quad a^2r = p$

$c+d=12$

$cd=q$

$a+ar=3 \Rightarrow a(1+r)=3 \Rightarrow a=\frac{3}{1+r}$

$ar^2+ar^3=12$

$ar^2(1+r)=12$

$\frac{3}{\cancel{1+r}} \times r^2 (\cancel{1+r}) = \overset{4}{\cancel{12}}$

$r=\pm 2$

$a \;, ar, ar^2, ar^3$

for $r=2$

$a=\frac{3}{3}=1$

for $r=-2$

$a=\frac{3}{1-2}=-3$

$p=ab=a\times ar=a^2r$

$=1\times 2=②$

$p=ab=a^2r=-3\times -2$

$=⑥$

$q=ar^2\times ar^3$

$=a^2r^5$

$=1\times 2^5$

$=32$ ✓

$q=ar^2\times ar^3$

$=a^2r^5$

$=9\times -32$

$=-288$

~~$\frac{q+p}{q-p}$~~ ~~15~~

~~$\frac{a^2r^5 + a^2r}{a^2r^5-a^2r}$~~ ~~$\frac{17}{15}$~~

$\frac{q+p}{q-p}=\frac{-288+6}{-288-6}=$

$\frac{q+p}{q-p}=\frac{32+2}{32-2}=\frac{\cancel{34}\,17}{\cancel{30}\,15}$

Seq. Series

1st term $\textcircled{A}$ C.R $\textcircled{R}$

Q. $a_p = a \Rightarrow A(R)^{p-1} = a$

$a_q = b \Rightarrow A(R)^{q-1} = b$

$a_r = c \Rightarrow A(R)^{r-1} = c$

$$L.H.S = (q-r)\log a + (r-p)\log b + (p-q)\log c$$

$$= (q-r)\log(AR^{(p-1)}) + (r-p)\log(AR^{(q-1)}) + (p-q)\log(AR^{(r-1)})$$

$$= (q-r)(\log A + \log R^{\textcircled{p-1}}) + (r-p)(\log A + \log R^{q-1}) + (p-q)(\log A + \log R^{r-1})$$

$$= (q-r)\log A + (q-r)(p-1)\log R + (r-p)\log A + (r-p)(q-1)\log R + (p-q)\log A + (p-q)(r-1)\log R$$

$$= \cancel{q\log A} - \cancel{r\log A} + (pq - pr - q + r)\log R + \cancel{r\log A} - \cancel{p\log A} + (rq - pq - r + p)\log R + \cancel{p\log A} - \cancel{q\log A} + (pr - rq - p + q)\log R$$

$$= \log R\left[\cancel{pq} - \cancel{pr} - \cancel{q} + \cancel{r} + \cancel{rq} - \cancel{pq} - \cancel{r} + \cancel{p} + \cancel{pr} - \cancel{rq} - \cancel{p} + \cancel{q}\right]$$

$$= \log R[0]$$

$$= 0 \checkmark$$

Q:

$$\frac{1}{3} + \frac{7}{3}i + 4 + \frac{1}{3}i + \frac{4}{3} - i$$

$$\left(\frac{1}{3} + \frac{4}{3} + 4\right) + \left(\frac{7}{3}i + \frac{1}{3}i - i\right)$$

$$\frac{17}{3} + i\left(\frac{7}{3} + \frac{1}{3} - 1\right) \checkmark$$

$$\frac{17}{3} + i\left(\frac{7 + 1 - 3}{3}\right) \checkmark$$

$$\frac{17}{3} + i\left(\frac{5}{3}\right) \quad \boxed{\frac{17}{3} + i\frac{5}{3}} \checkmark$$

Q. 2(xi)

$$x(x+y) + x^2(x^2+y^2) + x^3(x^3+y^3) + \dots n \text{ terms}$$

$$S = x^2 + xy + x^4 + x^2y^2 + x^6 + x^3y^3 + \dots$$

$\boxed{\times x^2}$ $\boxed{\times xy}$

$$S = \underbrace{(x^2 + x^4 + x^6 + \dots)}_{S_1} + \underbrace{(xy + x^2y^2 + x^3y^3 + \dots)}_{S_2}$$

$$= x^2\frac{((x^2)^n - 1)}{x^2 - 1} + xy\frac{((xy)^n - 1)}{xy - 1}$$

3.(ii)

$$S = (x+y) + (x^2+xy+y^2) + (x^3+x^2y+xy^2+y^3) \dots$$

$$= \frac{1}{x-y}\Big[$$

$$(a-b)(a+b) = a^2 - b^2$$

$$(a-b)(a^2+ba+b^2) = a^3 - b^3$$

$$a^4 - b^4$$

[illegible]

$$S = \frac{3}{5} + \frac{4}{5^2} + \frac{3}{5^3} + \frac{4}{5^4} + \frac{3}{5^5} + \frac{4}{5^6} + \dots$$

2n terms

$$= \left(\frac{3}{5} + \frac{3}{5^3} + \frac{3}{5^5} + \dots n \text{ terms}\right)$$

$$+ \left(\frac{4}{5^2} + \frac{4}{5^4} + \frac{4}{5^6} + \dots n \text{ terms}\right)$$

Q. $S = \sum_{n=1}^{10}(2+3^n)$

$$S = (2+3^1) + (2+3^2) + (2+3^3) + (2+3^4) + \dots + (2+3^{10})$$

$$= (2+2+\dots 10 \text{ terms}) + (3^1+3^2+3^3 \dots)$$

$$= 2\times 10 + 3\frac{(3^n - 1)}{3-1}$$

$$= 20 + \frac{3}{2}(3^n - 1)$$

$$8(1 + 11 + 111 + \cdots)$$

$$\frac{8}{9}(9 + 99 + 999 + \cdots)$$

$$\frac{8}{9}\left((10-1) + (10^2-1) + (10^3-1) + \cdots \underset{\text{times}}{n}\right)$$

$$\frac{8}{9}\left((10^1 + 10^2 + 10^3 + \cdots + 10^n) - (1+1+1+\cdots \underset{\text{tim}}{n})\right)$$

$$= \frac{8}{9}\left[\frac{10(10^n-1)}{10-1} - n\right]$$

$$= \frac{8}{9}\left[\frac{10(10^n-1)}{9} - n\right]$$

$$= \frac{8}{81}\underline{\left[10^{n+1} - 10 - 9n\right]}$$

*

$x \to \pi$

$\frac{0\pi}{12}, \frac{1\pi}{12}, \frac{2\pi}{12}$ 6, $\frac{3\pi}{12}$ 4, $\frac{4\pi}{12}, \frac{5\pi}{12}$

~~Sin~~ $\sin\frac{\pi}{12}$, $\sin(\pi-\theta) = \sin\theta$

$\downarrow$ $\frac{\pi}{12}$

$\sin\left(\frac{\pi}{6}\right) = \sin\left(\pi - \frac{\pi}{6}\right)$

$= \sin\left(\frac{5\pi}{6}\right)$

$\sin\left(\pi - \frac{\pi}{12}\right)$

$\sin\left(\frac{11\pi}{12}\right) = \sin\frac{\pi}{12}$

$\frac{\pi}{4} \to 45$

$\sin 2x = 2\sin x \cos x$

$\sin x \to$ half $\to (2\sin x)$

$\to \sin 2x \to$ half

$x = \pi$

$\frac{3\pi}{12}$ 4

(a) (b)

$A = \frac{a+b}{2}$, $G = \sqrt{ab}$

$(a+b) = 2A$ — ①, $ab = G^2$ — ②

$\to a^2+b^2+2ab - 4ab \to \sqrt{(a-b)^2}$

$(a-b) = \sqrt{(a+b)^2 - 4ab}$

$= \sqrt{(2A)^2 - 4\times G^2}$

$= \sqrt{4A^2 - 4G^2}$

$(a-b) = 2\sqrt{A^2-G^2} \to 2\sqrt{(A-G)(A+G)}$

① + ② $\to$

$2a = 2A + 2\sqrt{A^2-G^2}$

$$\boxed{a = A + \sqrt{A^2-G^2}}$$

$a+b = 2A$

$b = 2A - a = 2A - (A + \sqrt{A^2-G^2})$

$$\boxed{b = A - \sqrt{A^2-G^2}}$$

Q. ⓐ ⓑ , Ⓐ , Ⓖ

$a+b = 6G$

$a+b = 6\sqrt{ab}$

$(a+b)^2 = 36ab$

$a^2+b^2+2ab = 36ab$

$a^2+b^2-34ab = 0$

Divide both sides by b^2

$\frac{a^2}{b^2}+1-34\left(\frac{a}{b}\right)=0$

$\boxed{\frac{a}{b}=x}$

$x^2+1-34x=0$

$x^2-34x+1=0$

$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-34)+\sqrt{34^2-4\times1\times1}}{2\times1}$

$= \frac{34 + \sqrt{1152}}{2}$

2 | 1152
2 | 576
2 | 288
2 | 144
2 | 72
2 | 36
2 | 18
3 | 3

$\boxed{2^7 \times 3^2}$

~~= 34 +~~

$= \frac{34 + 3\times2^3\sqrt{2}}{2}$

$= \frac{\overset{17}{\cancel{34}} + \overset{12}{\cancel{24}}\sqrt{2}}{\cancel{2}} = \boxed{17+12\sqrt{2}}$ ← $\boxed{x}$

$\frac{a}{b} = \frac{3+2\sqrt{2}}{3-2\sqrt{2}}$

$= \frac{3+2\sqrt{2}}{3-2\sqrt{2}} \times \frac{3+2\sqrt{2}}{3+2\sqrt{2}}$

$= \frac{9+8+12\sqrt{2}}{9-8}$

$= \boxed{17+12\sqrt{2}}$ ✓

* Some Special Series

$\frac{n}{2}(2+\ldots)$

(i) $1+2+3+\ldots\ldots+n = \frac{n}{2}(1+n)$

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2} \qquad = \frac{n(n+1)}{2}$$

(ii) $1^2+2^2+3^2+\ldots+n^2 = \frac{n(n+1)(2n+1)}{6}$

$$\boxed{\sum_{x=1}^{n} x^2 = \frac{n(n+1)(2n+1)}{6}}$$

(iii) $1^3+2^3+3^3+\ldots+n^3 = \left(\frac{n(n+1)}{2}\right)^2$

$$\sum_{x=1}^{n} x^3 =$$

$a+b=6$

* $1+3+5+7+\ldots+(2n-1) = \frac{n}{\cancel{2}}(\cancel{1}+\cancel{2}n-\cancel{1})$

'n' no. of terms

$= n^2$

$n \to 1 \Rightarrow 2\times1-1 = 1$

$n \to 2 \to 2\times2-1 = 3$

$n \to n \to 2n-1$

$1+3+5+7 = 4^2 = 16$

$2\times50-1$

$1+3+5+7+\ldots+99 = 50^2$

$= 2500$

Ex-9.4

Q.1

$S = 1\times2 + 2\times3 + 3\times4 + 4\times5 + \dots$

$= n(n+1) = n^2 + n$

$\cancel{n}$ → Gen. exp.

nth term

nth term

$\sum_{n=1}^{n} n^2 + n = \sum_{n=1}^{n} n^2 + \sum_{n=1}^{n} n$

$= \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$

$2n+1$

$3\times1^2 + 5\times2^2 + 7\times3^2 + \dots$

Q.3 → Gen exp. →

$\sum_{n=1} (2n+1)\times n^2$

$a_n = a + (n-1)d$

$= 3 + (n-1)2$

$= 3 + 2n - 2$

$= 2n+1$

$\frac{5-3}{3\times5}$

$\frac{5}{3\times5} - \frac{3}{3\times5}$

$\frac{1}{3} - \frac{1}{5}$

$\frac{b-a}{ab}$

$\frac{b}{ab} - \frac{a}{ab}$

$\frac{1}{a} - \frac{1}{b}$

$S = \frac{1}{1\times3} + \frac{1}{3\times5} + \frac{1}{5\times7} + \dots$ nth term

$= \frac{1}{2}\left\{ \frac{2}{1\times3} + \frac{2}{3\times5} + \frac{2}{5\times7} + \dots \right\}$ $3-1$ $5-3$

$= \frac{1}{2}\left\{ 1 - \cancel{\frac{1}{3}} + \cancel{\frac{1}{3}} - \cancel{\frac{1}{5}} + \cancel{\frac{1}{5}} - \cancel{\frac{1}{7}} + \dots + \cancel{\frac{1}{n}} - \frac{1}{n+1} \right\}$

$= \frac{1}{2}\left(1 - \frac{1}{n+1}\right)$

$= \frac{1}{2}\cdot\frac{n}{n+1}$

$\frac{5-3}{3\times5}$ $②$

$\sum_{n=1}^{20} n^2 - \sum_{n=1}^{4} n^2$

$1^2 + 2^2 \dots 20^2 - 1^2 - 2^2 - 3^2 - 4^2$

$\frac{b-a}{ab}$

$\frac{1}{a} - \frac{1}{b}$

$\frac{1}{(b-a)}\cdot\frac{(b-a)}{a\times b}$

$S = \frac{1}{1\times2} + \frac{1}{2\times3} + \frac{1}{3\times4} + \dots$

$= \left(1 - \cancel{\frac{1}{2}}\right) + \left(\cancel{\frac{1}{2}} - \cancel{\frac{1}{3}}\right) + \left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{4}}\right) + \dots + \left(\cancel{\frac{1}{n}} - \frac{1}{n+1}\right)$

$= 1 - \frac{1}{n+1}$

$= \frac{n}{n+1}$

$\frac{3-2}{2\times3}$

$S = \frac{1}{1\times3} + \frac{1}{3\times5} + \frac{1}{5\times7} + \dots$

$$S = \frac{1}{1\times5} + \frac{1}{5\times9} + \frac{1}{9\times13} + \frac{1}{13\times17}$$

$$= \frac{1}{4}\left\{\frac{\overset{5-1}{④}}{1\times5} + \frac{\overset{9-5}{④}}{5\times9} + \frac{\overset{13-9}{④}}{9\times13} + \frac{\overset{17-13}{④}}{13\times17}\right\}$$

$$= \frac{1}{4}\left\{\frac{\cancel{5}}{1\times\cancel{5}} - \cancel{\frac{1}{1\times5}} + \cancel{\frac{9}{5\times9}} - \cancel{\frac{5}{5\times9}} + \cancel{\frac{13}{9\times13}} - \cancel{\frac{9}{9\times13}} - \cancel{\frac{1}{13}} + \cancel{\frac{17}{13\times17}} - \frac{\cancel{13}}{\cancel{13}\times17}\right\}$$

$$\cancel{\frac{1}{13}} \leftarrow$$

$$= \frac{1}{4}\left\{1 - \frac{1}{17}\right\} = \frac{1}{\cancel{4}} \times \frac{\overset{4}{\cancel{16}}}{17} = ④\frac{4}{17}$$