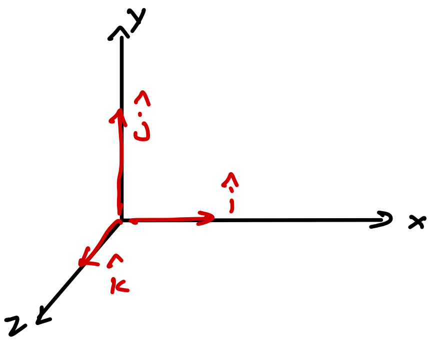
Vectors

* **unit vector** : A vector having magnitude is one is called unit vector.

If $\hat{a}$ is a unit vector along $\vec{A}$

$$\boxed{\hat{a} = \frac{\vec{A}}{|\vec{A}|}}$$

$$\underbrace{\vec{A}}_{\text{vector } \vec{A}} = \underbrace{|\vec{A}|}_{\text{Magnitude of } \vec{A}} \underbrace{\hat{a}}_{\text{unit vector along } \vec{A}}$$



an object is located at $(2, 3, 1)$

Position vector $\vec{r} = 2\hat{i} + 3\hat{j} + \hat{k}$

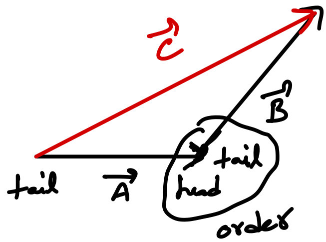
Magnitude of $\vec{r}$ $\quad |\vec{r}| = \sqrt{2^2 + 3^2 + 1^2} = \sqrt{4+9+1} = \sqrt{14}$

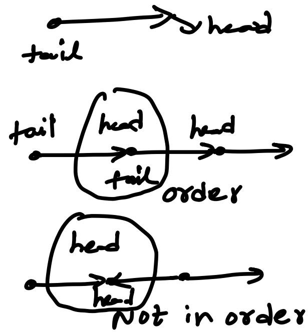
unit vector along $\vec{r}$ $\quad \hat{r} = \dfrac{\vec{r}}{|\vec{r}|} = \dfrac{2\hat{i} + 3\hat{j} + \hat{k}}{\sqrt{14}}$

* **Null vector** : A vector having zero magnitude is null vector

It's direction can't be determined.

Triangular law of vector addition





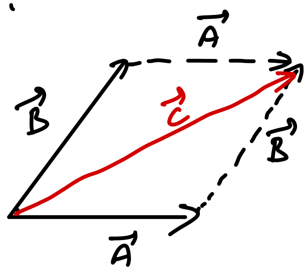
* $\vec{A}+\vec{B}+(-\vec{C})=\vec{0}$

$\vec{A}+\vec{B}=\vec{C}$

$\vec{A}+\vec{B}+\vec{C}$ = Resultant vector

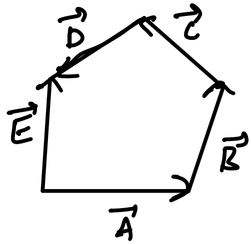
$\vec{A}-\vec{B}+\vec{C}$ = Resultant vector

Parallelogram law of vector addition:



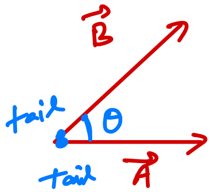
$\vec{B}+\vec{A}=\vec{C}$ (or) $\vec{A}+\vec{B}=\vec{C}$

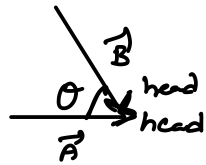
Polygon law of vector addition



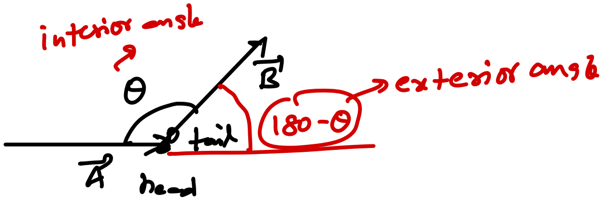
$\vec{A}+\vec{B}+\vec{C}+\vec{D}=\vec{E}$

angle b/n vectors :





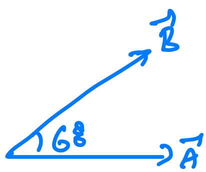
* when two tails (or) heads of vectors meet at a point. The interior angle gives angle b/n vectors

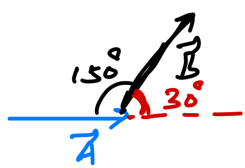


* when tail of one vector meets with head of another vector. the exterior angle gives angle b/n vectors.

$180-\theta$ is angle b/n $\vec{A}$ and $\vec{B}$

Ex:



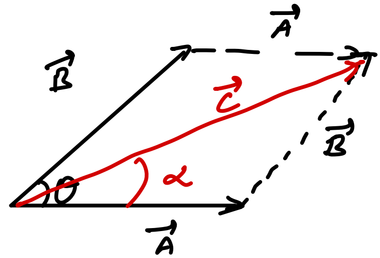


angle b/n $\vec{A}$ and $\vec{B}$ is $\underline{60^\circ}$

angle b/n vectors $\vec{A}$ & $\vec{B}$ is $\underline{30^\circ}$.

* Finding Magnitude of Resultant vector and it's direction

if $\vec{A}+\vec{B}=\vec{C}$



magnitude of Resultant vector

$$\boxed{C = \sqrt{A^2+B^2+2AB\cos\theta}}$$

$$\boxed{\alpha = \tan^{-1}\left[\frac{B\sin\theta}{A+B\cos\theta}\right]}$$ with $\vec{A}$.

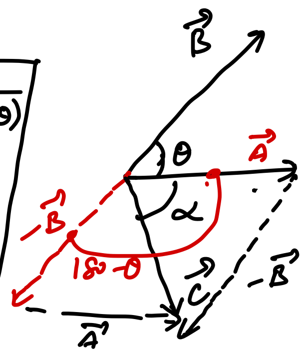
$$|\vec{C}| = |\vec{A}+\vec{B}|$$

* if $\vec{C} = \vec{A}-\vec{B}$

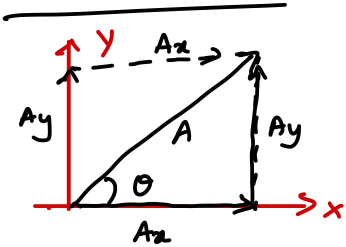
$$C = \sqrt{A^2+B^2+2AB\cos(180-\theta)}$$

$$C = \sqrt{A^2+B^2-2AB\cos\theta}$$

\# $$\alpha = \tan^{-1}\left[\frac{B\sin\theta}{A-B\cos\theta}\right]$$



* Resolution of vectors



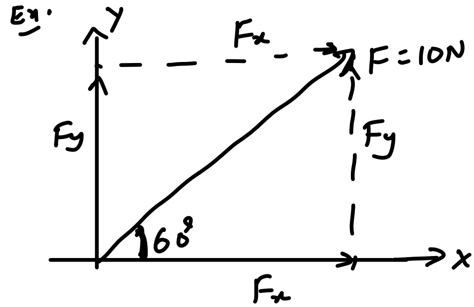
$$\vec{A} = A_x\hat{i} + A_y\hat{j}$$

$$\cos\theta = \frac{A_x}{A} \Rightarrow A_x = A\cos\theta$$

$$\sin\theta = \frac{A_y}{A} \Rightarrow A_y = A\sin\theta$$

$$\vec{A} = A\cos\theta\,\hat{i} + A\sin\theta\,\hat{j}$$

* Ex.



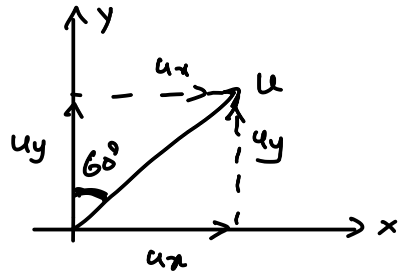
$F_x = F\cos 60^\circ = 10 \times \frac{1}{2} = 5N$

$F_y = F\sin 60^\circ = 10 \times \frac{\sqrt{3}}{2} = 5\sqrt{3}N$

$\vec{F} = F_x\hat{i} + F_y\hat{j}$

$\vec{F} = 5\hat{i} + 5\sqrt{3}\hat{j}$

2)

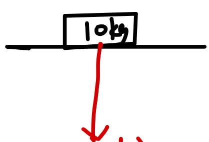


$u_y = u\cos 60^\circ = u/2$

$u_x = u\sin 60^\circ = \frac{\sqrt{3}u}{2}$

$\vec{u} = \frac{\sqrt{3}u}{2}\hat{i} + \frac{u}{2}\hat{j}$

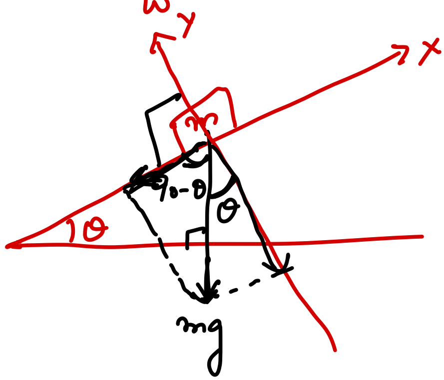
③

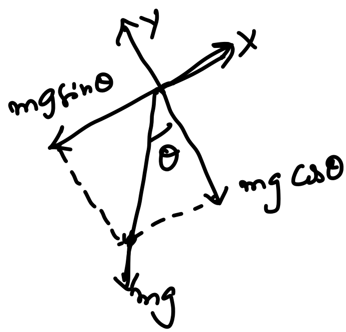


$\vec{W} = 10 \times 10(-\hat{j}) = -100\hat{j}\ N$

$g = 10 m/s^2$

④

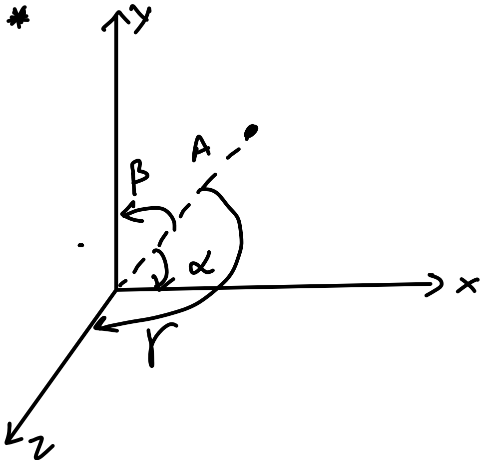




$W_x = -mg\sin\theta$

$W_y = -mg\cos\theta$

*



$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$$\cos\alpha = \frac{A_x}{A}$$

$$\cos\beta = \frac{A_y}{A}$$

$$\cos\gamma = \frac{A_z}{A}$$

$$\boxed{\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1}$$

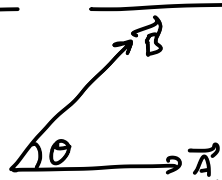
$$(1-\sin^2\alpha) + (1-\sin^2\beta) + (1-\sin^2\gamma) = 1$$

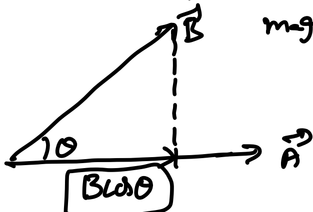
$$\boxed{\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2}$$

* Dot Product of vectors

$$\boxed{\vec{A}\cdot\vec{B} = AB\cos\theta}$$

Scalar Product.





magnitude of component of vector $\vec{B}$ along $\vec{A}$

$$B\cos\theta = \frac{\vec{A}\cdot\vec{B}}{A}$$

vector component of $\vec{B}$ along $\vec{A}$

$$\boxed{B\cos\theta\cdot\hat{a} = \frac{\vec{A}\cdot\vec{B}}{A}\cdot\left(\frac{\vec{A}}{A}\right) = \frac{(\vec{A}\cdot\vec{B})\vec{A}}{A^2}}$$

$$\boxed{\text{||y} \quad A\cos\theta\cdot\hat{b} = \frac{(\vec{A}\cdot\vec{B})\vec{B}}{B^2}}$$

Component of $\vec{B}$ $\perp r$ to $\vec{A}$

$$\vec{B} = \vec{B}_{\parallel} + \vec{B}_{\perp r}$$

$$\vec{B}_{\perp r} = \vec{B} - \vec{B}_{\parallel}$$

$$= \vec{B} - \frac{(\vec{A}\cdot\vec{B})\vec{A}}{A^2}$$

$$\hat{i}\cdot\hat{i} = 1 = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k}$$

$$\vec{A}\cdot\vec{A} = A^2$$

$$\hat{i}\cdot\hat{j} = 0$$

$$\hat{j}\cdot\hat{k} = 0$$

$$\hat{k}\cdot\hat{i} = 0$$

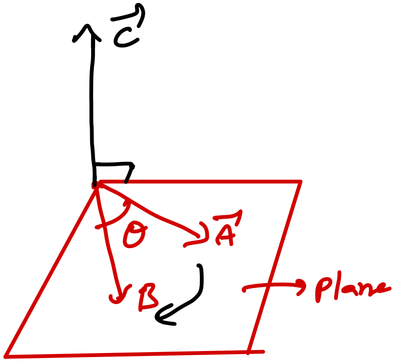
* $\vec{A} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\vec{B} = \hat{i} + \hat{j} - 2\hat{k}$

$\vec{A}\cdot\vec{B} = 2 + 3 - 8 = -3.$

* # If vectors are $\perp r$ to each other their Dot product is zero $\vec{A}\cdot\vec{B} = 0$

* vector Product (or) Cross product of vectors



$\vec{C} = \vec{A} \times \vec{B}$

$|\vec{C}| = AB\sin\theta = |\vec{A} \times \vec{B}|$

$\vec{C} = AB\sin\theta\,\hat{n}$

$\vec{C} = (\vec{A} \times \vec{B})\,\hat{n}$

$\hat{n}$ is unit vector along $\vec{C}$

$\vec{C} \perp \vec{A}$
$\vec{C} \perp \vec{B}$

$$\boxed{\hat{n} = \frac{\vec{C}}{|\vec{A} \times \vec{B}|} = \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|}}$$

$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A} \Rightarrow \vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

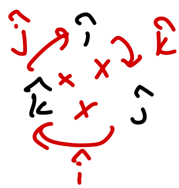
$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$

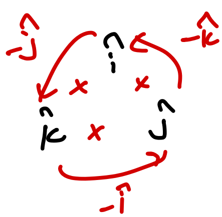
* $\hat{i} \times \hat{i} = 0$
$\hat{j} \times \hat{j} = 0$
$\hat{k} \times \hat{k} = 0$

$\vec{A} \times \vec{A} = 0$

if 2 vectors are Parallel to each other Their Cross product is Zero.

$\hat{i} \times \hat{j} = \hat{k}$
$\hat{j} \times \hat{k} = \hat{i}$
$\hat{k} \times \hat{i} = \hat{j}$





if $\vec{A} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\vec{B} = \hat{i} + \hat{j} - \hat{k}$

$$\vec{A} \times \vec{B} = \begin{array}{c} \quad + \qquad - \qquad + \\ \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 1 & 1 & -1 \end{vmatrix} \end{array}$$

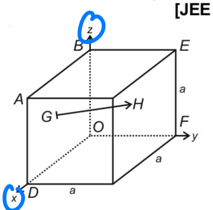
$$= \hat{i}\begin{vmatrix} 3 & 4 \\ 1 & -1 \end{vmatrix} - \hat{j}\begin{vmatrix} 2 & 4 \\ 1 & -1 \end{vmatrix} + \hat{k}\begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix}$$

$$= \hat{i}(-3-4) - \hat{j}(-2-4) + \hat{k}(2-3)$$

$$= \underline{-7\hat{i} + 6\hat{j} - \hat{k}}$$

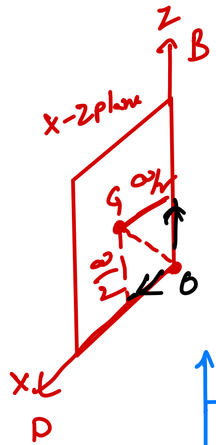
In the cube of side 'a' shown in the figure, the vector from the central point of the face $ABOD$ to the central point of the face $BEFO$ will be

[JEE (Main)-2019]

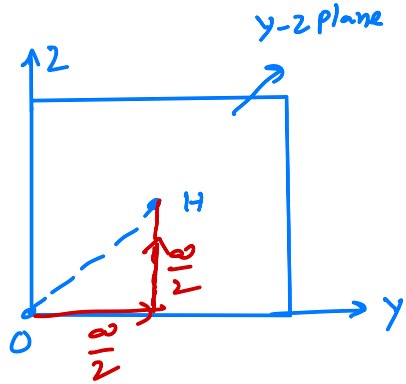


(1) $\frac{1}{2}a(\hat{j}-\hat{i})$ (2) $\frac{1}{2}a(\hat{i}-\hat{k})$

(3) $\frac{1}{2}a(\hat{j}-\hat{k})$ (4) $\frac{1}{2}a(\hat{k}-\hat{i})$



$\vec{OG} = \frac{a}{2}\hat{k} + \frac{a}{2}\hat{i}$



$\vec{OH} = \frac{a}{2}\hat{j} + \frac{a}{2}\hat{k}$

$$\vec{GH} = \vec{OH} - \vec{OG}$$

$$= \left(\frac{a}{2}\hat{j} + \frac{a}{2}\hat{k}\right) - \left(\frac{a}{2}\hat{k} + \frac{a}{2}\hat{i}\right)$$

$$= \frac{a}{2}\hat{j} - \frac{a}{2}\hat{i}$$

$$= \frac{a}{2}(\hat{j} - \hat{i})$$

Two vectors $\vec{A}$ and $\vec{B}$ have equal magnitudes. The magnitude of $(\vec{A}+\vec{B})$ is 'n' times the magnitude of $(\vec{A}-\vec{B})$. The angle between $\vec{A}$ and $\vec{B}$ is

[JEE (Main)-2019]

(1) $\cos^{-1}\left[\frac{n-1}{n+1}\right]$ ✓(2) $\cos^{-1}\left[\frac{n^2-1}{n^2+1}\right]$

(3) $\sin^{-1}\left[\frac{n-1}{n+1}\right]$ (4) $\sin^{-1}\left[\frac{n^2-1}{n^2+1}\right]$

$|\vec{A}+\vec{B}| = n\,|\vec{A}-\vec{B}|$

$\sqrt{A^2+B^2+2AB\cos\theta} = n\sqrt{A^2+B^2-2AB\cos\theta}$

$\theta = ?$

Sol: Consider $|\vec{A}| = |\vec{B}| = x$

$$|\vec{A}+\vec{B}| = n\,|\vec{A}-\vec{B}|$$

$$\sqrt{A^2+B^2+2AB\cos\theta} = n\sqrt{A^2+B^2-2AB\cos\theta}$$

$$A^2+B^2+2AB\cos\theta = n^2\left(A^2+B^2-2AB\cos\theta\right)$$

$$2x^2+2x^2\cos\theta = n^2\left(2x^2-2x^2\cos\theta\right)$$

$$(1+\cos\theta) = n^2(1-\cos\theta)$$

$$1+\cos\theta = n^2-n^2\cos\theta$$

$$n^2-1 = (n^2+1)\cos\theta$$

$$\cos\theta = \frac{n^2-1}{n^2+1}$$

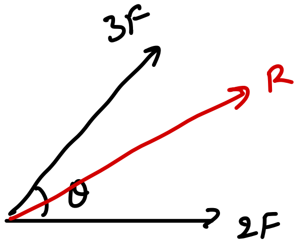
$$\theta = \cos^{-1}\left[\frac{n^2-1}{n^2+1}\right]$$

Two forces P and Q, of magnitude $2F$ and $3F$, respectively, are at an angle θ with each other. If the force Q is doubled, then their resultant also gets doubled. Then, the angle θ is

[JEE (Main)-2019]

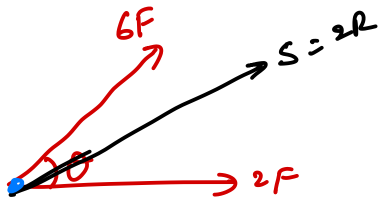
(1) 30° (2) 90°

(3) 60° (4) 120°



$R^2 = 4F^2 + 9F^2 + 2(2F)(3F)\cos\theta$

$R^2 = 13F^2 + 12F^2\cos\theta \rightarrow (1)$



$(2R)^2 = 4F^2 + 36F^2 + 2(2F)(6F)\cos\theta$

$4R^2 = 40F^2 + 24F^2\cos\theta$

$4(13F^2 + 12F^2\cos\theta) = 40F^2 + 24F^2\cos\theta$

$52F^2 + 48F^2\cos\theta = 40F^2 + 24F^2\cos\theta$

$\cancel{12F^2} = -\cancel{24}_{2}\,\cancel{F^2}\cos\theta$

$\cos\theta = \frac{-1}{2}$

$\boxed{\theta = 120^\circ}$

Let $|\vec{A_1}| = 3$, $|\vec{A_2}| = 5$ and $|\vec{A_1} + \vec{A_2}| = 5$. The value of $(2\vec{A_1} + 3\vec{A_2})\cdot(3\vec{A_1} - 2\vec{A_2})$ is : **[JEE (Main)-2019]**

(1) – 106.5

(2) – 118.5 ✓

(3) – 99.5

(4) – 112.5

$$(2\vec{A_1} + 3\vec{A_2})\cdot(3\vec{A_1} - 2\vec{A_2})$$

$$= (2\vec{A_1}\cdot 3\vec{A_1}) + (2\vec{A_1}\cdot -2\vec{A_2}) + (3\vec{A_2}\cdot 3\vec{A_1}) + (3\vec{A_2}\cdot -2\vec{A_2})$$

$$\vec{A}\cdot\vec{A} = A^2$$
$$\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$$

$$= 6\,\vec{A_1}\cdot\vec{A_1} - 4\,\vec{A_1}\cdot\vec{A_2} + 9\,\vec{A_2}\cdot\vec{A_1} - 6\,\vec{A_2}\cdot\vec{A_2}$$

$$= 6A_1^2 + \boxed{5\,\vec{A_1}\cdot\vec{A_2}} - 6A_2^2$$

$$|\vec{A_1} + \vec{A_2}| = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\theta}$$

$$\vec{A_1}\cdot\vec{A_2} = A_1A_2\cos\theta$$

$$5 = \sqrt{9 + 25 + 2(\vec{A_1}\cdot\vec{A_2})}$$

$$\cancel{25} = 9 + \cancel{25} + 2(\vec{A_1}\cdot\vec{A_2})$$

$$-\frac{9}{2} = \vec{A_1}\cdot\vec{A_2}$$

L.H.S $= 6\times 9 - 5\times\frac{9}{2} - 6\times 25$

$= 54 - \frac{45}{2} - 150$

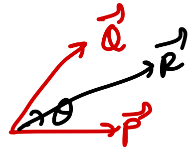
$= 45 - 22.5 - 150$

$= -118.5$

The sum of two forces $\vec{P}$ and $\vec{Q}$ is $\vec{R}$ such that $|\vec{R}| = |\vec{P}|$. The angle θ (in degrees) that the resultant of $2\vec{P}$ and $\vec{Q}$ will make with $\vec{Q}$ is, ________. **[JEE (Main)-2020]**

90

$|\vec{R}| = |\vec{P}| = x$



$\vec{R} = \vec{P} + \vec{Q}$

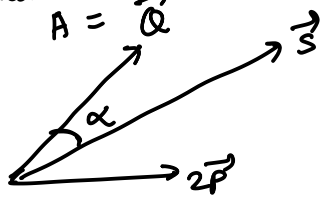
$\cancel{x^2} = \cancel{x^2} + Q^2 + 2PQ\cos\theta \rightarrow (1)$

$Q^{\cancel{2}} = -2P\cancel{Q}\cos\theta$

$Q = -2P\cos\theta = -2x\cos\theta$

$|\vec{Q}| = Q$

$A = \vec{Q}$



$|2\vec{P}| = 2x = B$

$\tan\alpha = \dfrac{2x\sin\theta}{Q + 2x\cos\theta}$

$= \dfrac{2x\sin\theta}{-2x\cos\theta + 2x\cos\theta}$

$= \dfrac{1}{0}$

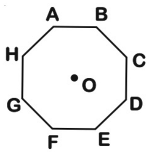
$= \infty$

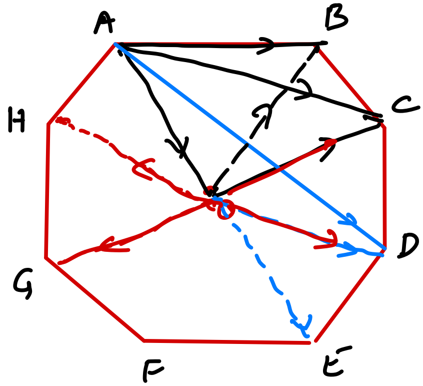
$\alpha = 90^\circ$

In an octagon ABCDEFGH of equal side, what is the sum of

$\overrightarrow{AB}+\overrightarrow{AC}+\overrightarrow{AD}+\overrightarrow{AE}+\overrightarrow{AF}+\overrightarrow{AG}+\overrightarrow{AH}$,

if $\overrightarrow{AO}=2\hat{i}+3\hat{j}-4\hat{k}$ **[JEE (Main)-2021]**





(1) $-16\hat{i}-24\hat{j}+32\hat{k}$

(2) $16\hat{i}+24\hat{j}+32\hat{k}$

(3) $16\hat{i}+24\hat{j}-32\hat{k}$ ✓

(4) $16\hat{i}-24\hat{j}+32\hat{k}$

$\overrightarrow{AB}=\overrightarrow{AO}+\cancel{\overrightarrow{OB}}$

$\overrightarrow{AC}=\overrightarrow{AO}+\cancel{\overrightarrow{OC}}$

$\overrightarrow{AD}=\overrightarrow{AO}+\cancel{\overrightarrow{OD}}$

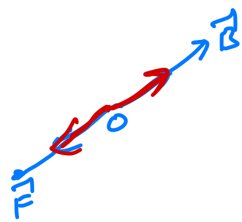
$\overrightarrow{AE}=\overrightarrow{AO}+\overrightarrow{OE}=2\overrightarrow{AO}$

$\overrightarrow{AF}=\overrightarrow{AO}+\overrightarrow{OF}=\overrightarrow{AO}-\cancel{\overrightarrow{OB}}$

$\overrightarrow{AG}=\overrightarrow{AO}+\overrightarrow{OG}=\overrightarrow{AO}-\cancel{\overrightarrow{OC}}$

$\overrightarrow{AH}=\overrightarrow{AO}+\overrightarrow{OH}=\overrightarrow{AO}-\cancel{\overrightarrow{OD}}$

$\overrightarrow{AO}+\overrightarrow{OE}=2\overrightarrow{AO}$



$\overrightarrow{OF}=-\overrightarrow{OB}$

$\overrightarrow{OG}=-\overrightarrow{OC}$

$\overrightarrow{AB}=\overrightarrow{AO}+\overrightarrow{OB}$

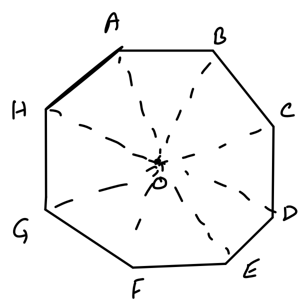
$=8\overrightarrow{AO}$

$=8(2\hat{i}+3\hat{j}-4\hat{k})$

$=16\hat{i}+24\hat{j}-32\hat{k}$

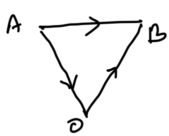
$$\vec{AE} = 2\vec{AO}$$

Sol:



$$\vec{AE} = \vec{AB}+\vec{BC}+\vec{CD}+\vec{DE}$$

$$\vec{AB}+\vec{AC}+\vec{AD}+\vec{AE}+\vec{AF}+\vec{AG}+\vec{AH} = 8\vec{AO}$$
$$= 8(2\hat{i}+3\hat{j}-4\hat{k})$$
$$= 16\hat{i}+24\hat{j}-32\hat{k}$$

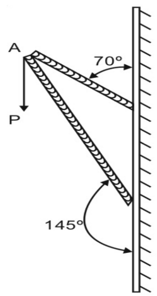


$$\left.\begin{aligned}
\vec{AB} &= \vec{AO}+\vec{OB}\\
\vec{AC} &= \vec{AO}+\vec{OC}\\
\vec{AD} &= \vec{AO}+\vec{OD}\\
\vec{AE} &= \vec{AO}+\vec{OE}\\
\vec{AF} &= \vec{AO}+\vec{OF}\\
\vec{AG} &= \vec{AO}+\vec{OG}\\
\vec{AH} &= \vec{AO}+\vec{OH}
\end{aligned}\right\}$$

$$\vec{OB} = -\vec{OF}$$
$$\vec{OC} = -\vec{OG}$$
$$\vec{OD} = -\vec{OH}$$
$$\vec{AO} = \vec{OE}$$

$$\Rightarrow 7\vec{AO}+\vec{OE} = 7\vec{AO}+\vec{AO}$$
$$= \underline{8\vec{AO}}$$

Consider a frame that is made up of two thin massless rods AB and AC as shown in the figure. A vertical force $\vec{P}$ of magnitude 100 N is applied at point A of the frame. **[JEE (Main)-2021]**



Suppose the force is $\vec{P}$ resolved parallel to the arms AB and AC of the frame. The magnitude of the resolved component along the arm AC is xN.

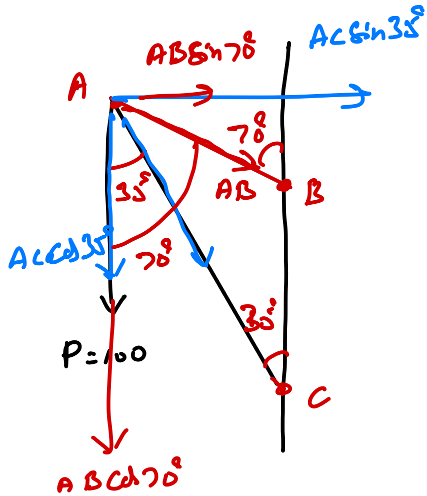
The value of x, to the nearest integer, is 164.

[Given : sin(35°) = 0.573,

cos(35°) = 0.819

sin(110°) = 0.939,

cos(110°) = –0.342]



$$AC\cos 35^\circ + AB\cos 70^\circ = P$$

$$AB\sin 70^\circ + AC\sin 35^\circ = 0$$

$$\Rightarrow AB\sin 70^\circ = -AC\sin 35^\circ$$

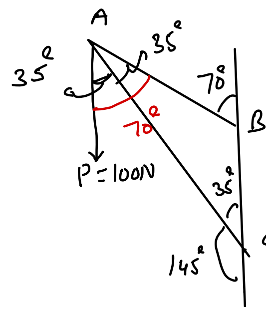
$$AB = -\frac{AC\sin 35^\circ}{\sin 70^\circ} = \_\_\_\_$$

$$\sin 70^\circ = \sin 2(35^\circ) = 2\sin 35^\circ\cdot\cos 35^\circ$$

$$\Rightarrow AC\cos 35^\circ - \frac{AC\,\cancel{\sin 35^\circ}}{2\,\cancel{\sin 35^\circ}\cdot\cos 35^\circ}\cdot\cos 70^\circ = P$$

$$AC = ?$$

Sol:



$$AC\cos 35^\circ + AB\cos 70^\circ = P$$

$$AC\sin 35^\circ + AB\sin 70^\circ = 0$$

$$AC\sin 35^\circ = -AB\sin 70^\circ$$

$$AC\cos 35^\circ - AC\frac{\sin 35^\circ}{\sin 70^\circ}\cdot\cos 70^\circ = P$$

$$AC\cos 35^\circ - AC\frac{\cancel{\sin 35^\circ}\,\cos 70^\circ}{2\cancel{\sin 35^\circ}\cos 35^\circ} = P$$

$$\frac{2AC\cos^2 35^\circ - AC\,(\cos^2 35^\circ - \sin^2 35^\circ)}{2\cos 35^\circ} = P$$

$$AC\cos^2 35^\circ + AC\sin^2 35^\circ = 2P\cos 35^\circ$$

$$AC = 2P\cos 35^\circ$$

$$= 2\times 100\times(0.819)$$

$$= \underline{163.8}$$

$81.9\times 2 = 163.8$

$$\underline{AC = 164\,N}$$

If $\vec{A}$ and $\vec{B}$ are two vectors satisfying the relation $\vec{A} \bullet \vec{B} = |\vec{A} \times \vec{B}|$. Then the value of $|\vec{A} - \vec{B}|$ will be

[JEE (Main)-2021]

(1) $\sqrt{A^2 + B^2}$

✓ (2) $\sqrt{A^2 + B^2 - \sqrt{2}AB}$

(3) $\sqrt{A^2 + B^2 + 2AB}$

(4) $\sqrt{A^2 + B^2 + \sqrt{2}AB}$

$$\vec{A} \cdot \vec{B} = |\vec{A} \times \vec{B}|$$

$$\cancel{AB}\cos\theta = \cancel{AB}\sin\theta$$

$$\frac{\sin\theta}{\cos\theta} = 1 \Rightarrow \tan\theta = 1$$

$$\Rightarrow \theta = 45^\circ$$

$$|\vec{A} - \vec{B}| = \sqrt{A^2 + B^2 - 2AB\cos 45^\circ} = \sqrt{A^2 + B^2 - \sqrt{2}AB}$$

What will be the projection of vector $\vec{A} = \hat{i} + \hat{j} + \hat{k}$ on vector $\vec{B} = \hat{i} + \hat{j}$? **[JEE (Main)-2021]**

(1) $\sqrt{2}\left(\hat{i} + \hat{j} + \hat{k}\right)$

(2) $\left(\hat{i} + \hat{j}\right)$

(3) $\sqrt{2}\left(\hat{i} + \hat{j}\right)$

(4) $2\left(\hat{i} + \hat{j} + \hat{k}\right)$

vector component of $\vec{A}$ along $\vec{B}$ = $A\cos\theta\,\hat{B}$

$$= \left(\frac{\vec{A}\cdot\vec{B}}{B}\right)\frac{\vec{B}}{B}$$

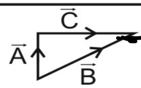
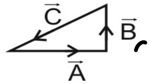
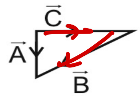
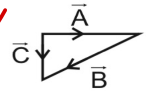
$$= \frac{(\vec{A}\cdot\vec{B})\vec{B}}{B^2}$$

$B = \sqrt{1^2+1^2} = \sqrt{2}$

$$= \frac{(1+1+0)(\hat{i}+\hat{j})}{2}$$

$$= (\hat{i}+\hat{j})$$

Match **List I** with **List II**. **[JEE (Main)-2021]**

List I	**List II**
(a) $\vec{C}-\vec{A}-\vec{B}=0$	(i) 
(b) $\vec{A}-\vec{C}-\vec{B}=0$	(ii) 
(c) $\vec{B}-\vec{A}-\vec{C}=0$	(iii) 
(d) $\vec{A}+\vec{B}=-\vec{C}$	(iv) 

Choose the correct answer from the options given below:

(1) (a) → (iv), (b) → (i), (c) → (iii), (d) → (ii)

(2) (a) → (i), (b) → (iv), (c) → (ii), (d) → (iii)

(3) (a) → (iii), (b) → (ii), (c) → (iv), (d) → (i)

(4) (a) → (iv), (b) → (iii), (c) → (i), (d) → (ii)

(i) $\vec{A}+\vec{C}=\vec{B}$

$\vec{A}+\vec{C}-\vec{B}=0$

$\vec{B}-\vec{A}-\vec{C}=0$

(ii) $\vec{A}+\vec{B}+\vec{C}=0$

$\vec{A}+\vec{B}=-\vec{C}$

(iii) $\vec{C}+\vec{B}=\vec{A}$

$\vec{A}-\vec{C}-\vec{B}=0$

(iv) $\vec{A}+\vec{B}=\vec{C}$

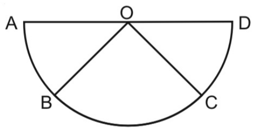
$\vec{C}-\vec{A}-\vec{B}=0$

Assertion A : If A, B, C, D are four points on a semi-circular arc with centre at 'O' such that $|\overrightarrow{AB}| = |\overrightarrow{BC}| = |\overrightarrow{CD}|$, then

$$\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} = 4\overrightarrow{AO} + \overrightarrow{OB} + \overrightarrow{OC}$$

Reason R : Polygon law of vector addition yields

$$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} = \overrightarrow{AD} = 2\overrightarrow{AO}$$



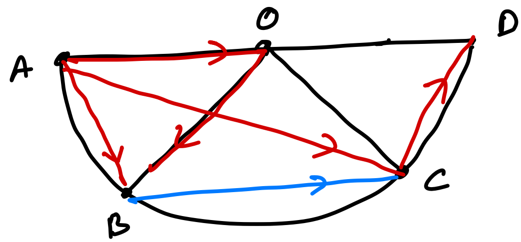
In the light of the above statements, choose the **most appropriate** answer from the options given below **[JEE (Main)-2021]**

(1) **A** is not correct but **R** is correct.

(2) **A** is correct but **R** is not correct.

(3) Both **A** and **R** are correct and **R** is the correct explanation of **A**.

(4) Both **A** and **R** are correct but **R** is not the correct explanation of **A**.



$\overrightarrow{AO} = \overrightarrow{OD}$

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC}$

$\overrightarrow{AD} = \overrightarrow{AO} + \overrightarrow{AO}$

$4\overrightarrow{AO} + \overrightarrow{OB} + \overrightarrow{OC}$

$\overrightarrow{AB} = \overrightarrow{AO} + \cancel{\overrightarrow{OB}}$

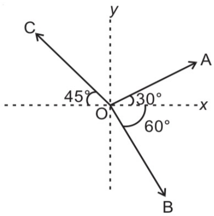
$\overrightarrow{BC} = \overrightarrow{BO} + \overrightarrow{OC} = -\cancel{\overrightarrow{OB}} + \cancel{\overrightarrow{OC}}$

$\overrightarrow{CD} = \overrightarrow{CO} + \overrightarrow{OD} = -\cancel{\overrightarrow{OC}} + \overrightarrow{OD}$

$\overrightarrow{AO} + \overrightarrow{OD} = 2\overrightarrow{AO} = \overrightarrow{AD}$

The magnitude of vectors $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$ in the given figure are equal. The direction of $\overrightarrow{OA}+\overrightarrow{OB}-\overrightarrow{OC}$ with x-axis will be

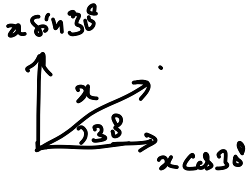
[JEE (Main)-2021]



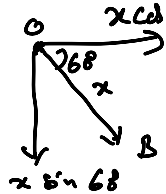
(1) $\tan^{-1}\dfrac{(1+\sqrt{3}-\sqrt{2})}{(1-\sqrt{3}-\sqrt{2})}$ (2) $\tan^{-1}\dfrac{(\sqrt{3}-1+\sqrt{2})}{(1+\sqrt{3}-\sqrt{2})}$

(3) $\tan^{-1}\dfrac{(\sqrt{3}-1+\sqrt{2})}{(1-\sqrt{3}+\sqrt{2})}$ (4) $\tan^{-1}\dfrac{(1-\sqrt{3}-\sqrt{2})}{(1+\sqrt{3}+\sqrt{2})}$

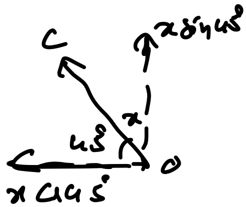
$$\overrightarrow{OA} = \frac{x\sqrt{3}}{2}\hat{i} + \frac{x}{2}\hat{j}$$



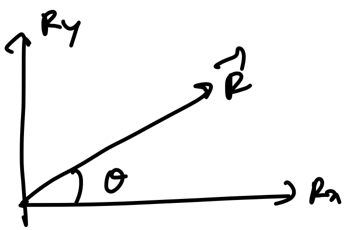
$$\overrightarrow{OB} = \frac{x}{2}\hat{i} - \frac{\sqrt{3}x}{2}\hat{j}$$



$$\overrightarrow{OC} = \frac{-x}{\sqrt{2}}\hat{i} + \frac{x}{\sqrt{2}}\hat{j}$$



$$\vec{R} = \overrightarrow{OA} + \overrightarrow{OB} - \overrightarrow{OC} = \rule{4cm}{0.4pt}$$



$$\tan\theta = \frac{R_y}{R_x}$$

$|\vec{OA}| = |\vec{OB}| = |\vec{OC}| = x$

Sol:

$$\vec{OA} = x\cos 30^\circ \hat{i} + x\sin 30^\circ \hat{j} = \frac{x}{2}(\sqrt{3}\hat{i} + \hat{j})$$

$$\vec{OB} = x\cos 60^\circ \hat{i} - x\sin 60^\circ \hat{j} = \frac{x}{2}(\hat{i} - \sqrt{3}\hat{j})$$

$$\vec{OC} = -x\cos 45^\circ \hat{i} + x\sin 45^\circ \hat{j} = \frac{x}{\sqrt{2}}(-\hat{i} + \hat{j})$$

$$\vec{OA} + \vec{OB} - \vec{OC} = \frac{x}{2}(\sqrt{3}\hat{i} + \hat{j} + \hat{i} - \hat{j}\sqrt{3}) - \frac{x}{\sqrt{2}}(-\hat{i} + \hat{j})$$

$$= \frac{x}{2}\left((\sqrt{3}+1)\hat{i} + (1-\sqrt{3})\hat{j}\right) + \frac{x}{\sqrt{2}}\hat{i} - \frac{x}{\sqrt{2}}\hat{j}$$

$$\vec{R} = \vec{OA} + \vec{OB} - \vec{OC} = \frac{x}{2\sqrt{2}}\left[\sqrt{3}+1+\sqrt{2}\right]\hat{i} + \frac{x}{2\sqrt{2}}\left[1-\sqrt{3}-\sqrt{2}\right]\hat{j}$$

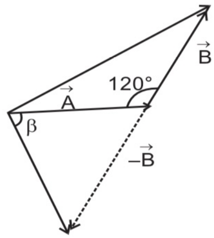
$$\tan\theta = \frac{R_y}{R_x} = \left(\frac{1-\sqrt{3}-\sqrt{2}}{1+\sqrt{3}+\sqrt{2}}\right)$$

$$\theta = \tan^{-1}\left[\frac{1-\sqrt{3}-\sqrt{2}}{1+\sqrt{3}+\sqrt{2}}\right]$$

option : 4

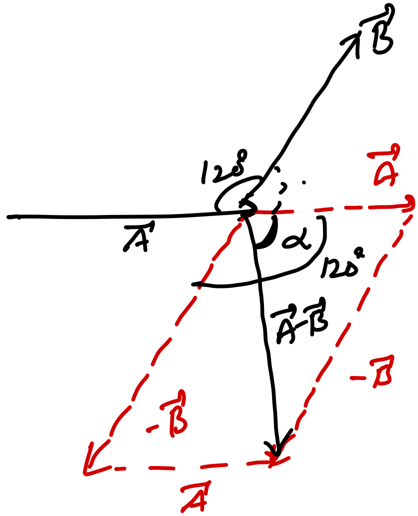
The angle between vector $(\vec{A})$ and $(\vec{A}-\vec{B})$ is:

[JEE (Main)-2021]



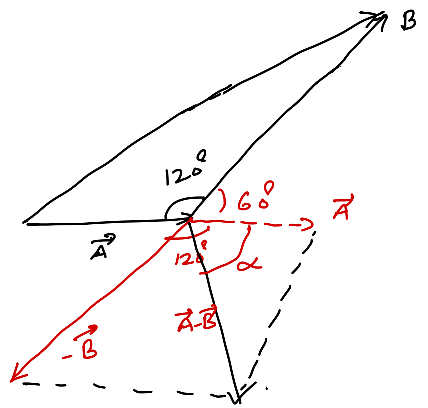
(1) $\tan^{-1}\left(\dfrac{-\frac{B}{2}}{A-B\frac{\sqrt{3}}{2}}\right)$ (2) $\tan^{-1}\left(\dfrac{\sqrt{3}B}{2A-B}\right)$

(3) $\tan^{-1}\left(\dfrac{B\cos\theta}{A-B\sin\theta}\right)$ (4) $\tan^{-1}\left(\dfrac{A}{0.7B}\right)$



$$\tan\alpha = \frac{B\sin 120^\circ}{A + B\cos 120^\circ}$$

Sol



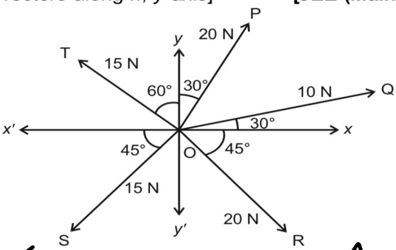
$$\alpha = \tan^{-1}\left[\frac{B\sin 120^\circ}{A + B\cos 120^\circ}\right]$$

$$= \tan^{-1}\left[\frac{B\left(\sqrt{3}/2\right)}{A - B/2}\right]$$

$$= \tan^{-1}\left[\frac{\sqrt{3}B}{2A - B}\right]$$

The resultant of these forces $\overrightarrow{OP}, \overrightarrow{OQ}, \overrightarrow{OR}, \overrightarrow{OS}$ and $\overrightarrow{OT}$ is approximately _______ N.

[Take $\sqrt{3} = 1.7$, $\sqrt{2} = 1.4$. Given $\hat{i}$ and $\hat{j}$ unit vectors along x, y axis] **[JEE (Main)-2021]**



(1) $9.25\hat{i} + 5\hat{j}$

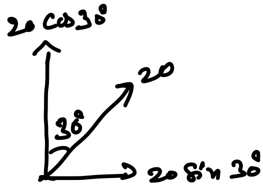
(2) $3\hat{i} + 15\hat{j}$

(3) $-1.5\hat{i} - 15.5\hat{j}$

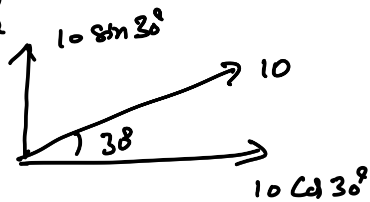
(4) $2.5\hat{i} - 14.5\hat{j}$

$9.32\hat{i} + 4.54\hat{j}$

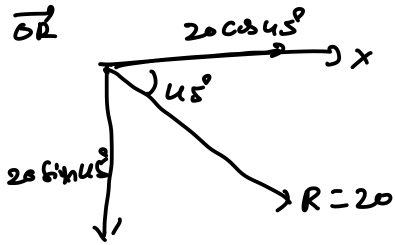
$\overrightarrow{OP} =$



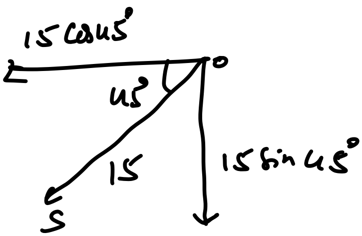
$\overrightarrow{OQ}$



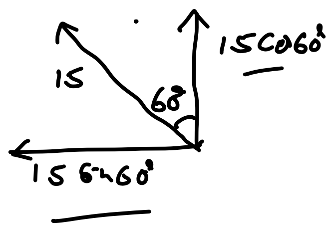
$\overrightarrow{OR}$



$\overrightarrow{OS}$



OT



Statement I: If three forces $\vec{F}_1, \vec{F}_2$ and $\vec{F}_3$ are represented by three sides of a triangle and $\vec{F}_1 + \vec{F}_2 = -\vec{F}_3$, then these three forces are concurrent forces and satisfy the condition for equilibrium.

Statement II: A triangle made up of three forces $\vec{F}_1, \vec{F}_2$ and $\vec{F}_3$ as its sides taken in the same order, satisfy the condition for translatory equilibrium.

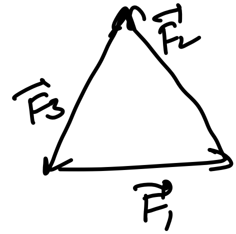
In the light of the above statements, choose the **most appropriate** answer from the options given below: **[JEE (Main)-2021]**

(1) Statement I is false but statement II is true

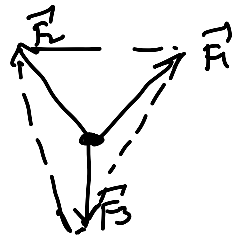
(2) Both statement I and statement II are false

(3) Both statement I and statement II are true

(4) Statement I is true but statement II is false



$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$



Statement I : Two forces $(\vec{P}+\vec{Q})$ and $(\vec{P}-\vec{Q})$ where $\vec{P} \perp \vec{Q}$, when act at an angle θ_1 to each other, the magnitude of their resultant is $\sqrt{3(P^2+Q^2)}$, when they act at an angle θ_2, the magnitude of their resultant becomes $\sqrt{2(P^2+Q^2)}$. This is possible only when $\theta_1 < \theta_2$.

Statement II : In the situation given above.

$\theta_1 = 60°$ and $\theta_2 = 90°$

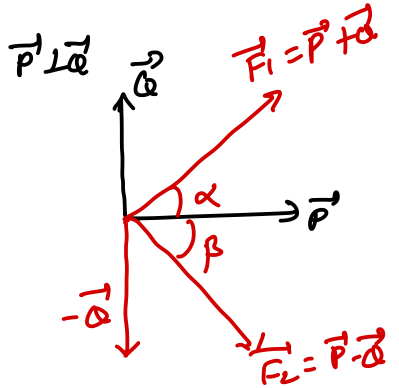
In the light of the above statements, choose the **most appropriate** answer from the options given below: **[JEE (Main)-2021]**

(1) Both Statement I and Statement II are true.

(2) Statement I is true but Statement II is false.

(3) Statement I is false but Statement II is true.

(4) Both Statement I and Statement II are false.



$$|\vec{F_1}| = |\vec{F_2}| = \sqrt{P^2+Q^2} = F$$

$$\sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta_1} = \sqrt{3(P^2+Q^2)}$$

$$\sqrt{2F^2(1+\cos\theta_1)} = \sqrt{3(P^2+Q^2)}$$

$$2F\cos\left(\frac{\theta_1}{2}\right) = \sqrt{3(P^2+Q^2)}$$

$$2\cancel{\sqrt{P^2+Q^2}}\cos\left(\frac{\theta_1}{2}\right) = \sqrt{3}\cancel{(P^2+Q^2)}$$

$$2\cos\left(\frac{\theta_1}{2}\right) = \sqrt{3}$$

$$\cos\left(\frac{\theta_1}{2}\right) = \frac{\sqrt{3}}{2}$$

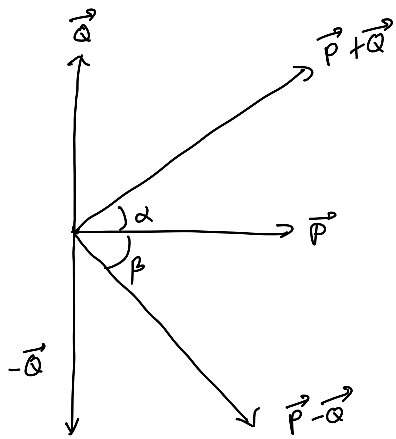
$$\frac{\theta_1}{2} = 30° \Rightarrow \underline{\theta_1 = 60°}$$

$$2\cancel{\sqrt{P^2+Q^2}}\cos\left(\frac{\theta_2}{2}\right) = \sqrt{2\cancel{(P^2+Q^2)}}$$

$$\cos\left(\frac{\theta_2}{2}\right) = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}} \Rightarrow \frac{\theta_2}{2} = 45°$$

$$\underline{\theta_2 = 90°}$$

sol:-



$$\tan\alpha = \frac{Q\sin 90^\circ}{P + Q\cos 90^\circ} = \frac{Q}{P}$$

$$\tan\beta = \frac{Q}{P}$$

$$\theta = \alpha + \beta$$

if $\theta_1 = 60^\circ$

$$|\vec{P}+\vec{Q}|^2 = P^2 + Q^2$$
$$|\vec{P}-\vec{Q}|^2 = P^2 + Q^2$$

$$R_1^2 = (P^2+Q^2) + (P^2+Q^2) + 2\sqrt{P^2+Q^2}\,\sqrt{P^2+Q^2}\cdot\cos 60^\circ$$
$$= 2(P^2+Q^2)\,[1 + 1/2)$$
$$= 3\,(P^2+Q^2)$$
$$R_1 = \sqrt{3(P^2+Q^2)}$$

if $\theta_2 = 90^\circ$ $\quad R_2 = \sqrt{2(P^2+Q^2)}$

both Statements are true.

If $\vec{A} = (2\hat{i} + 3\hat{j} - \hat{k})$ m and $\vec{B} = (\hat{i} + 2\hat{j} + 2\hat{k})$ m. The magnitude of component of vector $\vec{A}$ along vector $\vec{B}$ will be __2__ m

Magnitude of vector $\vec{A}$ along $\vec{B}$ = $A\cos\theta$

$$= \frac{\vec{A}\cdot\vec{B}}{B}$$

$$= \frac{2+6-2}{\sqrt{1+4+4}}$$

$$= \frac{6}{3}$$

$$= \underline{2}$$

A force $\vec{F} = (\hat{i} + 2\hat{j} + 3\hat{k})$ N acts at a point $(\underbrace{4\hat{i} + 3\hat{j} - \hat{k}}_{\vec{r_f}})$ m. Then the magnitude of torque about the point $(\underbrace{\hat{i} + 2\hat{j} + \hat{k}}_{\vec{r_i}})$ m will be $\sqrt{x}$ N-m. The value of x is ______.

$$\vec{r} = \vec{r_f} - \vec{r_i}$$

$$\vec{\tau} = \vec{r} \times \vec{F}$$

Sol:-

$$\vec{r} = (4-1)\hat{i} + (3-2)\hat{j} + (-\hat{k} - \hat{k}) = 3\hat{i} + \hat{j} - 2\hat{k}$$

$$\vec{F} = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\vec{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 1 & 2 & 3 \end{vmatrix}$$

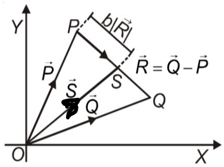
$$= \hat{i}(3+4) - \hat{j}(9+2) - \hat{k}(6-1)$$

$$\vec{\tau} = 7\hat{i} - 11\hat{j} - 5\hat{k}$$

$$|\vec{\tau}| = \sqrt{49+121+25} = \sqrt{195}$$

Three vectors $\vec{P}$, $\vec{Q}$ and $\vec{R}$ are shown in the figure. Let S be any point on the vector $\vec{R}$. The distance between the points P and S is $b|\vec{R}|$. The general relation among vectors $\vec{P}$, $\vec{Q}$ and $\vec{S}$ is

[JEE (Adv)-2017 (Paper-2)]

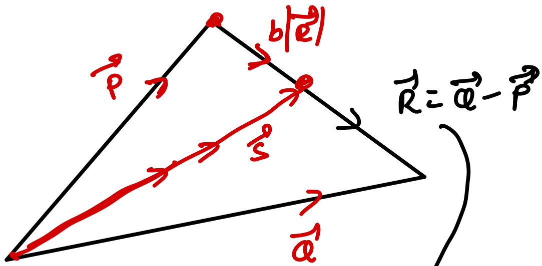


(A) $\vec{S} = (1-b^2)\vec{P} + b\vec{Q}$

(B) $\vec{S} = (b-1)\vec{P} + b\vec{Q}$

(C) $\vec{S} = (1-b)\vec{P} + b\vec{Q}$

(D) $\vec{S} = (1-b)\vec{P} + b^2\vec{Q}$



$$\vec{P} + b|\vec{R}|\hat{R} = \vec{S}$$

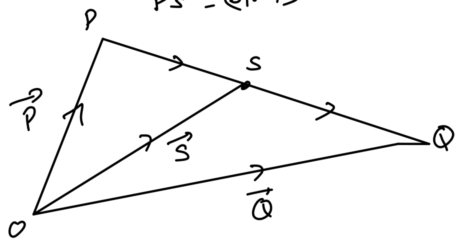
$$\vec{P} + b\cancel{|\vec{R}|}\cdot\frac{\vec{R}}{\cancel{|\vec{R}|}} = \vec{S}$$

$$\vec{P} + b(\vec{Q} - \vec{P}) = \vec{S}$$

$$(1-b)\vec{P} + b\vec{Q} = \vec{S}$$

Sol:-

$\vec{PS} = (b|\vec{R}|)\hat{R}$



$$\vec{S} = \vec{P} + b|\vec{R}| \cdot \hat{R}$$
$$= \vec{P} + b|\vec{R}| \cdot \frac{\vec{R}}{|\vec{R}|}$$
$$\vec{S} = \vec{P} + b\vec{R}$$

$\vec{R} = \vec{Q} - \vec{P}$

$$\vec{S} = \vec{P} + b(\vec{Q} - \vec{P})$$
$$\vec{S} = \vec{P}(1-b) + b\vec{Q}$$

Two vectors $\vec{A}$ and $\vec{B}$ are defined as $\vec{A} = a\hat{i}$ and $\vec{B} = a(\cos\omega t\hat{i} + \sin\omega t\hat{j})$, where a is a constant and $\omega = \frac{\pi}{6}$ rad s^{-1}. If $|\vec{A}+\vec{B}| = \sqrt{3}\,|\vec{A}-\vec{B}|$ at time t = τ for the first time, the value of τ, in seconds, is **2**.

[JEE (Adv)-2018 (Paper-1)]

$\vec{A} = a\hat{i}$

$\vec{B} = a\cos\omega t\,\hat{i} + a\sin\omega t\,\hat{j}$

$\vec{A}+\vec{B} = a[1+\cos\omega t]\hat{i} + a\sin\omega t\,\hat{j}$

$|\vec{A}+\vec{B}| =$ ______

$\vec{A}-\vec{B} = a[1-\cos\omega t]\hat{i} - a\sin\omega t\,\hat{j}$

$|\vec{A}-\vec{B}| =$ ______

$|\vec{A}+\vec{B}| = \sqrt{3}\,|\vec{A}-\vec{B}|$

$\omega = \pi/6$

Sol:-

$$\vec{A} = a\hat{i} \qquad \vec{B} = a(\cos\omega t\,\hat{i} + \sin\omega t)\hat{j}$$

$$\vec{A}+\vec{B} = a(1+\cos\omega t)\hat{i} + a\sin\omega t\,\hat{j}$$

$$|\vec{A}+\vec{B}| = a\sqrt{(1+\cos\omega t)^2 + \sin^2\omega t}$$

$$= a\sqrt{1+\cos^2\omega t + 2\cos\omega t + \sin^2\omega t}$$

$$= a\sqrt{2(1+\cos\omega t)}$$

$$\vec{A}-\vec{B} = a(1-\cos\omega t)\hat{i} - a\sin\omega t\,\hat{j}$$

$$|\vec{A}-\vec{B}| = a\sqrt{(1-\cos\omega t)^2 + \sin^2\omega t}$$

$$= a\sqrt{2(1-\cos\omega t)}$$

$$|\vec{A}+\vec{B}| = \sqrt{3}\,|\vec{A}-\vec{B}|$$

$$a\sqrt{2(1+\cos\omega t)} = \sqrt{3}\,a\sqrt{2(1-\cos\omega t)}$$

$$2(1+\cos\omega t) = 3(2(1-\cos\omega t))$$

$$1+\cos\omega t = 3-3\cos\omega t$$

$$4\cos\omega t = 2$$

$$\cos\omega t = \frac{1}{2}$$

$$\omega t = \pi/3$$

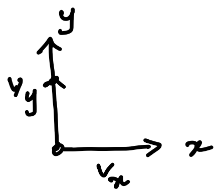
$$\omega = \frac{\pi}{6}, \qquad t = \tau$$

$$\frac{\pi}{6}\tau = \frac{\pi}{3}$$

$$\tau = 2\text{ sec}$$

Motion in 2-D

* if a body is moving with uniform velocity in x-y plane.



distance covered at any time t

$$x = v_x t$$
$$y = v_y t$$

* if a body is moving with uniform acceleration in x-y plane

$$v_x = u_x + a_x t$$
$$x = u_x t + \frac{1}{2} a_x t^2$$
$$v_x^2 - u_x^2 = 2 a_x \cdot x$$

$$v_y = u_y + a_y t$$
$$y = u_y t + \frac{1}{2} a_y t^2$$
$$v_y^2 - u_y^2 = 2 a_y \cdot y$$

$$\boxed{\begin{gathered} \vec{v} = \vec{u} + \vec{a} t \\ \vec{s} = \vec{u} t + \frac{1}{2} \vec{a} t^2 \\ \vec{v} \cdot \vec{v} - \vec{u} \cdot \vec{u} = 2 \vec{a} \cdot \vec{s} \end{gathered}}$$

$$\vec{v} = v_x \hat{i} + v_y \hat{j}$$
$$\vec{u} = u_x \hat{i} + u_y \hat{j}$$
$$\vec{s} = x \hat{i} + y \hat{j}$$
$$\vec{a} = a_x \hat{i} + a_y \hat{j}$$

* if a body is moving in Non-uniform motion

* $v_x = \dfrac{dx}{dt}$ * $v_y = \dfrac{dy}{dt}$

* $a_x = \dfrac{dv_x}{dt}$ * $a_y = \dfrac{dv_y}{dt}$

* $\int dx = \int v_x dt$
* $\int dy = \int v_y dt$
* $\int dv_x = \int a_x dt$
* $\int dv_y = \int a_y dt$
* $\int v_x \cdot dv_x = \int a_x dx$
* $\int v_y \cdot dv_y = \int a_y dy$

* To identify Path of Particle

equation should be in y and x Relation.

if $y = x + c$ st. line path

if $x^2 + y^2 = R^2$ (constant) path is circle

if $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ Path is ellipse

if $y = Ax^2 + Bx$ Path is parabola

if $y = \frac{k}{x}$ Path is rectangular hyperbola

A particle has an initial velocity of $3\hat{i}+4\hat{j}$ and an acceleration of $0.4\hat{i}+0.3\hat{j}$. Its speed after 10 s is **[AIEEE-2009]**

✓(1) $7\sqrt{2}$ units (2) 7 units

(3) 8.5 units (4) 10 units

$\vec{u} = 3\hat{i}+4\hat{j}$

$\vec{a} = 0.4\hat{i}+0.3\hat{j}$

$t = 10\,s$.

$\vec{v} = \vec{u}+\vec{a}t = (3\hat{i}+4\hat{j})+(4\hat{i}+3\hat{j})$

$\vec{v} = 7\hat{i}+7\hat{j}$

Speed $|\vec{v}| = \sqrt{7^2+7^2} = \underline{7\sqrt{2}}$

The position co-ordinates of a particle moving in a 3-D coordinate system is given by $x = a \cos \omega t$, $y = a \sin \omega t$ and $z = a\omega t$

The speed of the particle is **[JEE (Main)-2019]**

(1) $2a\omega$ (2) $\sqrt{2}\,a\omega$

(3) $\sqrt{3}\,a\omega$ (4) $a\omega$

$x = a \cos \omega t \qquad y = a \sin \omega t \qquad z = a\omega t$

$v_x = \frac{dx}{dt} = -a\omega \sin \omega t \;,\; v_y = \frac{dy}{dt} = a\omega \cos \omega t \;,\; v_z = \frac{dz}{dt} = a\omega$

$\vec{v} = v_x \hat{i} + v_y \hat{j} + v_z \hat{k}$

$\vec{v} = -a\omega \sin \omega t\, \hat{i} + a\omega \cos \omega t\, \hat{j} + a\omega\, \hat{k}$

$|\vec{v}| = \sqrt{a^2\omega^2 \sin^2 \omega t + a^2\omega^2 \cos^2 \omega t + a^2\omega^2}$

$= \sqrt{a^2\omega^2 + a^2\omega^2}$

$|\vec{v}| = \sqrt{2}\, a\omega$

A particle is moving with a velocity $\vec{v} = K(y\hat{i} + x\hat{j})$, where K is a constant. The general equation for its path is **[JEE (Main)-2019]**

(1) $y^2 = x$ + constant (2) $y = x^2$ + constant

(3) $y^2 = x^2$ + constant (4) xy = constant

$$\vec{v} = \underbrace{ky}_{V_x}\hat{i} + \underbrace{kx}_{V_y}\hat{j}$$

$$V_x = ky \qquad V_y = kx$$

$$\frac{dx}{dt} = ky \rightarrow (1) \qquad \frac{dy}{dt} = kx \rightarrow (2)$$

we have eliminate dt term.

$$\frac{(2)}{(1)} \Rightarrow \frac{\frac{dy}{\cancel{dt}}}{\frac{dx}{\cancel{dt}}} = \frac{\cancel{k}x}{\cancel{k}y}$$

$$\int y\,dy = \int x\,dx$$

$$\frac{y^2}{2} = \frac{x^2}{2} + C$$

$$y^2 = x^2 + 2C \rightarrow \text{Constant}$$

The position vector of a particle changes with time according to the relation $\vec{r}(t) = 15t^2\hat{i} + (4 - 20t^2)\hat{j}$. What is the magnitude of the acceleration at $t = 1$? **[JEE (Main)-2019]**

(1) 50

(2) 100

(3) 40

(4) 25

$$\vec{r} = 15t^2\,\hat{i} + (4 - 20t^2)\,\hat{j}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = 30t\,\hat{i} - 40t\,\hat{j}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = 30\hat{i} - 40\hat{j} \rightarrow \text{Constant}$$

$$|\vec{a}| = \sqrt{(30)^2 + (40)^2} = 50\ m/s^2$$

A particle starts from the origin at t = 0 with an initial velocity of $3.0\hat{i}$ m/s and moves in the x-y plane with a constant acceleration $\left(6.0\hat{i}+4.0\hat{j}\right)$ m/s^2. The x-coordinate of the particle at the instant when its y-coordinate is 32 m is D meters. The value of D is **[JEE (Main)-2020]**

(1) 60 (2) 32

(3) 40 (4) 50

at $t=0$, $x=0, y=0$

$\vec{u} = 3\hat{i}$

$u_x = 3$, $u_y = 0$

$\vec{a} = 6\hat{i} + 4\hat{j}$

$a_x = 6$, $a_y = 4$

if $y = 32$ m

$y = u_y t + \frac{1}{2} a_y t^2$

$32 = 0 + \frac{1}{\cancel{2}} \times \cancel{4}^{2} \times t^2$

$t = \sqrt{\frac{32}{2}} = 4$ sec

$x = u_x t + \frac{1}{2} a_x t^2$

$= 3 \times 4 + \frac{1}{\cancel{2}} \times 6 \times \cancel{16}^{8}$

$= 12 + 48$

$= \underline{60 \text{ m}}$

Starting from the origin at time $t = 0$, with initial velocity $5\hat{j}$ ms^{-1}, a particle moves in the x-y plane with a constant acceleration of $(10\hat{i} + 4\hat{j})$ ms^{-2}. At time t, its coordinates are (20 m, y_0 m). The values of t and y_0 are, respectively **[JEE (Main)-2020]**

(1) 4 s and 52 m

(2) 2 s and 24 m

(3) 5 s and 25 m

✓(4) 2 s and 18 m

at $t=0$, $x=0$, $y=0$

$\vec{u} = 5\hat{j}$ m/s

$u_x = 0$, $u_y = 5$

$\vec{a} = 10\hat{i} + 4\hat{j}$

$a_x = 10$, $a_y = 4$

at t $\quad (20, y_0)$
$\quad\quad\quad \downarrow_x \quad \downarrow_y$

$x = u_x t + \frac{1}{2} a_x t^2$

$20 = 0 + \frac{1}{\cancel{2}} \times \cancel{10}^{5} \times t^2$

$t^2 = \frac{20}{5} = 4 \Rightarrow \underline{t = 2\text{sec}}$ ✓

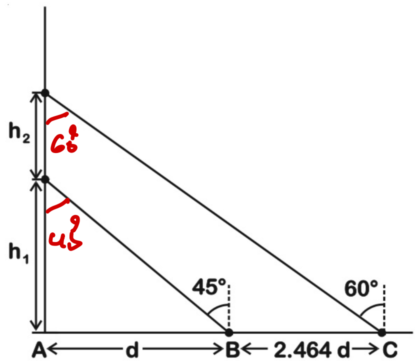
$y = u_y t + \frac{1}{2} a_y t^2$

$y_0 = 5 \times 2 + \frac{1}{\cancel{2}} \times 4 \times \cancel{4}^{2}$

$= 10 + 8$

$= \underline{18 \text{ m}}$ ✓

A balloon is moving up in air vertically above a point A on the ground. When it is at a height h_1, a girl standing at a distance d (point B) from A (see figure) sees it at an angle 45° with respect to the vertical. When the balloon climbs up a further height h_2, it is seen at an angle 60° with respect to the vertical if the girl moves further by a distance 2.464 d (point C). Then the height h_2 is (given tan30° = 0.5774) **[JEE (Main)-2020]**



(1) d (2) 0.732 d

(3) 1.464 d (4) 0.464 d

$$\tan 45^\circ = \frac{d}{h_1} \Rightarrow \underline{d = h_1}$$

$$\tan 60^\circ = \frac{d + 2.464d}{h_1 + h_2}$$

$$\sqrt{3} = \frac{d + 2.464d}{d + h_2}$$

$h_2 = ?$

$\underline{h_2 = d}$

A mosquito is moving with a velocity $\vec{v} = 0.5t^2\hat{i} + 3t\hat{j} + 9\hat{k}$ m/s and accelerating in uniform conditions. What will be the direction of mosquito after 2 s? **[JEE (Main)-2021]**

(1) $\tan^{-1}\left(\frac{\sqrt{85}}{6}\right)$ from y-axis

(2) $\tan^{-1}\left(\frac{5}{2}\right)$ from y-axis

(3) $\tan^{-1}\left(\frac{5}{2}\right)$ from x-axis

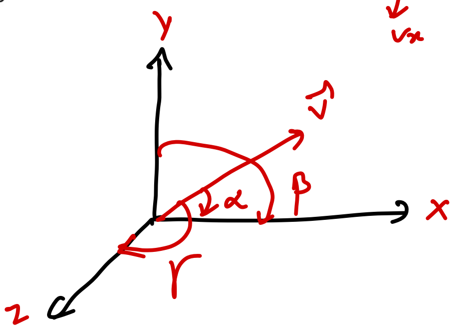
(4) $\tan^{-1}\left(\frac{2}{3}\right)$ from x-axis

$\vec{v} = 0.5\,t^2\hat{i} + 3t\hat{j} + 9\hat{k}$

at $t = 2s$

$\vec{v} = 0.5\times 4\,\hat{i} + 3\times 2\,\hat{j} + 9\hat{k}$

$\vec{v} = \underbrace{2\hat{i}}_{v_x} + \underbrace{6\hat{j}}_{v_y} + \underbrace{9\hat{k}}_{v_z}$



$\cos\alpha = \frac{v_x}{|\vec{v}|} = \frac{2}{\sqrt{4+36+81}}$

$\cos\alpha = \frac{2}{11}$

11, α, 2, $\sqrt{121-4} = \sqrt{117}$

$\tan\alpha = \frac{\sqrt{117}}{2}$

$\cos\beta = \frac{v_y}{|\vec{v}|} = \frac{6}{11}$

11, β, 6, $\sqrt{121-36} = \sqrt{85}$

$\tan\beta = \frac{\sqrt{85}}{6}$

$\beta = \tan^{-1}\left(\frac{\sqrt{85}}{6}\right)$ with y-axis

Motion of a particle in x-y plane is described by a set of following equations $x = 4\sin\left(\frac{\pi}{2} - \omega t\right)$ m and $y = 4\sin(\omega t)$ m. The path of the particle will be

[JEE (Main)-2022]

(1) Circular (2) Helical

(3) Parabolic (4) Elliptical

$x = 4\sin(90 - \omega t) = 4\cos\omega t$

$y = 4\sin\omega t$

$x^2 + y^2 = 4^2$ circle equation with radius '4'

At time $t = 0$ a particle starts travelling from a height $7\hat{z}$ cm in a plane keeping z coordinate constant. At any instant of time it's position along the $\hat{x}$ and $\hat{y}$ directions are defined at $3t$ and $5t^3$ respectively. At $t = 1$ s acceleration of the particle will be

[JEE (Main)-2022]

(1) $-30\hat{y}$

(2) $30\hat{y}$

(3) $3\hat{x}+15\hat{y}$

(4) $3\hat{x}+15\hat{y}+7\hat{z}$

$x = 3t \qquad y = 5t^3 \qquad z = 7$

$v_x = \frac{dv_x}{dt} = 3 \qquad v_y = \frac{dy}{dt} = 15t^2 \qquad v_z = 0$

$a_x = \frac{dv_x}{dt} = 0 \qquad a_y = 30t \qquad a_z = 0$

at $t = 1s \qquad a_y = 30$

$\vec{a} = 30\hat{y}$

Starting at time $t = 0$ from the origin with speed 1 ms^{-1}, a particle follows a two-dimensional trajectory in the x-y plane so that its coordinates are related by the equation $y = \frac{x^2}{2}$. The x and y components of its acceleration are denoted by a_x and a_y, respectively. Then **[JEE (Adv)-2020 (Paper-2)]**

(A) $a_x = 1$ ms^{-2} implies that when the particle is at the origin, $a_y = 1$ ms^{-2}

(B) $a_x = 0$ implies $a_y = 1$ ms^{-2} at all times

(C) at $t = 0$, the particle's velocity points in the x-direction

(D) $a_x = 0$ implies that at $t = 1$ s, the angle between the particle's velocity and the x axis is 45°

at $t=0$, $x=0$, $y=0$

$y = \frac{x^2}{2}$

diff. w.r.t time $\frac{dy}{dt} = \cancel{\frac{1}{2}}\cancel{2}x\frac{dx}{dt}$

$\boxed{v_y = x v_x}$

orgin at $x=0 \Rightarrow \boxed{v_y = 0}$

diff. w.r.t time 't'

$\frac{dv_y}{dt} = x\frac{dv_x}{dt} + \frac{dx}{dt}\cdot v_x$

$\boxed{a_y = x a_x + v_x^2}$

orgin at $t=0$, $x=0$, $a_y = v_x^2$

$a_y = x a_x + v_x^2$

$a_x = 0$

$\Rightarrow a_y = v_x^2 = 1\ m/s^2$

$y = \frac{1}{2}a_y t^2$

$y = \frac{1}{2}\times 1\times 1 = \frac{1}{2}$

$y = x^2/2$

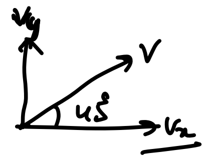
$\frac{1}{2} = \frac{x^2}{2}$

$\Rightarrow x = 1m$

$v_y = x v_x$

$v_y = 1\times 1$

$v_x = 1$



(C) at $t=0$, $x=0 \Rightarrow v_y = 0$

$u = 1\ m/s$ $\quad u_x = 1\ m/s$ $\quad v_x = u_x = 1\ m/s$

if $a_x = 0$ $\quad a_y = v_x^2 = 1\ m/s^2$

at origin $a_y = 1\ m/s^2$

Sol:

$$y = \frac{x^2}{2}$$

$$v_y = \frac{2x}{2}\frac{dx}{dt} = x v_x$$

$$a_y = x\frac{dv_x}{dt} + v_x \cdot \frac{dx}{dt}$$

$$= x \cdot a_x + v_x \cdot v_x$$

$$a_y = x a_x + v_x^2$$

* at $\underline{t = 0\,s}$ $\quad x = 0,\ y = 0 \Rightarrow v_y = 0$

$\Rightarrow v_x = 1\,m/s$ (given $v = 1\,m/s$)

$$a_y = 0 \cdot a_x + v_x^2 = 1\,m/s^2$$

* when particle is at origin, irrespective of values of a_x $\quad a_y = 1\,m/s^2$.

* when $a_x = 0$, $\quad a_y = 1\,m/s^2$ for all times.

* at $\underline{t = 1\,s}$ and $a_x = 0 \Rightarrow v_x = 1\,m/s$

$$v_y = x v_x = x$$

$$a_y = x a_x + v_x^2 = v_x^2 = 1\,m/s^2$$

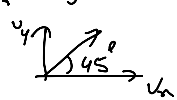
$$y = \frac{1}{2} a_y t^2 = \frac{1}{2} \times 1 \times 1 = \frac{1}{2}\,m.$$

$$y = \frac{x^2}{2}$$

$$x^2 = 2y = 2 \times \frac{1}{2} = 1 \Rightarrow x = 1\,m$$

* $v_y = x = 1\,m/s$ $\underline{\text{at } t = 1\,s}$

$v_x = v_y = 1\,m/s$



The coordinates of a particle moving in a plane are given by $x(t) = a\cos(pt)$ and $y(t) = b\sin(pt)$ where a, $b (< a)$ and p are positive constants of appropriate dimensions. Then **[1999S - 3 Marks]**

✓ (a) the path of the particle is an ellipse

✓ (b) the velocity and acceleration of the particle are normal to each other at $t = \pi/(2p)$

✓ (c) the acceleration of the particle is always directed towards a focus

✗ (d) the distance travelled by the particle in time interval $t = 0$ to $t = \pi/(2p)$ is a

at $t = \frac{\pi}{2p}$

$x = 0 \qquad y = b$

at $t = 0$

$x = a \qquad y = 0$

$t = 0 \rightarrow t = \frac{\pi}{2p}$

$a \rightarrow b$

$x = a\cos pt \qquad y = b\sin pt$

$\frac{x}{a} = \cos pt \qquad \frac{y}{b} = \sin pt$

$\cos^2 pt + \sin^2 pt = \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \leftarrow$ ellipse

$v_x = -ap\sin pt \qquad v_y = bp\cos pt$

$a_x = -ap^2\cos pt \qquad a_y = -bp^2\sin pt$

at $t = \frac{\pi}{2p}$ $\quad \boxed{v_x = -ap} \qquad v_y = 0$

$a_x = 0, \quad \boxed{a_y = -bp^2}$

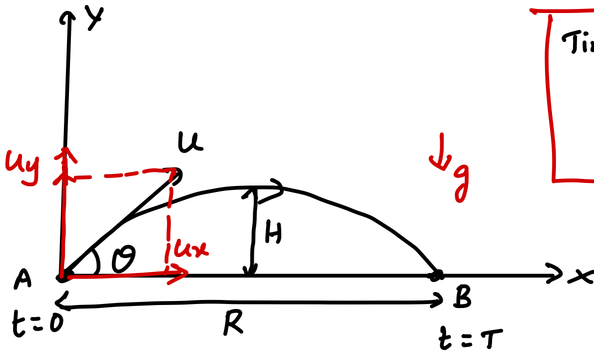
at $t = \frac{\pi}{2p}$ $\quad \vec{v} \perp r\ \vec{a}$

✓ $\vec{a} = -p^2(a\cos pt\,\hat{i} + b\sin pt\,\hat{j})$

towards focus

Projectile Motion - I

* ground to ground



Time of Flight

$$T = \frac{2u\sin\theta}{g} = \frac{2u_y}{|a_y|}$$

$u_x = u\cos\theta$ $a_x = 0$

$u_y = u\sin\theta$ $a_y = -g$

Horizontal Range (R) : It is Maximum horizontal distance covered by the projectile.

$$\# \quad \boxed{R = u_x T = \frac{2u_x u_y}{|a_y|} = \frac{2u^2\sin\theta\cos\theta}{g} = \frac{u^2\sin 2\theta}{g}}$$

Maximum height (H) :

$$\# \quad \boxed{H = \frac{u^2\sin^2\theta}{2g} = \frac{u_y^2}{2|a_y|}}$$

* velocity at any time $v_x = u_x + \cancel{a_x}t \Rightarrow \boxed{v_x = u_x}$

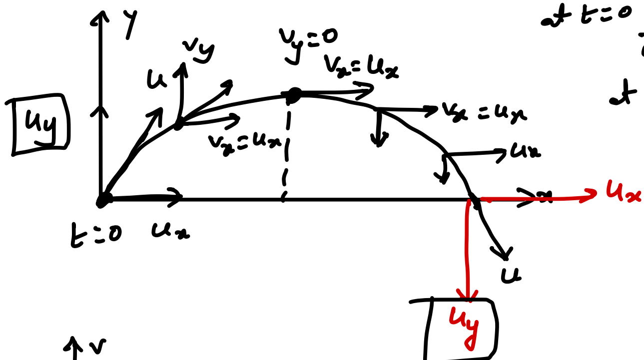
along horizontal direction velocity is constant

* velocity along y-axis (at any time t)

$$v_y = u_y + a_y t = u_y - gt$$

along Y-axis first body retard, then accelerate.

from point of projection to Maximum height speed decreases. Then from Max. height to ground speed increases.



at $t = 0$

$$\vec{u} = u_x \hat{i} + u_y \hat{j}$$

at $t = T$

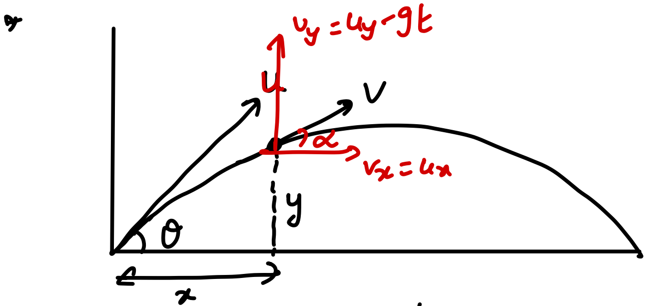
$$\vec{v} = u_x \hat{i} - u_y \hat{j}$$

body retards

body accelerates.

time of ascent (t_a) : time to reach Max. height

$$\boxed{t_a = \frac{T}{2} = \frac{u_y}{g} = \frac{u \sin\theta}{g}}$$



$$\alpha = \tan^{-1}\left[\frac{v_y}{v_x}\right]$$

$$\alpha = \tan^{-1}\left[\frac{u_y - gt}{u_x}\right]$$

at time 't' $\quad x = u_x t \Rightarrow t = \frac{x}{u_x}$

$$y = u_y t - \frac{1}{2} g t^2$$

$$\tan\theta = \frac{u_y}{u_x}$$

$$y = \left(\frac{u_y}{u_x}\right)x - \frac{1}{2}\frac{g}{u_x^2}x^2$$

$$y = (\tan\theta)\,x - \left(\frac{g}{2u_x^2}\right)x^2$$

$$\boxed{y = Ax - Bx^2} \rightarrow \underline{\text{Parabola}}$$

$A = \tan\theta \qquad B = \dfrac{g}{2u_x^2} \qquad (u_x = u\cos\theta)$

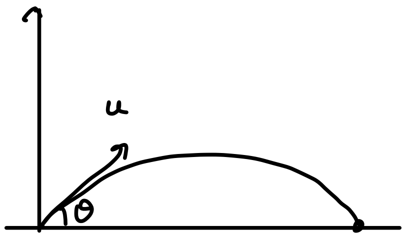
<u>Trajectory equation</u> : equation of path travelled by the projectile

here path is parabola

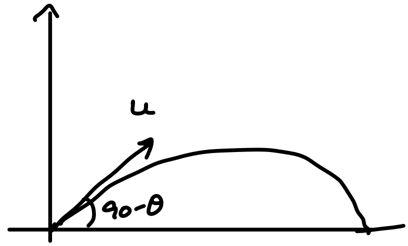
$$\# \quad \boxed{y = (\tan\theta)\,x - \left(\frac{g}{2u^2\cos^2\theta}\right)x^2 = x\tan\theta\left[1 - \frac{x}{R}\right]}$$

R is Range $\quad R = \dfrac{2u_x u_y}{g}$

*



$$R_1 = \frac{u^2 \sin 2\theta}{g}$$



$$R_2 = \frac{u^2 \sin 2(90-\theta)}{g}$$

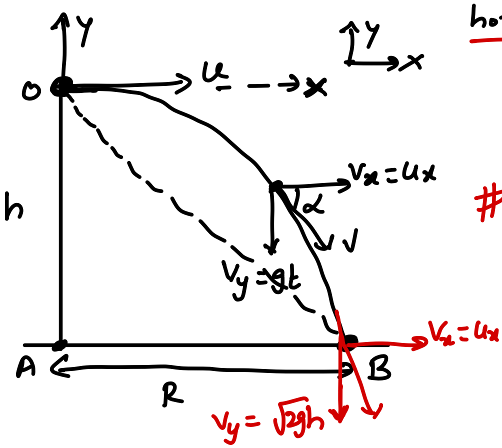
$$R_2 = \frac{u^2 \sin 2\theta}{g}$$

$R_1 = R_2$

for complementary angles (sum of the angles is 90°) Range is same

$(20^\circ, 70^\circ)$, $(45^\circ, 45^\circ)$, $(30^\circ, 60^\circ)$, $(40^\circ, 50^\circ)$

* Projectile Motion from Certain height above the ground



$u_x = u$, $u_y = 0$

$a_x = 0$, $a_y = -g$

horizontal Projectile motion

time of Flight

#

$$t = \sqrt{\frac{2h}{g}}$$

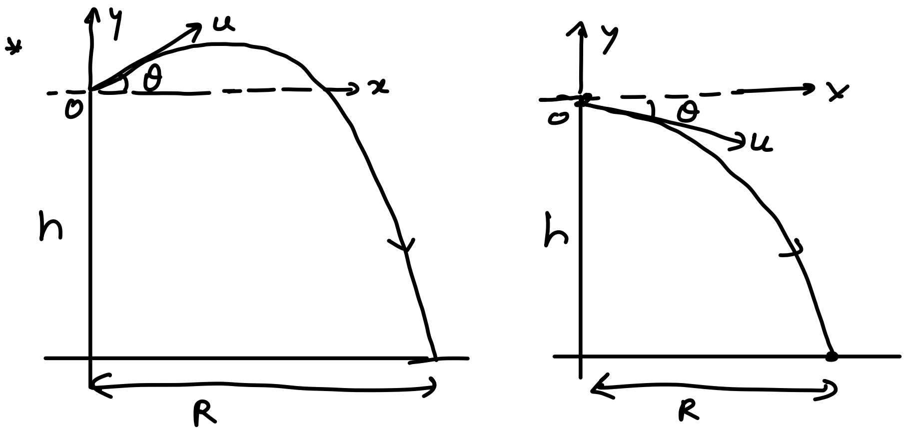
$$\text{Range } R = ut = u\sqrt{\frac{2h}{g}}$$

$$\alpha = \tan^{-1}\left[\frac{v_x}{v_y}\right]$$

$$\Rightarrow \boxed{\alpha = \tan^{-1}\left[\frac{u_x}{gt}\right]}$$

$$OB = \sqrt{h^2 + R^2}$$

trajectory is Parabola

* 

$u_x = u\cos\theta$, $a_x = 0$
$u_y = u\sin\theta$, $a_y = -g$

$u_x = u\cos\theta$, $a_x = 0$
$u_y = -u\sin\theta$, $a_y = -g$

To find Time of Flight

use $y = u_y t + \frac{1}{2} a_y t^2$

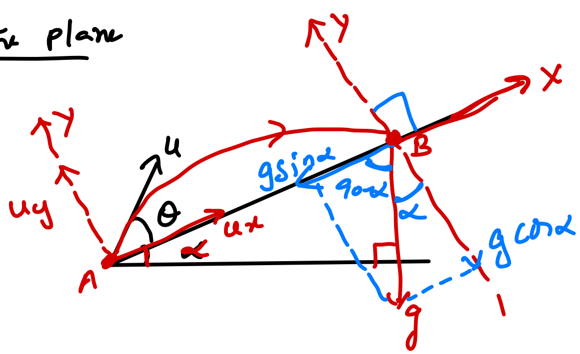
$-h = \underline{u_y t - \frac{1}{2} g t^2}$ ✓ $t = ?$

To Find Range (R) $x = u_x t + \frac{1}{2} \cancel{a_x} t^2$, $a_x = 0$

$\underline{R = x = u_x t}$ ✓

* Projectile motion on inclined plane :

① up the plane



$u_x = u\cos\theta$
$u_y = u\sin\theta$
$a_x = -g\sin\alpha$
$a_y = -g\cos\alpha$

To find Time of Flight $\quad (AB)_y = 0$

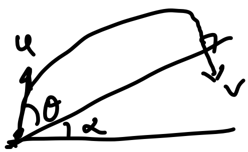
$$y = 0 \Rightarrow u_y T + \frac{1}{2} a_y T^2 = 0$$

$$\boxed{T = \frac{2u_y}{|a_y|} = \frac{2u\sin\theta}{g\cos\alpha}}$$

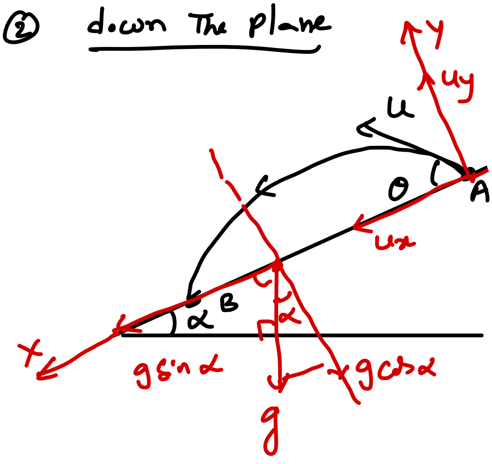
Range $(AB)_x = R$

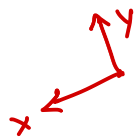
$$x = u_x T + \frac{1}{2} a_x T^2$$

$$V_x = 0 \Rightarrow u_x + a_x t = 0$$



② down The plane





$u_x = u\cos\theta$

$u_y = u\sin\theta$

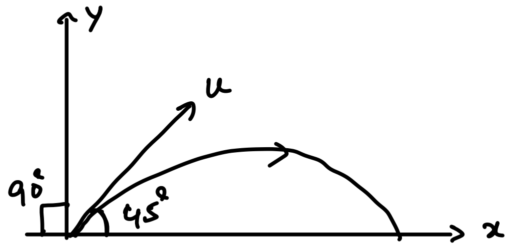
$a_x = g\sin\alpha$

$a_y = -g\cos\alpha$

$$\boxed{T = \frac{2u_y}{|a_y|} = \frac{2u\sin\theta}{g\cos\alpha}}$$

For Range $\quad R = u_x T + \frac{1}{2} a_x T^2$

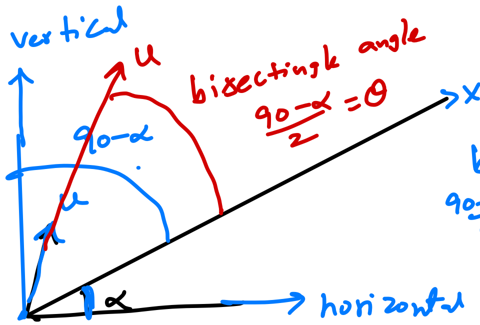
Finding Maximum Range

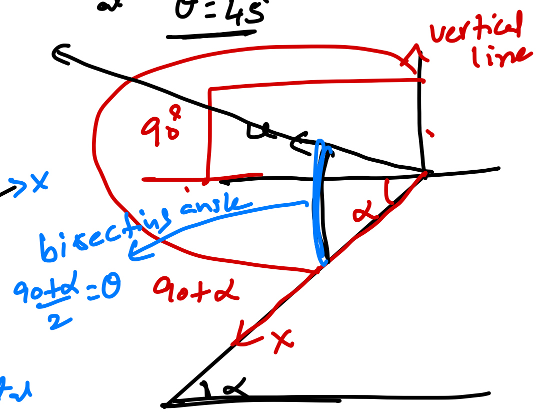


if you project the particle with bisecting angle Range will be maximum

$$\boxed{R_{max} = \frac{u^2}{g}}$$

at $\theta = 45°$





for R_{max} $\Rightarrow$ $\boxed{\theta = \frac{90-\alpha}{2} = 45° - \frac{\alpha}{2}}$

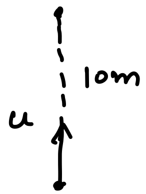
for R_{max}

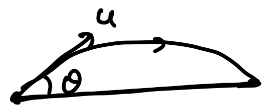
$$\boxed{\theta = \frac{90+\alpha}{2} = 45° + \frac{\alpha}{2}}$$

A boy can throw a stone up to a maximum height of 10 m. The maximum horizontal distance that the boy can throw the same stone up to will be

[AIEEE-2012]

(1) 10 m
(2) $10\sqrt{2}$ m
(3) 20 m
(4) $20\sqrt{2}$ m





$$h_{max} = \frac{u^2}{2g} = 10$$

$$\Rightarrow \frac{u^2}{g} = 20m$$

for R_{max}, $\theta = 45°$

$$R_{max} = \frac{u^2}{g} = 20m$$

Two guns *A* and *B* can fire bullets at speeds $\boxed{1\text{ km/s}}$ and $\boxed{2\text{ km/s}}$ respectively. From a point on a horizontal ground, they are fired in all possible directions. The ratio of maximum areas covered by the bullets fired by the two guns, on the ground is

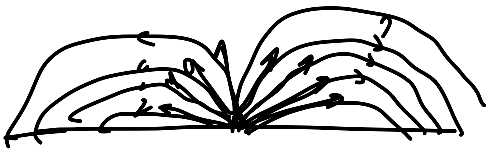
[JEE (Main)-2019]

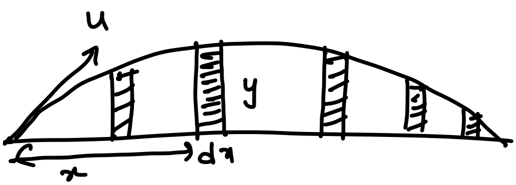
(1) 1 : 4 (2) 1 : 8

(3) 1 : 2 ✓(4) 1 : 16

$A \propto v^4$

\# Maximum Area of Path is not Maximum πR_{Max}^2 ↓ Range





$$y = u_y t + \frac{1}{2} a_y t^2$$

$$y = (\tan\theta)x - \frac{g}{2u^2\cos^2\theta} \cdot x^2$$

Area of strip $dA = y\,dx$

$$A = \int \left((u\sin\theta)t - \frac{1}{2}gt^2\right) dx$$

$$A_{Max} \propto v^4$$

$$\frac{A_1}{A_2} = \left(\frac{1}{2}\right)^4 = \frac{1}{16}$$

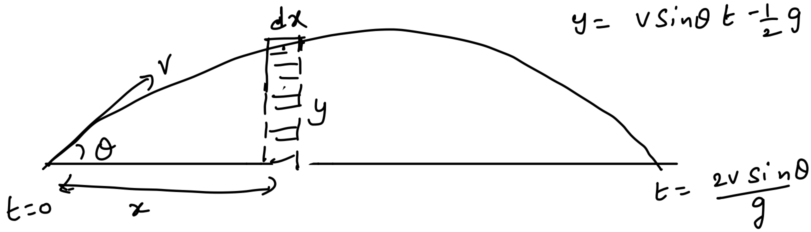
A ball is thrown at speed v from zero height on level ground. At what angle should it be thrown so that the area under the trajectory is maximum?

Sol^n:-

$x = v\cos\theta\, t$

$dx = v\cos\theta\, dt$

$y = v\sin\theta\, t - \frac{1}{2} g t^2$



Area of strip $\quad dA = y\,dx$

$$A = \int_0^{\frac{2v\sin\theta}{g}} \left(v\sin\theta\, t - \frac{1}{2} g t^2\right) v\cos\theta\, dt$$

$$A = v^2 \sin\theta \cdot \cos\theta \int_0^{\frac{2v\sin\theta}{g}} t\, dt - \frac{v}{2} g\cos\theta \int_0^{\frac{2v\sin\theta}{g}} t^2\, dt$$

$$= v^2\sin\theta\cos\theta \left[\frac{t^2}{2}\right]_0^{\frac{2v\sin\theta}{g}} - \frac{vg\cos\theta}{2}\left[\frac{t^3}{3}\right]_0^{\frac{2v\sin\theta}{g}}$$

$$= \frac{v^2\sin\theta\cos\theta}{\cancel{2}} \cdot \frac{\overset{2}{\cancel{4}}v^2\sin^2\theta}{g^2} - \frac{vg\cos\theta}{\underset{3}{\cancel{6}}} \frac{\overset{4}{\cancel{8}}v^3\sin^3\theta}{g^3}$$

$$A = \frac{2v^4\sin^3\theta \cdot \cos\theta}{3g^2}$$

$\boxed{A = \dfrac{2v^4}{3g^2} \sin^3\theta \cos\theta}$ $\qquad A \propto \dfrac{v^4}{g^2}$.

For Maximum Area $\dfrac{dA}{d\theta} = 0$

$$(3\sin^2\theta \cdot \cos\theta) \cdot \cos\theta + \sin^3\theta \cdot (-\sin\theta) = 0$$

$$3\sin^2\theta \cdot \cos^2\theta = \sin^4\theta$$

$$3 = \tan^2\theta$$

$$\Rightarrow \tan\theta = \sqrt{3}$$

$$\underline{\theta = 60^\circ}$$

at $\underline{\theta = 60^\circ}$ $\Rightarrow$ A is Maximum

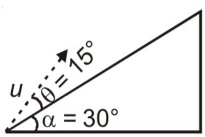
$$A_{max} = \frac{2v^4}{3g^2} \sin^3 60^\circ \cdot \cos 60^\circ$$

$$= \frac{\cancel{2}v^4}{\cancel{3}g^2} \frac{\cancel{3}}{4} \frac{\sqrt{3}}{2} \cdot \frac{1}{\cancel{2}}$$

$$\underline{A_{max}} = \underline{\frac{\sqrt{3}\,v^4}{8g^2}}$$

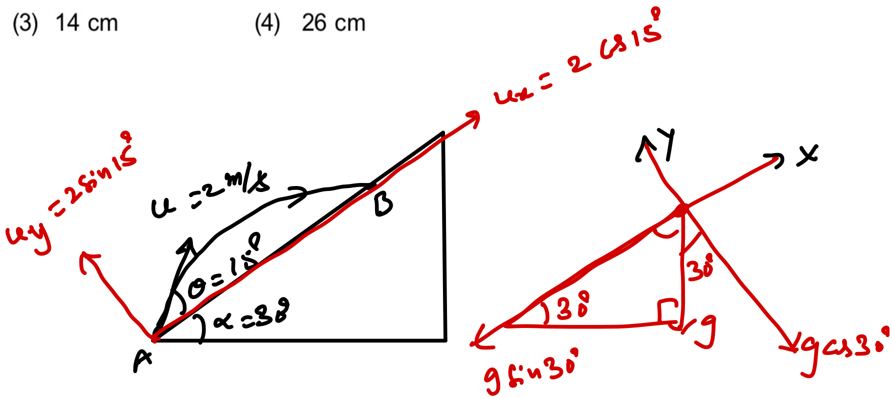
A plane is inclined at an angle $\alpha = 30°$ with respect to the horizontal. A particle is projected with a speed $u = 2\ ms^{-1}$, from the base of the plane, making an angle $\theta = 15°$ with respect to the plane as shown in the figure. The distance from the base, at which the particle hits the plane is close to :

(Take $g = 10\ ms^{-2}$) **[JEE (Main)-2019]**



(1) 18 cm ✓(2) 20 cm

(3) 14 cm (4) 26 cm



$u_x = 2\cos 15°$, $a_x = -g\sin 30°$

$u_y = 2\sin 15°$, $a_y = -g\cos 30°$

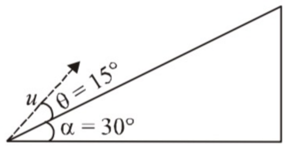
use

$\boxed{\begin{array}{l}\sin 15° = 0.26 \\ \cos 15° = 0.96\end{array}}$

$T = \dfrac{2u_y}{|a_y|} = \dfrac{2 \times 2\sin 15°}{g\sin 30°} =$ ______

Range $R = u_x T + \frac{1}{2} a_x T^2 =$ ______

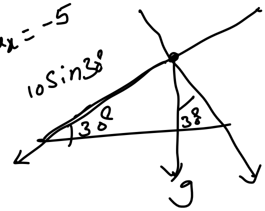
A plane is inclined at an angle $\alpha = 30^\circ$ with respect to the horizontal. A particle is projected with a speed $u = 2\ ms^{-1}$, from the base of the plane, as shown in figure. The distance from the base, at which the particle hits the plane is close to: (Take $g = 10\ ms^{-2}$)



(a) 20 cm (b) 18 cm (c) 26 cm (d) 14 cm

Sol:

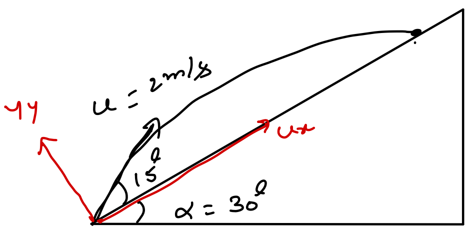
$a_x = -5$



$a_y = -5\sqrt{3}$

$\sin 15^\circ = 0.26$ ✓

$\cos 15^\circ = 0.96$ ✓



$$t = \frac{2u_y}{|a_y|} = \frac{2 \times 0.52}{5\sqrt{3}}$$

$$= \frac{1.04}{8.66}$$

$$= 0.12\ sec$$

$$\sin 15^\circ = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

$$u_x = 2 \times \cos 15^\circ = 2 \times 0.96 = 1.92$$

$$u_y = 2 \times \sin 15^\circ = 2 \times 0.26 = 0.52$$

$$x = u_x t + \frac{1}{2} a_x t^2$$

$$= 1.92 \times 0.12 - \frac{1}{2} \times 5 \times (0.12)^2$$

$$= 0.2304 - 0.036$$

$$= 0.194\ m$$

$$= 19.4\ cm$$

$$\underline{x \simeq 20\ cm}$$ ✓

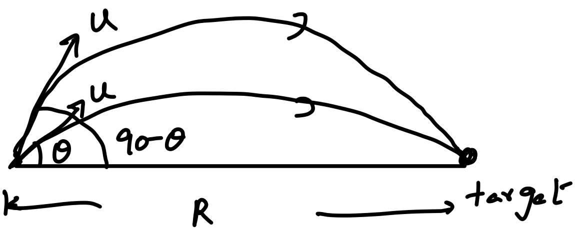
A shell is fired from a fixed artillery gun with an initial speed u such that it hits the target on the ground at a distance R from it. If t_1 and t_2 are the values of the time taken by it to hit the target in two possible ways, the product $t_1 t_2$ is :

[JEE (Main)-2019]

(1) $\frac{R}{2g}$ (2) $\frac{2R}{g}$

(3) $\frac{R}{g}$ (4) $\frac{R}{4g}$

to get same Range angle of Projections should be complementary



$$t_1 = \frac{2u\sin\theta}{g}$$

$$t_2 = \frac{2u\sin(90-\theta)}{g}$$

$$t_2 = \frac{2u\cos\theta}{g}$$

$$\left(R = \frac{2u^2\sin\theta\cos\theta}{g}\right)$$

$$t_1 \cdot t_2 = \frac{4u^2\sin\theta\cdot\cos\theta}{g^2}$$

$$\Rightarrow t_1 t_2 = \frac{2R}{g}$$

The trajectory of a projectile near the surface of the earth is given as $y = 2x - 9x^2$. If it were launched at an angle θ_0 with speed v_0 then ($g = 10\ \text{ms}^{-2}$): **[JEE (Main)-2019]**

(1) $\theta_0 = \sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$ and $v_0 = \frac{5}{3}\text{ms}^{-1}$

(2) $\theta_0 = \cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$ and $v_0 = \frac{3}{5}\text{ms}^{-1}$

(3) $\theta_0 = \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$ and $v_0 = \frac{5}{3}\text{ms}^{-1}$

(4) $\theta_0 = \sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$ and $v_0 = \frac{3}{5}\text{ms}^{-1}$

$y = 2x - 9x^2$

Trajectory eqn

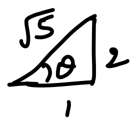
$y = (\tan\theta)\,x - \left(\frac{g}{2u^2\cos^2\theta}\right)x^2$

$\tan\theta = 2$

$\sin\theta = \frac{2}{\sqrt{5}}$

$\cos\theta = \frac{1}{\sqrt{5}}$

$\tan\theta = \frac{2}{1}$



$\frac{g}{2u^2\cos^2\theta} = 9$

$\frac{\cancel{10}\,5}{\cancel{2}\,u^2 \times \frac{1}{5}} = 9$

$u^2 = \frac{25}{9}$

$u = 5/3 = V_0$

Two particles are projected from the same point with the same speed u such that they have the same range R, but different maximum heights, h_1 and h_2. Which of the following is correct?

[JEE (Main)-2019]

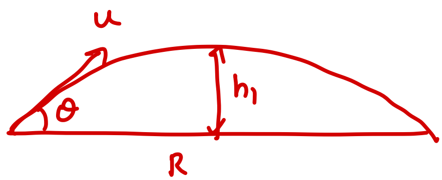
(1) $R^2 = 4\,h_1h_2$ (2) $R^2 = 16\,h_1h_2$

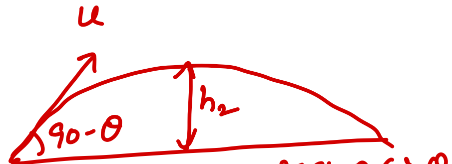
(3) $R^2 = 2\,h_1h_2$ (4) $R^2 = h_1h_2$

$$h_1 = \frac{u^2\sin^2\theta}{2g}$$

$$h_2 = \frac{u^2\sin^2(90-\theta)}{2g}$$

$$h_2 = \frac{u^2\cos^2\theta}{2g}$$





$$R = \frac{2u^2\sin\theta\cos\theta}{g}$$

$$R^2 = \frac{4u^4\sin^2\theta\cos^2\theta}{g^2}$$

$$\boxed{R^2 = 16\,h_1h_2}$$

$$R = 4\sqrt{h_1h_2}$$

Given below are two statements. One is labelled as **Assertion A** and the other is labelled as **Reason R**.

[JEE (Main)-2022]

Assertion A: Two identical balls A and B thrown with same velocity 'u' at two different angles with horizontal attained the same range R. If A and B reached the maximum height h_1 and h_2 respectively, then $R = 4\sqrt{h_1h_2}$.

Reason R: Product of said heights.

$$h_1h_2 = \left(\frac{u^2\sin^2\theta}{2g}\right)\cdot\left(\frac{u^2\cos^2\theta}{2g}\right)$$

Choose the correct answer :

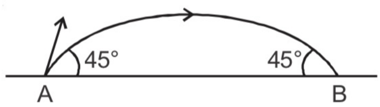
(1) Both **A** and **R** are true and **R** is the correct explanation of **A**.

(2) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**.

(3) **A** is true but **R** is false.

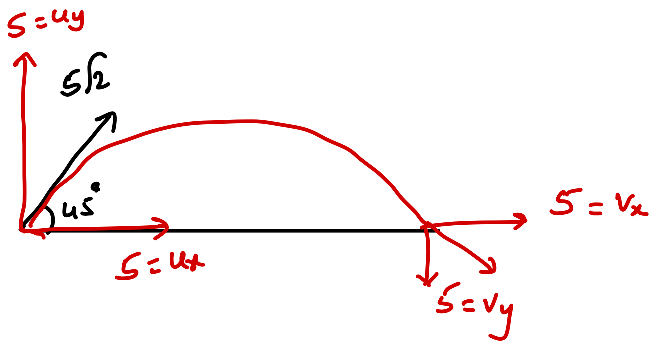
(4) **A** is false but **R** is true.

The projectile motion of a particle of mass 5 g is shown in the figure. **[JEE (Main)-2021]**



The initial velocity of the particle is $5\sqrt{2}$ ms^{-1} and the air resistance is assumed to be negligible. The magnitude of the change in momentum between the points A and B is $x \times 10^{-2}$ $kgms^{-1}$.

The value of x, to the nearest integer, is 5.



change in momentum $\vec{\Delta p} = m(\vec{v_f} - \vec{v_i})$

$$\vec{\Delta p} = 5 \times 10^{-3}\left((5\hat{i} - 5\hat{j}) - (5\hat{i} + 5\hat{j})\right)$$

$$= 5 \times 10^{-3}(-5\hat{j} - 5\hat{j})$$

$$= 5 \times 10^{-3}(-10\hat{j})$$

$$|\vec{\Delta p}| = 50 \times 10^{-3} = 5 \times 10^{-2} \ \frac{kg\text{-}m}{s}$$

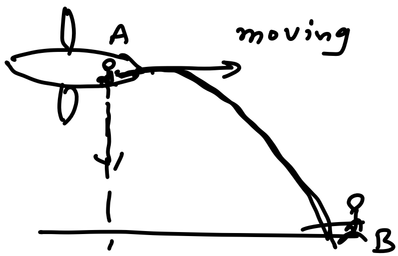
A bomb is dropped by a fighter plane flying horizontally. To an observer sitting in the plane, the trajectory of the bomb is a : **[JEE (Main)-2021]**

(1) Straight line vertically down the plane

(2) Hyperbola

(3) Parabola in the direction of motion of plane

(4) Parabola in a direction opposite to the motion of plane



w.r.t observer A in the plane path of bomb is st. line

w.r.t observer B (on the ground) path of bomb is parabola

option : 1

A player kicks a football with an initial speed of 25 ms^{-1} at an angle of 45° from the ground. What are the maximum height and the time taken by the football to reach at the highest point during motion? (Take g = 10 ms^{-2}) **[JEE (Main)-2021]**

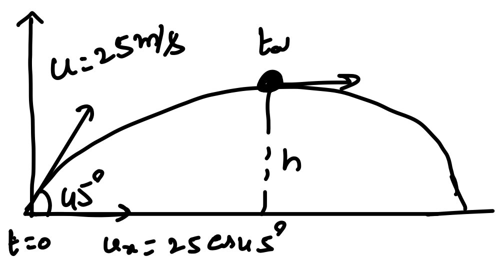
(1) h_{max} = 10 m, T = 2.5 s

(2) h_{max} = 15.625 m, T = 1.77 s

(3) h_{max} = 3.54 m, T = 0.125 s

(4) h_{max} = 15.625 m, T = 3.54 s

$u_y = 25 \sin 45° = \frac{25}{\sqrt{2}}$



$t_\omega = \frac{u_y}{g} = \frac{25/\sqrt{2}}{10}$

$= 2.5 \times 0.707$

$t_\omega \simeq 1.778$

$h_{max} = \frac{u_y^2}{2g} = 15.625$ m

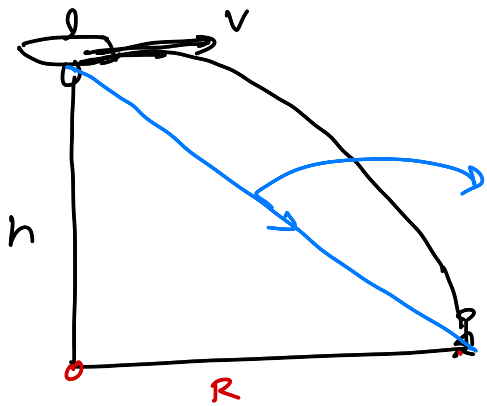
A helicopter is flying horizontally with a speed 'v' at an altitude 'h' has to drop a food packet for a man on the ground. What is the distance of helicopter from the man when the food packet is dropped? **[JEE (Main)-2021]**

(1) $\sqrt{\dfrac{2gh}{v^2}+h^2}$

(2) $\sqrt{2ghv^2+h^2}$

(3) $\sqrt{\dfrac{2v^2h}{g}+h^2}$

(4) $\sqrt{\dfrac{2ghv^2+1}{h^2}}$



$R = v\sqrt{\dfrac{2h}{g}}$

$S = \sqrt{h^2+R^2}$

A projectile is projected with velocity of 25 m/s at an angle θ with the horizontal. After t seconds its inclination with horizontal becomes zero. If R represents horizontal range of the projectile, the value of θ will be [use $g = 10$ m/s^2]

[JEE (Main)-2022]

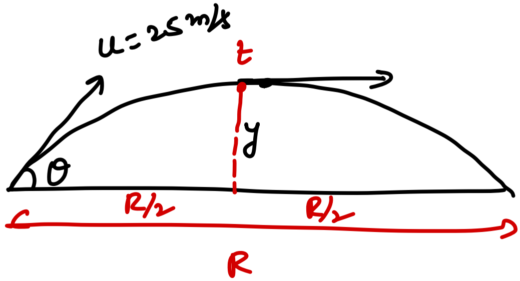
(1) $\frac{1}{2}\sin^{-1}\left[\frac{5t^2}{4R}\right]$

(2) $\frac{1}{2}\sin^{-1}\left[\frac{4R}{5t^2}\right]$

(3) $\tan^{-1}\left[\frac{4t^2}{5R}\right]$

(4) $\cot^{-1}\left[\frac{R}{20t^2}\right]$

$\boxed{\text{at max. height, velocity is parallel to horizontal inclination is zero}}$



$$t = \frac{u_y}{g} = \frac{25\sin\theta}{10} \Rightarrow t^2 = \frac{625}{100}\sin^2\theta$$

$$R = \frac{2u_x u_y}{g} = \frac{\cancel{2}\times \cancel{25}^{5}\cos\theta \times 25\sin\theta}{\cancel{10}\,\cancel{2}}$$

$$R = 125\cos\theta\cdot\sin\theta$$

$$\frac{R}{t^2} = \frac{125\cos\theta\cdot\cancel{\sin\theta}}{\frac{625}{100}\sin^{\cancel{2}}\theta} = \frac{\cancel{125}^{5}\times\cancel{100}^{4}}{\cancel{625}_{\cancel{25}}}\cot\theta$$

$$\frac{R}{t^2} = \frac{20}{1}\cot\theta$$

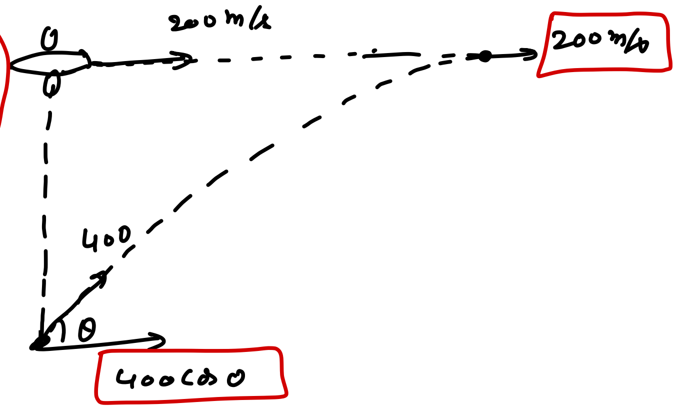
$$\cot\theta = \frac{R}{20t^2}$$

$$\theta = \cot^{-1}\left[\frac{R}{20t^2}\right]$$

A fighter jet is flying horizontally at a certain altitude with a speed of 200 ms^{-1}. When it passes directly overhead an anti-aircraft gun, a bullet is fired from the gun, at an angle θ with the horizontal, to hit the jet. If the bullet speed is 400 m/s, the value of θ will be ____°. **[JEE (Main)-2022]**

60°

bullet will hit the fighter jet. If their horizontal velocities are same



$200 = 400\cos\theta$

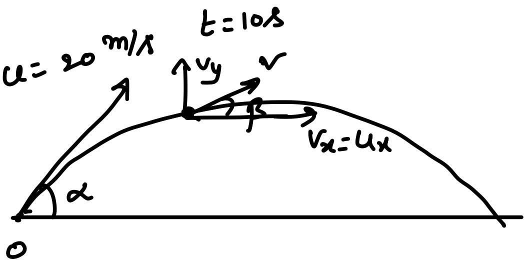
$\cos\theta = \frac{1}{2}$

$\theta = 60^\circ$

A projectile is launched at an angle 'α' with the horizontal with a velocity 20 ms^{-1}. After 10 s, its inclination with horizontal is 'β'. The value of tanβ will be (g = 10 ms^{-2}). **[JEE (Main)-2022]**

(1) $\tan\alpha + 5\sec\alpha$ (2) $\tan\alpha - 5\sec\alpha$

(3) $2\tan\alpha - 5\sec\alpha$ (4) $2\tan\alpha + 5\sec\alpha$



$V_x = u_x$

$v\cos\beta = u\cos\alpha = 20\cos\alpha$

$v_y = u_y - gt$

$v_y = 20\sin\alpha - 10\times10$

$v_y = 20\sin\alpha - 100$

$$\tan\beta = \frac{v_y}{V_x} = \frac{20\sin\alpha - 100}{20\cos\alpha}$$

$$= \tan\alpha - \frac{100}{20}\sec\alpha$$

$$\tan\beta = \tan\alpha - 5\sec\alpha$$

Two projectiles thrown at 30° and 45° with the horizontal respectively, reach the maximum height in same time. The ratio of their initial velocities is :

[JEE (Main)-2022]

(1) $1:\sqrt{2}$ (2) $2:1$

(3) $\sqrt{2}:1$ (4) $1:2$

$$t = \frac{u_y}{g}$$

$$t_1 = \frac{u_{1y}}{g} \quad , \quad t_2 = \frac{u_{2y}}{g}$$

$$t_1 = t_2 \Rightarrow u_{1y} = u_{2y}$$

$$u_1 \cdot \sin 30^\circ = u_2 \cdot \sin 45^\circ$$

$$u_1 \times \frac{1}{2} = u_2 \times \frac{1}{\sqrt{2}}$$

$$\frac{u_1}{u_2} = \frac{2}{\sqrt{2}} = \sqrt{2} : 1$$

If the initial velocity in horizontal direction of a projectile is unit vector $\hat{i}$ and the equation of trajectory is $y = 5x(1 - x)$. The y component vector of the initial velocity is 5 $\hat{j}$. **[JEE (Main)-2022]**

(Take $g = 10\ m/s^2$)

$$y = 5x - 5x^2$$

Trajectory eqn $\quad y = (\tan\theta)x - \left(\frac{g}{2u^2\cos^2\theta}\right)x^2$

$$\frac{g}{2u_x^2} = 5$$

$$\frac{10}{2u_x^2} = 5 \Rightarrow u_x = 1\ m/s$$

$$\tan\theta = 5$$

$$\frac{u_y}{u_x} = 5$$

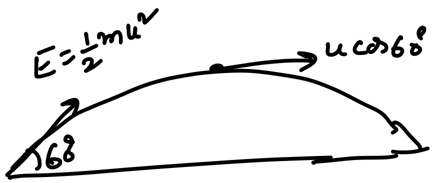
$$u_y = 5u_x = 5 \times 1 = 5$$

A ball is projected with kinetic energy E, at an angle of 60° to the horizontal. The kinetic energy of this hall at the highest point of its flight will become

[JEE (Main)-2022]

(1) Zero (2) $\frac{E}{2}$

(3) $\frac{E}{4}$ (4) E



K·E at highest point

$K = \frac{1}{2} m u^2 \cos^2 60°$

$= E \times \frac{1}{4}$

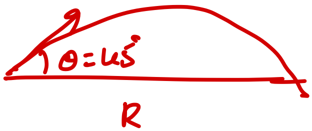
$= \frac{E}{4}$

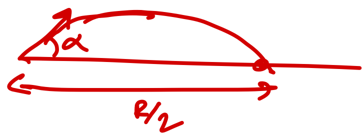
An object is projected in the air with initial velocity u at an angle θ. The projectile motion is such that the horizontal range R, is maximum. Another object is projected in the air with a horizontal range half of the range of first object. The initial velocity remains same in both the case. The value of the angle of projection, at which the second object is projected, will be 15 degree. **[JEE (Main)-2022]**

$\theta = 45^\circ$

$R_1 = R = \frac{u^2}{g}$

$R_2 = \frac{R}{2} = \frac{u^2}{2g}$





$$R_2 = \frac{1}{2}\left(\frac{u^2}{g}\right)$$

$$\frac{\cancel{u^2} \sin 2\alpha}{\cancel{g}} = \frac{1}{2} \frac{\cancel{u^2}}{\cancel{g}}$$

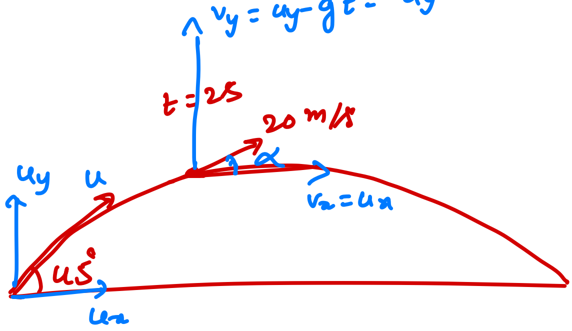
$$\sin 2\alpha = \frac{1}{2}$$

$$2\alpha = 30^\circ$$

$$\alpha = 15^\circ$$

A body is projected from the ground at an angle of 45° with the horizontal. Its velocity after 2 s is 20 ms^{-1}. The maximum height reached by the body during its motion is ___20___ m. (use g = 10 ms^{-2}) **[JEE (Main)-2022]**

$v_y = u_y - gt = u_y - 10\times 2 = u_y - 20$



$v^2 = v_x^2 + v_y^2$

$400 = u_x^2 + (u_y - 20)^2$

$\cancel{400} = \underbrace{u_x^2 + u_y^2}_{u^2} + \cancel{400} - 40u_y$

$u^2 = 40u_y$

$u^{\cancel{2}} = 40 \times \cancel{u} \sin 45°$

$u = 40 \times \frac{1}{\sqrt{2}} = \underline{20\sqrt{2}\ m/s}$

$H = \frac{u_y^2}{2g} = \frac{20\times 20}{2\times 10} = \underline{20 m}$

$u_y = 20 m/s$ ✓

$u_x = \underline{20 m/s}$ ✓

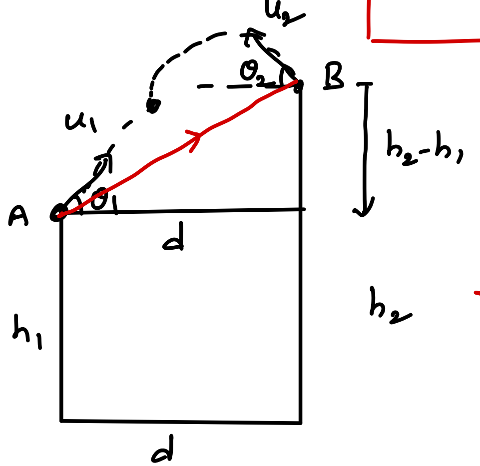
Projectile Motion - 2

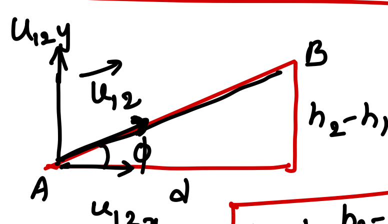
* Condition for collision between two Projectiles

To collide, There should not be any relative velocity along the line perpendicular to line joining b/w initial positions of the two Projectiles.

$\boxed{u_{1y} = u_{2y}}$

They collide at time $\boxed{t = \dfrac{\text{separation b/w initial positions of projectiles (AB)}}{|\vec{u}_{12}|_{\text{along AB}}}}$

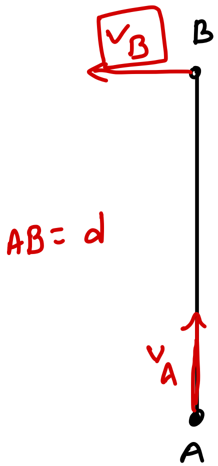




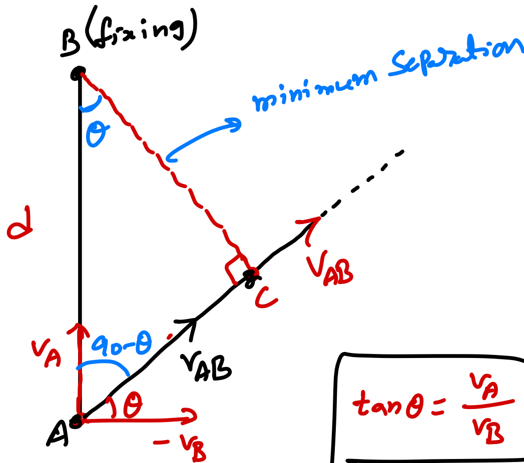
$$\boxed{t = \frac{AB}{|\vec{u}_{12}|}}$$

$$\boxed{\tan\phi = \frac{h_2-h_1}{d} = \frac{u_{12y}}{u_{12x}}}$$

→ Finding minimum separation b/n two moving particles and time for minimum separation :-



⇒



$$\boxed{\tan\theta = \frac{V_A}{V_B}}$$

$$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$$

$$|\vec{V}_{AB}| = \sqrt{V_A^2 + V_B^2}$$

drop a perpendicular line from one particle to relative velocity line

BC line gives minimum separation b/n two particle

$$\cos\theta = \frac{BC}{d}$$

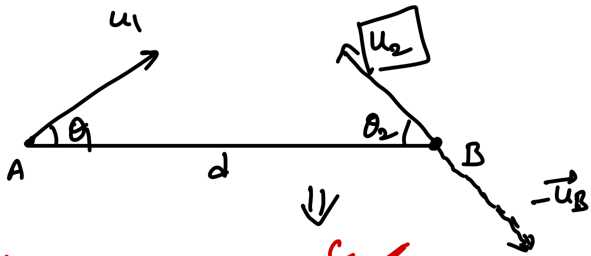
$$\Rightarrow \boxed{BC = d\cos\theta}$$

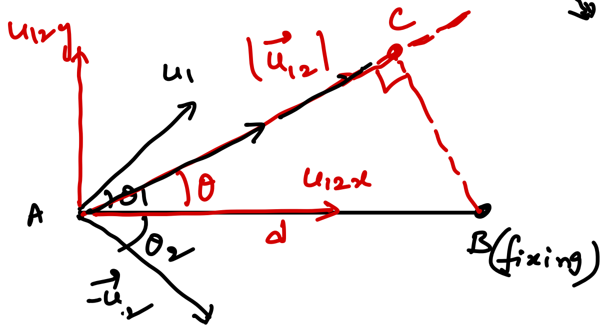
Time when separation b/n particles is minimum.

Time for minimum separation

$$\# \boxed{t = \frac{AC}{|\vec{V}_{AB}|} = \frac{d\sin\theta}{|\vec{V}_{AB}|}}$$

$$\sin\theta = \frac{AC}{d} \Rightarrow AC = d\sin\theta$$





$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$

$$\boxed{\tan\theta = \frac{u_{12y}}{u_{12x}}}$$

minimum separation $BC = d\cos\theta$ ✓

Time for minimum separation $t = \dfrac{AC}{|\vec{u}_{12}|} = \dfrac{d\sin\theta}{|\vec{u}_{12}|}$ ✓

$$|\vec{u}_{12}| = \sqrt{u_{12x}^2 + u_{12y}^2}$$

A train is moving along a straight line with a constant acceleration *a*. A boy standing in the train throws a ball forward with a speed of 10 m/s, at an angle of 60° to the horizontal. The boy has to move forward by 1.15 m inside the train to catch the ball back at the initial height. The acceleration of the train, in m/s^2, is

5

IIT-JEE-2011 (Paper-2)]

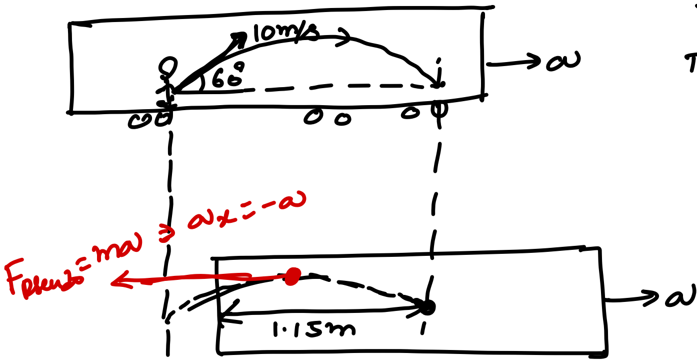
$u_x = 10\cos 60^\circ = 5$ m/s

$u_y = 10\sin 60^\circ = 5\sqrt{3}$ m/s

Time of Flight

$$T = \frac{2u_y}{g} = \frac{2\times 5\sqrt{3}}{10}$$

$T = \sqrt{3}$ sec



$$x = u_x t + \frac{1}{2} a_x t^2$$

$$1.15 = 5\times\sqrt{3} - \frac{1}{2} a (\sqrt{3})^2$$

$$1.15 = 5\sqrt{3} - \frac{3a}{2}$$

$$\frac{3a}{2} = 5\sqrt{3} - 1.15$$

$$\frac{5\times 1.7}{8.5}$$

$$1.5a = 8.5 - 1.15$$

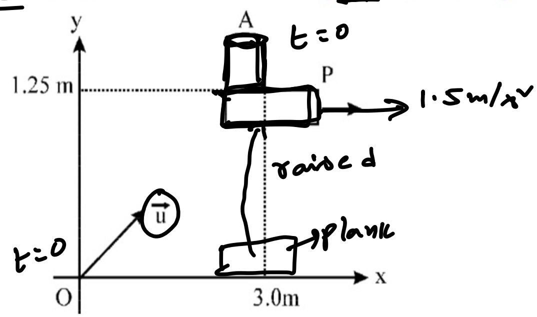
$$a = \frac{7.35}{1.5}$$

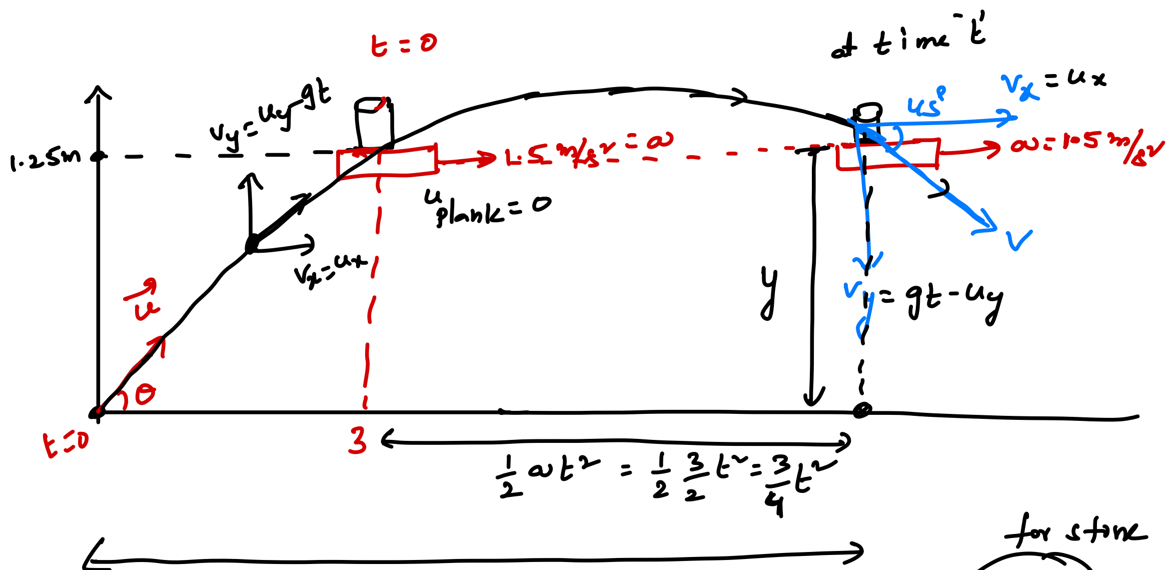
$$a \approx \sqrt{\frac{7.35}{1.5}} \approx 5 \text{ m/s}^2$$

An object A is kept fixed at the point $x = 3$ m and $y = 1.25$ m on a plank P raised above the ground. At time $t = 0$ the plank starts moving along the $+x$ direction with an acceleration $1.5\ m/s^2$. At the same instant a stone is projected from the origin with a velocity $\vec{u}$ as shown. A stationary person on the ground observes the stone hitting the object during its downward motion at an angle of 45° to the horizontal. All the motions are in the $X–Y$ plane. Find $\vec{u}$ and the time after which the stone hits the object. Take $g = 10$ m/s **[2000 - 10 Marks]**

Ans: $t = 1\ sec$

$\vec{u} = 3.75\,\hat{i} + 6.25\,\hat{j}$





for stone $a_y = -g$

for object $x = 3 + \frac{3}{4}t^2 \longrightarrow (1)$

for stone $y = u_y t + \frac{1}{2}a_y t^2 \Rightarrow 1.25 = u_y t - \frac{1}{2}g t^2 \longrightarrow (2)$

distance travelled by the stone in time 't'

$$x = u_x t + \frac{1}{2} a_x t^2$$ for stone $a_x = 0$

$$x = u_x t \longrightarrow 3$$

stone hitting the object with angle $45°$ with horizontal

$$\tan 45° = \frac{v_y}{v_x} = \frac{gt - u_y}{u_x} = 1$$

$$gt - u_y = u_x$$

$$gt = u_y + u_x \longrightarrow 4$$

t , u_x, u_y $\qquad \vec{u} = u_x \hat{i} + u_y \hat{j}$

by using 1, 2, 3 & 4 we can find $t, \vec{u}$

Calculations

* $x = 3 + \frac{1}{2} a t^2$

$u_x t = 3 + \frac{1}{2} \times 1.5 \, t^2 \rightarrow (1)$

0.75

* $y = u_y t - \frac{1}{2} g t^2$

$1.25 = u_y t - 5t^2$

$u_y t = 1.25 + 5t^2 \rightarrow (2)$

$(1) + (2) \Rightarrow$

$(u_x + u_y) t = 4.25 + 5.75 t^2$

$u_x + u_y = \frac{4.25}{t} + 5.75 t$

we have $gt = u_x + u_y$

$\Rightarrow u_x + u_y = 10t$

$\Rightarrow \frac{4.25}{t} + 5.75t = 10t$

$\Rightarrow \frac{4.25}{t} = 10t - 5.75t = 4.25t$

$\Rightarrow 4.25 = 4.25 t^2$

$\Rightarrow t^2 = 1 \Rightarrow \boxed{t = 1 \text{ sec}}$: Ans

$(1) \Rightarrow u_x \times 1 = 3 + 0.75 \times 1 = 3.75$

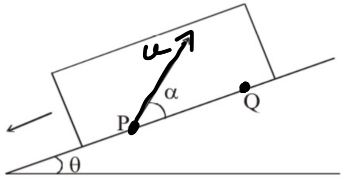
$\Rightarrow u_x = 3.75 \text{ m/s}$

$(2) \Rightarrow u_y \times 1 = 1.25 + 5 \times 1 = 6.25$

$\Rightarrow u_y = 6.25 \text{ m/s}$

Ans: $\boxed{\vec{u} = u_x \hat{i} + u_y \hat{j} = 3.75 \hat{i} + 6.25 \hat{j}}$

A large, heavy box is sliding without friction down a smooth plane of inclination θ. From a point P on the bottom of the box, a particle is projected inside the box. The initial speed of the particle with respect to the box is u, and the direction of projection makes an angle α with the bottom as shown in Figure. **[1998 - 8 Marks]**

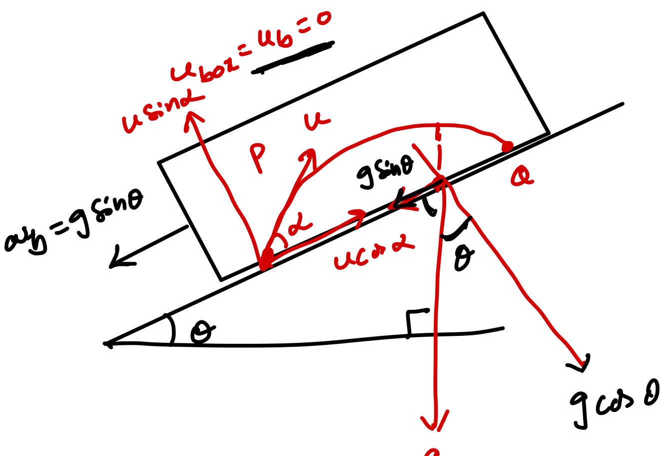


(a) Find the distance along the bottom of the box between the point of projection P and the point Q where the particle lands. (Assume that the particle does not hit any other surface of the box. Neglect air resistance.)

(b) If the horizontal displacement of the particle as seen by an observer on the ground is zero, find the speed of the box with respect to the ground at the instant when particle was projected.

Ans: (i) $\boxed{\dfrac{u^2 \sin 2\alpha}{g\cos\theta}}$

(ii) $\dfrac{u\cos(\theta+\alpha)}{\cos\theta}$



$u_{particle} = u_p$

$u_{pbx} = u_{px} = u\cos\alpha$

$u_{pby} = u_{py} = u\sin\alpha$

$a_{px} = -g\sin\theta$

$a_{py} = -g\cos\theta$

$a_{bx} = -g\sin\theta$

$a_{by} = 0$

$a_{pbx} = -g\sin\theta + g\sin\theta = 0$

$a_{pby} = -g\cos\theta - 0 = -g\cos\theta$

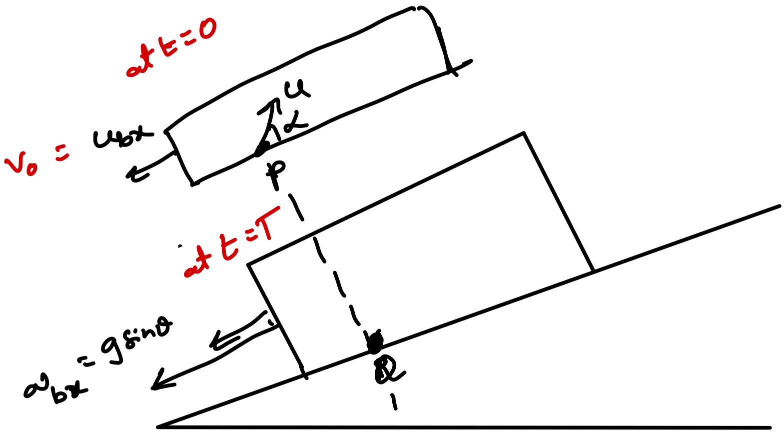
$y = 0$ for particle $P \to Q$

$$T = \frac{2u_y}{|a_y|} = \frac{2\times u\sin\alpha}{g\cos\theta}$$

Range PQ $\quad x_{Pb} = u_{p.bx}\, t + \frac{1}{2} a_{pbx} t^2 = u\cos\alpha \times \frac{2u\sin\alpha}{g\cos\theta}$

u_{px} $\quad 0$

$$PQ = \frac{u^2\sin 2\alpha}{g\cos\theta}$$



$$x = u_{bx} t + \frac{1}{2} a_{bx} t^2$$

$$-\frac{u^2 \sin 2\alpha}{g\cos\theta} = -v_0 \frac{2u\sin\alpha}{g\cos\theta} - \frac{1}{2} g\sin\theta \left(\frac{2u\sin\alpha}{g\cos\theta}\right)^2$$

$v_0 = ?$

(ii) To get zero horizontal displacement w.r.t ground.

box should move by $\underline{\dfrac{u^2 \sin 2\alpha}{g\cos\theta}}$ down the plane.

for that it should have initial velocity v_0

$$x = v_0 t + \frac{1}{2} a_x t^2 \qquad t = \frac{2u\sin\alpha}{g\cos\theta}$$

$$-\frac{u^2 \sin 2\alpha}{g\cos\theta} = -v_0 \cdot \frac{2u\sin\alpha}{g\cos\theta} - \frac{1}{2} g\sin\theta \left(\frac{2u\sin\alpha}{g\cos\theta}\right)^2$$

$$\Rightarrow \frac{u^2 \sin 2\alpha}{\cancel{g}\cos\theta} - \frac{1}{\cancel{2}} \cancel{g}\sin\theta \cdot \frac{\overset{2}{\cancel{4}} u^2 \sin^2\alpha}{g^{\cancel{2}}\cos^2\theta} = \frac{2v_0 u\sin\alpha}{\cancel{g}\cos\theta}$$

$$\Rightarrow \frac{\cancel{2}u^{\cancel{2}}\cancel{\sin\alpha}}{\cancel{\cos\theta}}\left[\cos\alpha - \frac{\sin\alpha \cdot \sin\theta}{\cos\theta}\right] = \frac{\cancel{2}v_0 \cancel{u}\cancel{\sin\alpha}}{\cancel{\cos\theta}}$$

$$\Rightarrow \frac{u}{\cos\theta}\left[\cos\theta\cos\alpha - \sin\theta\cdot\sin\alpha\right] = v_0$$

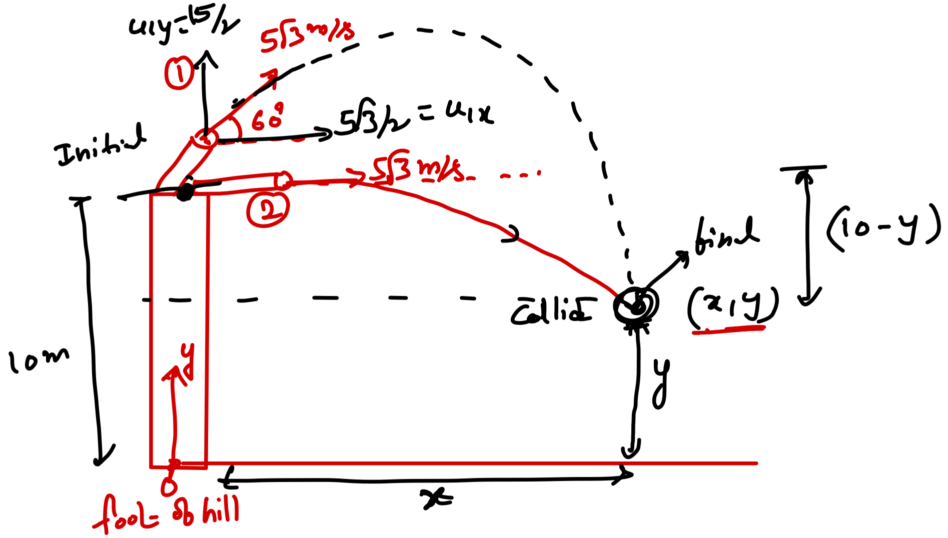
$$\Rightarrow \frac{u\cos(\theta+\alpha)}{\cos\theta} = v_0$$

Ans : $\boxed{v_0 = \dfrac{u\cos(\theta+\alpha)}{\cos\theta}}$ ✓

Two guns, situated on the top of a hill of height 10 m, fire one shot each with the same speed $5\sqrt{3}$ m s^{-1} at some interval of time. One gun fires horizontally and other fires upwards at an angle of 60° with the horizontal. The shots collide in air at a point P. Find (i) the time-interval between the firings, and (ii) the coordinates of the point P. Take origin of the coordinate system at the foot of the hill right below the muzzle and trajectories in x-y plane. **[1996 - 5 Marks]**

Ans : (i) 1sec

(ii) $(x, y) = (5\sqrt{3}, 5)$ m



when both bullets meets $x_1 = x_2$ and $y_1 = y_2$

bullet-1

$u_{1x} = \frac{5\sqrt{3}}{2}$

$u_{1y} = 5\sqrt{3} \times \frac{\sqrt{3}}{2} = \frac{15}{2}$

$a_{1x} = 0$

$a_{1y} = -10\ m/s^2$

bullet-2

$u_{2x} = 5\sqrt{3}$ m/s

$u_{2y} = 0$

$a_{1x} = 0$

$a_{1y} = -10\ m/s^2$

$x_1 = u_{1x} t_1 + \frac{1}{2}\cancel{a_{1x}} t^2$ (0)

$x_2 = u_{2x} t_2 + \frac{1}{2}\cancel{a_{2x}} t^2$

$x_1 = x_2 \Rightarrow \frac{5\sqrt{3}}{2} t_1 = 5\sqrt{3} t_2 \Rightarrow t_1 = 2t_2$ (1)

$y_1 = u_{1y} t + \frac{1}{2} a_{1y} t^2$

$-(10-y) = \frac{15}{2} t_1 - 5t_1^2$

$y_2 = u_{2y} t + \frac{1}{2} a_{2y} t^2$

$-(10-y) = 0 - 5t_2^2$

$\frac{15}{2} t_1 - 5t_1^2 = -5t_2^2$ (2)

$y_1 = u_y t_1 + \frac{1}{2} a_y t_1^2$

$-(10-y) = \frac{15}{2} t_1 - 5 t_1^2$

$y_2 = u_y t_2 + \frac{1}{2} a_y t_2^2$

$-(10-y) = 0 - 5t_2^2$

$\Rightarrow \quad \frac{15}{2} t_1 - 5t_1^2 = -5t_2^2$

$\Rightarrow \frac{15}{2} t_1 = 5(t_1^2 - t_2^2)$

$\Rightarrow \frac{15}{2} \times 2t_2 = 5(4t_2^2 - t_2^2)$

$\Rightarrow \quad 15 t_2 = 15 t_2^2$

$\Rightarrow$ $t_2 = 1\,s$ and $t_1 = 2\,sec$

(i) The time interval is $t_1 - t_2 = 1\,sec$ Ans: 1 sec

(ii) the co-ordinate of P $x = 5\sqrt{3}\, t_2 = 5\sqrt{3}\, m$

$-(10-y) = -5t_2^2$

$\Rightarrow 10 - y = 5 \times 1$

$\Rightarrow y = 5\,m$

Ans: $(x, y) = (5\sqrt{3}, 5)$

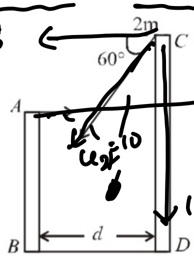
Two towers AB and CD are situated a distance d apart as shown in figure.

AB is 20 m high and CD is 30 m high from the ground. An object of mass m is thrown from the top of AB horizontally with a velocity of 10 m/s towards CD. **[1994 - 6 Marks]**

Simultaneously another object of mass 2 m is thrown from the top of CD at an angle of 60° to the horizontal towards AB with the same magnitude of initial velocity as that of the first object. The two objects move in the same vertical plane, collide in mid-air and stick to each other.

$$\boxed{\text{Ans: (i) } d = 10\sqrt{3}\,m \quad \text{(ii) } \frac{20}{\sqrt{3}}\,m}$$

$5 = 10\cos 60°$



$u_{1x} = 10\ m/s$

$u_{1y} = 0$

$10\sin 60° = 5\sqrt{3}$

(i) Calculate the distance 'd' between the towers and,

(ii) Find the position where the objects hit the ground.

$u_{2x} = -5$

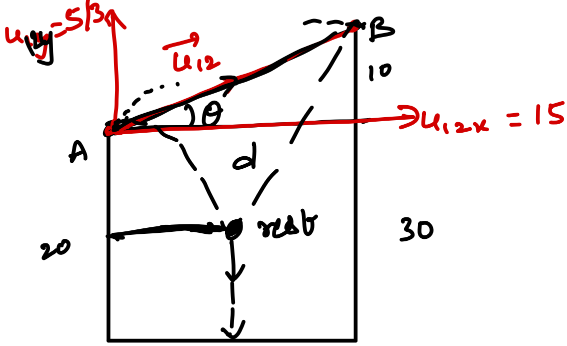
$u_{2y} = -5\sqrt{3}$

$u_{12x} = u_{1x} - u_{2x} = 10 - (-5)$

$u_{12x} = 15\ m/s$

$u_{12y} = u_{1y} - u_{2y} = 0 - (-5\sqrt{3})$

$u_{12y} = 5\sqrt{3}\ m/s$



$$\tan\theta = \frac{u_{12y}}{u_{12x}} = \frac{10}{d}$$

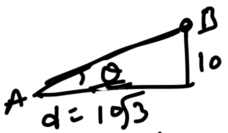
$$\frac{5\sqrt{3}}{15} = \frac{10}{d}$$

$$\frac{1}{\sqrt{3}} = \frac{10}{d}$$

$$d = 10\sqrt{3}\ m$$

Time for Collision

$$t = \frac{AB}{|\vec{u}_{12}|} = \frac{20}{10\sqrt{3}} = \frac{2}{\sqrt{3}}\ sec$$



$AB = \sqrt{10^2 + (10\sqrt{3})^2} = 20\ m$

$|\vec{u}_{12}| = \sqrt{15^2 + (5\sqrt{3})^2} = \sqrt{225 + 75} = 10\sqrt{3}\ m/s$

along horizontal direction no F_{ext} $\Rightarrow$ $P_{ix} = P_{fx}$

$m \times \vec{u}_{1x} + 2m\,\vec{u}_{2x} = (m + 2m)\,\vec{v}$

$m\,10\hat{i} + 2m(-5\hat{i}) = 3m\,\vec{v}$

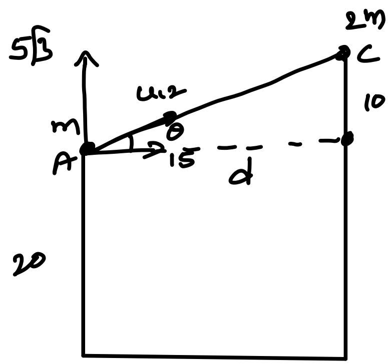
$0 = \vec{v}$

$\vec{v} = 0$

(i)

$$u_{12x} = u_{1x} - u_{2x} = 10-(-5) = 15\ m/s$$

$$u_{12y} = u_{1y} - u_{2y} = 0-(-5\sqrt{3}) = 5\sqrt{3}\ m/s$$



$$\tan\theta = \frac{5\sqrt{3}}{15} = \frac{1}{\sqrt{3}}$$

$$\theta = 30^\circ$$

$$\tan\theta = \frac{10}{d} = \frac{1}{\sqrt{3}}$$

Ans: $d = 10\sqrt{3}\ m$

$$AC = \sqrt{10^2 + (10\sqrt{3})^2} = 20m$$

$$d = 10 \times 1.732 = 17.32m$$

(ii) Time of collision $t = \frac{AC}{|\vec{u}_{12}|} = \frac{20}{10\sqrt{3}} = \frac{2}{\sqrt{3}}$ sec.

along horizontal direction no external force $\Rightarrow$ P is conserved

$$P_i = P_f$$

$$m \times 10\,\hat{i} - 2m \times 5\,\hat{i} = (m+2m)\vec{v}$$

$$0 = 3m\vec{v} \Rightarrow \vec{v} = 0$$

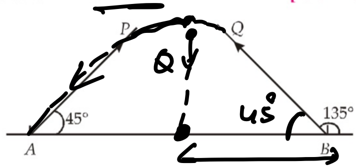
after collision. $(m+2m)$ system comes to rest and fall vertically down ward.

Collision takes place at $x = u_{1x}t = 10 \times \frac{2}{\sqrt{3}} = \frac{20}{\sqrt{3}}$

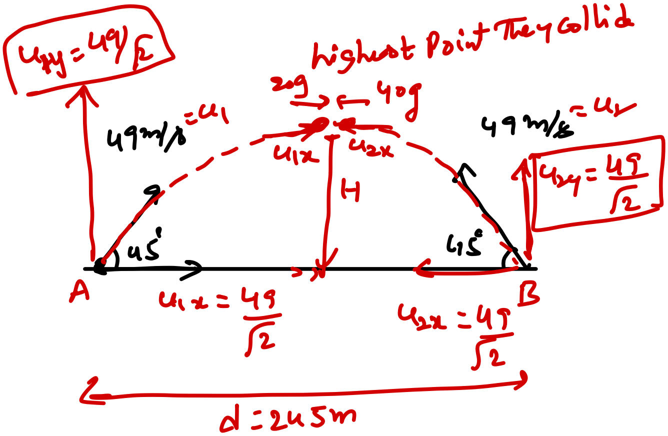
Ans: $x = 11.547m$

Particles P and Q of mass 20 gm and 40 gm respectively are simultaneously projected from points A and B on the ground. The initial velocities of P and Q make 45° and 135° angles respectively with the horizontal AB as shown in the figure. Each particle has an initial speed of 49 m/s. The separation AB is 245 m. **[1982 - 8 Marks]**

$$\boxed{\text{Ans}: \; t = \frac{5}{\sqrt{2}}\text{ sec}}$$



Both particle travel in the same vertical plane and undergo a collision After the collision, P retraces its path, Determine the position of Q when it hits the ground. How much time after the collision does the particle Q take to reach the ground? Take $g = 9.8$ m/s².



$$u_{12x} = \frac{49}{\sqrt{2}} \times 2 = 49\sqrt{2}$$

$$t = \frac{d}{|\vec{u}_{12}|_x} = \frac{245}{49\sqrt{2}} = \frac{5}{\sqrt{2}}$$

$$t = \frac{5}{\sqrt{2}} \text{ sec}$$

$$H = \frac{u_{y1}^2}{2g} = \frac{\left(\frac{49}{\sqrt{2}}\right)^2}{2\times 9.8} = \frac{49\times 49}{4\times 9.8} = \_\_\_\_$$

$$P_{i_x} = P_{f_x}$$

$$20\times \vec{u}_{1x} + 40(\vec{u}_{2x}) = 20\,\vec{v}_{1x} + 40\,\vec{v}_{2x}$$

$$20\times \frac{49}{\sqrt{2}}\hat{i} - 40\times\frac{49}{\sqrt{2}}\hat{i} = 20\times\left(-\frac{49}{\sqrt{2}}\hat{i}\right) + 40\,\vec{v}_{2x}$$

$$\cancel{40\times\frac{49}{\sqrt{2}}}\hat{i} - \cancel{40\times\frac{49}{\sqrt{2}}}\hat{i} = 40\,\vec{v}_{2x}$$

$$\Rightarrow v_{2x} = 0$$

Q Particle Comes to rest just after Collision It fall freely

$$t = \sqrt{\frac{2H}{g}}$$

conservation of linear momentum along horizontal direction

$$20\times u_{1x} + 40\times u_{2x} = 20(-u_{1x}) + 40 V_{2x}$$

$$40 u_{1x} + 40 u_{2x} = 40 V_{2x}$$

$$40\left(\frac{49}{\sqrt{2}}\right) - 40\times\frac{49}{\sqrt{2}} = 40 V_{2x}$$

$$\Rightarrow V_{2x} = 0$$

so after collision 2nd particle fall vertically down.

\# given 1st particle return back to original projection

$$H = \frac{u_y^2}{2g} = \frac{\left(49/\sqrt{2}\right)^2}{2\times 9.8} = \frac{49\times \cancel{49}}{2\times 2\times \cancel{9.8}_{0.2}} = \frac{490}{8}\,m$$

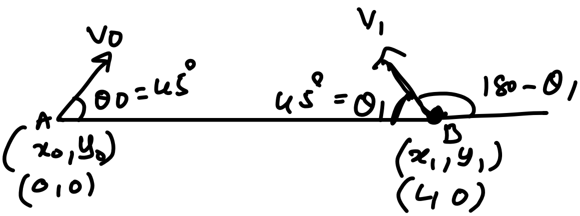
time to reach ground is $t = \sqrt{\frac{2H}{g}} = \sqrt{\frac{\cancel{2}\times \cancel{490}^{100}}{8\times \cancel{9.8}_{\cancel{4.9}}}}$

Ans:

$$\boxed{t = \frac{10}{2\sqrt{2}} = \frac{5}{\sqrt{2}}\,sec \qquad t = 5\times 0.707 = 3.53\,sec}$$

A ball is thrown from the location $(x_0, y_0) = (0,0)$ of a horizontal playground with an initial speed v_0 at an angle θ_0 from the $+x$-direction. The ball is to be hit by a stone, which is thrown at the same time from the location $(x_1, y_1) = (L, 0)$. The stone is thrown at an angle $(180 - \theta_1)$ from the $+x$-direction with a suitable initial speed. For a fixed v_0, when $(\theta_0, \theta_1) = (45^\circ, 45^\circ)$, the stone hits the ball after time T_1, and when $(\theta_0, \theta_1) = (60^\circ, 30^\circ)$, it hits the ball after time T_2. In such a case, $(T_1/T_2)^2$ is 2.

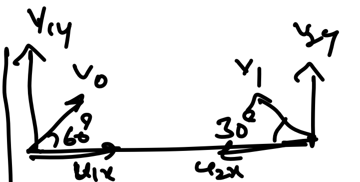
2024 - AdV (P2)



To Collide There should not be any Relative velocity in Perpendicular line to AB

$$v_0 \times \frac{1}{\sqrt{2}} = \frac{v_1}{\sqrt{2}} \Rightarrow v_1 = v_0$$

$$T_1 = \frac{L}{u_{12x}} = \frac{L}{\frac{v_0}{\sqrt{2}} - \left(-\frac{v_0}{\sqrt{2}}\right)} = \frac{L}{\sqrt{2}\, v_0}$$



$$v_0 \sin 60^\circ = v_1 \sin 30^\circ$$

$$v_0 \frac{\sqrt{3}}{2} = v_1 \times \frac{1}{2}$$

$$v_1 = \sqrt{3}\, v_0$$

$$v_{12x} = v_0 \cos 60^\circ + v_1 \cos 30^\circ$$

$$= \frac{v_0}{2} + \sqrt{3}\, v_0 \times \frac{\sqrt{3}}{2}$$

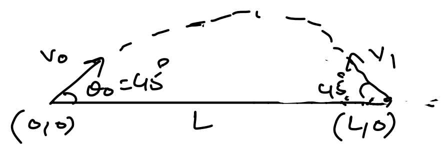
$$= \frac{4 v_0}{2}$$

$$= 2 v_0$$

$$T_2 = \frac{L}{2 v_0}$$

$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{\frac{\cancel{L}}{\sqrt{2}\,\cancel{v_0}}}{\frac{\cancel{L}}{2\cancel{v_0}}}\right)^2 = \left(\frac{2}{\sqrt{2}}\right)^2 = (\sqrt{2})^2 = 2$$

Sol:



time for collision $t = \dfrac{S_{rel}}{(V_{rel})_x}$

for collision there should not be relative motion $\perp r$ to line joining between their initial positions.

Case (i)

$$\frac{v_0}{\sqrt{2}} = \frac{v_1}{\sqrt{2}} \Rightarrow v_1 = v_0$$

$$v_{1x} = \frac{v_0}{\sqrt{2}}$$

$$T_1 = \frac{L}{\frac{v_0}{\sqrt{2}} + \frac{v_0}{\sqrt{2}}} = \frac{L}{\sqrt{2}v_0}$$

Case (ii)

$$v_{0y} = v_{1y}$$

$$v_0 \sin 60° = v_1 \sin 30°$$

$$\frac{\sqrt{3}v_0}{2} = \frac{v_1}{2}$$

$$\Rightarrow v_1 = \sqrt{3}v_0$$

$$v_{1x} = v_1 \cos 30° = \sqrt{3}v_0 \cdot \frac{\sqrt{3}}{2}$$

$$v_{1x} = \frac{3v}{2}$$

$$v_{0x} = v_0 \cos 60° = \frac{v}{2}$$

$$T_2 = \frac{L}{v_{rel\,x}} = \frac{L}{\frac{3v}{2} + \frac{v}{2}} = \frac{L}{2v}$$

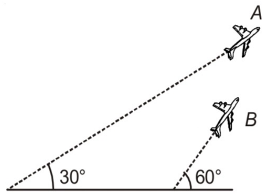
$$\frac{T_1}{T_2} = \left(\frac{\frac{L}{\sqrt{2}v_0}}{\frac{L}{2v}}\right) = \sqrt{2}$$

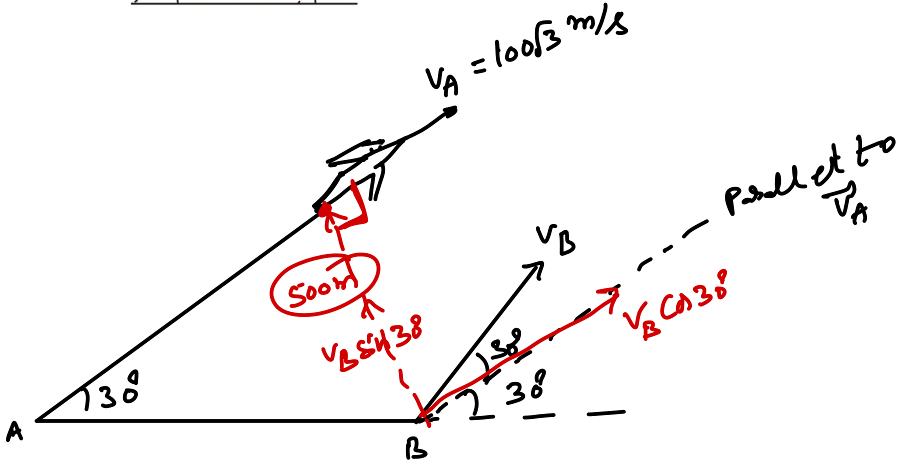
$$\left(\frac{T_1}{T_2}\right)^2 = 2$$

Ans : 2

Airplanes A and B are flying with constant velocity in the same vertical plane at angles 30° and 60° with respect to the horizontal respectively as shown in figure. The speed of A is $100\sqrt{3}$ m/s. At time $t = 0$ s, an observer in A finds B at a distance of 500 m. The observer sees B moving with a constant velocity perpendicular to the line of motion of A. If at $t = t_0$, A just escapes being hit by B, t_0 in seconds is

5 sec





There is no relative motion along Parallel to line of A

$$100\sqrt{3} = V_B \cos 30°$$

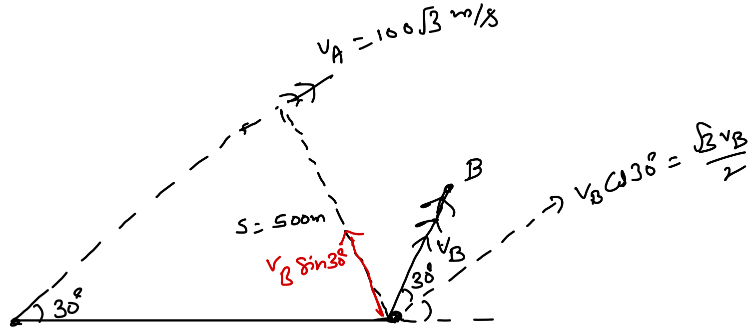
$$100\sqrt{3} = V_B \times \frac{\sqrt{3}}{2} \Rightarrow V_B = 200 \text{ m/s}$$

$$V_\perp = V_B \sin 30° = 200 \times \frac{1}{2} = 100 \text{ m/s}$$

B Can reach A in time $t = \frac{500}{V_\perp} = \frac{500}{100} = 5$ sec

$t = t_0 = 5$ sec

sol:-



at critical case, There should not be a relative motion along direction of A.

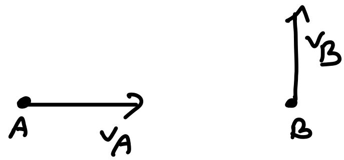
$$\frac{\sqrt{3}\, v_B}{2} = 100\sqrt{3}$$

$$v_B = 200 \text{ m/s}$$

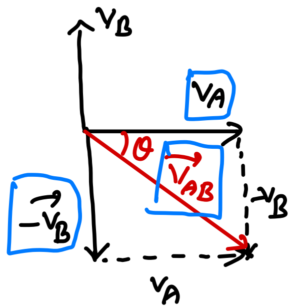
$$t = \frac{S}{v_B \sin 30^\circ} = \frac{500}{200 \times \frac{1}{2}} = \frac{500}{100} = 5 s.$$

Relative Motion in 2D

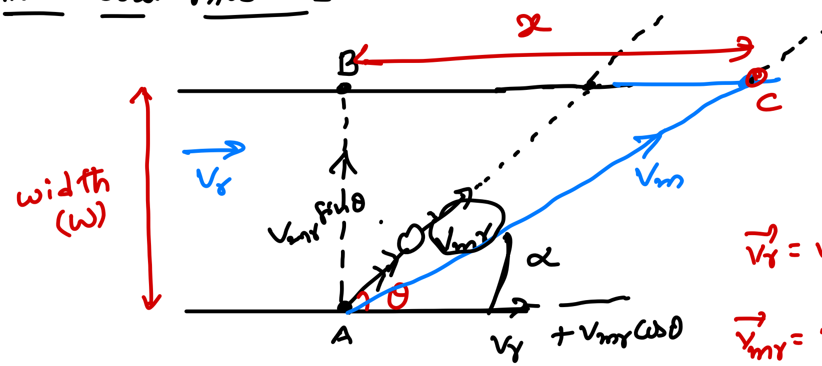
* Relative velocity is $\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$



$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$



River-boat Problems



$\vec{V}_r$ = velocity of River

$\vec{V}_{mr}$ = velocity of man w.r.t river

$\vec{V}_m$ is velocity of man.

$\vec{V}_{mr} = \vec{V}_m - \vec{V}_r$

$\Rightarrow \quad \vec{V}_m = \vec{V}_{mr} + \vec{V}_r$

$$\tan\alpha = \frac{V_{my}}{V_{mx}} = \frac{V_{mr}\sin\theta}{V_{mr}\cos\theta + V_r}$$

drift → horizontal distance covered by man

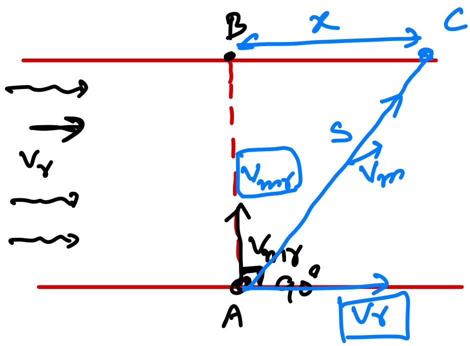
$x = V_{mx} \cdot t$

$$\boxed{x = (V_{mr}\cos\theta + V_r)\frac{w}{V_{my}}}$$

$$\boxed{t = \frac{w}{V_{my}}}$$

$$\boxed{V_{my} = V_{mr}\sin\theta}$$

Condition to cross the river in minimum time



he should swim $\perp_r$ to River flow.

$\theta = 90^\circ$

$$t_{min} = \frac{w}{(V_{my})_{Maximum}}$$

$V_{my} = V_{mr}\sin\theta$

$\sin\theta$ → Maximum

$\sin\theta = 1$

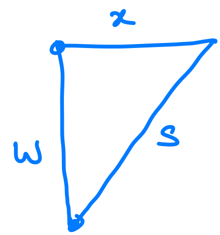
$\theta = 90^\circ$

$$\boxed{t_{min} = \frac{w}{V_{mr}}}$$

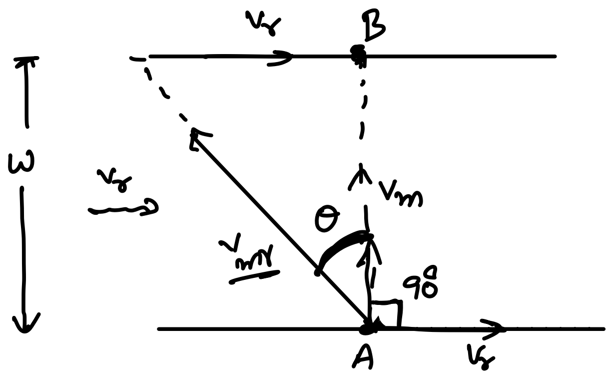
drift $\boxed{(x) = V_{mx}\, t = \frac{V_r w}{V_{br}}}$

$$\boxed{V_m = \sqrt{V_r^2 + V_{mr}^2}}$$

displacement $s = \sqrt{w^2 + x^2}$



Condition to cross the River in shorter Path (minimum drift)



to get minimum drift, Man should swim in upstream.

upstream means. Swimmer is moving opposite to river flow

for zero drift $(x=0)$

exactly he will reach to B.

down stream: Swimmer is moving along the direction of river flow

$V_m = \sqrt{V_{mr}^2 - V_r^2}$

$$t = \frac{W}{V_m} = \frac{W}{\sqrt{V_{mr}^2 - V_r^2}}$$

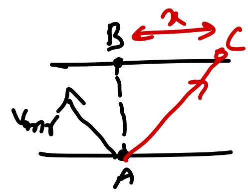
$$\sin\theta = \frac{V_r}{V_{mr}}$$

Person should swim an angle $\boxed{90^\circ + \sin^{-1}\left[\frac{V_r}{V_{mr}}\right]}$ with river flow.

this relative is Possible only when $V_r < V_{mr}$

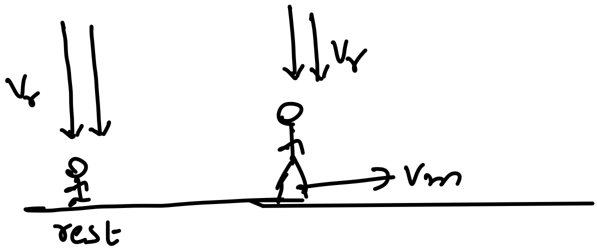
if $V_r > V_{mr}$ $\quad \theta = \sin^{-1}\left(\frac{V_{mr}}{V_r}\right)$

if $V_r > V_{mr}$ → he never Reach B

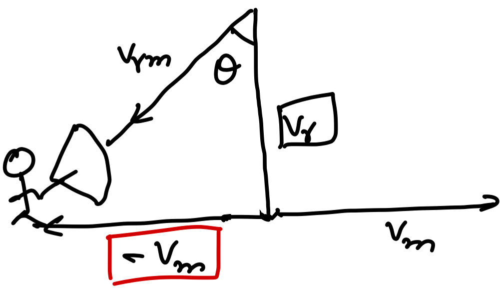


Rain - umbrella Problems

$V_r \rightarrow$ velocity of rain
$V_m \Rightarrow$ velocity of man
$V_{rm} \Rightarrow$ velocity of rain w.r.t. man.



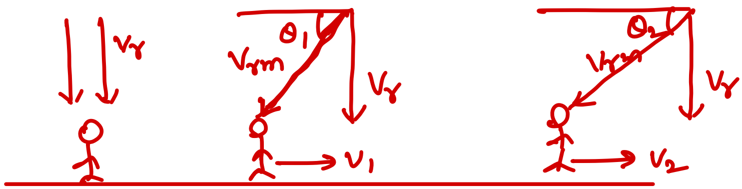
To protect himself from rain he should umbrella with an angle 'θ' with verticle



$$\vec{V}_{rm} = \vec{V}_r - \vec{V}_m$$

$$\tan\theta = \frac{V_m}{V_r}$$

$$\boxed{\theta = \tan^{-1}\left(\frac{V_m}{V_r}\right)}$$



man at rest

man increases speed to V_2

$$\tan\theta_1 = \frac{V_r}{V_1}$$

$$\tan\theta_2 = \frac{V_r}{V_2}$$

* wind - aircraft Problems

$$\vec{V}_{aw} = \vec{V}_a - \vec{V}_w$$

↙ velocity of air craft w.r.t wind.

$\vec{V}_a$ velocity of air craft

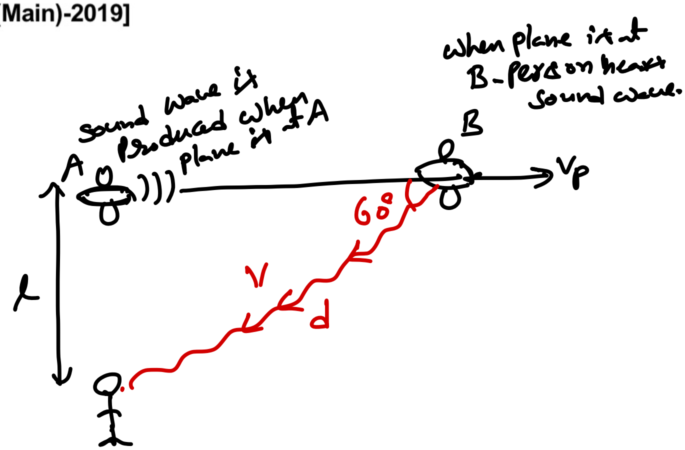
$\vec{V}_w$ velocity of wind

A person standing on an open ground hears the sound of a jet aeroplane, coming from north at an angle 60° with ground level. But he finds the aeroplane right vertically above his position. If v is the speed of sound, speed of the plane is

[JEE (Main)-2019]

(1) $\frac{2v}{\sqrt{3}}$ (2) $\frac{\sqrt{3}}{2}v$

✓(3) $\frac{v}{2}$ (4) v



$d \rightarrow$ distance travelled by sound wave

$v_p \rightarrow$ velocity of plane

$v \rightarrow$ velocity of sound

time for sound wave $t = \frac{d}{v} = \frac{l \operatorname{cosec} 60°}{v} \rightarrow (1)$

time for plane from $A \rightarrow B$ $t = \frac{AB}{v_p} = \frac{l \cot 60°}{v_p} \rightarrow (2)$

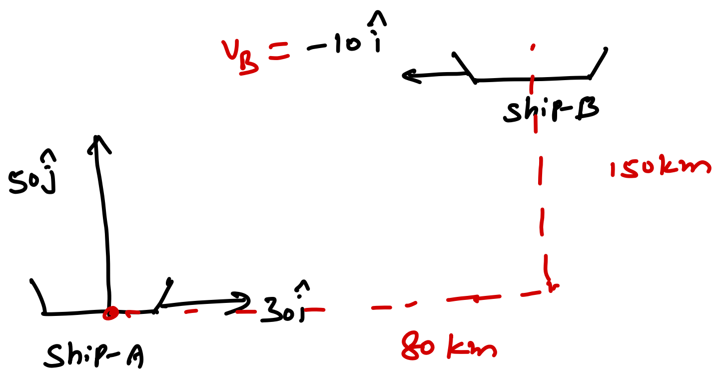
$$\frac{\cancel{l} \times \frac{2}{\cancel{\sqrt{3}}}}{v} = \frac{\cancel{l} \times \frac{1}{\cancel{\sqrt{3}}}}{v_p}$$

$$v_p = \frac{v}{2}$$

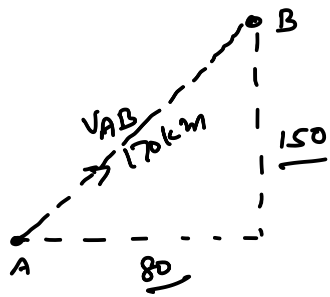
Ship A is sailing towards north-east with velocity $\vec{v} = 30\hat{i} + 50\hat{j}$ km / hr where $\hat{i}$ points east and $\hat{j}$, north. Ship B is at a distance of 80 km east and 150 km north of Ship A and is sailing towards west at 10 km/hr. A will be at minimum distance from B in **[JEE (Main)-2019]**

(1) 2.2 hrs. (2) 4.2 hrs.

(3) 3.2 hrs. (4) 2.6 hrs.



$$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B = (30\hat{i} + 50\hat{j}) - (-10\hat{i}) = 40\hat{i} + 50\hat{j}$$



$$t = \frac{AB}{|\vec{V}_{AB}|}$$

$$t = \frac{170}{\sqrt{40^2 + 50^2}} \text{ hrs}$$

$$t = \frac{170}{\sqrt{4100}} = \frac{17}{\sqrt{41}}$$

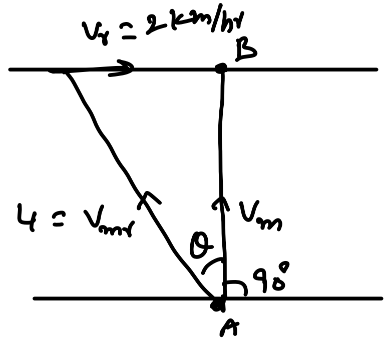
$$AB = \sqrt{(150)^2 + (80)^2} = \sqrt{22500 + 6400}$$
$$= \sqrt{28900} = 170\text{ km}$$

$$\underline{t \simeq 2.6\text{ hr}}$$

The stream of a river is flowing with a speed of 2 km/h. A swimmer can swim at a speed of 4 km/h. What should be the direction of the swimmer with respect to the flow of the river to cross the river straight? **[JEE (Main)-2019]**

(1) 60° (2) 90°

(3) 150° ✓(4) 120°



$$\alpha = 90° + \sin^{-1}\left(\frac{V_r}{V_{mr}}\right)$$

$$\alpha = 90° + \sin^{-1}\left(\frac{2}{4}\right)$$

$= 90° + 30°$

$= 120°$ with river flow

When a car is at rest, its driver sees rain drops falling on it vertically. When driving the car with speed v, he sees that rain drops are coming at an angle 60° from the horizontal. On further increasing the speed of the car to $(1 + \beta)v$, this angle changes to 45°. The value of β is close to

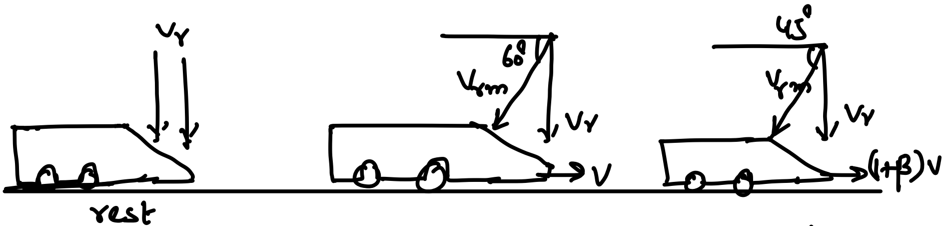
[JEE (Main)-2020]

(1) 0.37

(2) 0.41

(3) 0.73

(4) 0.50



$$\tan 60° = \frac{V_r}{V}$$

$$\sqrt{3} = \frac{V_r}{V}$$

$$V_r = \sqrt{3}V$$

$$\tan 45° = \frac{V_r}{V(1+\beta)} = 1$$

$$V_r = V(1+\beta)$$

$$\sqrt{3}\cancel{V} = \cancel{V}(1+\beta)$$

$$\Rightarrow \beta = \sqrt{3}-1$$

$$= 1.732-1$$

$$\beta = \underline{0.732}$$

A particle is moving along the x-axis with its coordinate with time 't' given by $x(t) = 10 + 8t - 3t^2$. Another particle is moving along the y-axis with its coordinate as a function of time given by $y(t) = 5 - 8t^3$. At $t = 1$ s, the speed of the second particle as measured in the frame of the first particle is given as $\sqrt{v}$. Then v (in m/s) is 580. **[JEE (Main)-2020]**

$$x(t) = 10 + 8t - 3t^2$$

$$v_x = \frac{dx}{dt} = 8 - 6t$$

$t = 1s$

$$v_x = 8 - 6 = 2 \text{ m/s}$$

$$\vec{v}_A = 2\hat{i}$$

$$y(t) = 5 - 8t^3$$

$$\frac{dy}{dt} = v_y = -24t^2$$

$$t = 1 \Rightarrow v_y = -24 \text{ m/s}$$

$$\vec{v}_B = -24 \text{ m/s } \hat{j}$$

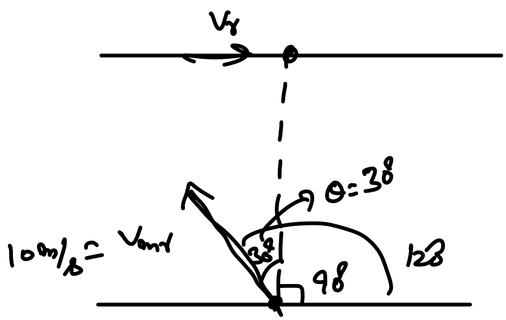
$$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A = -24\hat{j} - 2\hat{i}$$

$$|\vec{v}_{BA}| = \sqrt{24^2 + 2^2} = \sqrt{580}$$

$\underline{580}$

A person is swimming with a speed of 10 m/s at an angle of 120° with the flow and reaches to a point directly opposite on the other side of the river. The speed of the flow is ‘x’ m/s. The value of ‘x’ to the nearest integer is 5.

[JEE (Main)-2021]



$$\sin\theta = \frac{V_r}{V_{man}}$$

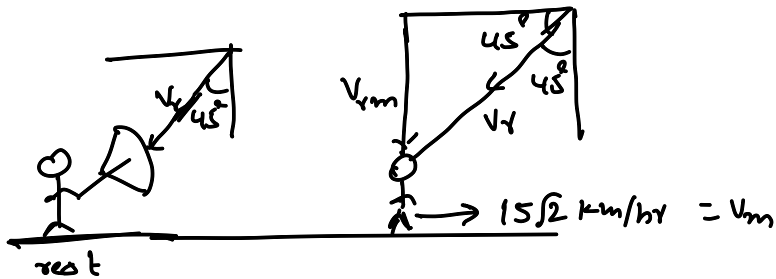
$$\frac{1}{2} = \frac{x}{10}$$

$$x = \frac{10}{2} = \underline{5\,m/s}$$

A girl standing on road holds her umbrella at 45° with the vertical to keep the rain away. If she starts running without umbrella with a speed of $15\sqrt{2}$ kmh^{-1}, the rain drops hit her head vertically. The speed of rain drops with respect to the moving girl is **[JEE (Main)-2022]**

(1) 30 kmh^{-1} (2) $\frac{25}{\sqrt{2}}$ kmh^{-1}

✓(3) $\frac{30}{\sqrt{2}}$ kmh^{-1} (4) 25 kmh^{-1}



$$\tan 45^\circ = \frac{V_{rm}}{V_m} = 1$$

$$V_{rm} = 15\sqrt{2} \text{ km/hr}$$

$$= \underline{\frac{30}{\sqrt{2}} \text{ km/hr}}$$

A boat which has a speed of 5 km/hr in still water crosses a river of width 1 km along the shortest possible path in 15 minutes. The velocity of the river water in km/hr is

[1988 - 1 Mark]

(a) 1 (b) 3 (c) 4 (d) $\sqrt{41}$

$V_{br} = 5$ km/hr

$w = 1$ km

$t = 15$ min $= \frac{1}{4}$ hr

$t = \dfrac{w}{\sqrt{v_{br}^2 - V_r^2}}$

$\frac{1}{4} = \dfrac{1}{\sqrt{25 - v_r^2}}$

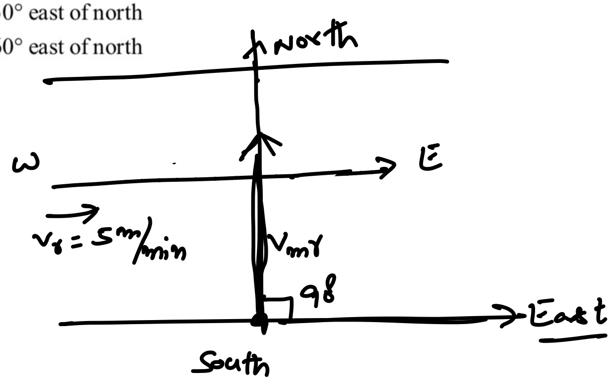
$25 - v_r^2 = 16 \Rightarrow v_r^2 = 9$

$V_r = 3$ km/hr

A river is flowing from west to east at a speed of 5 metres per minute. A man on the south bank of the river, capable of swimming at 10 metres per minute in still water, wants to swim across the river in the shortest time. He should swim in a direction **[1983 - 1 Mark]**

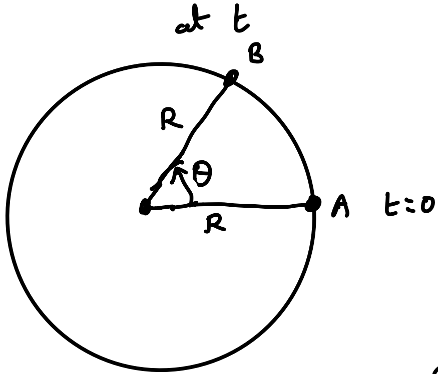
(a) due north (b) 30° east of north

(c) 30° west of north (d) 60° east of north



Circular kinematics

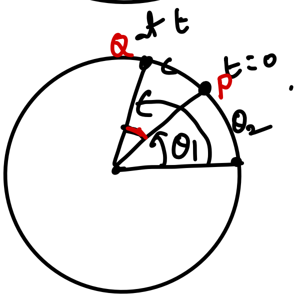
*



θ is angular position

S.I unit is rad

angular displacement $\Delta\theta = \theta_2 - \theta_1$



for small angles
$\vec{d\theta} = \vec{d\theta_2} - \vec{d\theta_1}$

angular velocity $\omega_{inst} = \frac{d\theta}{dt}$

$\omega_{avg} = \frac{\Delta\theta}{\Delta t}$ S.I unit rad/s

angular acceleration $\alpha_{inst} = \frac{d\omega}{dt}$

$\alpha_{avg} = \frac{\Delta\omega}{\Delta t} = \frac{\omega_f - \omega_i}{\Delta t}$

1 revolution = 1 rotation = 2π rad

1 rpm = $\frac{2\pi}{60} \frac{rad}{sec} = \frac{\pi}{30} \frac{rad}{sec}$

S.I unit rad/s^2

* for one full rotation $T = \frac{2\pi}{\omega}$ (or) $\omega = \frac{2\pi}{T}$

* for second's hand $T = 60\ sec$

$\omega = \frac{2\pi}{60}\ rad/sec$

* for minutes hand $T = 60\ min = 60 \times 60\ sec = 3600\ sec$

$\omega = \frac{2\pi}{3600}\ rad/s$

* for 12' hr's clock (hour's hand) $T = 12\ hr = 12 \times 3600\ sec$

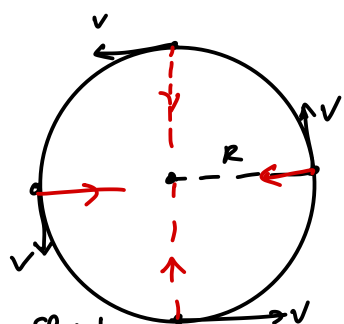
$\omega = \frac{2\pi}{12 \times 3600}\ rad/sec$

* for 24 hr's clock (hour's hand) $T = 24\ hr = 86400\ sec$

$\omega = \frac{2\pi}{86,400}\ rad/sec$

* uniform circular motion :- particle moves in a circle with constant speed

* velocity direction changes
* acceleration direction changes



$$\boxed{a_c = \frac{v^2}{R} = R\omega^2}$$

$a_t = 0$

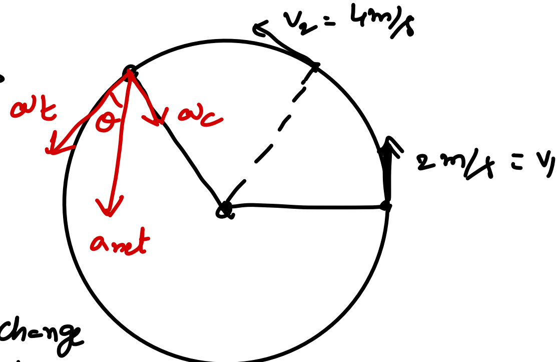
$a_t \Rightarrow$ tangential acceleration

$a_t =$ rate of change in speed

$$\boxed{a_t = \frac{d|\vec{v}|}{dt} = \frac{dv}{dt} = R\alpha}$$

* Non-Uniform circular Motion : speed of Particle changes

* speed changes
* velocity changes
* acceleration changes



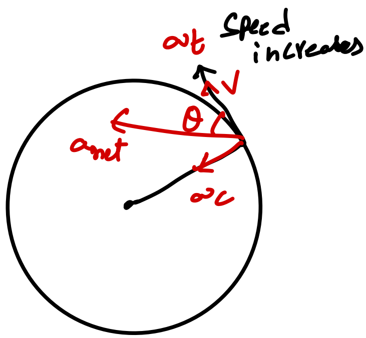
$$\boxed{a_c = \frac{v^2}{R} = R\omega^2}$$

$$\boxed{a_t = R\alpha = \frac{dv}{dt}}$$

↳ rate of change in speed

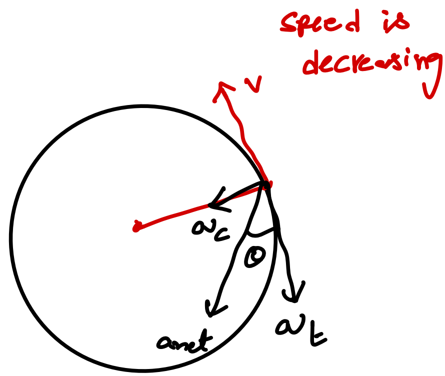
$$\boxed{a_{net} = \sqrt{a_c^2 + a_t^2}}$$

*



$\theta = \tan^{-1}\left(\frac{a_c}{a_t}\right)$

$\vec{a}_t$ and $\vec{v}$ are in same direction



$\theta = \tan^{-1}\left(\frac{a_c}{a_t}\right)$

$\vec{a}_t$ is opposite to $\vec{v}$

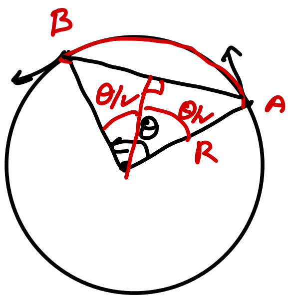
* relation b/w kinematic terms

distance Covered

$S = R\theta$

if $\theta = 30^\circ$ $\Rightarrow$ $S = R \times 30 \times \frac{\pi}{180}$

$S = \frac{R\pi}{6}$ ✓



displacement $|\vec{AB}| = 2R\sin\left(\frac{\theta}{2}\right)$

$1^\circ = \frac{\pi}{180}$ rad

* $S = R\theta$

diff. w.r.t time

$\frac{ds}{dt} = R\frac{d\theta}{dt}$

speed $\boxed{v = R\omega}$

diff. w.r.t time 't' (R is constant)

$\frac{dv}{dt} = R\frac{d\omega}{dt}$

rate of change in speed

$\boxed{a_t = R\alpha}$

* equations of Motion in circular motion

(i) if a particle is moving with constant angular velocity

$$\boxed{\theta = \omega t} \qquad t = \frac{\theta}{\omega}$$

(ii) if a particle is moving with uniform angular acceleration

$$\omega = \omega_0 + \alpha t \qquad (\omega - t \text{ eqn})$$

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 \qquad (\theta - t \text{ eqn})$$

$$\omega^2 - \omega_0^2 = 2\alpha\theta \qquad (\omega - \theta \text{ eqn})$$

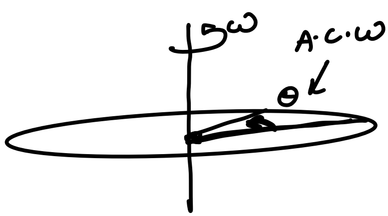
$$\theta_{n^{th}} = \omega_0 + \frac{\alpha}{2}(2n-1)$$

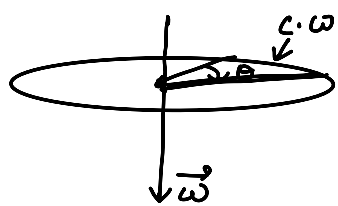
(iii) In Non-uniform motion

$$\omega = \frac{d\theta}{dt}$$

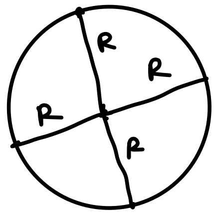
$$\alpha = \frac{d\omega}{dt}$$

$$\omega\, d\omega = \alpha\, d\theta$$





Finding Radius of Curvature

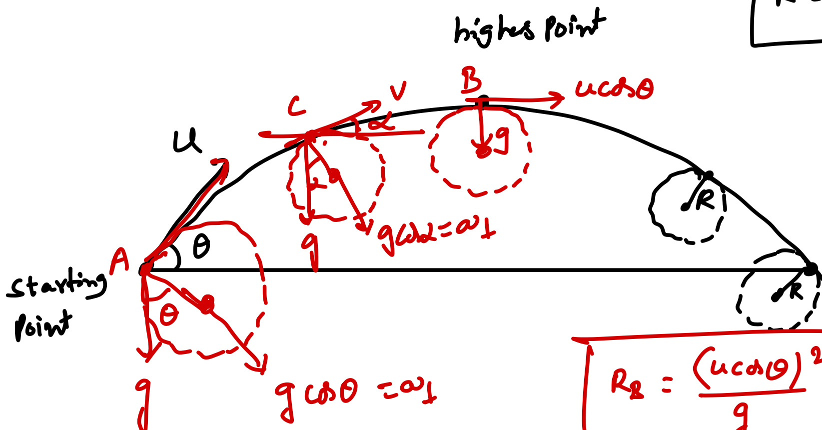


In circular path (fixed circle)
R is constant

In case of parabola

$$a_\perp = \frac{v^2}{R}$$

$$\boxed{R = \frac{v^2}{a_\perp}}$$



$$\boxed{R_B = \frac{(u\cos\theta)^2}{g}}$$

$$\boxed{R_A = \frac{u^2}{g\cos\theta}}$$

$$\# \quad \boxed{R_C = \frac{v^2}{g\cos\alpha} = \frac{u^2\cos^2\theta}{g\cos^3\alpha}}$$

$v_x = u_x$

$v\cos\alpha = u\cos\theta$

$\Rightarrow v = \dfrac{u\cos\theta}{\cos\alpha}$

if $y = f(x)$ is given
to find Radius of Curvature

$$\boxed{R = \frac{\left(1 + \left(\frac{dy}{dx}\right)^2\right)^{3/2}}{\frac{d^2y}{dx^2}}}$$

* Relative angular velocity when Time periods are given

$$\omega_A - \omega_B = \frac{2\pi}{T_A} - \frac{2\pi}{T_B}$$

(or) relative Time interval $\Delta T = \frac{2\pi}{\omega_{rel}} = \frac{2\pi}{\omega_A - \omega_B}$

For a particle in uniform circular motion, the acceleration $\vec{a}$ at a point $P(R, \theta)$ on the circle of radius R is (Here θ is measured from the x-axis)

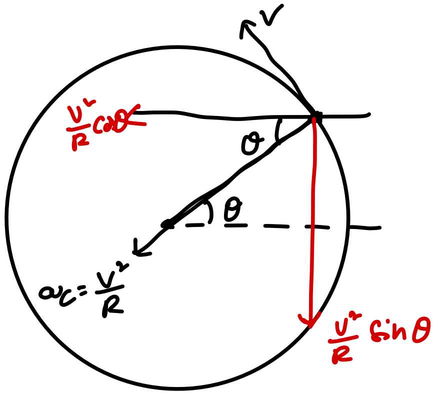
[AIEEE-2010]

(1) $\frac{v^2}{R}\hat{i} + \frac{v^2}{R}\hat{j}$

(2) $-\frac{v^2}{R}\cos\theta\,\hat{i} + \frac{v^2}{R}\sin\theta\,\hat{j}$

(3) $-\frac{v^2}{R}\sin\theta\,\hat{i} + \frac{v^2}{R}\cos\theta\,\hat{j}$

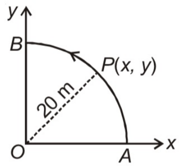
(4) $-\frac{v^2}{R}\cos\theta\,\hat{i} - \frac{v^2}{R}\sin\theta\,\hat{j}$



$$\vec{a} = -\frac{v^2}{R}\cos\theta\,\hat{i} - \frac{v^2}{R}\sin\theta\,\hat{j}$$

A point P moves in counter-clockwise direction on a circular path as shown in the figure. The movement of P is such that it sweeps out a length $s = t^3 + 5$, where s is in metres and t is in seconds. The radius of the path is 20 m. The acceleration of P when $t = 2$ s is nearly

[AIEEE-2010]



(1) 14 m/s^2 (2) 13 m/s^2

(3) 12 m/s^2 (4) 7.2 m/s^2

$S = t^3 + 5$

Speed $V = 3t^2 \longrightarrow$ $t = 2s$, $V = 3(2^2) = 12$ m/s

tangent acceleration $a_t = \frac{dV}{dt} = 6t$

at $t = 2s$

$a_t = 6 \times 2$
$= 12$ m/s^2

$a_c = \frac{V^2}{R} = \frac{12 \times 12}{20}$

$a_{net} = \sqrt{a_c^2 + a_t^2}$

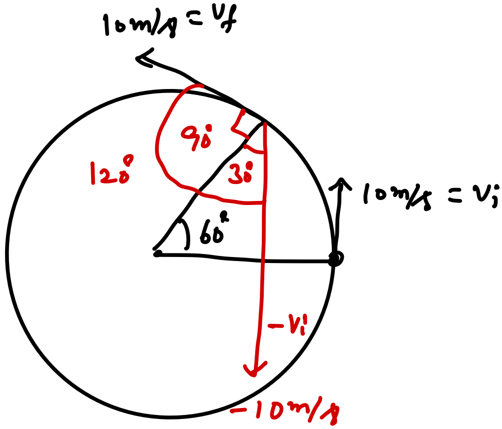
$= \sqrt{\left(\frac{12 \times 12}{20}\right)^2 + (12)^2}$

$\simeq 14$ m/s^2

A particle is moving along a circular path with a constant speed of 10 ms^{-1}. What is the magnitude of the change in velocity of the particle, when it moves through an angle of 60° around the centre of the circle? **[JEE (Main)-2019]**

(1) 10 m/s ✓ (2) Zero

(3) $10\sqrt{3}$ m/s (4) $10\sqrt{2}$ m/s



$$\vec{\Delta v} = \vec{v_f} - \vec{v_i}$$

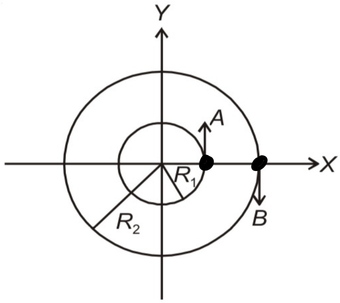
$$|\vec{\Delta v}| = \sqrt{10^2 + 10^2 + 2 \times 10 \times 10 \times \cos 120^\circ}$$

$$= \sqrt{100 + 100 + 200\left(-\frac{1}{2}\right)}$$

$$= \sqrt{100}$$

$$= 10 \text{ m/s}$$

Two particles A, B are moving on two concentric circles of radii R_1 and R_2 with equal angular speed ω. At $t = 0$, their positions and direction of motion are shown in the figure **[JEE (Main)-2019]**



The relative velocity $\vec{v}_A - \vec{v}_B$ at $t = \frac{\pi}{2\omega}$ is given by

(1) $\omega(R_2 - R_1)\hat{i}$ (2) $\omega(R_1 - R_2)\hat{i}$

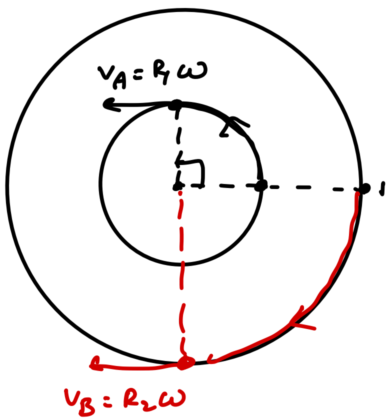
(3) $-\omega(R_1 + R_2)\hat{i}$ (4) $\omega(R_1 + R_2)\hat{i}$

$\vec{V}_A = -R_1\omega\,\hat{i}$

$\vec{V}_B = -R_2\omega\,\hat{i}$

$\vec{V}_A - \vec{V}_B = -R_1\omega\hat{i} - (-R_2\omega\hat{i})$

$= (R_2 - R_1)\omega\hat{i}$



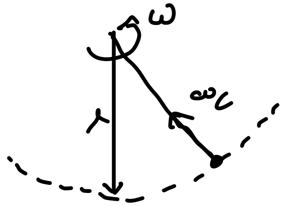
A clock has a continuously moving second's hand of 0.1 m length. The average acceleration of the tip of the hand (in units of ms^{-2}) is of the order of

[JEE (Main)-2020]

(1) 10^{-1} (2) 10^{-2}

(3) 10^{-4} (4) 10^{-3}

$\omega = \frac{2\pi}{60}$ rad/s



$a_c = l\omega^2$

$= 0.1 \times \left(\frac{2\pi}{60}\right)^2$

$= 0.1 \times \frac{\cancel{4}\pi^2}{\cancel{36}00}$ 9

$\pi^2 \simeq 10$

$= \frac{1}{900}$

$\simeq 10^{-3}$

A body rotating with an angular speed of 600 rpm is uniformly accelerated to 1800 rpm in 10 sec. The number of rotations made in the process is 200.
[JEE (Main)-2021]

$$\omega_i = \frac{600}{60}\frac{\text{rot}}{s} = 10\ \frac{\text{rot}}{s}$$

$$\omega_f = \frac{1800}{60}\frac{\text{rot}}{s} = 30\ \frac{\text{rot}}{s}$$

$$\alpha = \frac{\omega_f - \omega_i}{\Delta t} = \frac{30-10}{10} = 2\ \text{rot}/\text{sec}^2$$

$\Delta t = 10$ sec

$\theta = ?$

$$\omega_f^2 - \omega_i^2 = 2\alpha\theta$$

$$900 - 100 = 2 \times 2\ \theta$$

$$\theta = \frac{800}{4} = 200 \text{ rotations}$$

A ball is spun with angular acceleration $\alpha = 6t^2 - 2t$, where t is in second and α is in rads^{-2}. At $t = 0$, the ball has angular velocity of 10 rads^{-1} and angular position of 4 rad. The most appropriate expression for the angular position of the ball is :**[JEE (Main)-2022]**

(1) $\frac{3}{4}t^4 - t^2 + 10t$

(2) $\frac{t^4}{2} - \frac{t^3}{3} + 10t + 4$ ✓

(3) $\frac{2t^4}{3} - \frac{t^3}{6} + 10t + 12$

(4) $2t^4 - \frac{t^3}{2} + 5t + 4$

$$\alpha = 6t^2 - 2t$$

$$\frac{d\omega}{dt} = \alpha$$

$$d\omega = \alpha\, dt$$

$$\int_{10}^{\omega} d\omega = \int_0^t (6t^2 - 2t)\, dt$$

$$\omega = 10 + \frac{6t^3}{3} - \frac{2t^2}{2}$$

$$\frac{d\theta}{dt} = \omega$$

$$\int d\theta = \int \omega\, dt$$

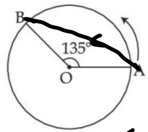
$$\int_4^{\theta} d\theta = \int_0^t (10 + 2t^3 - t^2)\, dt$$

$$\theta = 4 + 10t + \frac{2t^4}{4} - \frac{t^3}{3}$$

$$\theta = \frac{t^2}{2} - \frac{t^3}{3} + 10t + 4$$

A person moved from A to B on a circular path as shown in figure. If the distance travelled by him is 60 m, then the magnitude of displacement would be

(Given $\cos 135° = -0.7$) **[JEE (Main)-2022]**



(1) 42 m
(2) 47 m
(3) 19 m
(4) 40 m

$$S = R\theta$$

$$60 = R \times \frac{135 \times \pi}{180}$$

$$R = \frac{60 \times 180}{135 \times \pi}$$

$$\text{displacement} = 2R \sin\left(\frac{\theta}{2}\right)$$

$$= \frac{2 \times 60 \times 180}{135 \times \pi} \sin\left(\frac{135°}{2}\right)$$

$$= \underline{47\ m}$$

A fly wheel is accelerated uniformly from rest and rotates through 5 rad in the first second. The angle rotated by the fly wheel in the next second, will be:

[JEE (Main)-2022]

(1) 7.5 rad

(2) 15 rad

(3) 20 rad

(4) 30 rad

$\omega_i = 0$

$\theta = 5\text{ rad} \quad , \quad t = 1\text{ sec}$

$\theta = \frac{1}{2}\alpha t^2$

$\theta_1 = 5$

$t_1 = 1s$

$t_1 + t_2 = 2s$

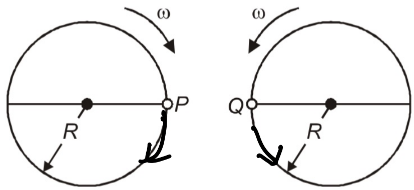
$5 = \frac{1}{2}\alpha (1)^2 \Rightarrow \frac{\alpha}{2} = 5$

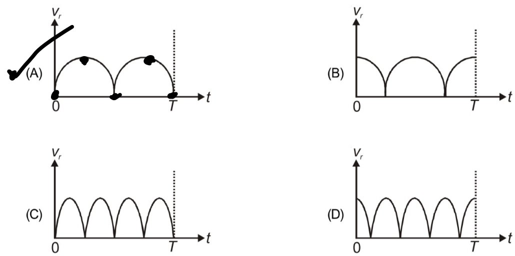
$\theta_1 + \theta_2 = \frac{1}{2}\alpha (t_1 + t_2)^2$

$5 + \theta_2 = 5 (2^2)$

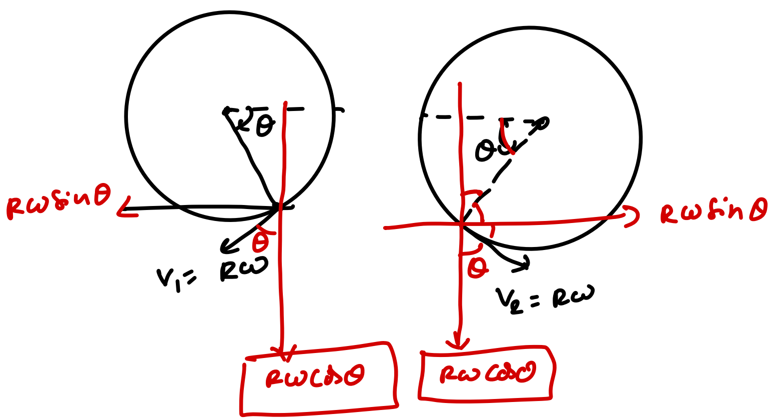
$\theta_2 = 20 - 5 = \underline{15\text{ rad}}$

Two identical discs of same radius R are rotating about their axes in opposite directions with the same constant angular speed ω. The discs are in the same horizontal plane. At time $t = 0$, the points P and Q are facing each other as shown in the figure. The relative speed between the two points P and Q is v_r. In one time period (T) of rotation of the discs, v_r as a function of time is best represented by **[IIT-JEE-2012 (Paper-2)]**





speed is always +ve



$(v_{rel})_y = 0$

$(v_{rel})_x = R\omega\sin\theta + R\omega\sin\theta = 2R\omega\sin\theta$

if $\theta = 0$ $v_{rel} = 0$

if $\theta = 90^\circ$ $v_{rel} = 2R\omega$

if $\theta = 180^\circ$ $v_{rel} = 0$

if $\theta = 270$ $v_{rel} = -2R\omega$

$|v_{rel}| = 2R\omega$

if $\theta = 360^\circ$

$v_{rel} = 0$