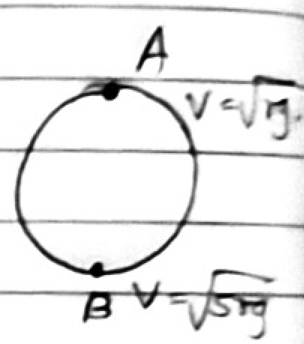
Problems on R.D

* A motorcyclist (as a particle) is undergoing vertical circles inside a sphere of death. The speed of motorcycle varies between 6 m/s to 10 m/s. Calculate diameter of the sphere of death. How much minimum values are possible for these two speeds?

Given - $V_B = 10\,m/s$, $V_A = 6\,m/s$.



To find - ① Diameter of sphere of death

② $(V_B)_{min}$ & $(V_A)_{min}$

Formula. ① $V_B^2 - V_A^2 = 4rg$

② $V_B(min) = \sqrt{5rg}$

③ $(V_A)_{min} = \sqrt{rg}$

Sol^n :- ① $(10)^2 - (6)^2 = 4r \times 10$

$r = \frac{64}{40} = 1.6\,m$

$\therefore d = 2r = 2 \times 1.6\,m = \boxed{3.2m} \rightarrow$ diameter

& $(V_B)_{min} = \sqrt{5 \times 1.6 \times 10} = 4\sqrt{5}\,m/s$.

& $(V_A)_{min} = \sqrt{1.6 \times 10} = 4\,m/s$

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* A flywheel used to prepare earthenwere pots is set into rotation at 100 rpm. It is in the form of disc of mass 10 kg. and radius 0.4m. A lump of clay (to be taken equivalent to a particle) of mass 1.6 kg falls on it and adheres to it at a certain distance x from the centre. Calculate 'x' if the wheel now rotates at 80 rpm.

→ **Given,** $n_1 = 100$ rpm $\quad n_2 = 80$ rpm.

$$\omega_1 = \frac{100 \times 2\pi}{60} = \frac{10\pi}{3} \text{ rad/s}$$

$$\omega_2 = \frac{80 \times 2\pi}{60} = \frac{8\pi}{3} \text{ rad/s}$$

$M = 10$ kg, $\quad R = 0.4$ m, $\quad m = 1.6$ kg

To find Distance from centre of fly wheel where lump of clay falls (x).

Formula $I_1\omega_1 = I_2\omega_2$

$$\left(\frac{MR^2}{2}\right)\omega_1 = \left(\frac{MR^2}{2} + mx^2\right)\omega_2$$

$$\left[\frac{10 \times (0.4)^2}{2}\right] \times \left(\frac{10\pi}{3}\right) = \left[\frac{10 \times (0.4)^2}{2} + (1.6 \times x^2)\right] \times \left(\frac{8\pi}{3}\right)$$

$\therefore 0.8 \times 5 = (0.8 + 1.6x^2) \times 4$

$\therefore 0.8 \times 5 = (0.8 \times 4) + (1.6x^2 \times 4)$

$\therefore (0.8 \times 5) - (0.8 \times 4) = 1.6x^2 \times 4$

$0.8 = 1.6 \times 4x^2 \qquad x^2 = \sqrt{\frac{1.6}{0.8 \times 4}}$

$\therefore x = \boxed{\frac{1}{\sqrt{8}}\text{ m}}$ or $= \boxed{0.35 \text{ m}}$

What will be duration of day, if earth suddenly shrinks to 1/64 th of its original volume, mass remaining unchanged ?

→ Acc. to. Law of Conservation of momentum

$$I_1 \omega_1 = I_2 \omega_2 .$$

$$\therefore \frac{2}{5} M R_1^2 \times \left(\frac{2\pi}{T_1}\right) = \frac{2}{5} M R_2^2 \times \left(\frac{2\pi}{T_2}\right)$$

$$\therefore \frac{T_2}{T_1} = \left(\frac{R_2}{R_1}\right)^2 \quad \ldots (1).$$

V_1 = original volume of earth &

V_2 = Reduced —II———

$$\therefore V_2 = \frac{V_1}{64} \quad \ldots \text{(Given)}$$

$$\therefore \frac{4}{3}\pi R_2^3 = \frac{1}{64} \times \left(\frac{4}{3}\pi R_1^3\right).$$

$$\therefore \left(\frac{R_2}{R_1}\right)^3 = \frac{1}{64} . \implies \therefore \frac{R_2}{R_1} = \left(\frac{1}{64}\right)^{1/3}.$$

$$\therefore \frac{R_2}{R_1} = \frac{1}{4}$$ — put in eq (1)

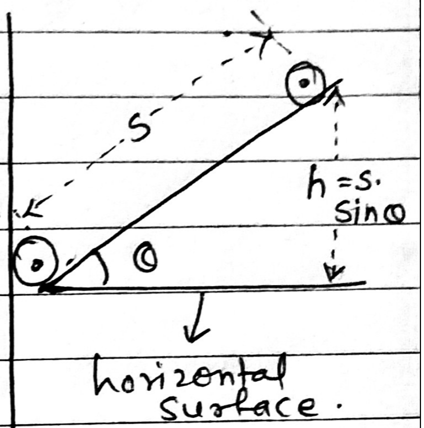
$$\therefore \frac{T_2}{T_1} = \left(\frac{1}{4}\right)^2 \implies T_2 = 24 \times \left(\frac{1}{4}\right)^2$$

$$T_2 = \frac{3}{2} \text{ hrs.} = \underline{1.5 \text{ hrs.}}$$

* Linear acceleration and speed while pure rolling:

Q. A rigid object is rolling down an inclined plane. Derive expression for acceleration along track & speed after falling through certain vertical distance.

Consider object purely rolling without slipping.
inclination angle with horizontal $= \theta$
As object starts rolling down grav. P.E converts into K.E.



Total K.E.

$\therefore E_{total} = E_{trans.} + E_{rot.}$

$$E = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2.$$

$$\therefore \quad = \frac{1}{2}mv^2\left[1 + \frac{K^2}{R^2}\right].$$

$$I\omega^2 = \left(\because mK^2 \times \frac{v^2}{R^2}\right).$$

$$= mv^2 \times \left(\frac{K^2}{R^2}\right)$$

$\therefore$ But K.E. = P.E.

$$\therefore \frac{1}{2}\cancel{m}v^2\left[1 + \frac{K^2}{R^2}\right] = \cancel{m}gh$$

$$\therefore \boxed{v = \sqrt{\frac{2gh}{\left(1+\frac{K^2}{R^2}\right)}}}$$

$\therefore$ Now, a = linear accⁿ along plane

$$v^2 = u^2 + 2as$$

$$\therefore 2as = v^2 - u^2.$$

$$\therefore \cancel{2}a\left(\frac{h}{\sin\theta}\right) = \frac{\cancel{2}gh}{\left(1+\frac{K^2}{R^2}\right)} - 0.$$

$$\therefore \boxed{a = \frac{g\sin\theta}{\left(1+\frac{K^2}{R^2}\right)}}$$