

FORMULAE BOOK - SUDHANSHU SIR

REAL NUMBERS

1. **LCM** : Product of the greatest power of each (ALL) prime factor, involved in the numbers.

$$P = a^3 b^4 c^5 k^2, \quad Q = a^4 b^2 c^3 d^4 \quad \mathbf{LCM} = \mathbf{a^4 b^4 c^5 k^2 d^4}$$

2. **HCF** : Product of the smallest power of each common prime factor in the numbers.

$$P = a^3 b^4 c^5 k^2, \quad Q = a^4 b^2 c^3 d^4 \quad \mathbf{HCF} = \mathbf{a^3 b^2 c^3}$$

3. For any two positive integers a and b,

$$\mathbf{HCF} (a, b) \times \mathbf{LCM} (a, b) = \mathbf{a \times b}$$

FORMULAE BOOK - SUDHANSHU SIR

REAL NUMBERS

SOME IMPORTANT FACTS

IF **X**, **Y** AND **Z** are three numbers and **X** is a factor of **Y** & **Z** is a multiple of **Y**.

$$\text{HCF (X,Y)} = \text{X}, \quad \text{HCF(Y,Z)} = \text{Y}, \quad \text{HCF (X,Z)} = \text{X}$$

$$\text{HCF (X,Y,Z)} = \text{X}$$

$$\text{LCM (X,Y)} = \text{Y}, \quad \text{LCM(Y,Z)} = \text{Z}, \quad \text{LCM(X,Z)} = \text{Z}$$

$$\text{LCM (X,Y,Z)} = \text{Z}$$

FORMULAE BOOK - SUDHANSHU SIR

REAL NUMBERS

SOME IMPORTANT FACTS

- ▶ **THE LCM OF TWO COPRIME NUMBERS IS THEIR PRODUCT**
- ▶ **TWO PRIME NUMBERS ARE ALWAYS COPRIME**
- ▶ **TWO CONSECUTIVE NUMBERS ARE ALWAYS COPRIME**
- ▶ **TWO CONSECUTIVE ODD NUMBERS ARE ALWAYS COPRIME**

FORMULAE BOOK - SUDHANSHU SIR

REAL NUMBERS

Theorem 1.2 (Fundamental Theorem of Arithmetic) : Every composite number can be Expressed (factorised) as a product of primes, and this factorisation is unique, apart from the order in which the prime factors occur.

1. *** USED IN proving 6^n or $(2 \times 3)^n$, 15^n or $\{3 \times 5\}^n$, 4^n or $(2 \times 2)^n$ does not end with the digit 0.

2. *** USED IN PROVING A NUMBER IS COMPOSITE BY TAKING COMMON. (DISTRIBUTIVE PROPERTY)

FORMULAE BOOK - SUDHANSHU SIR

REAL NUMBERS

***** IF 'p' IS PRIME & 'p' DIVIDES a^2 THEN p DIVDES 'a'**

(IT IS USED IN PROVING $\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{7}, \sqrt{11}$ SUCH NUMBERS AS IRRATIONAL USING CONTRADICTION METHOD OF DIVISIBILITY BY CO PRIME NUMBERS.

*** In complex irrational numbers $3\sqrt{2}-6$, $2\sqrt{3}$, $3+5\sqrt{7}$ it is assumed that the square root of the given number is already irrational and proved by simple transposing and proving $LHS \neq RHS$

FORMULAE BOOK - SUDHANSHU SIR

REAL NUMBERS

SOME FACTS ON IRRATIONAL NUMBERS IN RELATION WITH RATIONAL NUMBERS

- The sum of rational and irrational number is always irrational.
- The difference of rational and irrational number is always irrational.

SOME FACTS ON IRRATIONAL NUMBERS IN RELATION WITH RATIONAL NUMBERS

- **The product of non zero rational and irrational number is always irrational**
- **The quotient of non zero rational and irrational number is always irrational.**

FORMULAE BOOK - SUDHANSHU SIR

REAL NUMBERS

SOME FACTS ON IRRATIONAL NUMBERS IN RELATION WITH IRRATIONAL NUMBERS

- **The sum of two irrational numbers can either be rational or irrational.**
- **The difference of two irrational numbers can either be rational or irrational.**
- **The product of two irrational number can either be rational or irrational.**
- **The ratio of two irrational number can either be rational or irrational.**

FORMULAE BOOK - SUDHANSHU SIR

REAL NUMBERS

★ If a and b are rationals and \sqrt{a} and \sqrt{b} are irrationals then $\sqrt{a+b}$ can be rational or irrational.

★ If p and q are prime positive integers, then prove that $\sqrt{p} + \sqrt{q}$ is irrational.

SOME IMPORTANT MULTIPLES :

$$7 \times 13 = 91$$

$$13 \times 17 = 221$$

$$17 \times 23 = 391$$

$$19 \times 29 = 551$$

$$7 \times 11 \times 13 = 1001$$

$$13 \times 19 = 247$$

$$13 \times 29 = 377$$

$$23 \times 29 = 667$$

$$19 \times 23 = 437$$

$$17 \times 31 = 527$$

$$11 \times 13 = 143$$

$$17 \times 19 = 323$$

$$13 \times 31 = 403$$

$$17 \times 29 = 493$$

$$23 \times 31 = 713$$

$$19 \times 31 = 589$$

$$13 \times 23 = 299$$

$$29 \times 31 = 899$$

$$13 \times 17 \times 19 = 4199$$

$$17 \times 19 \times 23 = 7429$$

REAL NUMBERS

- 1. 2 IS THE ONLY EVEN PRIME NUMBER**
- 2. 2 & 3 ARE THE ONLY CONSECUTIVE PRIME NUMBERS (OF THE FORM $N, N+1$)**
- 3. 4 IS THE LEAST COMPOSITE NUMBER**
- 4. 9 IS THE HIGHEST ONE DIGIT COMPOSITE NUMBER**
- 5. The product of 3 consecutive integers is divisible by 6.**
- 6. 10 IS THE LOWEST TWO DIGIT COMPOSITE NUMBER**
- 7. m^2-1 is divisible by 8 if m is an odd number**
- 8. THE TOTAL NUMBERS BETWEEN TWO NUMBERS 'A' & 'B' = $B-A-1$**
- 9. THE TOTAL NUMBERS FROM A TO B = $B-A+1$**

**FIND THE LARGEST LENGTH WHICH CAN EXACTLY MEASURE
THE GIVEN LENGTH = HCF**

**FIND THE MINIMUM NUMBER OF SQUARE TILES
REQUIRED TO COVER THE RECTANGULAR FLOOR.**

**FIND THE NUMBER OF CHAIRS REQUIRED IF THE
NUMBER OF TEACHERS ON EACH CHAIR OF
DIFFERENT SUBJECT SHOULD BE SAME.**

**FIND THE NUMBER OF SHELF REQUIRED TO KEEP THE
BOOKS OF DIFFERENT SUBJECT IF THE BOOKS IN
EACH SHELF HAS TO BE SAME.**

**FIND THE TIME WHEN THE BELL WILL RING AGAIN
SIMULTANEOUSLY AND HOW MANY TIMES**

**FIND THE TIME WHEN THE THREE DIFFERENT
RUNNER WILL STEP TOGETHER AGAIN**

QUESTIONS BASED ON HCF

FIND THE NUMBER OF TILES REQUIRED IF EACH TILE IS A SQUARE TO COVER A RECTANGULAR HALL.

FIND THE NUMBER OF SHELF REQUIRED TO KEEP THE BOOKS OF DIFFERENT SUBJECT IF THE BOOKS IN EACH SHELF HAS TO BE SAME.

If a & b are two prime numbers of the form p and $p+1$, and $M = a^a + b^b$; $N = a^b + b^a$, then M & N are primes.

Find the largest number that divides 515,748 & 897 leaving remainders 11,13 & 15 respectively.

SPECIAL QUESTIONS

If the LCM of two numbers 216 and k is 648 then how many values of k are possible

SPECIAL QUESTIONS

If the LCM of two numbers $A = 2^{14} \times 3^{19} \times 7^{23}$ and k is $2^{14} \times 3^{19} \times 7^{35}$ then how many values of k are possible?

- (a) 266 (b) 33
(c) 300 (d) 35

SPECIAL QUESTIONS

If the LCM of two numbers 1728 and k is 5184, then how many values of k are possible?

- (a) 6 (b) 7**
(c) 11 (d) 8

SPECIAL QUESTIONS

If the LCM of two natural numbers is 200. How many set of values of two natural numbers are possible?

LCM of two numbers A and B is $N = p^x q^y$ then, possible values of A and B is $(x+1)(y+1) + xy$

A library buys 36 English books, 48 Science books & 72 Mathematics books. The thickness of each book is the same. Now, the Librarian wants the books to be placed in stacks, such that each stack has books of the same subject, and the height of each stack is the same. Also, the librarian wants as few stacks as possible. How many books will there be in each stack? How many stacks of books will there be?

Ann has swimming lessons every fifth day and diving lessons every third day. If she has a swimming lesson and a diving lesson today, after how many days will she once again have both swimming and diving lessons. ?

Find the largest dimensions of the square tiles which can be used to floor a hall of dimension 416 m x 234 m completely.

Shelly wants to tile a rectangular floor that measures 132m x 150m using square tiles that are all of the same size, with no overlap and no squares hanging over the edge.

What is the measurement of the largest square tiles she can use?

Two buckets contain 54 & 90 Itrs of water respectively. Find the maximum capacity of a jug which can measure the water of either bucket an exact number of times.

A juice seller has 1120ml of orange juice, 1400ml of grape juice and 1680ml of watermelon juice. he packs each kind of juice in small jars of equal capacity. What is the maximum capacity of the each of the small jars such that no juice is left over.

How many jars will be required to pack the three different types of juice?

An Army contingent of 1000 members is to march behind army band of 56 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march.

The Traffic lights at three different road crossings change after every 75 seconds, 100 seconds, & 150 seconds respectively. If they all change simultaneously at 09:10:00 Hrs, at what time will they again change simultaneously?

Boxes of height 15 cm are stacked next to boxes of height 24 cm each. What is the shortest height at which the two stacks will be of the same height?

The HCF of two numbers is 18 and their LCM is 324. If one of the numbers is 162, find the other number.

If p and q are two coprime numbers, then find the HCF and LCM of p and q .

If l and m are two numbers such that l is a factor of m , find the HCF and LCM of l and m .

In a school there are two Sections A and B of class X. There are 48 students in Section A and 60 students in Section B. Determine the least number of books required for the library of the school so that the books can be distributed equally among all students of each section.

VVIQ :

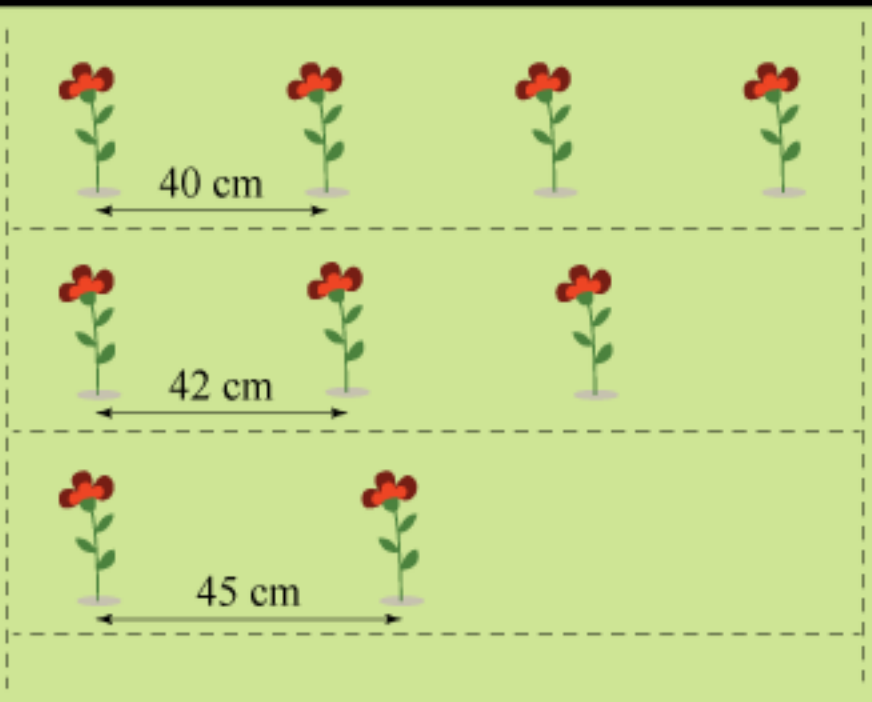
Find the least number of square tiles by which the floor of a room of dimensions 16.58m and 8.32m can be covered completely.

VVIQ:

The sum of the two numbers is 588 and their HCF is 49. How many such pairs of numbers can be formed?

**Find the largest number
which divides 92 & 74
leaving 2 as remainder.**

The Traffic lights at three different road crossings change after every 48 seconds, 72 seconds, & 108 seconds respectively. If they all change simultaneously at 08:20:00 Hrs, at what time will they again change simultaneously?



Meher is planting saplings in her garden. In one part of the garden, she makes three parallel rows. Starting from the left end of each row, she plants saplings at equal distances. In the first row, the distance between adjacent saplings is 40cm. This distance is 42 cm in the second row, and 45cm in the third row. Meher observes that the leftmost saplings in the three rows are aligned. At what distance from the left end will he again find aligned saplings?

Two numbers are in the ratio 2:3 and their LCM is 180. What is the HCF of these numbers.

The ratio of HCF to LCM of the least composite number and the least prime number is

The LCM of the smallest 2 digit number and smallest composite number is :

REAL NUMBERS- CASE STUDY

CASE STUDY 1.

To enhance the reading skills of grade X students, the school nominates you and two of your friends to set up a class library. There are two sections- section A and section B of grade X. There are 32 students in section A and 36 students in section B.



1. What is the minimum number of books you will acquire for the class library, so that they can be distributed equally among students of Section A or Section B?

a) 144

b) 128

c) 288

d) 272

2. If the product of two positive integers is equal to the product of their HCF and LCM is true then, the HCF (32 , 36) is

- a) 2
- b) 4
- c) 6
- d) 8

3. 36 can be expressed as a product of its primes as

a) $2^2 \times 3^2$

b) $2^1 \times 3^3$

c) $2^3 \times 3^1$

d) $2^0 \times 3^0$

4. $7 \times 11 \times 13 \times 15 + 15$ is a

- a) Prime number
- b) Composite number
- c) Neither prime nor composite
- d) None of the above

5. If p and q are positive integers such that $p = ab^2$ and $q = a^2b$, where a, b are prime numbers, then the LCM (p, q) is

- a) ab
- b) a^2b^2
- c) a^3b^2
- d) a^3b^3

ANSWERS

- 1. c) 288
- 2. b) 4
- 3. a) $2^2 \times 3^2$
- 4. b) composite number
- 5. b) a^2b^2

CASE STUDY 2:

A seminar is being conducted by an Educational Organisation, where the participants will be educators of different subjects. The number of participants in Hindi, English and Mathematics are 60, 84 and 108 respectively.



1. In each room the same number of participants are to be seated and all of them being in the same subject, hence maximum number participants that can accommodated in each room are

- a) 14
- b) 12
- c) 16
- d) 18

2. What is the minimum number of rooms required during the event?

a) 11

b) 31

c) 41

d) 21

3. The LCM of 60, 84 and 108 is

a) 3780

b) 3680

c) 4780

d) 4680

4. The product of HCF and LCM of 60,84 and 108 is

a) 55360

b) 35360

c) 45500

d) 45360

5. 108 can be expressed as a product of its primes as

a) $2^3 \times 3^2$

b) $2^3 \times 3^3$

c) $2^2 \times 3^2$

d) $2^2 \times 3^3$

ANSWERS

1. b) 12

2. d) 21

3. a) 3780

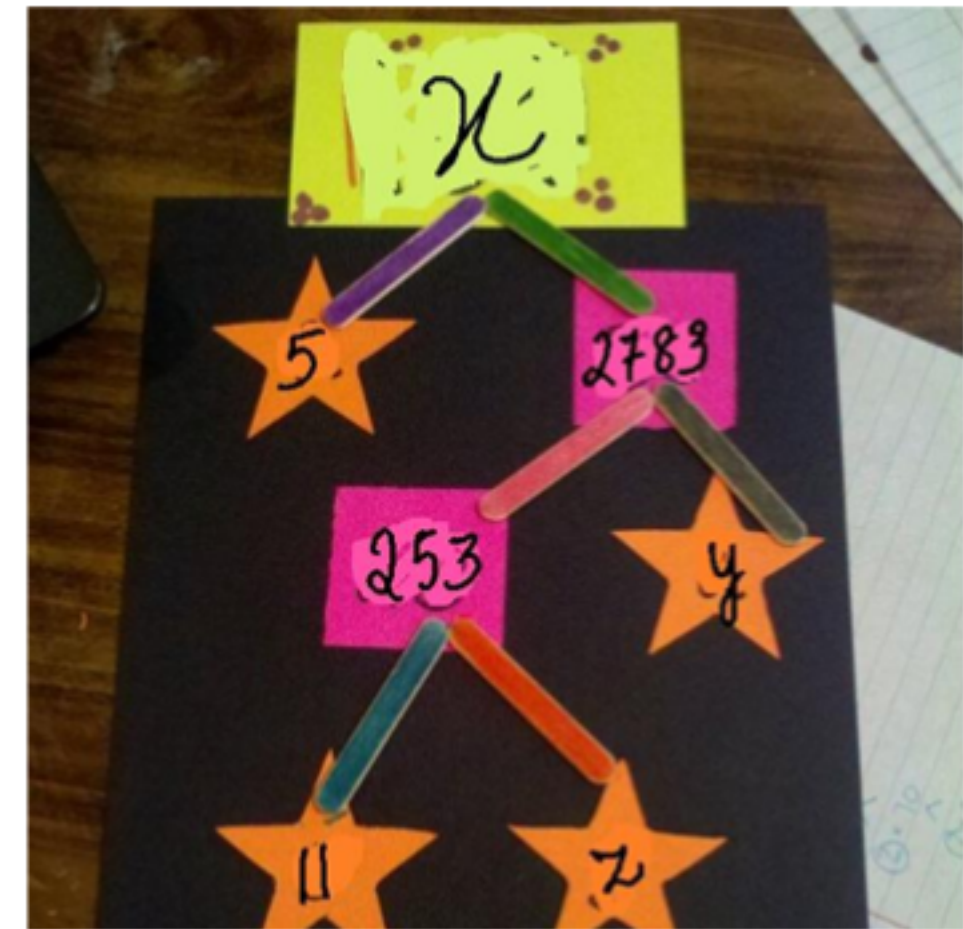
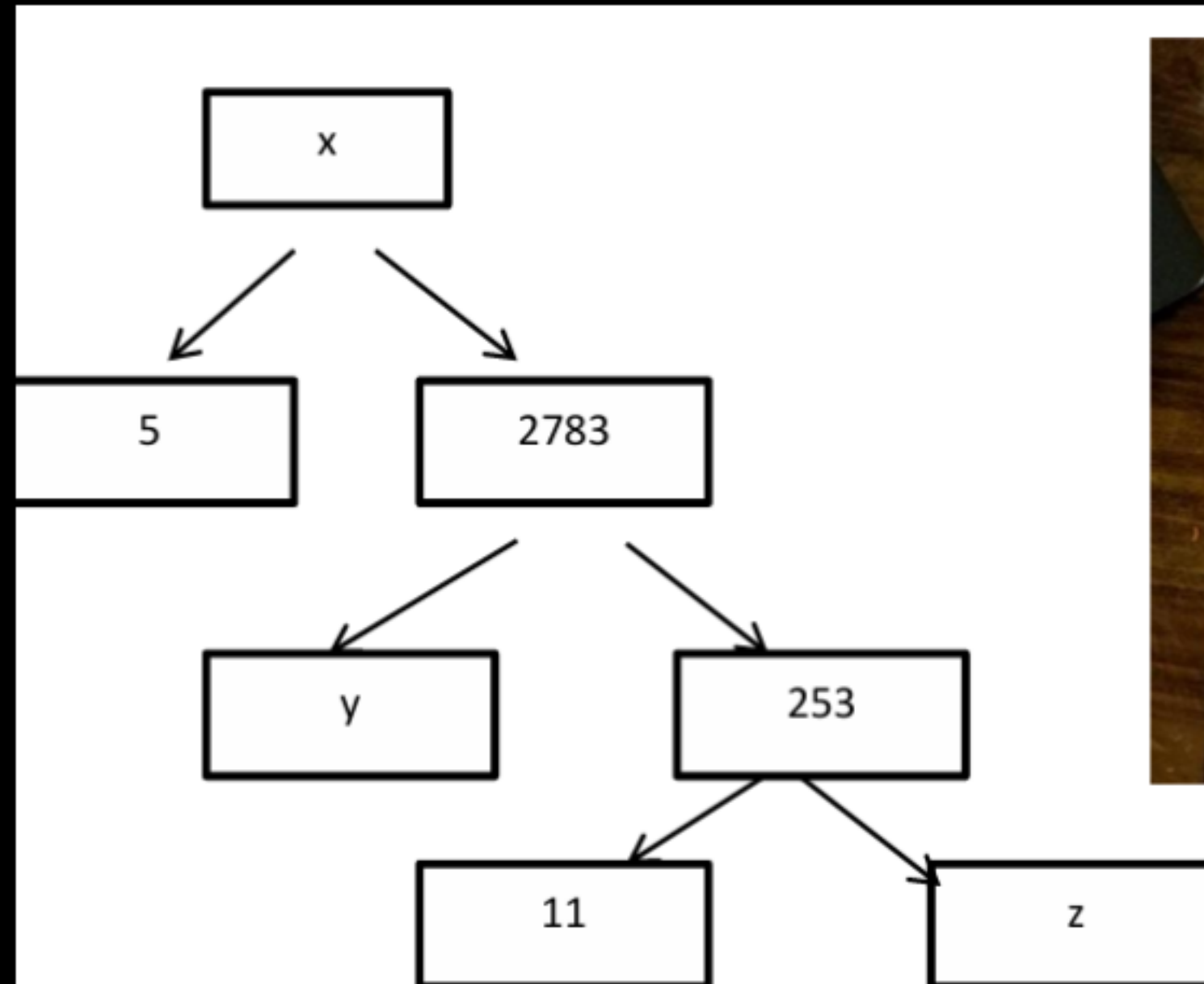
4. d) 45360

5. d) $2^2 \times 3^3$

CASE STUDY 3:

A Mathematics Exhibition is being conducted in your School and one of your friends is making a model of a factor tree. He has some difficulty and asks for your help in completing a quiz for the audience.

Observe the following factor tree and answer the following:



1. What will be the value of x ?

a) 15005

b) 13915

c) 56920

d) 17429

2. What will be the value of y ?

a) 23

b) 22

c) 11

d) 19

3. What will be the value of z ?

a) 22

b) 23

c) 17

d) 19

4. According to Fundamental Theorem of Arithmetic 13915 is a
- a) Composite number
 - b) Prime number
 - c) Neither prime nor composite
 - d) Even number

5. The prime factorisation of 13915 is

a) $5 \times 11^3 \times 13^2$

b) $5 \times 11^3 \times 23^2$

c) $5 \times 11^2 \times 23$

d) $5 \times 11^2 \times 13^2$

ANSWERS

1. b) 13915

2. c) 11

3. b) 23

4. a) composite number

5. c) $5 \times 11^2 \times 23$

For 71st republic day Parade on 26/01/2023 in Delhi, Captain RS Meel is planning for Parade of following two groups:

1. First group of Army contingent of 624 members behind an army band of 32 members.

2. Second group of CRPF troops with 468 soldiers behind the 228 members of bikers.

These two groups are to march in the same number of columns. This sequence of soldiers is followed by Jhankis of different states which are showing the culture of the respective states.

(i) What is the maximum number of columns in which the army troops can march?

(a) 8 (b) 16 (c) 4 (d) 32

(ii) What is the maximum number of columns in which CRPF troops can march?

(a) 4 (b) 8 (c) 12 (d) 16

(iii) What is the maximum number of columns in which total army troops and CRPF troops together can march past?

(a) 2 (b) 4 (c) 6 (d) 8

(iv) What should be subtracted from the numbers of CRPF soldiers and the number of bikers so that their maximum number of columns is equal to the maximum number of columns of army troops?

- (a) 4 soldiers & 4 bikers**
- (b) 4 soldiers & 2 bikers**
- (a) 2 soldiers & 4 bikers**
- (a) 2 soldiers & 2 bikers**