1. $\int_1^2 \left(4x^3 - x\right) dx =$ $\int_1^2 (4x^3 - x)\, dx$

(A) $\frac{27}{2}$ (B) 27 (C) 36 (D) 57

$\Rightarrow \left[x^4 - \frac{x^2}{2}\right]_1^2 \Rightarrow 16 - 2 - 1 + \frac{1}{2}$

$\Rightarrow 16 - 5/2 \Rightarrow 27/2$

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2. Let f be the function defined by $f(x) = x^3 - 3x^2 - 9x + 11$. At which of the following values of x does f attain a local minimum?

(A) 3 (B) 1 (C) -1 (D) -3

$f(x) = x^3 - 3x^2 - 9x + 11$

$f'(x) = 3x^2 - 6x - 9$

for minima & maxima $f'(x) = 0$

So, $3x^2 - 6x - 9 = 0$ $x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = 3, -1$

at $x = 3$ f will give minima

3. $\frac{d}{dx}\left(2(\sin\sqrt{x})^2\right) =$

(A) $4\cos\left(\frac{1}{2\sqrt{x}}\right)$ (B) $4\sin\sqrt{x}\cos\sqrt{x}$ (C) $\frac{2\sin\sqrt{x}}{\sqrt{x}}$ (D) $\frac{2\sin\sqrt{x}\cos\sqrt{x}}{\sqrt{x}}$

$4\sin(\sqrt{x})\cdot\cos(\sqrt{x})\cdot\frac{1}{2\sqrt{x}}$

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4. The position of a particle is given by the parametric equations $x(t) = \ln\left(t^2+1\right)$ and $y(t) = e^{3-t}$. What is the velocity vector at time $t = 1$?

(A) $\langle 1, e^2\rangle$ (B) $\langle 1, -e^2\rangle$ (C) $\left\langle \frac{1}{2}, e^2\right\rangle$ (D) $\left\langle \frac{1}{2}, -e^2\right\rangle$

$V_x = \frac{dx}{dt}$ $V_y = \frac{dy}{dt}$

$V_x = \frac{1}{t^2+1}(2t)$ $V_y = e^{3-t}(-1)$

$V_x\Big|_{t=1} = 1$ $V_y\Big|_{t=1} = -e^2$

$(V_x, V_y) = (1, -e^2)$

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5. $\sum_{n=1}^{\infty} \frac{e^n}{\pi^n}$ is

(A) $\frac{\pi}{\pi - e}$ (B) $\frac{e}{\pi - e}$ (C) $\frac{e}{\pi \ln\left(\frac{\pi}{e}\right)}$ (D) divergent

$$S_n = \frac{a}{1-r} = \frac{e/\pi}{1 - e/\pi} \Rightarrow \frac{e}{\pi - e}$$

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x	$f(x)$	$f'(x)$	$f''(x)$	$g(x)$	$g'(x)$	$g''(x)$
2	4	-3	3	-2	5	1

6. The table above gives values of the twice-differentiable functions f and g and their derivatives at $x = 2$. If h is the function defined by $h(x) = \frac{f'(x)}{g(x)}$, what is the value of $h'(2)$?

(A) $\frac{9}{4}$ (B) $\frac{3}{5}$ (C) $-\frac{3}{2}$ (D) $-\frac{21}{4}$

$$h'(x) = \frac{g(x)f''(x) - f'(x)g'(x)}{(g(x))^2}$$

$$h'(2) = \frac{g(2)f''(2) - f'(2)g'(2)}{(g(2))^2}$$

$$\frac{-2 \times 3 - (-3)(5)}{4}$$

$h'(2) \Rightarrow 9/4$

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7. Which of the following is the Maclaurin series for $x\cos\left(x^2\right)$?

(A) $x - \dfrac{x^5}{2!} + \dfrac{x^9}{4!} - \dfrac{x^{13}}{6!} + \cdots$

(B) $x - \dfrac{x^3}{2!} + \dfrac{x^5}{4!} - \dfrac{x^7}{6!} + \cdots$

(C) $x^3 - \dfrac{x^7}{3!} + \dfrac{x^{11}}{5!} - \dfrac{x^{15}}{7!} + \cdots$

(D) $x^3 - \dfrac{x^5}{3!} + \dfrac{x^7}{5!} - \dfrac{x^9}{7!} + \cdots$

$\cos(x^2) = 1 - \dfrac{x^4}{2!} + \dfrac{x^8}{4!} - \dfrac{x^{12}}{6!} +$

$x\cos(x^2) = x - \dfrac{x^5}{2!} + \dfrac{x^9}{4!} - \dfrac{x^{13}}{6!} +$

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8. $\displaystyle\lim_{x\to\infty} \frac{10 - 6x^2}{5 + 3e^x}$ is

(A) -2 (B) 0 (C) 2 (D) nonexistent

LH $\rightarrow \dfrac{-12x}{3e^x} \rightarrow \dfrac{-12}{3e^x}$ $\qquad \dfrac{1}{e^x} \rightarrow 0$

$\dfrac{-12}{3} \times 0 = 0$

9. The function f is not differentiable at $x = 5$. Which of the following statements must be true?

(A) f is not continuous at $x = 5$.

(B) $\lim_{x \to 5} f(x)$ does not exist.

(C) $\lim_{x \to 5} \dfrac{f(x) - f(5)}{x - 5}$ does not exist.

(D) $\int_0^5 f(x)\,dx$ does not exist.

$x=5$ is Not Diff.

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10. The second derivative of a function f is given by $f''(x) = x(x-3)^5(x-10)^2$. At which of the following values of x does the graph of f have a point of inflection?

(A) 3 only

(B) 0 and 3 only

(C) 3 and 10 only

(D) 0, 3, and 10

+ − + +

0 3 10

Sign Changes accross 0 & 3

11. $\displaystyle\int_0^{\pi} \frac{e^x - 1}{e^x - x}\,dx =$

(A) $e^{\pi} - \pi - 1$ (B) $\ln(e^{\pi} - \pi) - 1$ (C) $\pi - \ln \pi$ (D) $\ln(e^{\pi} - \pi)$

$e^x - x = t$

$(e^x - 1)\,dx = dt$

$\displaystyle\int_1^{e^\pi - \pi} \frac{1}{t}\,dt \Rightarrow \Big[\ln(t)\Big]_1^{e^\pi - \pi}$

$\ln(e^\pi - \pi)$

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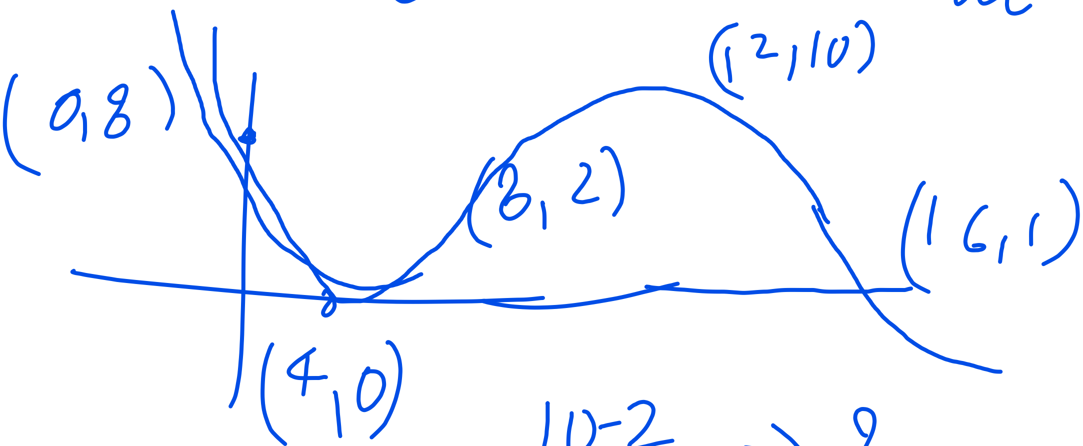
x	0	4	8	12	16
$f(x)$	8	0	2	10	1

12. The table above gives selected values for the differentiable function f. In which of the following intervals must there be a number c such that $f'(c) = 2$?

(A) (0, 4) (B) (4, 8) (C) (8, 12) (D) (12, 16)

at point c function Tangents slope will be 2



$\dfrac{10-2}{12-8} \Rightarrow \dfrac{8}{4} = 2$

13. Let $y = f(x)$ be the solution to the differential equation $\dfrac{dy}{dx} = x + 2y$ with initial condition $f(0) = 2$. What is the approximation for $f(-0.4)$ obtained by using Euler's method with two steps of equal length starting at $x = 0$?

(A) 0.76 (B) 1.20 (C) 1.29 (D) 3.96

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14. What is the slope of the line tangent to the curve $\sqrt{x} + \sqrt{y} = 2$ at the point $\left(\dfrac{9}{4}, \dfrac{1}{4}\right)$?

(A) -3 (B) $-\dfrac{1}{3}$ (C) 1 (D) $\dfrac{4}{3}$

$$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} \Rightarrow -\frac{\sqrt{y}}{\sqrt{x}} \Rightarrow \frac{-\frac{1}{2}}{3/2} \Rightarrow -\frac{1}{3}$$

15. $\int_1^{\infty} \frac{6}{(x+3)^{3/2}}\, dx$ is

(A) $\frac{3}{4}$ (B) 3 (C) 6 (D) divergent

$x+3=t$

$x=1 \quad t=4$

$\int_4^{\infty} \frac{6}{t^{3/2}}\, dt \Rightarrow \left[-12 \frac{1}{t^{1/2}}\right]_4^{\infty}$

$\Rightarrow \left[\frac{-12}{\infty} + \frac{12}{2}\right] \Rightarrow 6$

16. If $\frac{dy}{dx} = 2 - y$, and if $y = 1$ when $x = 1$, then $y =$

(A) $2 - e^{x-1}$ (B) $2 - e^{1-x}$ (C) $2 - e^{-x}$ (D) $2 + e^{-x}$

$-\ln(2-y) = x + C$

$x = 1, y = 1$

$-\ln(1) = 1 + C$

$\boxed{C = -1}$

$-\ln(2-y) = x - 1$

$2 - y = e^{-x+1}$

$y = 2 - e^{-x+1}$

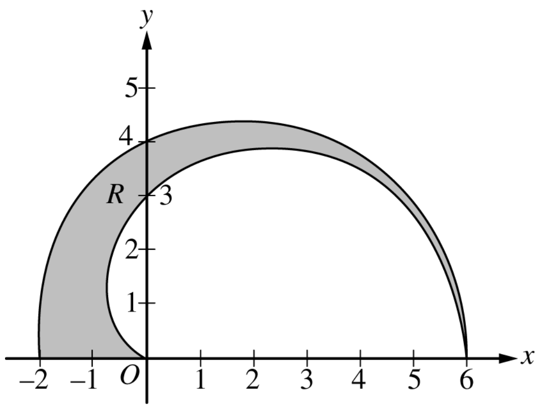
17. Which of the following series converges?

(A) $\sum_{n=1}^{\infty} (-1)^n \left(\frac{1-n}{n}\right)$ $= 1 \neq 0$

(B) $\sum_{n=1}^{\infty} (-1)^n \left(\frac{n+1}{2n}\right)$ $= \frac{1}{2} \neq 0$

(C) $\sum_{n=1}^{\infty} (-1)^n \left(\frac{n^2}{3\sqrt{n}}\right)$ $\to \infty \neq 0$

(D) $\sum_{n=1}^{\infty} (-1)^n \left(\frac{2\sqrt{n}}{n}\right)$ $\to 0$



18. Let R be the region in the first and second quadrants between the graphs of the polar curves $f(\theta) = 3 + 3\cos\theta$ and $g(\theta) = 4 + 2\cos\theta$, as shaded in the figure above. Which of the following integral expressions gives the area of R ?

(A) $\int_{-2}^{6} (g(\theta) - f(\theta))\, d\theta$

(B) $\int_{0}^{\pi} (g(\theta) - f(\theta))\, d\theta$

(C) $\frac{1}{2}\int_{0}^{\pi} (g(\theta) - f(\theta))^2\, d\theta$

(D) $\frac{1}{2}\int_{0}^{\pi} \left((g(\theta))^2 - (f(\theta))^2\right) d\theta$

19. Which of the following statements about the series $\sum_{n=1}^{\infty} \frac{(-1)^n n}{n^2 + 1}$ is true?

(A) The series can be shown to diverge by comparison with $\sum_{n=1}^{\infty} \frac{1}{n}$.

(B) The series can be shown to diverge by limit comparison with $\sum_{n=1}^{\infty} \frac{1}{n}$.

(C) The series can be shown to converge by comparison with $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

(D) The series can be shown to converge by the alternating series test.

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20. If $\int_1^4 f(x)\,dx = 8$ and $\int_1^4 g(x)\,dx = -2$, which of the following cannot be determined from the information given?

(A) $\int_4^1 g(x)\,dx$

(B) $\int_1^4 3f(x)\,dx$

(C) $\int_1^4 3f(x)\,g(x)\,dx$

(D) $\int_1^4 (3f(x) + g(x))\,dx$

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