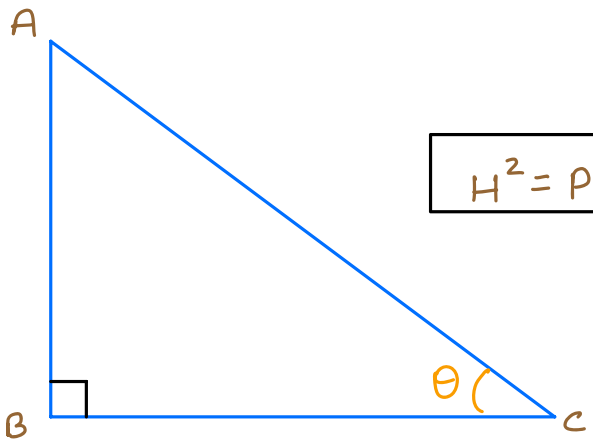
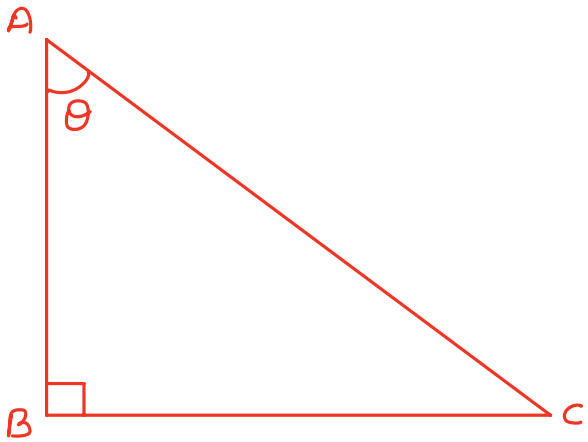
TRIGONOMETRY



$$H^2 = P^2 + B^2$$



AC — Hypotenuse
AB — Perpendicular
BC — Base

$$\sin\theta = \frac{AB}{AC}$$

$$\cos\theta = \frac{BC}{AC}$$

$$\tan\theta = \frac{AB}{BC}$$

$$\operatorname{cosec}\theta = \frac{AC}{AB}$$

$$\sec\theta = \frac{AC}{BC}$$

$$\cot\theta = \frac{BC}{AB}$$

$$\sin\theta = \frac{P}{H}$$

$$\cos\theta = \frac{B}{H}$$

$$\tan\theta = \frac{P}{B}$$

$\sin\theta \longleftrightarrow \operatorname{cosec}\theta$

$\cos\theta \longleftrightarrow \sec\theta$

$\tan\theta \longleftrightarrow \cot\theta$

AC — Hypotenuse
BC — Perpendicular
AB — Base

$$\sin\theta = \frac{BC}{AC}$$

$$\cos\theta = \frac{AB}{AC}$$

$$\tan\theta = \frac{BC}{AB}$$

$$\operatorname{cosec}\theta = \frac{AC}{BC}$$

$$\sec\theta = \frac{AC}{AB}$$

$$\cot\theta = \frac{AB}{BC}$$

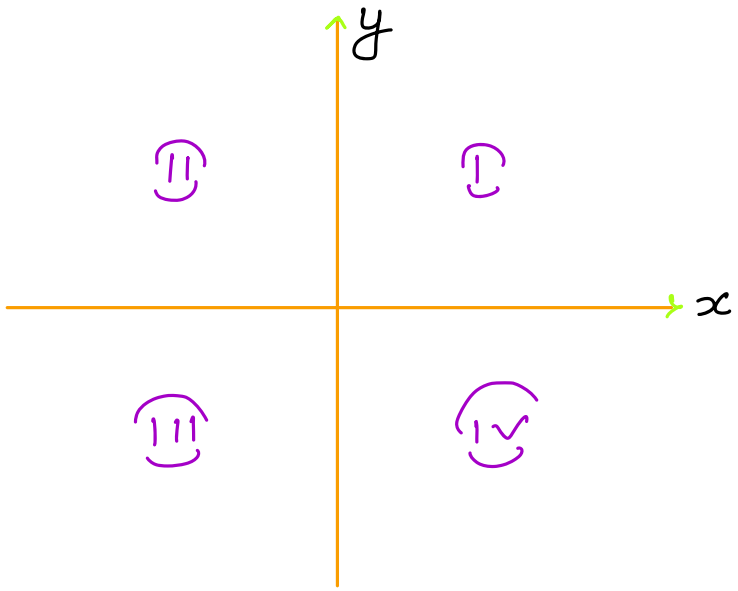
$$\sin^2\theta + \cos^2\theta = 1$$

$$\sec^2\theta - \tan^2\theta = 1$$

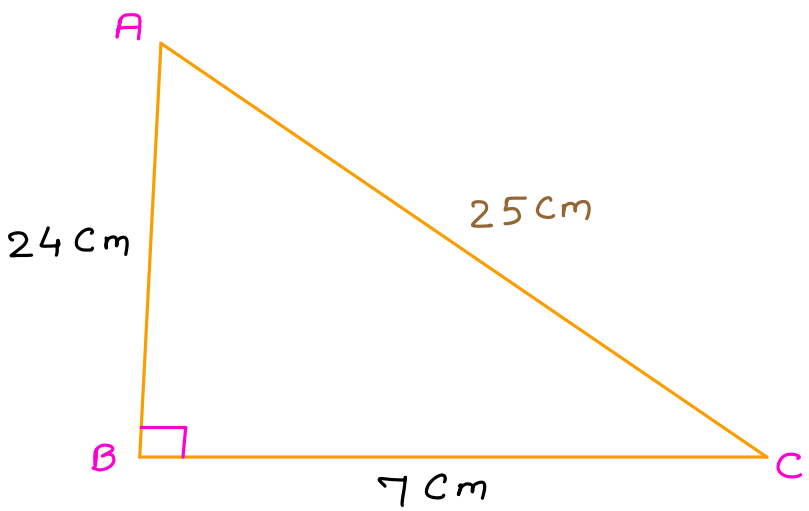
$$\operatorname{cosec}^2\theta - \cot^2\theta = 1$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$



Q-1

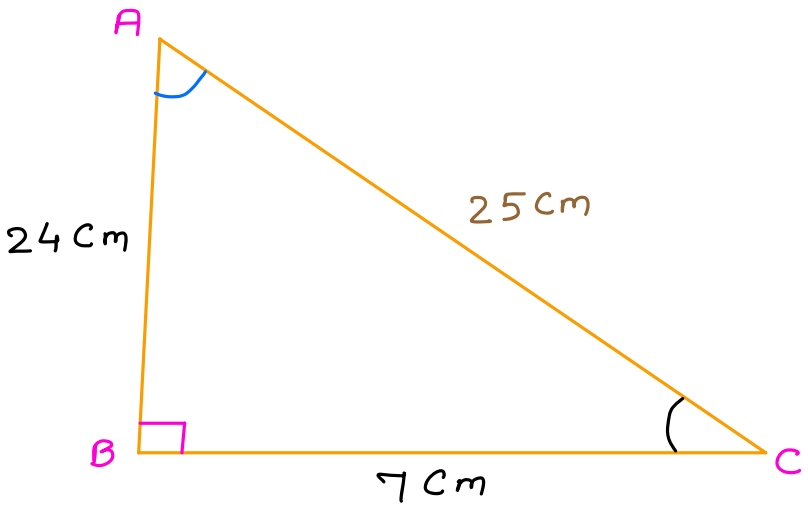


$$AC^2 = (24)^2 + (7)^2$$

$$AC^2 = 576 + 49$$

$$AC^2 = 625$$

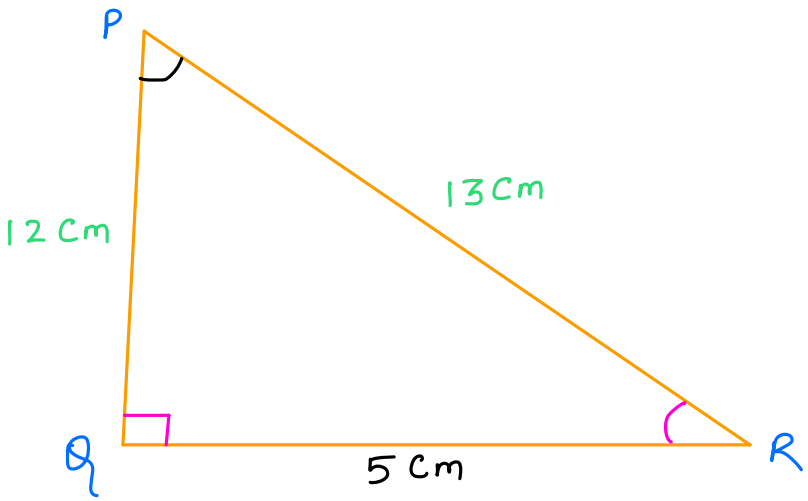
$$AC = \sqrt{625} = 25\text{cm}$$



$\sin A = \frac{7}{25}$ $\quad \cos A = \frac{24}{25}$

$\sin C = \frac{24}{25}$ $\quad \cos C = \frac{7}{25}$

Q-2



$PR^2 = PQ^2 + QR^2$

$169 = 144 + QR^2$

$QR^2 = 169 - 144 = 25$

$QR = 5$ Cm

$\tan P = \frac{5}{12}$

~~$\cot R = \frac{12}{5}$~~ $\quad \cot R = \frac{5}{12}$

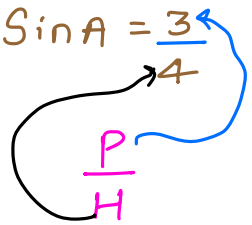
$\cot P = \frac{12}{5}$ $\quad \cot P = \frac{12}{5}$

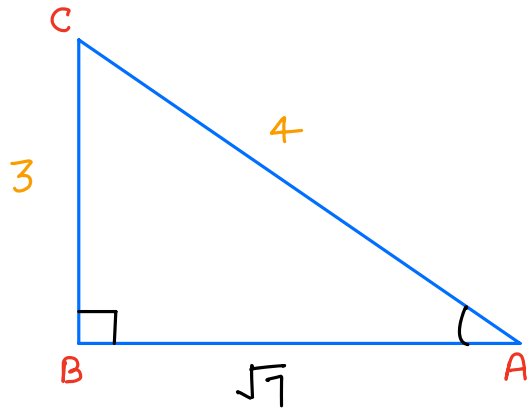
$\tan P - \cot R = \frac{5}{12} - \frac{5}{12} = 0$

Q-3

$\sin A = \frac{3}{4}$ $\cos A$ $\tan A$

$\frac{P}{H} = \frac{3}{4}$





$\cos A = \frac{\sqrt{7}}{4}$ ✓

$\tan A = \frac{3}{\sqrt{7}}$ ✓

$16 = 9 + AB^2$

$AB^2 = 7$

$AB = \sqrt{7}$

Method-2

$\sin A = \frac{3}{4}$

$\sin^2 A + \cos^2 A = 1$

$\left(\frac{3}{4}\right)^2 + \cos^2 A = 1$

$\frac{9}{16} + \cos^2 A = 1$

$\cos^2 A = 1 - \frac{9}{16}$

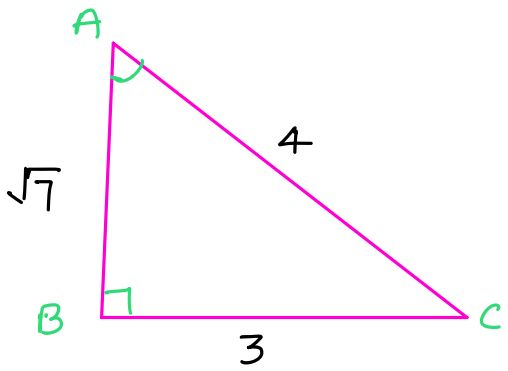
$\cos^2 A = \frac{16-9}{16} = \frac{7}{16}$

$\cos A = \frac{\sqrt{7}}{4}$ ✓

$$\tan A = \frac{\sin A}{\cos A}$$

$$\tan A = \frac{\frac{3}{4}}{\frac{\sqrt{7}}{4}}$$

$$\frac{3}{4} \times \frac{4}{\sqrt{7}} = \frac{3}{\sqrt{7}}$$



$$\sin A = \frac{3}{4} = \frac{P}{H}$$

$$16 = 9 + AB^2$$

$$AB^2 = 7$$

$$AB = \sqrt{7}$$

$$\cos A = \frac{\sqrt{7}}{4} \checkmark$$

$$\tan A = \frac{3}{\sqrt{7}} \checkmark$$

	0°	30°	45°	60°	90°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
cosec θ	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec θ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
cot θ	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\tan 0^\circ = \frac{\sin 0^\circ}{\cos 0^\circ} = \frac{0}{1} = 0$$

$$\tan 30^\circ = \frac{\sin 30^\circ}{\cos 30^\circ} = \frac{\frac{1}{\sqrt{2}}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{2}} \times \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\tan 60^\circ = \frac{\sin 60^\circ}{\cos 60^\circ} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{\sqrt{3}}{2} \times \frac{2}{1} = \sqrt{3}$$

$$\tan 45^\circ = \frac{\sin 45^\circ}{\cos 45^\circ} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = 1$$

$$\tan 90^\circ = \frac{\sin 90^\circ}{\cos 90^\circ} = \frac{1}{0} = \infty$$

Not defined

$$\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2} = \frac{3}{4} + \frac{1}{4} = 1$$

$$\frac{5\cos^2 60^\circ + 4\sec^2 30 - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$$

$$\frac{5\left[\frac{1}{2}\right]^2 + 4\left[\frac{2}{\sqrt{3}}\right]^2 - 1}{\frac{1}{4} + \frac{3}{4}} = \frac{\frac{5}{4} + 4 \times \frac{4}{3} - 1}{1}$$

$$\frac{5}{4} + \frac{16}{3} - 1 = \frac{15 + 64 - 12}{12} = \frac{67}{12}$$

Q-2

(iii)

$\sin 2A = 2\sin A$

Let $A = 0^\circ$

$\sin 0^\circ = 2\sin 0^\circ$

$0 = 0$

Q-3

IMPORTANT

$\tan(A+B) = \sqrt{3}$

$\tan(A-B) = \frac{1}{\sqrt{3}}$

$\tan(A+B) = \tan 60^\circ$

$\tan(A-B) = \tan 30^\circ$

$A+B = 60$

$A-B = 30$

$2A = 90$

$A = 45^\circ$

$45^\circ + B = 60^\circ$

$B = 15^\circ$

$A+B = \tan^{-1}\sqrt{3}$

$A-B = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$A+B = \tan^{-1}(\tan 60^\circ)$

$A-B = \tan^{-1}(\tan 30^\circ)$

$A+B = 60^\circ$

$A-B = 30^\circ$

Q-4

(i)

$\sin(A+B) = \sin A + \sin B$

Let $A = 0^\circ$ $B = 90^\circ$

$\sin(90) = \sin 0 + \sin 90$

$1 = 0 + 1$

$1 = 1$

Let $A = 30^\circ$ $B = 60^\circ$

$\sin(90) = \sin 30^\circ + \sin 60^\circ$

$1 = \frac{1}{2} + \frac{\sqrt{3}}{2}$

$1 \neq \frac{1+\sqrt{3}}{2}$

FALSE

$\sin\theta$ Increase as θ Increase
$\cos\theta$ Decrease as θ Increase

Q-7

$$\cot\theta = \frac{7}{8}$$

$$\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$$

$$(a+b)(a-b) = a^2 - b^2$$

METHOD :- 1

$$\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{1-\sin^2\theta}{1-\cos^2\theta} = \frac{\cos^2\theta}{\sin^2\theta} = \left[\frac{\cos\theta}{\sin\theta}\right]^2$$

$$= (\cot\theta)^2$$

$$= \frac{49}{64}$$

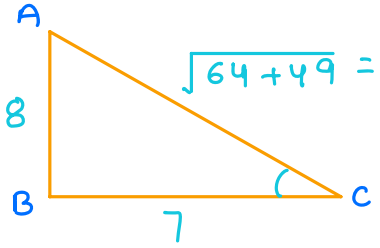
$$\sin^2\theta + \cos^2\theta = 1$$

$$1 - \sin^2\theta = \cos^2\theta$$

$$1 - \cos^2\theta = \sin^2\theta$$

METHOD :- 2

$$\frac{B}{P} = \frac{7}{8}$$



$$\sin\theta = \frac{8}{\sqrt{113}}$$

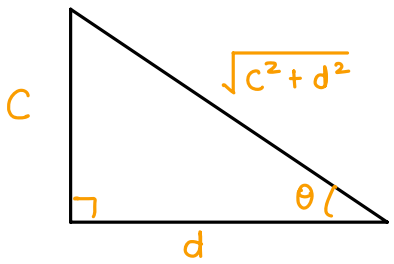
$$\cos\theta = \frac{7}{\sqrt{113}}$$

$$\frac{\left(1+\frac{8}{\sqrt{113}}\right)\left(1-\frac{8}{\sqrt{113}}\right)}{\left(1+\frac{7}{\sqrt{113}}\right)\left(1-\frac{7}{\sqrt{113}}\right)} = \frac{1-\frac{64}{113}}{1-\frac{49}{113}} = \frac{\frac{113-64}{113}}{\frac{113-49}{113}} = \frac{49}{64}$$

Q-7

RS AGR.

$$\sin\theta = \frac{c}{\sqrt{c^2+d^2}} = \frac{P}{h}$$



$$\cos\theta = \frac{b}{h} = \frac{d}{\sqrt{c^2+d^2}}$$

$$\tan\theta = \frac{P}{b} = \frac{c}{d}$$

$$h^2 = P^2 + b^2$$

$$c^2 + d^2 = c^2 + b^2$$

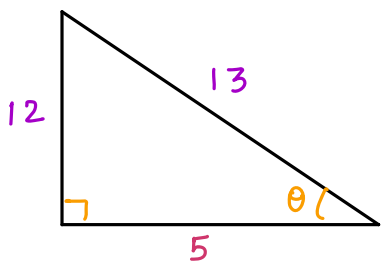
$$b^2 = d^2$$

$$b = \sqrt{d^2}$$

$$b = d$$

Q-12

$$\sin\theta = \frac{12}{13}$$



$$\cos\theta = \frac{5}{13}$$

$$169 = 144 + b^2$$

$$b^2 = 25$$

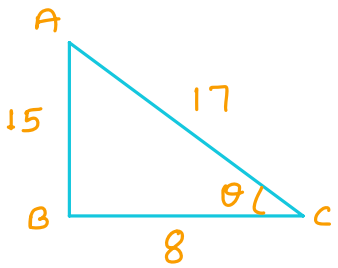
$$b = 5$$

SOLVE IT

Q-16

$$\sec\theta = \frac{17}{8} = \frac{h}{B}$$

$$\cos\theta = \frac{B}{H}$$



$$\sin\theta = \frac{15}{17} \qquad \cos\theta = \frac{8}{17}$$

$$\tan\theta = \frac{15}{8}$$

$$H^2 = P^2 + B^2$$

$$289 = P^2 + 64$$

$$P^2 = 225$$

$$P = 15$$

$$\frac{3 - 4 \times \frac{225}{289}}{4 \times \frac{64}{289} - 3} = \frac{3 - \frac{900}{289}}{\frac{256}{289} - 1}$$

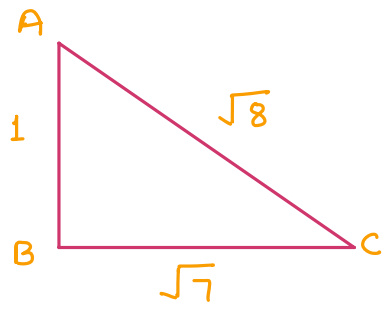
$$\frac{\frac{867 - 900}{289}}{\frac{256 - 867}{289}} = \frac{-\frac{33}{289} \times \frac{289}{-611}}{}$$

$$= \frac{33}{611}$$

$$\frac{3 - \tan^2\theta}{1 - 3\tan^2\theta} = \frac{3 - \frac{225}{64}}{1 - \frac{3 \times 225}{64}} = \frac{\frac{192 - 225}{64}}{1 - \frac{675}{64}}$$

$$\frac{\frac{-33}{64}}{\frac{64 - 675}{64}} = -\frac{33}{\cancel{64}} \times \frac{\cancel{64}}{-611} = \frac{33}{611}$$

Q-18



$\sin\theta = \frac{1}{2\sqrt{2}}$ $\quad \operatorname{Cosec}\theta = 2\sqrt{2}$

$\cos\theta = \frac{\sqrt{7}}{2\sqrt{2}}$ $\quad \sec\theta = \frac{2\sqrt{2}}{\sqrt{7}}$

$$\frac{8 + \frac{8}{7}}{8 - \frac{8}{7}} = \frac{\frac{56+8}{7}}{\frac{56-8}{7}} = \frac{\cancel{64}^{8}}{\cancel{7}} \times \frac{\cancel{7}}{\cancel{48}_{6}} = \frac{4}{3}$$

Proved

Q-25

$$x - y = \cot A + \cos A - \cot A + \cos A = 2\cos A$$

$$x + y = \cot A + \cos A + \cot A - \cos A = 2\cot A$$

$$\frac{4\cos^2 A}{4\cot^2 A} + \frac{4\cos^2 A}{4}$$

$$\frac{16\cos^2 A + 16\cos^2 A \cot^2 A}{16\cot^2 A}$$

$$\frac{16\cos^2 A + 16\cos^2 A \frac{\cos^2 A}{\sin^2 A}}{16\frac{\cos^2 A}{\sin^2 A}}$$

$\tan A = \frac{\sin A}{\cos A}$

$\cot A = \frac{\cos A}{\sin A}$

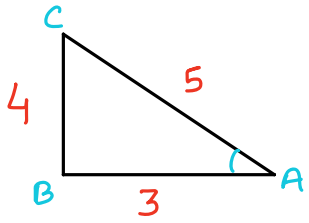
$$\frac{16\cos^2 A \sin^2 A + 16\cos^2 A \cos^2 A}{\cancel{\sin^2 A}} \Big/ 16\frac{\cos^2 A}{\cancel{\sin^2 A}}$$

$$= \frac{\cancel{16\cos^2 A}\,(\sin^2 A + \cos^2 A)}{\cancel{16\cos^2 A}}$$

$= 1$ Ans

Q-20

$\tan A = \frac{4}{3}$



$H^2 = P^2 + B^2$

$H^2 = 16 + 9 = 25$

$H = 5$

$\sec A = \frac{5}{3}$

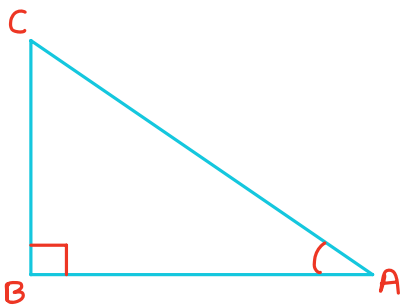
$\operatorname{Cosec} A = \frac{5}{4}$

$$\sqrt{\frac{\frac{5}{3} - \frac{5}{4}}{\frac{5}{3} + \frac{5}{4}}} = \sqrt{\frac{\frac{20-15}{12}}{\frac{20+15}{12}}}$$

$$\sqrt{\frac{5}{12} \times \frac{12}{35_7}} = \frac{1}{\sqrt{7}}$$

Proved

Q-22



$\tan A = 1$

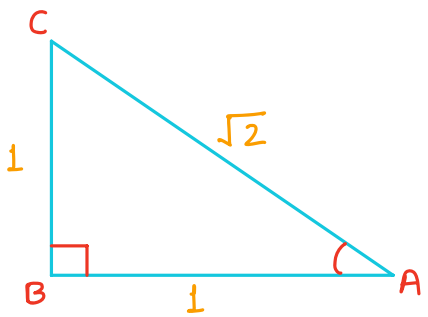
$\tan 45^\circ = 1$

$A = 45^\circ$

$2 \sin 45^\circ \cos 45^\circ$

$2 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{2}{2} = 1$

OR



$\tan A = 1 = \frac{P}{B}$

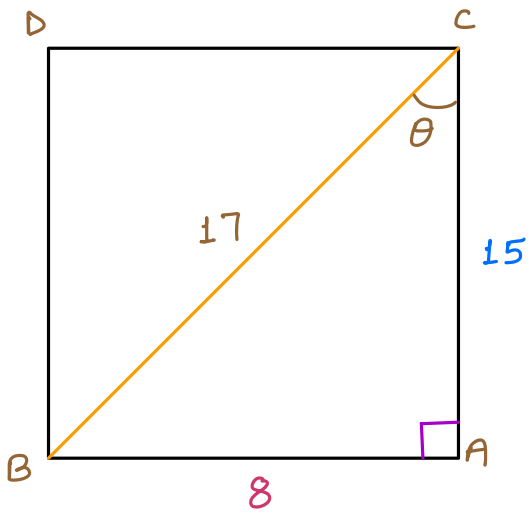
$H^2 = 1 + 1 = 2$

$H = \sqrt{2}$

$\sin A = \frac{1}{\sqrt{2}}$ $\cos A = \frac{1}{\sqrt{2}}$

$2 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{2}{2} = 1$

Q-21



$\sin\theta = \frac{8}{17} = \frac{P}{H}$

$H^2 = P^2 + B^2$

$289 = 64 + B^2$

$B^2 = 225$

$B = 15$

$A = 15 \times 8 = 120\ cm^2$

$P = 2(l+b)$

$P = 2(15+8) = 46\ cm$

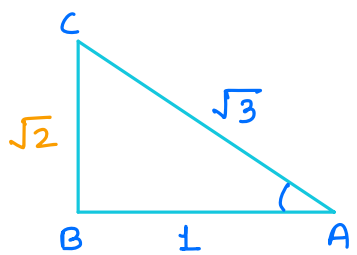
Q-28

$\sqrt{3}\tan\theta = 3\sin\theta$

$\sqrt{3}\frac{\sin\theta}{\cos\theta} = 3\sin\theta$

$\sqrt{3}\sin\theta = 3\sin\theta\cos\theta$

$\cos\theta = \frac{\sqrt{3}\cancel{\sin\theta}}{3\cancel{\sin\theta}} = \frac{1}{\sqrt{3}}$



$3 = P^2 + 1$

$P^2 = 2$

$P = \sqrt{2}$

$\sin\theta = \frac{\sqrt{2}}{\sqrt{3}}$

Q-27

$$\cot A + \frac{1}{\cot A} = 2$$

Squaring Both Sides

$$\left(\cot A + \frac{1}{\cot A}\right)^2 = 4$$

$$\cot^2 A + \frac{1}{\cot^2 A} + 2 \cot A \times \frac{1}{\cot A} = 4$$

$$\cot^2 A + \frac{1}{\cot^2 A} = 2$$

Q-31

LOGICAL THINKING

BEST QUESTION

$$1 + \sin^2\theta = 3 \sin\theta \cos\theta$$

$$\sin^2\theta = 3 \sin\theta \cos\theta - 1$$

Dividing each Side by $\cos^2\theta$

DONOT ATTEMPT DIRECTLY IN EXAM

$$\frac{\sin^2\theta}{\cos^2\theta} = \frac{3 \sin\theta \cos\theta}{\cos^2\theta} - \frac{1}{\cos^2\theta}$$

$\sec^2\theta - \tan^2\theta = 1$

$$\tan^2\theta = 3\tan\theta - \sec^2\theta$$

$$\tan^2\theta + \sec^2\theta = 3\tan\theta$$

$$\tan^2\theta + 1 + \tan^2\theta = 3\tan\theta$$

$$2\tan^2\theta - 3\tan\theta + 1 = 0 \longrightarrow \text{Quadratic Eq}^n$$

$$2\tan^2\theta - (2+1)\tan\theta + 1 = 0$$

$$(2\tan\theta - 1)(\tan\theta - 1) = 0$$

$$2\tan^2\theta - 2\tan\theta - \tan\theta + 1 = 0$$

$$2\tan\theta(\tan\theta - 1) - 1(\tan\theta - 1) = 0$$

$\tan\theta = 1$

$\tan\theta = 1/2$

Q-15

(iii) $\tan(60-30) = \dfrac{\tan 60 - \tan 30}{1 + \tan 60 \tan 30}$

$$\tan 30^\circ = \frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1 + \sqrt{3} \times \frac{1}{\sqrt{3}}} = \frac{\frac{3-1}{\sqrt{3}}}{2} = \frac{2}{\sqrt{3}} \times \frac{1}{2} = \frac{1}{\sqrt{3}}$$

$$\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

Q-16

$$\tan(A+B) = \frac{\frac{1}{3} + \frac{1}{2}}{1 - \frac{1}{3} \times \frac{1}{2}}$$

$\sin(A+B) \neq \sin A + \sin B$

$$= \frac{\frac{2+3}{6}}{1 - \frac{1}{6}} = \frac{\frac{5}{6}}{\frac{6-1}{6}} = \frac{\frac{5}{6}}{\frac{5}{6}} = 1$$

$\tan(A+B) = 1$

$\tan(A+B) = \tan 45^\circ$

$A+B = 45^\circ$

Q-17

$$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$$

$A = 30^\circ$

$$\tan 60^\circ = \frac{2 \times \frac{1}{\sqrt{3}}}{1-\left(\frac{1}{3}\right)} = \frac{\frac{2}{\sqrt{3}}}{\frac{3-1}{3}} = \frac{2}{\sqrt{3}} \times \frac{3}{2} = \sqrt{3}$$

$2\sqrt{3} + \sqrt{3}$

$\sqrt{3}(2+1) = 3\sqrt{3}$

$\sqrt{3} \times \sqrt{3} = \sqrt{9} = 3$

$2\sqrt{3} + 1$

Q-21

$$\tan(x+30^\circ) = 1$$

$$\tan(x+30) = \tan 45$$

$$x + 30 = 45$$

$$x = 15^\circ$$

Q-22

$$\frac{\cot\theta - 1}{\cot\theta + 1} = \frac{1-\sqrt{3}}{1+\sqrt{3}}$$

$$(1+\sqrt{3})(\cot\theta - 1) = (1-\sqrt{3})(\cot\theta + 1)$$

$$\cot\theta - 1 + \sqrt{3}\cot\theta - \sqrt{3} = \cot\theta + 1 - \sqrt{3}\cot\theta - \sqrt{3}$$

$$\sqrt{3}\cot\theta + \sqrt{3}\cot\theta = \cot\theta + 1 - \sqrt{3} + \sqrt{3} + 1 - \cot\theta$$

$$2\sqrt{3}\cot\theta = 2$$

$$\cot\theta = \frac{2}{2\sqrt{3}}$$

$$\cot\theta = \frac{1}{\sqrt{3}}$$

$$\boxed{\theta = 60^\circ}$$

(OR)

Q-22

$$\frac{\cot\theta - 1}{\cot\theta + 1} = \frac{1-\sqrt{3}}{1+\sqrt{3}}$$

Componendo & Dividendo

$$\boxed{\frac{a-b}{a+b} = \frac{a-b+a+b}{a-b-a-b}}$$

$$\frac{\cot\theta - 1 + \cot\theta + 1}{\cot\theta - 1 - \cot\theta - 1} = \frac{1-\sqrt{3}+1+\sqrt{3}}{1-\sqrt{3}-1-\sqrt{3}}$$

$$\frac{2\cot\theta}{-2} = \frac{2}{-2\sqrt{3}}$$

$$\cot\theta = \frac{1}{\sqrt{3}}$$

$$\theta = 60^\circ$$