8

PERMUTATIONS AND COMBINATIONS

FUNDAMENTAL PRINCIPLE OF COUNTING

Multiplication Principle : If an operation can be performed in 'm' different ways; following which a second operation can be performed in 'n' different ways, then the two operations in succession can be performed in m × n ways. This can be extended to any finite number of operations.

Addition principle : If an operation can be performed in 'm' different ways and another operation, which is independent of the first operation, can be performed in 'n' different ways. Then either of the two operations can be performed in (m + n) ways. This can be extended to any finite number of mutually exclusive operations.

Example 1 :
A person wants to go from station P to station R via station Q. There are 4 routes from P to Q and 5 routes from Q to R.In how many ways can he travel from P to R -

Sol. He can go from P to Q in 4 ways and Q to R in 5 ways
So number of ways of travel from P to R is $4 \times 5 = 20$

Example 2 :
A college offers 6 courses in the morning and 4 in the evening. Find the possible number of choices with the student if he wants to study one course in the morning and one in the evening.

Sol. The student has 6 choices from the morning courses out of which he can select one course in 6 ways.
For the evening course, he has 4 choices out of which he can select one in 4 ways.
Hence the total number of ways $= 6 \times 4 = 24$

Example 3 :
A person wants to leave station Q there are 4 routes from station Q to P and 5 routes from Q to R. In how many ways can he travel the station Q.

Sol. He can go from Q to P in 4 ways and Q to R in 5 ways
he can leave station Q in 4 + 5 = 9 ways.

Example 4 :
A college offers 6 courses in the morning and 4 in the evening. Find the number of ways a student can select exactly one course, either in the morning or in the evening.

Sol. The student has 6 choices from the morning courses out of which he can select one course in 6 ways.
For the evening course, he has 4 choices out of which he can select one in 4 ways.
Hence the total number of ways = 6 + 4 = 10.

FACTORIALS

If n is a natural number then the product of all natural number upto n is called factorial n and it is denoted by -

$n\,!$ or $\lfloor\underline{n}$

Thus, $n\,! = n\,(n-1)\,(n-2)\,.....\,3.2.1.$

It is obvious to note that

$n! = n\,(n-1)!$

$= n\,(n-1)\,(n-2)!$

$= n\,(n-1)\,(n-2)\,(n-3)!$ etc.

Note : $0\,! = 1,\ 1\,! = 1,\ 2\,! = 2,\ 3\,! = 6,\ 4\,! = 24,\ 5\,! = 120,\ 6\,! = 720$

PERMUTATION

Each of the different arrangements which can be made by taking some or all of a number of things is called a permutation.

COUNTING FORMULAS FOR PERMUTATIONS

Without Repetition :

(i) The number of permutations of n different things, taking r at a time is denoted by ${}^{n}P_{r}$ or P(n, r)

then ${}^{n}P_{r} = \dfrac{n!}{(n-r)!}\ (0 \le r \le n)$

$= n(n-1)\,(n-2)\,.......\,(n-r+1),\ n \in N$ and $r \in W$

(ii) The number of arrangements of n different objects taken all at a time is ${}^{n}P_{n} = n\,!$

Note : ${}^{n}P_{1} = n,\quad {}^{n}P_{r} = n.\,{}^{n-1}P_{r-1},\quad {}^{n}P_{r} = (n-r+1).\,{}^{n}P_{r-1},$

${}^{n}P_{n} = {}^{n}P_{n-1}$

Example 5 :
Find the number of ways in which four persons can sit on six chairs.

Sol. ${}^{6}P_{4} = 6.5.4.3 = 360$

With Repetition :

(i) The number of permutations of n things taken all at a time, p are alike of one kind, q are alike of second kind and r are alike of a third kind and the rest n – (p + q + r) are all different is

$$\frac{n!}{p!\,q!\,r!}$$

(ii) The number of permutations of n different things taken r at a time when each thing may be repeated any number of times is n^{r}.

Example 6 :

In how many ways can 5 prizes be distributed among 4 boys when every boy can take one or more prizes ?

Sol. First prize may be given to any one of the 4 boys, hence first prize can be distributed in 4 ways.
similarly every one of second, third, fourth and fifth prizes can also be given in 4 ways.
$\therefore$ The number of ways of their distribution

$$= 4 \times 4 \times 4 \times 4 \times 4 = 4^5 = 1024$$

Example 7 :

Find the number of words that can be formed out of the letters of the word COMMITTEE.

Sol. There are 9 letters in the given word in which two T's, two M's and two E's are identical. Hence the required number of words $= \frac{9!}{2!2!2!} = \frac{9!}{(2!)^3}$

Number of Permutations under certain conditions:

(a) The number of permutation of n different things taken all together when r particular things are to be place at some r given places $= {}^{n-r}P_{n-r} = (n-r)!$

(b) The number of permutations of n different things taken r at a time when m particular things are to be placed at m given places $= {}^{n-m}P_{r-m}$.

(c) Number of permutations of n different things, taken r at a time, when a particular things is to be always included in each arrangement, is r. ${}^{n-1}P_{r-1}$

(d) Number of permutations of n different things, taken all at a time, when m specified things always come together is
$m! \times (n - m + 1)!$

(e) Number of permutations of n different things, taken all at a time, when m specified things never come together is
$n! - m! \times (n - m + 1)!$

Example 8 :

How many different 4-letter words can be formed with the letters of the word 'JAIPUR' when A and I are always to be included ?

Sol. Since A and I are always to be included, so first we select 2 letters from the remaining 4, which can be done in ${}^4C_2 = 6$ ways. Now these 4 letters can arranged 4! = 24 ways, so the required number $= 6 \times 24 = 144$.

Example 9 :

How many different words can be formed with the letter of the word 'JAIPUR' which start with 'A' and end with 'I'.

Sol. After putting A and I at their respective places (only in one way) we shall arrange the remaining 4 different letters at 4 places in 4! ways. Hence the required number = 1 × 4! = 24

Circular Permutations :

(i) Arrangement round a circular table :
The number of circular permutations of n different things taken all at a time is (n –1)!, if clockwise and anticlockwise orders are taken as different.

(ii) Arrangement of beads or flowers (all different) around a circular necklace or garland :
The number of circular permutations of n different things taken all at a time is $\frac{1}{2}(n-1)!$, if clockwise and anticlockwise orders are taken as not different.

(iii) Number of circular permutations of n different things taken r at a time :
Case - I : If clockwise and anticlockwise orders are taken as different, then the required number of circular permutations $= \frac{{}^nP_r}{r}$.
Case- II : If clockwise and anticlockwise orders are taken as not different, then the required number of circular permutations $= \frac{{}^nP_r}{2r}$

(iv) Restricted Circular Permutation
When there is a restriction in a circular permutation then first of all we shall perform the restricted part of the operation and then perform the remaining part treating it similar to a linear permutation.

Example 10 :

In how many ways can 5 boys and 5 girls be seated at a round table so that no two girls may be together ?

Sol. Leaving one seat vacant between two boys, 5 boys may be seated in 4! ways. Then at remaining 5 seats, 5 girls any sit in 5! ways. Hence the required number = 4! × 5!

Example 11 :

In how many ways can 4 beads out of 6 different beads be strung into a ring ?

Sol. In this case a clockwise and corresponding anticlockwise ordered will give the same circular permutation. So the required number $= \frac{{}^6P_4}{4.2} = \frac{6.5.4.3}{4.2} = 45$

Example 12 :

Find the number of ways in which 10 persons can sit round a circular table so that none of them has the same neighbours in any two arrangements.

Sol. 10 persons can sit round a circular table in 9! ways. But here clockwise and anticlockwise orders will give the same neighbours. Hence the required number of ways $= \frac{1}{2}9!$

COMBINATION

Each of the different groups or selections which can be made by some or all of a number of given things without reference to the order of the things in each group is called a Combination.

Difference between permutation and combination :

Problems of Permutations	Problems of Combinations
1 Arrangements	Selections, choose
2 Standing in a line seated in a row	Distributed group is formed
3 Problems on digits	Committee
4 Problems on letters from a word	Geometrical problems

COUNTING FORMULAS FOR COMBINATION

1. Selection of objects without Repetition

The number of combinations of n different things taken r at a time is denoted by ^nC_r or C (n, r) or $\binom{n}{r}$

Then $^nC_r = \frac{n!}{r!(n-r)!}$; $(0 \le r \le n)$

$= \frac{^nP_r}{r!} = \frac{n(n-1)(n-2)....(n-r+1)}{r(r-1)(r-2).........2.1}$; $n \in N$ and $r \in W$

If r > n, then $^nC_r = 0$

Some Important Results :

(i) $^nC_n = 1$, $^nC_0 = 1$, (ii) $^nC_r = \frac{^nP_r}{r!}$

(iii) $^nC_r = {^nC_{n-r}}$, (iv) $^nC_x = {^nC_y} \Rightarrow x + y = n$

(v) $^nC_r + {^nC_{r-1}} = {^{n+1}C_r}$ (vi) $^nC_r = \frac{n}{r} . {^{n-1}C_{r-1}}$

(vii) $^nC_r = \frac{1}{r}(n-r+1)\, {^nC_{r-1}}$ (viii) $^nC_1 = {^nC_{n-1}} = n$

Example 13 :

If $^{20}C_r = {^{20}C_{r-10}}$, then find the value of $^{18}C_r$

Sol. $^{20}C_r = {^{20}C_{r-10}} \Rightarrow r + (r - 10) = 20 \Rightarrow r = 15$

$\therefore {^{18}C_r} = {^{18}C_{15}} = {^{18}C_3} = \frac{18.17.16}{1.2.3} = 816$

Example 14 :

How many combination of 4 letters can be made of the letters of the word 'JAIPUR' ?

Sol. Here 4 things are to be selected out of 6 different things.

So the number of combinations $= {^6C_4} = \frac{6.5.4.3}{4.3.2.1} = 15$

2. Selection of Objects with Repetition :

The total number of selections of r things from n different things when each thing may be repeated any number of times is $^{n+r-1}C_r$

Example 15 :

8 pens are to be selected from pens of 3 colours (pens of each colour being available any number of times), then find the total number of selections.

Sol. $^{3+8-1}C_8 = {^{10}C_8} = \frac{10.9}{2} = 45$

3. Restricted Selection/Arrangement

(i) The number of combinations of n different things taken r at a time when k particular objects occurs is $^{n-k}C_{r-k}$. If k particular objects never occur is $^{n-k}C_r$.

(ii) The number of arrangements of n distinct objects taken r at a time so that k particular object are always included $= {^{n-k}C_{r-k}}$. r ! and never included $= {^{n-k}C_r}$. r !

(iii) The number of combinations of n objects, of which p are non-identical, taken r at a time is

$^{n-p}C_r + {^{n-p}C_{r-1}} + {^{n-p}C_{r-2}} + ... + {^{n-p}C_0}$ if $r \le p$.

$^{n-p}C_r + {^{n-p}C_{r-1}} + {^{n-p}C_{r-2}} + ... + {^{n-p}C_{r-p}}$ if $r > p$.

Example 16 :

From 4 gentlemen and 6 ladies a committee of five is to be selected. Find the number of ways in which the committee can be formed so that gentlemen are in majority

Sol. The committee will consist of 4 gentlemen and 1 lady or 3 gentlemen and 2 ladies

$\therefore$ the number of committees $= {^4C_4} \times {^6C_1} + {^4C_3} \times {^6C_2} = 66$

4. Selection from distinct objects :

The number of ways (or combinations) of n different things selecting at least one of them is

$^nC_1 + {^nC_2} + {^nC_3} + ... + {^nC_n} = 2^n - 1$.

This can also be stated as the total number of combination of n different things.

Example 17 :

Ramesh has 6 friends. In how many ways can he invite one or more of them at a dinner ?

Sol. He can invite one, two three, four, five or six friends at the dinner. So total number of ways of his invitation

$= {^6C_1} + {^6C_2} + {^6C_4} + {^6C_5} + {^6C_6} = 2^6 - 1 = 63$

5. Selection from identical objects

The number of combination of n identical things taking $r(r \le n)$ at a time is 1.

The number of ways of selecting r things out of n alike things is n + 1 (where r = 0, 1, 2 .....n).

The number of ways to select some or all out of (p + q + r) things where p are alike of first kind, q are alike of second kind and r are alike of third kind is $= (p + 1)(q + 1)(r + 1) - 1$

Example 18 :

There are n different books and p copies of each in a library. Find the number of ways in which one or more than one book can be selected.

Sol. Total cases = p + 1 (if selected or not)

Required number of ways

$= (p + 1)(p + 1)......n \text{ terms} - 1 = (p + 1)^n - 1$

Example 19 :

A bag contains 3 one rupee coins, 4 fifty paise coins and 5 ten paise coins. How many selection of money can be formed by taking atleast one coin from the bag.

Sol. Here are 3 things of first kind, 4 things of second kind and 4 things of third kind so the total number of selections
$= (3 + 1)(4 + 1)(5 + 1) – 1 = 119$

6. Selection when both identical and distinct objects are present: If out of (p + q + r + t) things, p are alike one kind, q are alike of second kind, r are alike of third kind and t are different, then the total number of combinations is

$(p + 1)(q + 1)(r + 1)\, 2^t – 1$

DIVISION AND DISTRIBUTION OF OBJECTS

1. The number of ways in which (m + n) different things can be divided into two groups which contain m and n things respectively is

$${}^{m+n}C_m\, {}^nC_n = \frac{(m+n)!}{m!n!},\ m \neq n$$

Particular case :

When m = n, then total number of combination is

$\frac{(2m)!}{(m!)^2}$ when order of groups is considered.

$\frac{(2m)!}{2!(m!)^2}$ when order of groups is not considered.

2. The number of ways in which (m + n + p) different things can be divided into three groups which contain m, n and p things respectively is

$${}^{m+n+p}C_m . {}^{n+p}C_p . {}^pC_p = \frac{(m+n+p)!}{m!n!p!},\ m \neq n \neq p$$

Particular case :

When m = n = p, then total number of combination is

$\frac{(3m)!}{(m!)^3}$ when order of groups is considered.

$\frac{(3m)!}{3!(m!)^3}$ when order of groups is not considered.

3. Total number of ways to divide n identical things among r person is ${}^{n+r-1}C_{r-1}$
Also total number of ways to divide n identical things among r persons so that each gets atleast one is ${}^{n-1}C_{r-1}$

Example 20 :

In how many ways 20 identical mangoes may be divided among 4 persons and if each person is to be given atleast one mango, then find the number of ways .

Sol. 20 identical mangoes may be divided among 4 persons in ${}^{20+4-1}C_{4-1} = {}^{19}C_3 = 1771$ ways.
If each person is to be given atleast one mango, then number of ways will be ${}^{20-1}C_{4-1} = {}^{19}C_3 = 969$.

Example 21 :

In how many ways can a pack of 52 cards be divided in 4 sets, three of them having 17 cards each and fourth just one card?

Sol. Since the cards are to be divided into 4 sets, 3 of them having 17 cards each and 4th just one card, so number of ways

$$= \frac{52!}{(17!)^3 3!1!} = \frac{52!}{(17!)^3 3!}$$

NEGATIVE BINOMIAL EXPANSION

$(1 – x)^{–n} = 1 + {}^nC_1\, x + {}^{n+1}C_2\, x^2 + {}^{n+2}C_3\, x^3 + ...$ to ∞, if $– 1 < x < 1$

Coefficient of x^r in this expansion ${}^{n+r-1}C_1$ $(n \in N)$

Result : Number of ways in which it is possible to make a selection form m + n + p = N things, where p are alike of one kind, m alike of second kind & n alike of third kind taken r at a time is given by coefficient of x^r in the expansion of
$(1 + x + x^2 + ... + x^p)(1 + x + x^2 + ... + x^m)(1 + x + x^2 + ... + x^n)$.

For example the number of ways in which a selection of four letters can be made from the letters of the word PROPORTION is given by coefficient of x^4 in
$(1 + x + x^2 + x^3)(1 + x + x^2)(1 + x + x^2)(1 + x)(1 + x)(1 + x)$.

METHOD OF FICTION PARTITION

Number of ways in which n identical things may be distributed among p persons if each person may receive none, one or more things is, ${}^{n+p-1}C_n$.

Results : Let $N = p^a q^b r^c$....... where p, q, r....... are distinct primes and a, b, c.......... are natural numbers then :

(a) The total numbers divisors of N including 1 & N is $= (a + 1)(b + 1)(c + 1)$.........

(b) The sum of these divisors
$(p^0 + p^1 + p^2 + ... + p^a)(q^0 + q^1 + q^2 + ... + q^b)(r^0 + r^1 + r^2 + ... + r^c)$..........

(c) Number of ways in which N can be resolved as a product of two factors is

$$= \begin{cases} \frac{1}{2}(a+1)(b+1)(c+1)........... & \text{If N is not a perfect square} \\ \frac{1}{2}[(a+1)(b+1)(c+1)......+1] & \text{If N is perfect square} \end{cases}$$

(d) Numbers of ways in which a composite number N can be resolved into two factors which are relatively prime (or coprime) to each other is equal to 2^{n-1} where n is the number of different prime factors in N.

Result : Let there be n types of objects with each type containing atleast r objects. Then the number of ways of arranging r objects in a row is n^r.

DERANGEMENT THEOREM

Any change in the given order of the thing is called a Derangement.

(i) If n items are arranged in a row, then the number of ways in which they can be rearranged so that no one of them occupies the place assigned to it is

$$n!\ 1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}+....+(-1)^n\frac{1}{n!}$$

(ii) If n things are arranged at n places then the number of ways to rearrange exactly r things at right places is

$$\frac{n!}{r!}\ 1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}+....+(-1)^{n-r}\frac{1}{(n-r)!}$$

Example 22 :

There are 3 letters and 3 envelopes. Find the number of ways in which all letters are put in the wrong envelopes.

Sol. The required number of ways

$$=3!\ 1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}\ =3-1=2$$

Example 23 :

There are 4 balls of different colour and 4 boxes of colours same as those of the balls. Then find the number of ways to place two balls in boxes with respect to their colour.

Sol. The required number of ways

$$\frac{4!}{2!}\ 1-\frac{1}{1!}+\frac{1}{2!}\ =4.3\ \ 1-1+\frac{1}{2}\ =6$$

DIVISIBILITY OF NUMBERS

The following table shows the conditions of divisibility of same numbers

[redacted]

SUM OF NUMBERS

(i) For given n different digits a_1, a_2, a_3 ......a_n the sum of the digits in the unit place of all numbers formed (if numbers are not repeated) is $(a_1 + a_2 + a_3 + ...+ a_n)\ (n-1)!$
i.e. (sum of the digits) $(n-1)!$

(ii) Sum of the total numbers which can be formed with given n different digits a_1, a_2, ...a_n is
$(a_1 + a_2 + a_3 + ...+ a_n)\ (n-1)!$. (111 ....n times)

Example 24 :

Find the sum of all 4 digit numbers formed with the digits 1,2,4 and 6.

Sol. Sum = $(a_1 + a_2 + a_3 + ...+ a_n)\ (n-1)!$ (111 ....n times)
Using formula Sum

$$=(1+2+4+6).3!.(1111)=13\times 6\,1111=86658$$

Second Method : Here total 4-digit numbers will be 4! = 24. So every digit will occur 6 times at every one of the four places. Now since the sum of the given digits = 1 + 2 + 4 + 6= 13. So the sum of all the digits at every place of all 24 numbers = 13 x 6 = 78

So the sum of the values of all digits
at first place =78
at ten place = 780
at hundred place = 7800
at thousand place = 78000
$\therefore$ The required sum = 78 + 780 + 7800 + 78000 = 86658

IMPORTANT RESULTS ABOUT POINTS

If there are n points in a plane of which m (< n) are collinear, then

* Total number of different straight lines obtain by joining these n points is ${}^nC_2 - {}^mC_2 + 1$
* Total number of different triangles formed by joining these n points is ${}^nC_3 - {}^mC_3$
* Number of diagonals in polygon of n sides is ${}^nC_2 - n$ i.e. $\frac{n(n-3)}{2}$
* If m parallel lines in a plane are intersected by a family of other n parallel lines. Then total number of parallelograms so formed is ${}^mC_2 \times {}^nC_2$ i.e $\frac{mn(m-1)n-1)}{4}$

Example 25 :

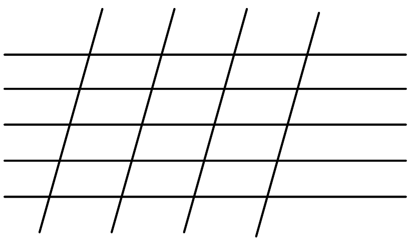
There are 10 points in a plane and 4 of them are collinear. Find the number of straight lines joining any two of them.

Sol. A straight line can be drawn joining two points, so there will be ${}^{10}C_2$ straight lines joining 10 points. But 4 of them are collinear, so we shall get only one line joining any two of these 4 points.
Hence the total number of lines = ${}^{10}C_2 - {}^4C_2 + 1 = 40$

Example 26 :

If 5 parallel straight lines are intersected by 4 parallel straight lines, then find the number of parallelograms thus formed.

Sol.



Number of parallelograms = ${}^5C_2 \times {}^4C_2 = 60$

ADDITIONAL EXAMPLES

Example 1 :

There are 13 players of cricket, out of which 4 are bowlers. In how many ways a team of eleven be selected from them so as to include atleast two bowlers ?

Sol. A team of eleven may be selected in the following three ways
(i) 2 bowlers + 9 others (ii) 3 bowlers + 8 others
(iii) 4 bowlers + 7 others
$\therefore$ There are 4 bowlers and 9 other, so the total number of selection
$= ({}^4C_2 \times {}^9C_9) + ({}^4C_3 \times {}^9C_8) + ({}^4C_4 \times {}^9C_7)$
$= 6 + 36 + 36 = 78$

Example 2 :

There are 10 points in a plane of which no three points are collinear and 4 points are concyclic. Find the number of different circles that can be drawn through at least 3 points of these points.

Sol. The number of circles $= ({}^{10}C_3 - {}^4C_3) + 1 = 117$

Example 3 :

To fill up 12 vacancies, there are 25 candidates of which 5 are from SC. If 3 of these vacancies are reserved for SC candidates while the remaining are open to all then find the number of ways in which the selection can be made.

Sol. 3 vacancies for SC candidates can be filled up from 5 candidates in ${}^5C_3$ ways.
After this for remaining $12 - 3 = 9$ vacancies, there will be $25 - 3 = 32$ candidates. These vacancies can be filled up in ${}^{22}C_9$ ways. Hence required number of ways $= {}^5C_3 \times {}^{22}C_9$

Example 4 :

Find the number of 6 digit numbers that can be made with the digits 1, 2, 3 and 4 and having exactly two pairs of digits.

Sol. The number will have 2 pairs and 2 different digits
The number of selections $= {}^4C_2 \times {}^2C_2$, and for each selection, number of arrangements $= \frac{6!}{2!2!}$.

Therefore, the required number of numbers

$$= {}^4C_2 \times {}^2C_2 \times \frac{6!}{2!2!} = 1080$$

Example 5 :

A bag contains 3 black, 4 white and 2 red balls, all the balls being different. Find the number of selections of atmost 6 balls containing balls of all the colours.

Sol. The required number of selections

$$= {}^3C_1 \times {}^4C_1 \times {}^2C_1 ({}^6C_3 + {}^6C_2 + {}^6C_0) = 42 \times 4!$$

Example 6 ;

Find the number of ways in which 20 different pearls of two colours can be set alternately on a necklace, there being. 10 pearls of each colour.

Sol. Ten pearls of one colour can be arranged in $\frac{1}{2}.(10 - 1)!$ ways.

The number of arrangements of 10 pearls of the other colour in 10 places between the pearls of the first colour = 10!

$\therefore$ the required number of ways $= \frac{1}{2} \times 9! \times 10! = 5\,(9!)^2$

Example 7 :

Find the number of ways of arranging six persons (having A, B, C and D among them) in a row so that A, B, C and D are always in order ABCD (not necessarily together).

Sol. The number of ways of arranging ABCD is 4!. For each arrangement of ABCD, the number of ways of arranging six persons is same. Hence required number is $\frac{6!}{4!} = 30$

Example 8 :

Find the number of words each containing 3 consonants and 2 vowels that can be formed out of 5 consonants and 4 vowels.

Sol. 3 consonants and 2 vowels from 5 consonants and 4 vowels can be selected in ${}^5C_3 \times {}^4C_2 = 60$ ways.
But total number of words with $3 + 2 = 5$ letters $= 5! = 120$
$\therefore$ The required number of words $= 60 \times 120 = 7200$

Example 9 :

Find the number of numbers less than 1000 that can be formed out of the digits 0,1,2,4 and 5 , no digit being repeated.

Sol. ${}^4C_1 + {}^4C_1 \times {}^4C_1 + {}^4C_1 \times {}^4C_1 \times {}^3C_1 = 4 + 16 + 48 = 68$

QUESTION BANK

EXERCISE - 1

Q.1 If $\frac{1}{9!}+\frac{1}{10!}=\frac{x}{11!}$, then the value of x is -
(A) 123 (B) 125
(C) 121 (D) None of these

Q.2 The number of different words (meaningful or meaningless) can be formed by taking four different letters from English alphabets is-
(A) $(26)^4$ (B) 358800
(C) $(25)^4$ (D) 15600

Q.3 In how many ways can a committee of 6 persons be made out of 10 persons ?
(A) 210 (B) 300
(C) 151200 (D) None of these

Q.4 In how many ways a committee of 5 members can be selected from 6 men and 5 women, consisting of 3 men and 2 women?
(A) 200 (B) 100
(C) 300 (D) None of these

Q.5 Out of 5 men and 2 women, a committee of 3 is to be formed. In how many ways can it be formed if atleast one woman is to be included?
(A) 20 (B) 30
(C) 25 (D) None of these

Q.6 In how many ways 11 players can be selected out of 15 players when (a) one particular player is always to be selected.(b) one particular player is never to be selected.
(A) 364,1365 (B) 1001,364
(C) 3003, 364 (D) 3003,1001

Q.7 In how many ways can I purchase one or more shirts if 6 different shirts are available ?
(A) 64 (B) 62
(C) 63 (D) 126

Q.8 A bag contains 3 one rupee coins, 4 fifty paise coins and 5 ten paise coins. How many selections of money can be formed by taking atleast one coin from the bag ?
(A) 120 (B) 60
(C) 119 (D) 59

Q.9 The value of $^8P_3$ is -
(A) 336 (B) 56
(C) 386 (D) None of these

Q.10 The number of numbers which can be formed with the digits 2, 3, 4, 5, 6 by taking 4 digits at a time are-
(A) 135 (B) 120
(C) 150 (D) None of these

Q.11 In how many ways can three persons sit on 6 chairs?
(A) 150 (B) 140
(C)120 (D) 110

Q.12 How many different signals can be made by 5 flags from 8 flags of different colours?
(A) 6720 (B) 5720
(C) 4720 (D) None of these

Q.13 How many numbers lying between 100 and 1000 can be formed with the digits 1,2,3,4,5,6 if the repetition of digits is not allowed?
(A) 30 (B) 120
(C) 50 (D) None of these

Q.14 How many four digit numbers are there with distinct digits?
(A) 4536 (B) 4526
(C) 4516 (D) None of these

Q.15 How many different words can be formed with the letters of the word “ALLAHABAD” ?
(A) 10080 (B) 8640
(C) 15120 (D) 7560

Q.16 How many numbers can be formed with the digits 2,3,3,4,2,3 taken all at a time.
(A) 460 (B) 60
(C) 260 (D) None of these

Q.17 There are 6 pockets in the coat of a person. In how many ways can he put 4 pens in these pockets ?
(A) 360 (B) 1296
(C) 4096 (D) None of these

Q.18 The number of three digit numbers can be formed without using the digits 0,2,3,4, 5 and 6 is (if repetition of digit is allowed)–
(A) 54 (B) 64
(C) 44 (D) None of these

Q.19 The number of numbers are there between 100 and 1000 in which all the digits are distinct is –
(A) 648 (B) 548
(C) 448 (D) None of these

Q.20 The number of three digit numbers greater than 600 can be formed by using the digits 2,3,4, 6,7 if repetition of digits is allowed-
(A) 50 (B) 20
(C) 30 (D) None of these

Q.21 In how many ways 3 prizes can be distributed among 5 students, when-
(a) no student receives more than a prize.
(b) a student can receive any number of prizes.
(c) a student does not get all prizes.
(A) 60,125,120 (B) 125,60,120
(C) 125,120,60 (D) None of these

Q.22 How many numbers lying between 1000 and 2000 can be formed with the digits 1,2,3,4, 5 which are divisible by 5.
(A) 3 (B) 6
(C) 12 (D) 18

Q.23 How many different words beginning with S and ending with K can be made by using the letters of the word ‘SIKAR’?
(A) 6 (B) 12
(C) 48 (D) 60

Q.24 How many different 3 letter words can be formed with the letters of the word 'JAIPUR' when A and I are always to be excluded ?

(A) 12 (B) 24
(C) 48 (D) 60

Q.25 How many six digit numbers can be formed by using the digits 0,1,2,3,4,5 and 6?

(A) 5040 (B) 4320
(C) 720 (4) 5760

Q.26 If ${}^{56}P_{r+6} : {}^{54}P_{r+3} = 30800 : 1$ then the value of r is -

(A) 14 (B) 41
(C) 51 (D) 10

Q.27 The number of ways in which 2 vacancies can be filled up by 13 candidates is-

(A) 25 (B) 78
(C) 156 (D) 169

Q.28 How many different words beginning with A and ending with L can be formed by using the letters of the word "ANILMANGAL'?

(A) 10080 (B) 40320
(C) 20160 (D) None of these

Q.29 How many numbers can be formed between 20000 and 30000 by using digits 2, 3, 5, 6, 9 when digits may be repeated?

(A) 125 (B) 24
(C) 625 (D) 1250

Q.30 The number of three letters words can be formed from the letters of word 'SACHIN' when I do not come in any word is-

(A) 120 (B) 60
(C) 24 (D) 48

Q.31 The number of numbers lying between 100 and 1000 which can be formed with the digits 0, 1, 2, 3, 4, 5, 6 is-

(A) 180 (B) 216
(C) 200 (D) None of these

Q.32 How many numbers between 1000 and 4000 (including 4000) can be formed with the digits 0,1,2,3,4 if each digit can be repeated any number of times?

(A) 125 (B) 275
(C) 375 (D) 500

Q.33 There are 3 letters and 3 envelopes. Find the number of ways in which all letters are put in the wrong envelopes.

(A) 6 (B) 4
(C) 2 (D) None of these

Q.34 There are 4 balls of different colour and 4 boxes of colours same as those of the balls. Then find the number of ways to place two balls in the boxes with respect to their colour.

(A) 6 (B) 4
(C) 2 (D) None of these

Q.35 In how many way can 52 playing cards be distributed into 3 groups of 17 cards each and one group of one card.

(A) $\frac{52!}{(17!)^3\ 3!}$ (B) $\frac{52!}{(17!)^3}$

(C) $\frac{52!}{(17!)^3\ 3!\ 2!}$ (D) None of these

Q.36 3 copies each of 4 different books are available. The number of ways in which these can be arranged on the shelf is-

(A) 12! (B) $\frac{12!}{3!\ 4!}$

(C) 369,600 (D) 369,000

Q.37 The number of ways of dividing 20 persons into 10 couples is-

(A) $\frac{20!}{2^{10}}$ (B) ${}^{20}C_{10}$

(C) $\frac{20!}{(2!)^9}$ (D) None of these

Q.38 How many words can be formed containing 4 consonants and 3 vowels out of 6 consonants and 5 vowels ?

(A) ${}^6C_4 \times {}^5C_3$ (B) ${}^6C_4 \times {}^5C_3 \times 7!$
(C) ${}^6P_4 \times {}^5P_3$ (D) ${}^6P_4 \times {}^5P_3 \times 7!$

Q.39 In how many ways can 7 persons be seated round two circular tables when 4 persons can sit on the first table and 3 can sit on the other ?

(A) 420 (B) 35
(C) 210 (D) 2520

Q.40 The number of words by taking 4 letters out of the letters of the word 'COURTESY', when T and S are always included are-

(A) 120 (B) 720
(C) 360 (D) None of these

EXERCISE - 2

Q.1 The number of ways in which 7 persons be seated at 5 places round a table are-
(A) 252 (B) 504
(C) 2520 (D) None of these

Q.2 In how many ways can 5 beads out 7 different beads be strung into a ring ?
(A) 504 (B) 2520
(C) 252 (D) None of these

Q.3 In how many ways can 6 persons be seated round a circular table when two particular persons sit together ?
(A) 120 (B) 240
(C) 48 (D) 24

Q.4 In how many ways can 15 students
(i) be divided into 3 groups of 5 each
(ii) be sent to three different colleges in groups of 5 each.

(A) $\frac{15!}{3!(5!)^3}, \frac{15!}{(5!)^3}$ (B) $\frac{15!}{(5!)^3}, \frac{15!}{(5!)^3}$

(C) $\frac{15!}{3!(5!)^3}, \frac{15!}{3!(5!)^3}$ (D) $\frac{15!}{(5!)^3}, \frac{15!}{3!(5!)^3}$

Q.5 Find the number of ways in which 16 identical toys are to be distributed among 3 children such that each child does not receive less than 3 toys.
(A) 36 (B) 18
(C) 72 (D) 54

Q.6 Find the number of non-negative integral solutions of $x_1 + x_2 + x_3 + 4x_4 = 20$.
(A) 436 (B) 418
(C) 536 (D) 318

Q.7 In how many ways can 10 identical toys be distributed among 3 children such that the first receives a maximum of 6 toys, the second receives a maximum of 7 toys and the third receives a maximum of 8 toys.
(A) 51 (B) 37
(C) 27 (D) 47

Q.8 In how many ways 5 identical balls can be distributed into 3 different boxes so that no box remains empty?
(A) 36 (B) 18
(C) 6 (D) 12

Q.9 Find the number of permutation of 4 letters taken from the word EXAMINATION.
(A) 1504 (B) 2520
(C) 2552 (D) 2454

Q.10 The sum of all numbers which can be formed with digits 1,2 and 3 is-
(A) 716 (B) 1432
(C) 2148 (D) None of these

Q.11 The number of ways in which 7 girls can be stand in a circle so that they do not have the same neighbour in any two arrangements?
(A) 720 (B) 380
(C) 360 (D) None of these

Q.12 The number of ways in which 7 men and 7 women can sit on a circular table so that no two women sit together is
(A) 7! . 7! (B) 7! . 6!
(C) $(6!)^2$ (D) 7!

Q.13 There are four balls of different colours and four boxes of colours same as those of the balls. The number of ways in which the balls, one each box, could be placed such that a ball does not go to box of its own colour is-
(A) 8 (B) 7
(C) 9 (D) None of these

Q.14 The number of positive integral solution of the equation $x_1 x_2 x_3 x_4 x_5 = 1050$ is –
(A) 1800 (B) 1600
(C) 1400 (D) None of these

Q.15 The number of ways of dividing 15 men and 15 women into 15 couples, each consisting of a man and a woman is –
(A) 1240 (B) 1840
(C) 1820 (D) 2005

Q.16 A seven digit number divisible by 9 is to be formed by using 7 out of number {1, 2, 3, 4, 5, 6, 7, 8, 9}. The number of ways in which this can be done is –
(A) 7! (B) 2 (7)!
(C) 3 (7)! (D) 4 (7)!

Q.17 A student is allowed to select at most n books from a collection of (2n + 1) books. If the total number of ways in which he can select a book is 63, then the value of n is –
(A) 6 (B) 3
(C) 4 (D) None of these

Q.18 There are three copies each of 4 different books. The number of ways in which they can be arranged on a shelf is –

(A) $\frac{12!}{(3!)^4}$ (B) $\frac{11!}{(3!)^2}$

(C) $9!/(3!)^2$ (D) None of these

Q.19 A boat is to be manned by eight men of whom 2 can only row on bow side and 3 can only row on stroke side, the number of ways in which the crew can be arranged is –
(A) 4360 (B) 5760
(C) 5930 (D) None of these

Q.20 There are 10 lamps in a hall. Each one of them can be switched on independently. The number of ways in which the hall can be illuminated is –
(A) 10^2 (B) 1023
(C) 10! (D) 2^{10}

Q.21 The number of ways in which the letters of the word 'VOWEL' can be arranged so that the letters O, E occupy only even places is –
(A) 12 (B) 24
(C) 18 (D) 36

Q.22 If 7 points out of 12 are in the same straight line, then number of triangles formed is –
(A) 19 (B) 185
(C) 201 (D) None of these

Q.23 A cricket team of 11 players is to be selected from 13 players of which 4 are bowlers and 2 wicket keepers. The number of ways to select the team, consisting 1 wicket keeper and atleast 3 bowlers is –

(A) 8 (B) 22
(C) 112 (D) None of these

Q.24 The number of ways in which an examiner can assign 30 marks to 8 questions, giving not less than 2 marks to any questions, is–

(A) $^{21}C_7$ (B) $^{10}C_7$
(C) $^{22}C_7$ (D) None of these

Q.25 A dictionary is printed consisting of 7 lettered words only that can be made with the letters of work CRICKET. If words are printed in the alphabetic order as in the dictionary then the number of words before the word CRICKET, is –

(A) 530 (B) 531
(C) 480 (D) 481

Q.26 Number of ways of selecting 5 letters from letters of word INDEPENDENT, is –

(A) 72 (B) 68
(C) 60 (D) 52

Directions : Assertion-Reason type questions.

This question contains Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.

(A) Statement- 1 is True, Statement-2 is True, Statement-2 is a correct explanation for Statement -1
(B) Statement -1 is True, Statement -2 is True ; Statement-2 is NOT a correct explanation for Statement - 1
(C) Statement - 1 is True, Statement- 2 is False
(D) Statement -1 is False, Statement -2 is True

Q.27 **Statement 1 :** The maximum number of points of intersection of 8 circles of unequal radii is 56.

Statement 2 : The maximum number of points into which 4 circles of unequal radii and 4 non coincident straight lines intersect, is 50.

Q.28 **Statement 1 :** A five digit number divisible by 3 is to be formed using the digits 0, 1, 2, 3, 4 and 5 with repetition. The total number formed are 216.

Statement 2 : If sum of digits of any number is divisible by 3 then the number must be divisible by 3.

Passage (Q.29-Q.30)

Let Set S = {1, 2, 3, ...n} be a set of first n natural numbers and A $\subseteq$ S. Suppose n (A) represents cardinal number and min (A) represents least number among the elements of set A.

Q.29 The greatest value of min (A), where A $\subseteq$ S and n (A) = r, $1 \le r \le n$ is –

(A) r (B) n – r
(C) n – r + 1 (D) r + 1

Q.30 The value of $\underset{n(A)=r}{}$ (min (A)) = k is –

(A) $n.^{n-k}C_{r-1}$
(B) $(n + 1)\ ^{n-k}C_{r-1} - r\ ^{n-k+1}C_r$
(C) $k\ .\ ^{n-k}C_{r-1} + n.\ ^{n-k+1}C_r$
(D) ^nC_r

EXERCISE - 3

PREVIOUS YEAR AIEEE QUESTIONS

Q.1 Find the no. of numbers which can be formed with digits 0,1,2,3,4 greater than 1000 and less than 4000 if repetition is allowed- **[AIEEE 2002]**

(A) 125 (B) 400
(C) 375 (D) 374

Q.2 If repetition of the digits is allowed, then the number of even natural numbers having three digits is- **[AIEEE-2002]**

(A) 250 (B) 350
(C) 450 (D) 550

Q.3 If ^nC_r denotes the number of combinations of n things taken r at a time, then the expression $^nC_{r+1} + {^nC_{r-1}} + 2 \times {^nC_r}$ equals- **[AIEEE 2003]**

(A) $^{n+1}C_{r+1}$ (B) $^{n+2}C_r$
(C) $^{n+2}C_{r+1}$ (D) $^{n+1}C_r$

Q.4 A student is to answer 10 out of 13 questions, an examination such that he must choose least 4 from the first five questions. The number of choices available to him, is- **[AIEEE 2003]**

(A) 346 (B) 140
(C) 196 (D) 280

Q.5 The number of ways in which 6 men and 5 women can dine at a round table if no two women are to sit together is given by **[AIEEE 2003]**

(A) $(7!) \times (5!)$ (B) $(6!) \times (5!)$
(C) 30 (D) $(5!) \times (4!)$

Q.6 How many ways are there to arrange the letters in the word GARDEN with the vowels in alphabetical order ? **[AIEEE 2004]**

(A) 120 (B) 240
(C) 360 (D) 480

Q.7 The number of ways of distributing 8 identical balls in 3 distinct boxes so that none of the boxes is empty is- **[AIEEE 2004]**

(A) 5 (B) 21
(C) 3^8 (D) $^8C_3$

Q.8 If the letters of the word SACHIN are arranged in all possible ways and these words are written out as in dictionary, then the word SACHIN appears at serial number - **[AIEEE-2005]**

(A) 601 (B) 600
(C) 603 (D) 602

Q.9 The value of $^{50}C_4 + \sum_{r=1}^{6} {^{56-r}C_3}$ is - **[AIEEE-2005]**

(A) $^{55}C_4$ (B) $^{55}C_3$
(C) $^{56}C_3$ (D) $^{56}C_4$

Q.10 At an election, a voter may vote for any number of candidates, not greater than the number to be elected. There are 10 candidates and 4 are to be elected. If a voter votes for at least one candidate, then the number of ways in which he can vote is **[AIEEE 2006]**

(A) 6210 (B) 385
(C) 1110 (D) 5040

Q.11 How many different words can be formed by jumbling the letters in the word MISSISSIPPI in which not two S are adjacent ? **[AIEEE 2008]**

(A) $6 . 7 . {^8C_4}$ (B) $6 . 8 . {^7C_4}$
(C) $7 . {^6C_4} . {^8C_4}$ (D) $8 . {^6C_4} . {^7C_4}$

Q.12 The set S : {1, 2, 3, …, 12} is to be partitioned into three sets A, B, C of equal size. Thus, $A \cup B \cup C = S$, $A \cap B = B \cap C = A \cap C = \phi$. The number of ways to partition S is- **[AIEEE 2007]**

(A) $12!/3!(4!)^3$ (B) $12!/3!(3!)^4$
(C) $12!/(4!)^3$ (D) $12!/(3!)^4$

Q13. In a shop there are five types of ice-creams available . A child buys six ice-creams.

Statement-1: The number of different ways the child can buy the six ice-creams is $^{10}C_5$

Statement -2: The number of different ways the child can buy the six ice-creams is equal to the number of different ways of arranging 6 A's and 4 B's in a row. **[AIEEE 2008]**

(A) Statement-1 is true, Statement -2 is true; Statement-2 is a correct explanation for Statement-1
(B) Statement-1 is true, Statement -2 is true; Statement-2 is not a correct explanation for Statement-1
(C) Statement-1 is true, Statement -2 is false
(D) Statement-1 is false, Statement-2 is true

Q.14 From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. Then the number of such arrangements is - **[AIEEE 2009]**

(A) Less than 500
(B) At least 500 but less than 750
(C) At least 750 but less than 1000
(D) At least 1000

Q.15 There are two urns. Urn A has 3 distinct red balls and urn B has 9 distinct blue balls. From each urn two balls are taken out at random and then transferred to the other. The number of ways in which this can be done is –**[AIEEE 2010]**

(A) 36 (B) 66
(C) 108 (D) 3

ANSWER KEY

EXERCISE - 1											
Q	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
A	C	B	A	A	C	B	C	C	A	B	C
Q	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**
A	A	B	A	D	B	B	B	A	A	A	B
Q	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**
A	A	B	B	B	C	A	C	B	A	C	C
Q	**34**	**35**	**36**	**37**	**38**	**39**	**40**				
A	A	A	C	D	B	A	C				

EXERCISE - 2											
Q	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
A	B	C	C	A	A	C	D	C	D	B	C
Q	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**
A	B	C	D	A	D	B	A	B	B	A	B
Q	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**			
A	B	A	A	A	B	D	C	B			

EXERCISE-3															
Q.No.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
Ans.	D	C	C	C	B	C	B	A	D	B	C	A	D	D	C