

**Problem 1.**

$$f\left(x + \sqrt{x^2 + 1}\right) = x - \sqrt{x^2 + 1}$$
$$f(2) + f(3) + f(6) = ?$$

**Problem 2.**

$$\int \frac{\sin x + x}{1 + \cos x} dx$$

**Problem 3.** Solve the equation

$$(x^2 + y^2 + y) dx - xdy = 0$$

**Problem 4.** Solve the equation

$$x^2(xdx + ydy) + 2y(xdy - ydx) = 0$$

**Problem 5.** Solve

$$\left[\frac{x}{2}\right]^2 + \frac{2}{[x]} = 5x$$

Where  $[.]$  is Greatest Integer Function.

**Problem 6.**

$$\sqrt{x + \sqrt{x}} - \sqrt{x - \sqrt{x}} = \frac{4}{5} \sqrt{\frac{x}{x + \sqrt{x}}}$$

Find  $x$ .

**Problem 7.** DBSP :

$$\sum_{r=0}^9 {}^{20}C_{2r} {}^{20}C_{2r+2} = \frac{(\alpha + 1)}{\alpha} ({}^{39}C_{22} - {}^{19}C_{11})$$

Find  $\alpha$ .

**Problem 8.** Find the number of four digit natural numbers with two even digits and two odd digits. Repetition allowed.

**Problem 9.** Find the value of

$$\int_{-2024\pi}^{2024\pi} \cot^{-1}(\cot x) dx$$

**Problem 10.** Find the Range of the function

$$f(x) = \frac{x}{\sqrt{x^2 + 1}}$$

**Problem 11.** Is 1280000401 a Prime?

**Problem 12.** Given that

$$f(x) = \frac{x+2}{2x+3}$$

Evaluate

$$I = \int \sqrt{\frac{f(x)}{x^2}} dx$$

**Problem 13.** Given that for  $a, b, c$  and  $\omega$  is the Complex cube root of unity

$$\begin{aligned} \frac{1}{a+\omega} + \frac{1}{b+\omega} + \frac{1}{c+\omega} &= 2\omega^2 \\ \frac{1}{a+\omega^2} + \frac{1}{b+\omega^2} + \frac{1}{c+\omega^2} &= 2\omega \\ \frac{1}{a+1} + \frac{1}{b+1} + \frac{1}{c+1} &=? \end{aligned}$$

Note: Make sure reasoning is proper.

**Problem 14.** Find the Maximum area of the Rectangle inscribed in a 3 – 4 – 5 Right triangle, with two vertices of the Rectangle on the Hypotenuse and other two each on legs of the triangle.

**Problem 15.** Repeat the above problem, if one vertex of the Rectangle is on Hypotenuse and other three on legs of the triangle.

**Problem 16.** DBSP: Solve for  $x$

$$x + \frac{x}{\sqrt{x^2-1}} = \frac{35}{12}$$

**Problem 17.** Given that  $A$  and  $B$  are Independent events and the events  $C$  and  $D$  are defined as:

$$C = A' \cap B, D = A \cap B'$$

Then find the value of

$$\frac{P(A) + P(B) - P(C \cup D)}{P(A \cap B)}$$

**Problem 18.**

$$\int_0^1 \frac{x^3 - x^2 - 1}{x^2 - x - 1} dx$$

**Problem 19.** BSP

$$\int \frac{x^2 + \sin^2 x}{(x \cos x + \sin x)^2} dx$$

**Problem 20.** Which is bigger  $9^{\sqrt{2}}$  OR  $(\sqrt{2})^9$ ? Estimation and Approximations not allowed.

**Problem 21.** *Solve the Trigonometric equation*

$$\sin^2 x + \frac{\sin^2 3x}{4} = \sin x \sin^2 3x$$

**Problem 22.** *ABCD is a Square. An Equilateral Triangle is formed with one vertex at A and the other two vertices on the sides BC and CD of the Square. Find the Ratio of their areas.*