QUADRATIC EQUATIONS

GENERAL FORM : $k(ax^2+bx+c) = 0$, a, b and c are integers and $a \neq 0$,

a = coefficient of x^2 ,
b = coefficient of x or linear part of the quadratic polynomial
c = constant

Number of Zeroes : Maximum '2',

Graph : **Parabola**
Graph intersects x axis atmost at two points.
Graph touches the x axis at 1 point.

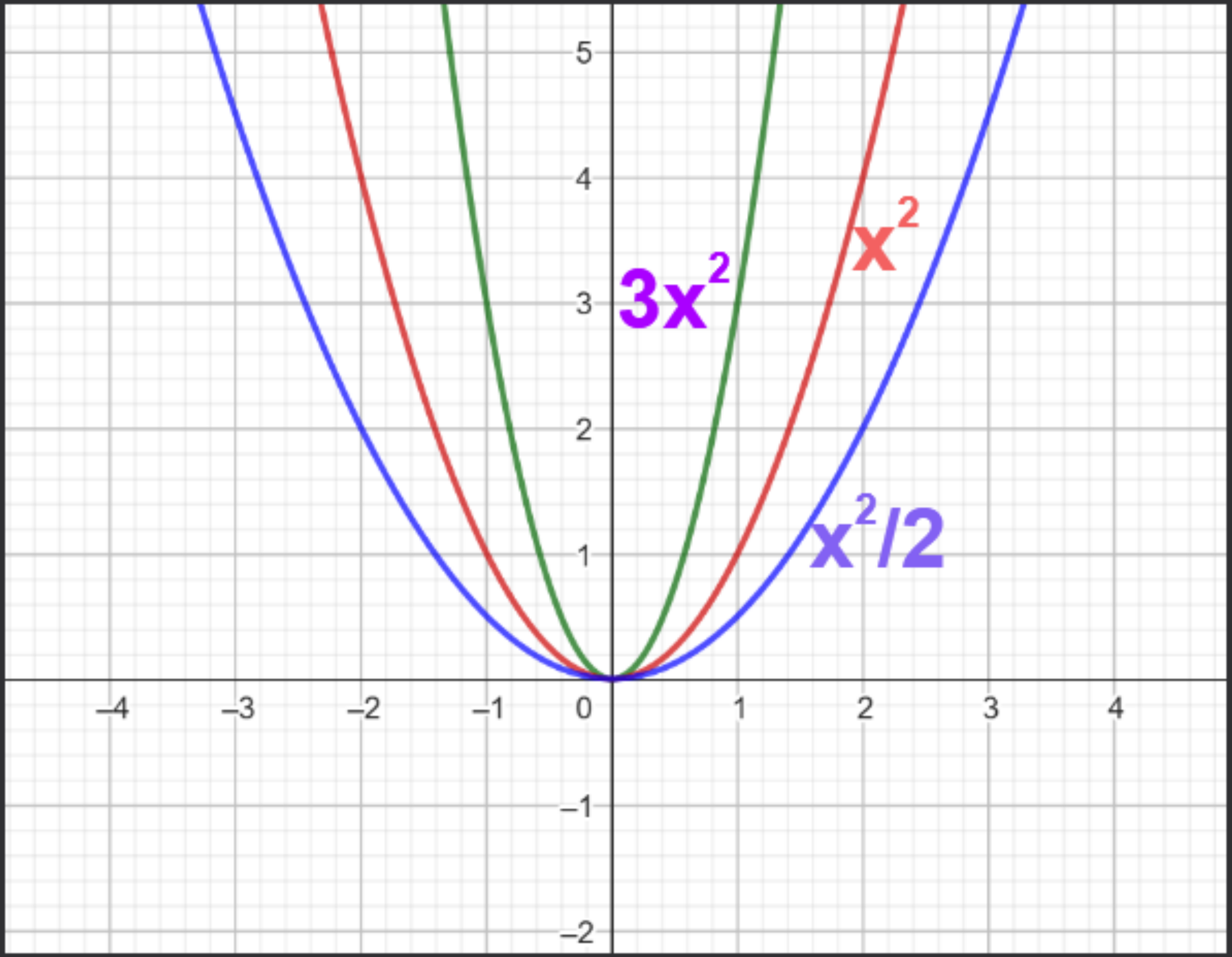
TYPES OF QUADRATIC EQUATIONS

1. $ax^2 = 0$, $b = 0$, $c = 0$

In the quadratic equation where both linear and constant part is '0', the graph passes through origin and quadratic equation has one root / solution/ zeroes and that is equal to 0.

$x^2/2$ =  = $3x^2$

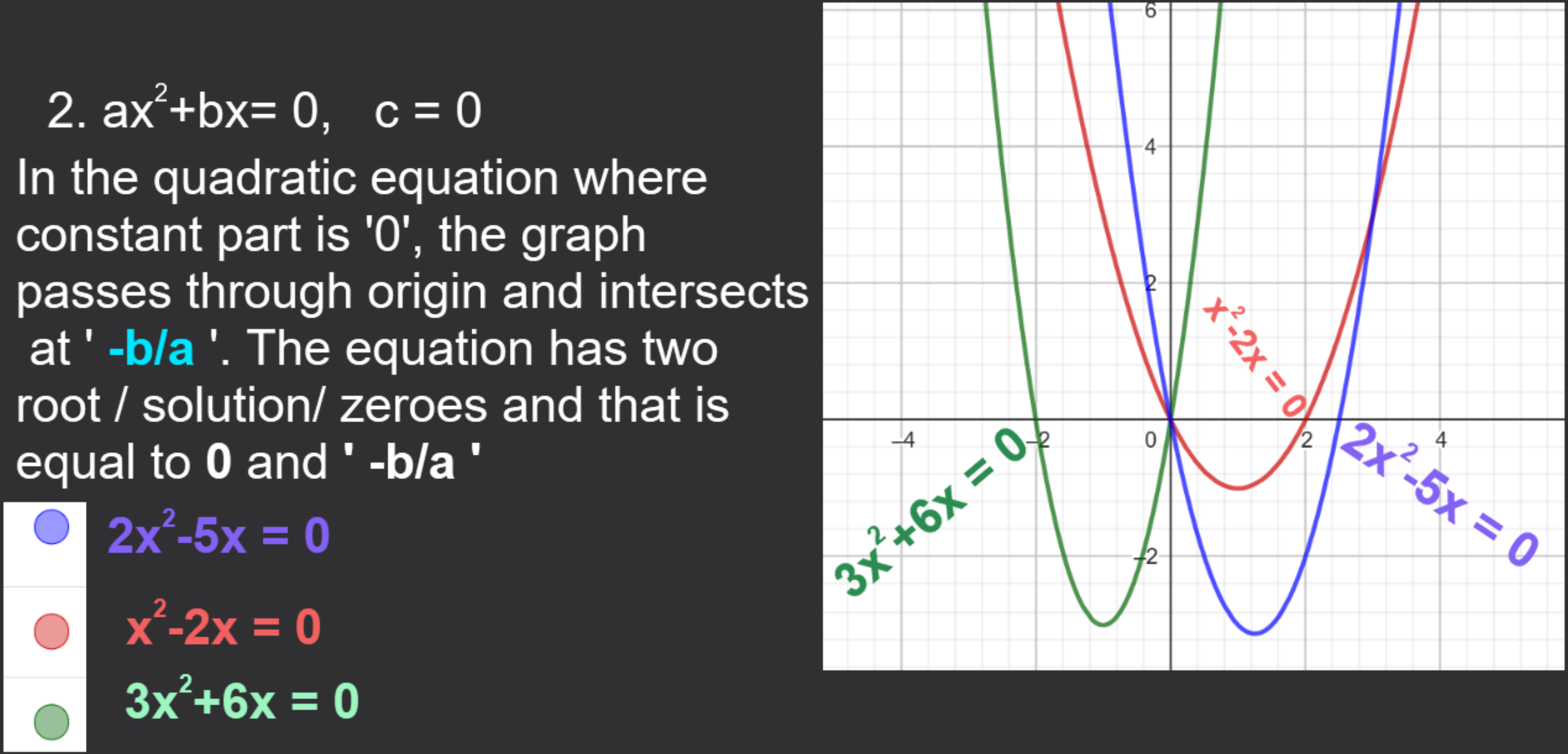
= x^2



TYPES OF QUADRATIC EQUATIONS

2. $ax^2+bx=0$, $c = 0$

In the quadratic equation where constant part is '0', the graph passes through origin and intersects at ' -b/a '. The equation has two root / solution/ zeroes and that is equal to **0** and **' -b/a '**



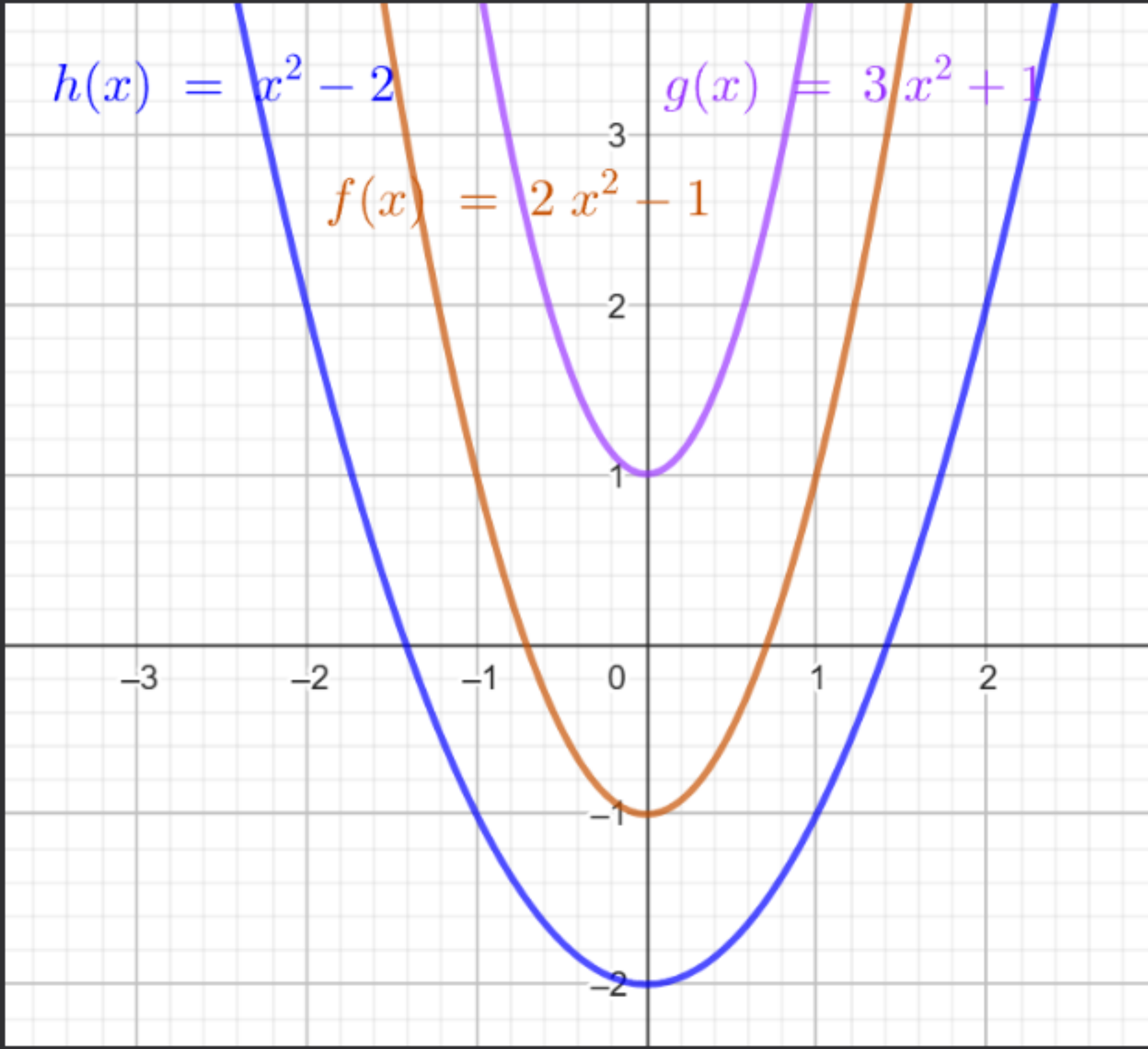
TYPES OF QUADRATIC EQUATIONS

3. $ax^2 + c = 0$, $b = 0$

In the quadratic equation where linear part is '0', the graph may has two zeroes which are **additive inverse** of each other or may not have any zero. If $c > 0$, then no zeroes. If $c < 0$, then both the zeroes are

$$\pm\sqrt{\frac{-c}{a}}$$

y axis is the axis of symmetry.



4. STANDARD FORM: $ax^2+bx+c = 0$

Methods to solve Quadratic equation :

1. Splitting the Middle Term:

$ax^2+bx+c = 0$

Step1: Write product ac in its prime factor form and look for sign.

Step2: If ac is positive then look for pair of factors among the prime factors of ac which will add up to give 'b' and split 'b' in those two factors.

Step3: If ac is negative then look for pair of factors among the prime factors of ac which will subtract to give 'b' and split 'b' in those two factors.

4. STANDARD FORM: $ax^2+bx+c = 0$

Methods to solve Quadratic equation :

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

2. USING QUADRATIC FORMULA

$ax^2+bx+c = 0$,

Step 1: Find Discriminant : $D = b^2-4ac$

Step 2(a): If $D > 0$, then $\alpha = \frac{-b+\sqrt{b^2-4ac}}{2a}$

$\beta = \frac{-b-\sqrt{b^2-4ac}}{2a}$

Step 2(b): If $D = 0$, then, roots are equal $\alpha = \beta = \frac{-b}{2a}$

Step 2(c): If $D < 0$, then no real roots exist.

4. STANDARD FORM: $ax^2+bx+c = 0$

Methods to solve Quadratic equation :

3. By Completing The Square:

3a.) $ax^2+bx+c = 0$ (multiply equation by a)

3b.) $(ax)^2 + abx + ac = 0$,

3c.) $(ax)^2 + 2(ax)(b/2) + \frac{b^2}{4} - \frac{b^2}{4} + ac = 0$

3d.) $(ax + b/2)^2 = (b^2-4ac)/4$

3e.) $(ax + b/2) = \frac{\pm\sqrt{b^2-4ac}}{2}$

3f.) $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

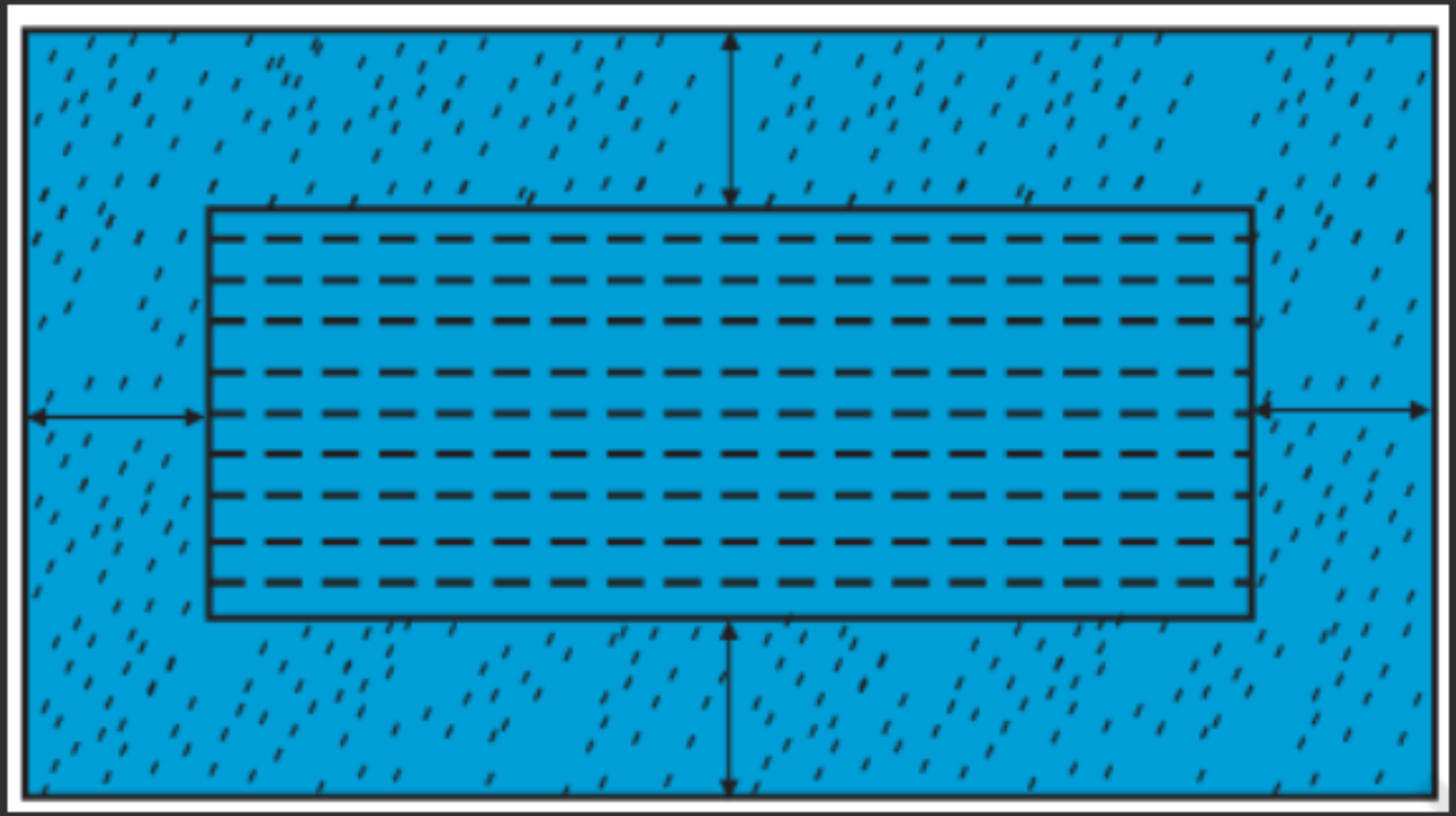
Find a natural number whose square diminished by 84 is equal to thrice of 8 more than the given number.

A natural number, when increased by 12, equals 160 times its reciprocal. Find the number.

A train, travelling at a uniform Speed for 360 km, would have taken 48 minutes less to travel the same distance if its speed were 5 km/h more. Find the original speed of the train.

If Zeba were younger by 5 years than what she really is, then the square of her age (in years) would have been 11 more than five times her actual age. What is her age now?

At present Asha’s age (in years) is 2 more than the square of her daughter Nisha’s age. When Nisha grows to her mother’s present age, Asha’s age would be one year less than 10 times the present age of Nisha. Find the present ages of both Asha and Nisha



In the centre of a rectangular lawn of dimensions 50 m × 40 m, a rectangular pond has to be constructed so that the area of the grass surrounding the pond would be 1184 m^2 [see Fig. 4.1]. Find the length and breadth of the pond.

At t minutes past 2 pm, the time needed by the minutes hand of a clock to show 3 pm was found to be 3 minutes less than $t^2/4$ minutes. Find t

A train travels at a certain average speed for a distance of 63 km and then travels a distance of 72 km at an average speed of 6 km/h more than its original speed. If it takes 3 hours to complete the total journey, what is its original average speed?

Had Ajita scored 10 more marks in her mathematics test out of 30 marks, 9 times these marks would have been the square of her actual marks. How many marks did she get in the test?

Check whether the equation $6x^2 - 7x + 2 = 0$ has real roots, and if it has, find them by the method of completing the squares.

A) Every quadratic equation has exactly one root.

R) $b^2 - 4ac = 0$, then both roots are equal.

A.) Every quadratic equation has at least one real root.

R) $b^2 -4ac > 0$, then roots are real and distinct.

A.) Every quadratic equation has at least two roots.

R) $b^2 - 4ac > 0$, then roots are real and distinct.

A.) Every quadratic equations has at most two roots.

R) b^2 -4ac >0, then roots are real and distinct.

A.) If the coefficient of x^2 and the constant term of a quadratic equation have opposite signs, then the quadratic equation has real roots

R.) When discriminant is always positive, it has always real roots, $ac < 0$ and $b^2 - 4ac > 0$.

A.) If the coefficient of x^2 and the constant term have the same sign and if the coefficient of x term is zero, then the quadratic equation has no real roots.

R.) When discriminant is always negative, it has no real roots, $ac > 0$ and $b^2 - 4ac < 0$.

A quadratic equation with integral coefficient has integral roots. Justify your answer.

Does there exist a quadratic equation whose coefficients are rational but both of its roots are irrational? Justify your answer

Does there exist a quadratic equation whose coefficients are all distinct irrationals but both the roots are rationals? Why

A.) 0.2 a root of the equation $x^2 - 0.4 = 0$?

R.) The equation becomes 0 at $x=0.2$

If $b = 0$, $c < 0$, is it true that the roots of $x^2 + bx + c = 0$ are numerically equal and opposite in sign? Justify