Chapter 1: Sets, Relations and Functions

Example 1.1: Find the number of subsets of A if A = {x: x = 4n + 1; 2 ≤ n ≤ 5; n ϵ N}.

Solution:

Given A - {x: x = 4n + 1; 2 ≤ n ≤ 5; n ϵ N}.

⇒ A = {9, 13, 17, 21}

Here, n = 4

Therefore, number of subsets = $2^n = 2^4 = 16$

Example 1.2: In a survey of 5000 persons in a town, it was found that 45% of the persons know Language A; 25% know Language B; 10% know Language C; 5% know Languages A and B; 4% know Languages B and C; and 4% know Languages A and C. If 3% of the persons know all the three Languages, find the number of persons who knows only Language A.

Solution:

Method 1: Using Cardinality:

Given: Total = 5000

n(A) = 45% = 0.45 x 5000 = 2250

n(B) = 25% = 0.25 x 5000 = 1250

n(C) = 10% = 0.10 x 5000 = 500

n(A ∩ B) = 5% = 0.05 x 5000 = 250

n(B ∩ C) = 4% = 0.04 x 5000 = 200

n(C ∩ A) = 4% = 0.04 x 5000 = 200

n(A ∩ B ∩ C) = 3% = 0.03 x 5000 = 150

Now to find the number of persons who knows only A = Number of persons who knows A – number of persons who knows A & B – number of persons who knows A & C + number of persons who knows all the three languages

Cardinality Rule: n(A ∩ B′ ∩ C′) = n[A ∩ (B U C)′] = n(A) – n[A ∩ (B U C)]

= n(A) – n(A ∩ B) – n(A ∩ C) + n(A ∩ B ∩ C)

= 2250 – 250 – 200 + 150 = 1950

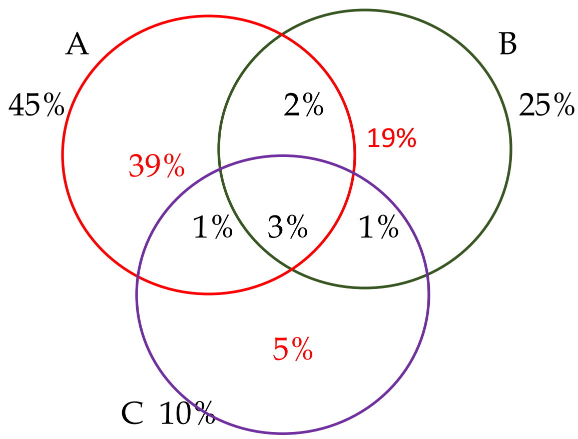
Method 2: Using Percentage

Now to find the % of persons who knows only A = % of persons who knows A – % of persons who knows A & B – % of persons who knows A & C + % of persons who knows all the three languages

= 45% - 5% - 4% + 3% = 39%

So, Number of persons who knows only A is = 0.39 x 5000 = 1950

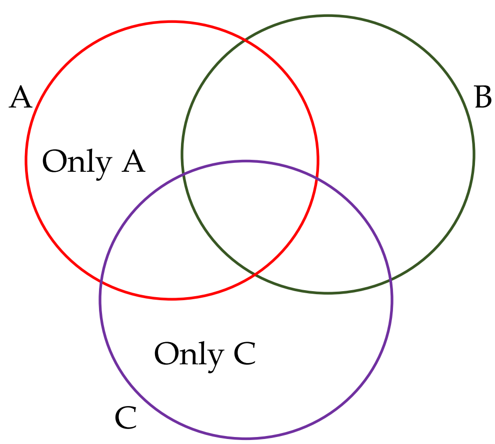
Venn Diagram:



Example 1.3: Prove that

((A U B′ U C) ∩ (A ∩ B′ ∩ C′)) U ((A U B U C′) ∩ (B′ ∩ C′)) = B′ ∩ C′

Solution:



As, A U B′ U C = A U C, A will be a subset of A U B′ U C

In $A \cap B' \cap C'$, as $B' \cap C'$ is the area of region "Only A", thus $A \cap B' \cap C'$ will be again "Only A" portion. Thus, $A \cap B' \cap C'$ will be a subset of A.

Thus, $A \cap B' \cap C' \subseteq A \subseteq A \cup B' \cup C$

Hence, $(A \cup B' \cup C) \cap (A \cap B' \cap C') = A \cap B' \cap C' = B' \cap C'$

Now, $A \cup B \cup C'$ will cover all region except "Only C".

$B' \cap C'$ will be the region only A.

Thus, $((A \cup B \cup C') \cap (B' \cap C')) = B' \cap C'$

Hence, $((A \cup B' \cup C) \cap (A \cap B' \cap C')) \cup ((A \cup B \cup C') \cap (B' \cap C')) = B' \cap C'$

Example 1.4

If X = {1, 2, 3,… 10} and A = {1, 2, 3, 4, 5} find the number of sets $B \subseteq X$ such that

A – B = {4}

Example 1.5

If A and B are two sets so that n(B – A) = 2n(A – B) = 4n(A ∩ B) and if n(A U B) = 14, find n(℘(A))

Solution:

Let $n(A \cap B) = k$

Then, $n(A - B) = 2k$; $n(B - A) = 4k$

By law,

$n(A \cup B) = n(A - B) + n(B - A) + n(A \cap B)$

$14 = 2k + 4k + k$

$7k = 14$

$k = 2$

So, $n(A \cap B) = 2$; $n(A - B) = 4$; $n(B - A) = 8$

But, $n(A) = n(A - B) + n(A \cap B) = 4 + 2 = 6$

Therefore, $n(\wp(A)) = 2^{n(A)} = 2^6 = 64$

Example 1.6

Two sets have m and k elements. If the total number of subsets of the first set is 112 more than that of the second set, Find the values of m and k.

Given: Let A and B be two sets.

$n(A) = m; n(B) = k$ where $m > k$

Moreover, $2^m - 2^k = 112$

$\Rightarrow 2^k(2^{m-k} - 1) = 16 \times 7$

$\Rightarrow 2^k(2^{m-k} - 1) = 2^4 \times 7$

This is possible only when

$2^k = 2^4$; $2^{m-k} - 1 = 7$

Therefore, $k = 4$

And, $2^{m-k} = 8$

$2^{m-k} = 2^3$

$m - k = 3$

$m - 4 = 3$

$m = 7$

Example 1.7:

If $n(A) = 10$ and $n(A \cap B) = 3$, find $n((A \cap B)' \cap A)$.

Solution:

$(A \cap B)' \cap A = (A' \cup B') \cap A = (A' \cap A) \cup (B' \cap A)$

$\Rightarrow \varnothing \cup (B' \cap A) = (B' \cap A) = (A - B) = A - (A \cap B)$

Therefore, $n((A \cap B)' \cap A) = n(A) - n(A \cap B) = 10 - 3 = 7$

Example 1.8:

If $A = \{1; 2; 3; 4\}$ and $B = \{3; 4; 5; 6\}$, find $n((A \cup B) \times (A \cap B) \times (A \Delta B))$.

Solution:

Now $A \cup B = \{1, 2, 3, 4, 5, 6\}$; $n(A \cup B) = 6$

$A \cap B = \{3, 4\}$; $n(A \cap B) = 2$

A Δ B = (A U B) – (A ∩ B) = {1, 2, 5, 6}; n(A Δ B) = 4

Therefore, n((A U B) x (A ∩ B) x (A Δ B)) = n(A U B) x n(A ∩ B) x n(A Δ B)

= 6 x 2 x 4 = 48

Example 1.9:

If ℘(A) denotes the power set of A, then find n(℘(℘(℘(∅)))).

Solution:

As n(∅) = 0, ℘(∅) = 2^0 = 1

℘(℘(∅)) = 2^1 = 2

℘(℘(℘(∅))) = 2^2 = 4

Therefore, **n(℘(℘(℘(∅)))) = 4**

Exercise – 1.1

1. **Write the following in roster form**

(i) {x ϵ N: x^2 < 121 and x is a prime}

= {1, 3, 5, 7, 9 |

(ii) The set of all positive roots of the equation (x – 1)(x + 1)(x^2 – 1) = 0

The roots of the equations are -1 and 1

So, the set is {1}

(iii) {x ϵ N: 4x + 9 < 52}

Given 4x + 9 < 52

4x < 52 – 9 = 43

x < 43 / 4 = 10.75

As x ϵ N, the set is {1, 2, 3, … 10}

(iv) {x : $\frac{x-4}{x+2}$ = 3, x ϵ R – {-2}}

$$\frac{x-4}{x+2} = 3$$

$$x - 4 = 2x + 6$$

$$2x - x = -4 - 6$$

$$x = -10$$

Therefore, the set is {-10}

2. **Write the set {-1, 1} in set builder form**

   {x : x is the root of the equation $x^2 - 1 = 0$}

3. **State whether the following sets are finite or infinite**

   (i) {x ∊ N : x is an even prime number} = {2} – Finite

   (ii) {x ∊ N : x is an odd prime number} = {3, 5, 7, 11 ….} – Infinite

   (iii) {x ∊ Z : x is even and less than 10} = {,,, 0, 2, 4, 6, 8} – Infinite

   (iv) {x ∊ R : x is a rational number} – Infinite

   (v) {x ∊ N : x is a rational number} – Infinite