10th CBSE Math

8. Introduction to Trigonometry

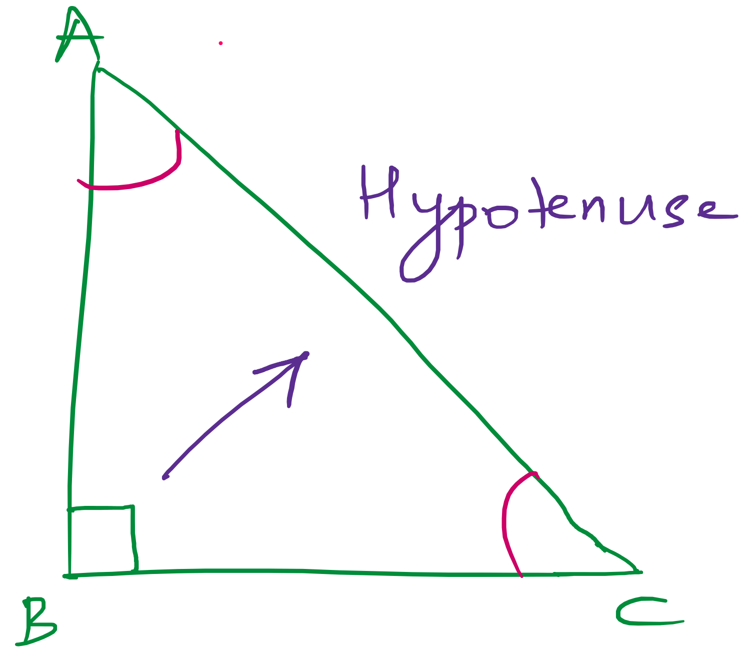
One Shot Revision

Trigonometry

It is used to study the relationship b/w the sides & angles of a triangle.

AB, BC, AC ⟶ Sides

∠A, ∠B, ∠C ⟶ Angles

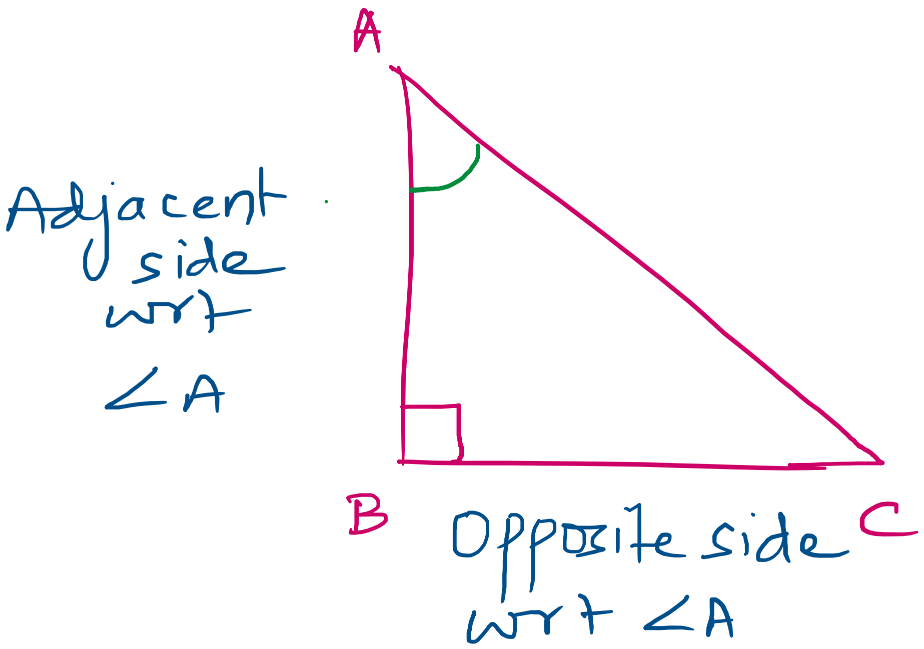


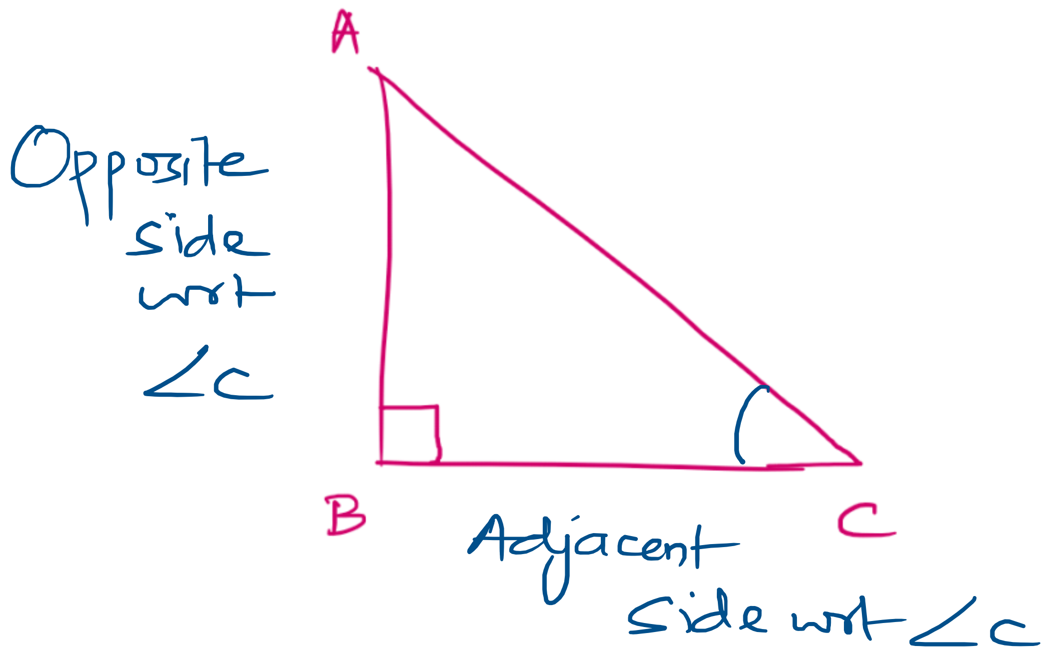
Pythagoras Theorem

$$AC^2 = AB^2 + BC^2$$

Trigonometric Ratios

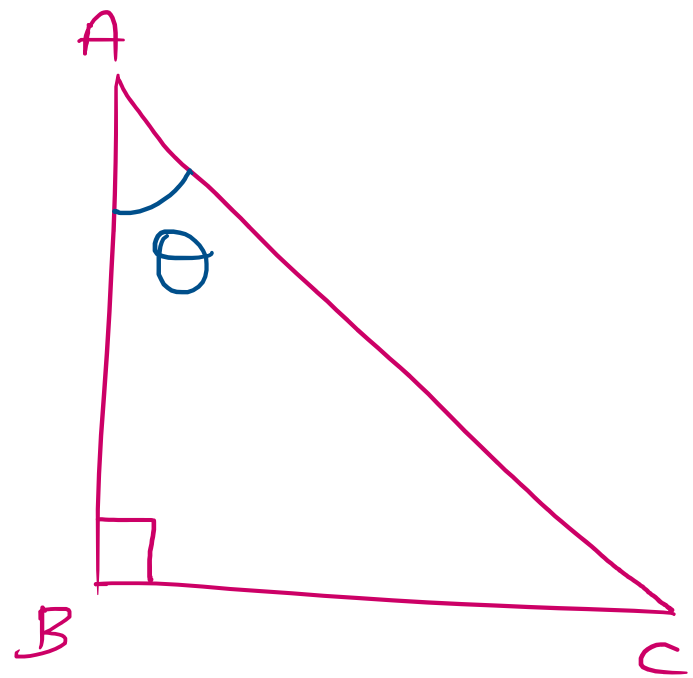
sine	secant
cosine	cosecant
tangent	cotangent


A
Adjacent side wrt ∠A
B
Opposite side wrt ∠A
C


A
Opposite side wrt ∠C
B
Adjacent side wrt ∠C
C

$$\text{Sine of } \angle A = \text{sine of } \theta = \sin\theta = \frac{\text{Opposite side wrt } \angle A}{\text{Hypotenuse}} = \frac{BC}{AC}$$

$$\sin\theta = \frac{BC}{AC}$$



$$\text{Cosine of } \angle A = \text{cosine of } \theta = \cos\theta = \frac{\text{Adjacent}}{\text{Hyp.}} = \frac{AB}{AC}$$

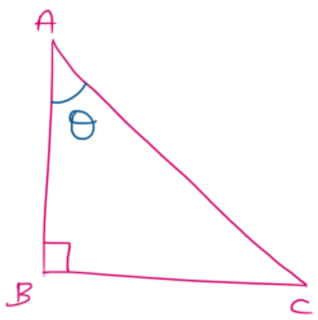
$$\cos\theta = \frac{AB}{AC}$$

$$\text{tangent of } \angle A = \text{tangent of } \theta = \tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}$$

$$\tan\theta = \frac{BC}{AB}$$

Sine of ∠A = Sine of θ = $\sin\theta = \frac{\text{Opposite side wrt } \angle A}{\text{Hypotenuse}} = \frac{BC}{AC}$

$$\sin\theta = \frac{BC}{AC}$$



Cosine of ∠A = Cosine of θ = $\cos\theta = \frac{\text{Adjacent}}{\text{Hyp.}}$

$$= \frac{AB}{AC}$$

$$\cos\theta = \frac{AB}{AC}$$

tangent of ∠A = tangent of θ = $\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}$

$$\tan\theta = \frac{BC}{AB}$$

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{AC}{BC}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{AC}{AB}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{AB}{BC} = \frac{\cos\theta}{\sin\theta}$$

$$\tan\theta = \frac{BC/AC}{AB/AC} = \frac{\sin\theta}{\cos\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$\frac{2}{3}$

$\frac{2 \times 5}{3 \times 5} = \frac{10}{15}$

$\frac{2 \times 7}{3 \times 7} = \frac{14}{21}$

Equivalent Fractions

$\frac{AB}{BC} = \frac{2}{3}$

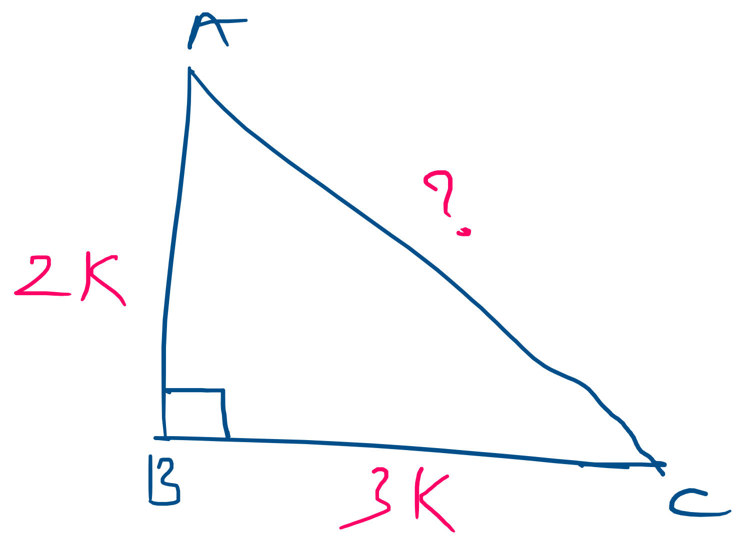
$AB = 2$

$BC = 3$

$\frac{AB}{BC} = \frac{2K}{3K}$

$AB = 2K$

$BC = 3K$



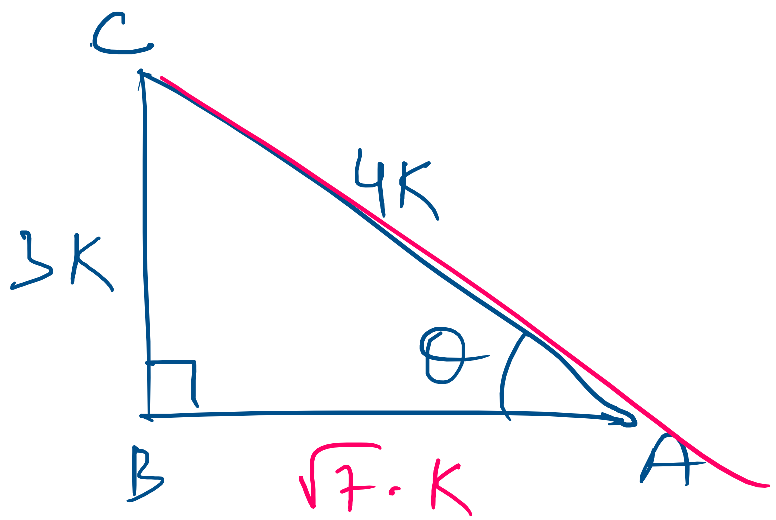
$\frac{10/5}{15/5} = \frac{2}{3}$

$\frac{14/7}{21/7} = \frac{2}{3}$

If ~~sin A~~ tan A $= \frac{3}{4}$, calculate cos A and tan A.

$$\cos A = \frac{AB}{AC} = \frac{\sqrt{7}\cdot \cancel{K}}{4\cancel{K}}$$

$$\cos A = \frac{\sqrt{7}}{4}$$



$$\sin A = \frac{3}{4}$$

$$\sin A = \frac{BC}{AC} = \frac{3K}{4K}$$

$$BC = 3K, \quad AC = 4K$$

where K is any positive integer.

$$\tan A = \frac{BC}{AB} = \frac{3\cancel{K}}{\sqrt{7}\cancel{K}} = \frac{3}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{3\sqrt{7}}{7}$$

$$4K^2 = 4K^2$$

Applying P.T.

$$AC^2 = AB^2 + BC^2$$

$$(4K)^2 = AB^2 + (3K)^2$$

$$16K^2 - 9K^2 = AB^2$$

$$AB^2 = 16K^2 - 9K^2$$

$$a = b$$

$$b = a$$

$$(4K)^2 = 16K^2$$

$$-1^2 = -1$$

$$(-1)^2 = 1$$

$$AB^2 = 16K^2 - 9K^2$$

$$(-1)^2 = 1$$

$$AB^2 = 16K^2 - 9K^2$$

$$AB^2 = 7K^2$$

$$AB = \pm\sqrt{7K^2}$$

$$AB = \pm\sqrt{7} \cdot \sqrt{K^2}$$

$$AB = \pm\sqrt{7} \cdot K$$

$AB = \sqrt{7} \cdot K$, $AB = -\sqrt{7} \cdot K$

$\because$ length of a side of a triangle cannot be negative.

$\therefore AB = \sqrt{7} \cdot K$

$$x^2 = 9$$

$$x = \pm\sqrt{9}$$

$$x = \pm 3$$

$x = 3$, $x = -3$

$$3^2 = 9$$

$$(-3)^2 = 9$$

$$\frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$\frac{5}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{5(2-\sqrt{3})}{2^2-(\sqrt{3})^2} = \frac{5(2-\sqrt{3})}{4-3} = \frac{5(2-\sqrt{3})}{1}$$

$$= 5(2-\sqrt{3})$$

$$= 10-5\sqrt{3}$$

$$(a+b)(a-b) = a^2-b^2$$

$$\frac{p}{a+\sqrt{b}} \times \frac{a-\sqrt{b}}{a-\sqrt{b}} =$$

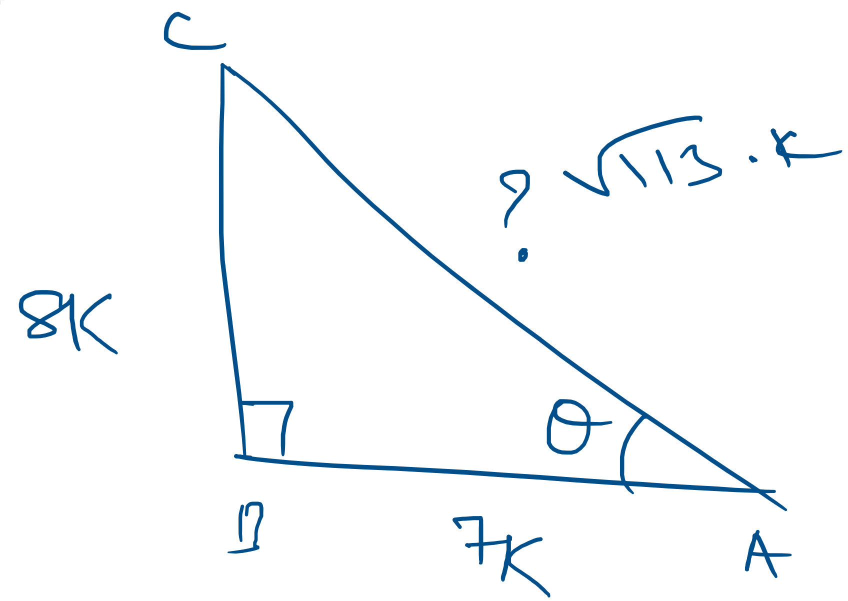
$$\frac{7}{2+3\sqrt{2}} \times \frac{2-3\sqrt{2}}{2-2\sqrt{2}}$$

$$\frac{p}{a-\sqrt{b}} \times \frac{a+\sqrt{b}}{a+\sqrt{b}} =$$

$$\frac{8}{4-6\sqrt{7}} \times \frac{4+6\sqrt{7}}{4+6\sqrt{7}}$$

If $\cot\theta = \frac{7}{8}$, evaluate : (i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$, (ii) $\cot^2\theta$

$$\cot\theta = \frac{7}{8}$$



$$\cot\theta = \frac{AB}{BC} = \frac{7K}{8K}$$

$$AB = 7K \quad \& \quad BC = 8K$$

$$\sin\theta = \frac{8K}{\sqrt{113}\cdot K} = \frac{8}{\sqrt{113}}$$

$$\frac{\left(1+\frac{8}{\sqrt{113}}\right)\left(1-\frac{8}{\sqrt{113}}\right)}{\left(1+\frac{7}{\sqrt{113}}\right)\left(1-\frac{7}{\sqrt{113}}\right)} = \frac{1^2-\left(\frac{8}{\sqrt{113}}\right)^2}{1^2-\left(\frac{7}{\sqrt{113}}\right)^2}$$

$$\frac{1^2 - \left(\frac{8}{\sqrt{113}}\right)^2}{1^2 - \left(\frac{7}{\sqrt{113}}\right)^2} = \frac{1 - \frac{8^2}{(\sqrt{113})^2}}{1 - \frac{7^2}{(\sqrt{113})^2}} = \frac{\frac{1}{1} - \frac{64}{113}}{\frac{1}{1} - \frac{49}{113}}$$

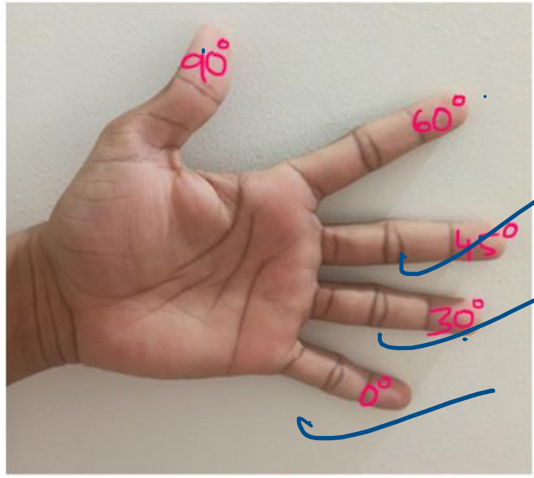
$$= \frac{\left(\frac{113-64}{113}\right)}{\left(\frac{113-49}{113}\right)} = \frac{\frac{49}{113}}{\frac{64}{113}} \qquad \left(\frac{a}{b}\right)^2 = \frac{a^2}{b^2}$$

$$= \frac{49}{113} \div \frac{64}{113} = \frac{49}{\cancel{113}} \times \frac{\cancel{113}}{64}$$

$$= \frac{49}{64}$$

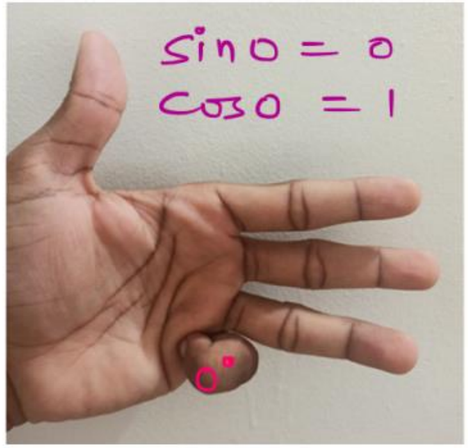
Trigonometric Ratios of some specific angles

$0^\circ, 30^\circ, 45^\circ, 60^\circ, 90^\circ$



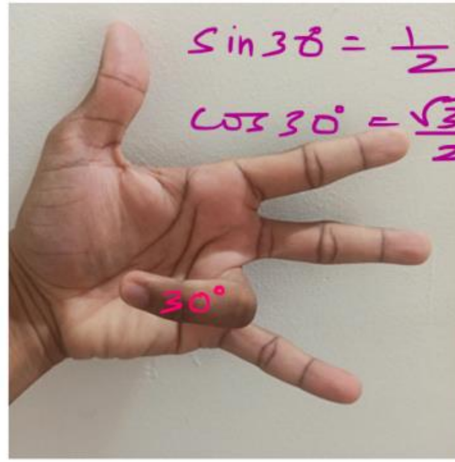
$$\sin\theta = \frac{\sqrt{\text{no. of fingers below}}}{2}$$

$$\cos\theta = \frac{\sqrt{\text{no. of fingers above}}}{2}$$



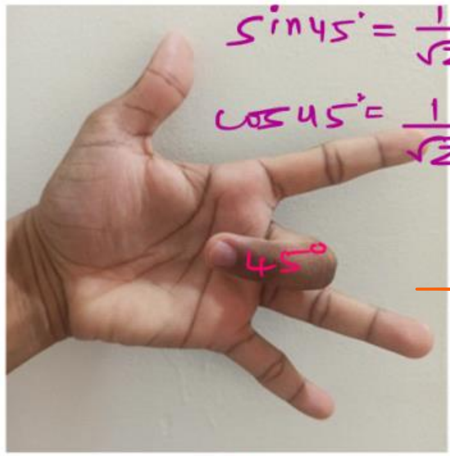
$$\sin 0° = \frac{\sqrt{0}}{2} = \frac{0}{2} = 0$$

$$\cos 0° = \frac{\sqrt{4}}{2} = \frac{2}{2} = 1$$



$$\sin 30° = \frac{\sqrt{1}}{2} = \frac{1}{2}$$

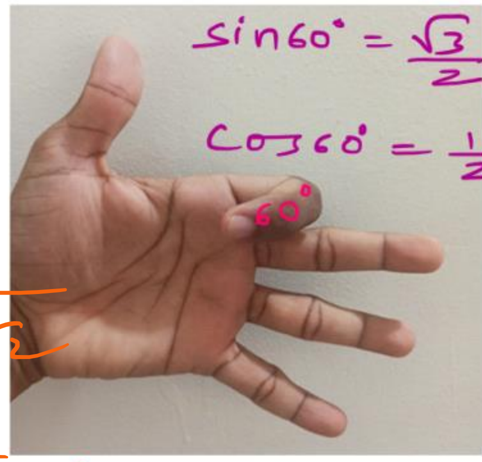
$$\cos 30° = \frac{\sqrt{3}}{2}$$



$$\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{\sqrt{2}\times\sqrt{2}} = \frac{1}{\sqrt{2}}$$

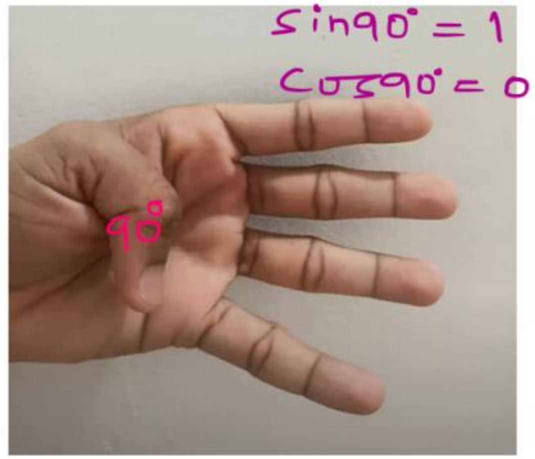
$$\sin 45° = \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{\sqrt{2}\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\cos 45° = \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{\sqrt{2}\sqrt{2}} = \frac{1}{\sqrt{2}}$$



$$\sin 60° = \frac{\sqrt{3}}{2}$$

$$\cos 60° = \frac{\sqrt{1}}{2} = \frac{1}{2}$$



$$\sin 90° = \frac{\sqrt{4}}{2} = \frac{2}{2} = 1$$

$$\cos 90° = \frac{\sqrt{0}}{2} = 0$$

$\angle$ A	0°	30°	45°	60°	90°
sin A	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos A	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan A	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
cosec A	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec A	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
cot A	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

$$\frac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$$

$$\frac{\frac{1}{2} + \frac{1}{1} - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{1}{2} + \frac{1}{1}}$$

$\sin^2 30^\circ + \tan^2 45^\circ - \operatorname{cosec}^2 60^\circ$

$(\sin 30^\circ)^2 \quad (\tan 45^\circ)^2 \quad (\operatorname{cosec} 60^\circ)^2$

$\sin^2\theta = (\sin\theta)^2$

∠ A	0°	30°	45°	60°	90°
sin A	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos A	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan A	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
cosec A	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec A	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
cot A	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

$$\frac{\frac{1}{2}+\frac{1}{1}-\frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}}+\frac{1}{2}+\frac{1}{1}} = \frac{\left(\frac{\sqrt{3}+2\sqrt{3}-4}{2\sqrt{3}}\right)}{\left(\frac{4+\sqrt{3}+2\sqrt{3}}{2\sqrt{3}}\right)}$$

$$= \frac{3\sqrt{3}-4}{2\sqrt{3}} \div \frac{3\sqrt{3}+4}{2\sqrt{3}}$$

$$3 \times 5 \times 7$$

$$= \frac{3\sqrt{3}-4}{2\sqrt{3}} \times \frac{2\sqrt{3}}{3\sqrt{3}+4} = \frac{3\sqrt{3}-4}{3\sqrt{3}+4}$$

$$1\sqrt{3} + 2\sqrt{3} = \sqrt{3}(1+2) = \sqrt{3}\cdot 3$$

$$\frac{2}{3}+\frac{4}{5} = \frac{(2\times 5)+(4\times 3)}{3\times 5}$$

$$= 5\sqrt{3}$$

$$\frac{2}{3}+\frac{4}{5}+\frac{6}{7} = \frac{2\times 5\times 7}{3\times 5\times 7}$$

$$= \frac{3\sqrt{3}-4}{3\sqrt{3}+4} \times \frac{3\sqrt{3}-4}{3\sqrt{3}-4} = \frac{(3\sqrt{3}-4)(3\sqrt{3}-4)}{(3\sqrt{3})^2 - 4^2}$$

$$= \frac{(3\sqrt{3}-4)^2}{3^2 \times (\sqrt{3})^2 - 16} = \frac{(3\sqrt{3})^2 + 4^2 - 2(3\sqrt{3})(4)}{27-16}$$

$$= \frac{27+16-24\sqrt{3}}{11} = \frac{43-24\sqrt{3}}{11}$$

$$(ab)^m = a^m \cdot b^m$$

$$2 \times 2 = 2^2$$

$$3 \times 3 = 3^2$$

$$(a+b)(a+b) = (a+b)^2$$

$$\frac{(3\sqrt{3}-4)^2}{\quad}$$

$$(x-y)^2 = x^2 + y^2 - 2xy$$

$$\frac{5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$$

$$\frac{5(\cos 60^\circ)^2 + 4(\sec 30^\circ)^2 - (\tan 45^\circ)^2}{(\sin 30^\circ)^2 + (\cos 30^\circ)^2}$$

$$= \frac{5\left(\frac{1}{2}\right)^2 + 4\left(\frac{2}{\sqrt{3}}\right)^2 - (1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{5 \times \frac{1}{4} + 4\left(\frac{4}{3}\right) - \frac{1}{1}}{\frac{1}{4} + \frac{3}{4}}$$

$\angle$ A	0°	30°	45°	60°	90°
sin A	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos A	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan A	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
cosec A	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec A	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
cot A	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

$$= \frac{5 \times \frac{1}{4} + 4\left(\frac{4}{3}\right) - \frac{1}{1}}{\frac{1}{4} + \frac{3}{4}}$$

a,

$$= \frac{\frac{5}{4} + \frac{16}{3} - \frac{1}{1}}{\left(\frac{1+3}{4}\right)} = \frac{\frac{+15+64-12}{4 \times 3}}{\left(\frac{4}{4}\right)} = \frac{79-12}{12}$$

$$= \frac{67}{12}$$

$$12 \overline{)67(} 5$$

$$\frac{60}{7}$$

$$= 5\frac{7}{12}$$

Trigonometric Identities

Identity

$$(x+y)^2 = x^2 + y^2 + 2xy$$

$x = 0, \; y = 1$

$x = 5, \; y = 9$

$x = -2, \; y = -3$

Equation

$$x + y = 10$$

$x = 9, \; y = 5$

LHS	RHS
$(0+1)^2$	$0^2 + 1^2 + 2(0)(1)$
1^2	$= 0 + 1 + 0$
1	$= 1$

Trigonometric Identities

$$AC^2 = AB^2 + BC^2 \quad \text{—— (1)}$$

Dividing (1) by AC^2

$$\frac{AC^2}{AC^2} = \frac{AB^2}{AC^2} + \frac{BC^2}{AC^2}$$

$$1 = \left(\frac{AB}{AC}\right)^2 + \left(\frac{BC}{AC}\right)^2$$

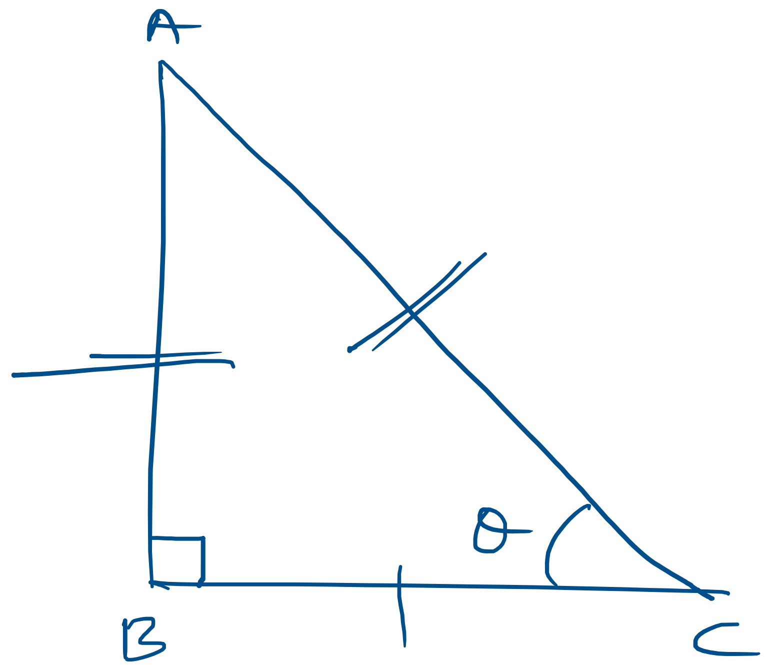
$$1 = (\sin\theta)^2 + (\cos\theta)^2$$

$$1 = \sin^2\theta + \cos^2\theta$$

$$\boxed{\sin^2\theta + \cos^2\theta = 1}$$

$a = b$

$b = a$



$$\frac{a^2}{b^2} = \left(\frac{a}{b}\right)^2$$

Trigonometric Identities

$$AC^2 = AB^2 + BC^2 \quad \text{—— (1)}$$

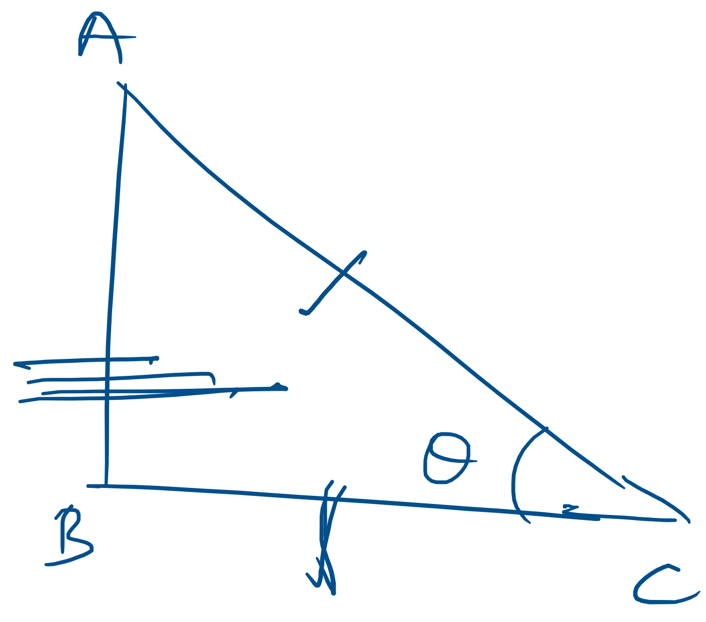
Dividing (1) by AB^2

$$\frac{AC^2}{AB^2} = \frac{AB^2}{AB^2} + \frac{BC^2}{AB^2}$$

$$\left(\frac{AC}{AB}\right)^2 = 1 + \left(\frac{BC}{AB}\right)^2$$

$$(\operatorname{cosec}\theta)^2 = 1 + (\cot\theta)^2$$

$$1 + \cot^2\theta = \operatorname{cosec}^2\theta$$



$$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$\left\{\begin{array}{l} \sin^2\theta + \cos^2\theta = 1 \\ 1 + \tan^2\theta = \sec^2\theta \\ 1 + \cot^2\theta = \operatorname{cosec}^2\theta \end{array}\right\}$$

Find the value of x if

$$2\ \text{cosec}^2 30 + x \sin^2 60 - \frac{3}{4} \tan^2 30 = 10$$

$$2(\text{cosec}\,30^\circ)^2 + x(\sin 60^\circ)^2 - \frac{3}{4}(\tan 30^\circ)^2 = 10$$

$$2(2)^2 + x\left(\frac{\sqrt{3}}{2}\right)^2 - \frac{3}{4}\left(\frac{1}{\sqrt{3}}\right)^2 = 10$$

$$8 + x \times \frac{3}{4} - \frac{3}{4} \times \frac{1}{3} = 10$$

$$+8 + \frac{3x}{4} - \frac{1}{4} = 10$$

$$\frac{3x}{4} - \frac{1}{4} = 10 - 8$$

$$\frac{3x}{4} - \frac{1}{4} = 2$$

$$\frac{3x - 1}{4} = 2$$

$$3x - 1 = 8$$

$$3x = 9$$

$$x = 3$$

Thank You !!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