Maxwell's Equations and the Displacement Current

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Outline

Gauss's Law for Electric Fields

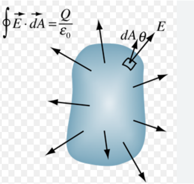
Integral Form:

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enclosed}}}{\varepsilon_0}$$

Differential Form:

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

Physical Significance: The electric flux through a closed surface is proportional to the charge enclosed within the surface.



Gauss's Law for Magnetic Fields

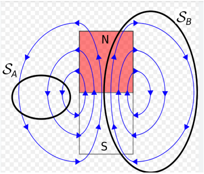
Integral Form:

$$\oint \vec{B} \cdot d\vec{A} = 0$$

Differential Form:

$$\nabla \cdot \vec{B} = 0$$

Physical Significance: Magnetic monopoles do not exist; the net magnetic flux through any closed surface is zero.



Faraday's Law of Electromagnetic Induction

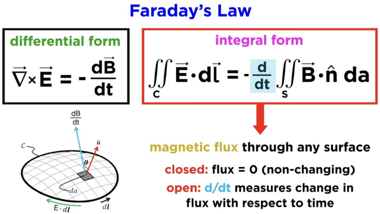
Integral Form:

$$\oint \vec{E} \cdot d\vec{\ell} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{A}$$

Differential Form:

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Physical Significance: A time-varying magnetic field induces a circulating electric field.



Ampere's Law (Without Displacement Current)

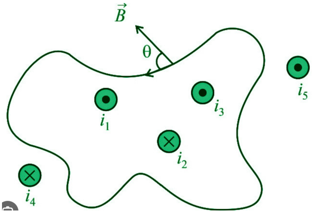
Integral Form:

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{\text{enclosed}}$$

Differential Form:

$$\nabla \times \vec{B} = \mu_0 \vec{J}$$

Physical Significance: Magnetic fields are generated by steady electric currents.



Introduction to Displacement Current

The Problem with Ampere's Law: Ampere's Law fails in regions with a time-varying electric field. For example, inside a capacitor during charging, there is no conduction current in the gap, yet a magnetic field exists.
Maxwell's Contribution: Maxwell introduced the *Displacement Current*, given by:

$$I_d = \varepsilon_0 \frac{d\Phi_E}{dt},$$

where $\Phi_E = \int \vec{E} \cdot d\vec{A}$ is the electric flux.
Significance: The displacement current ensures consistency in the magnetic field generation even in the absence of conduction currents.

Modified Ampere's Law (With Displacement Current)

Integral Form:

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 \left(I_{\text{enclosed}} + \varepsilon_0 \frac{d\Phi_E}{dt} \right)$$

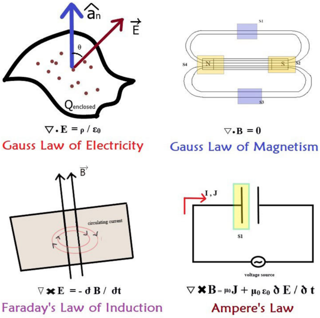
Differential Form:

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

Physical Significance: Both conduction currents ($\vec{J}$) and time-varying electric fields ($\frac{\partial \vec{E}}{\partial t}$) contribute to the generation of magnetic fields.

Unified Framework of Electromagnetism

Maxwell's Equations form the foundation of classical electrodynamics and explain: - Electric and magnetic fields and their sources. - The interdependence of $\vec{E}$ and $\vec{B}$ in time-varying scenarios. - The existence of electromagnetic waves as solutions.



Summary of Maxwell's Equations (1/2)

1. Gauss's Law for Electric Fields: Integral Form:

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enclosed}}}{\varepsilon_0}$$

Differential Form:

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

2. Gauss's Law for Magnetic Fields: Integral Form:

$$\oint \vec{B} \cdot d\vec{A} = 0$$

Differential Form:

$$\nabla \cdot \vec{B} = 0$$

Summary of Maxwell's Equations (2/2)

3. Faraday's Law of Induction: Integral Form:

$$\oint \vec{E} \cdot d\vec{\ell} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{A}$$

Differential Form:

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

4. Modified Ampere's Law: Integral Form:

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 \left(I_{\text{enclosed}} + \varepsilon_0 \frac{d\Phi_E}{dt} \right)$$

Differential Form:

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$