Current Electricity

* Electric Current-

- Definition: The net amount of charge flowing across the area in the forward direction in the time interval t, then $I = \frac{q}{t}$

- In SI units, the unit of current is ampere $= 1\,C\,s^{-1}$

* Electric current in Conductors

- The current carriers in metals or solid conductors is free electrons.
- The current carriers in liquid conductors or electrolytic solutions are ions.
- The current carriers in discharge tubes or gaseous conductors are free electrons.

* When there is no electric field, the electrons are moving due to thermal motion during which they collide with fixed ions.
- The direction of velocity after collision is completely random. The number of electrons travelling in any direction will be equal to the number of electrons travelling in the opposite direction, hence there is no net electric current.

* Ohm's Law.

- Statement - The potential difference across the ends of the conductor is directly proportional to current flowing through the conductor provided temperature and other physical conditions remains constant.

- Consider a conductor of length 'l' maintained at a potential difference V carrying a current 'I'.

$$V \propto I$$
$$V = RI$$

- R is a constant of proportionality.
- $R = \frac{V}{I}$ → is resistance of conductor.
- The SI unit of resistance is ohm (Ω).
- Resistance of a conductor depends on

a) material of the conductor

b) dimensions of the conductor.

1) R directly proportional to length of the conductor 'l'

2) R is inversely proportional to area of cross-section 'A'

- $\therefore R \propto \frac{l}{A}$

- For a given conductor $R = \rho \frac{l}{A}$
- Where constant of proportionality ρ depends on the material of the conductor.
- ρ is called resistivity.
- According to Ohm's law.

$$V = I \times R = \frac{I \rho l}{A} \qquad \left[R = \frac{\rho l}{A}\right]$$

$$V = \rho l \frac{I}{A} = \rho l j$$

- $\frac{I}{A}$ is called current density or current per unit area
- current density is 'j'
- SI unit of current density is (A/m^2) ampere per square metre
- If E is the uniform electric field in the conductor whose length is l, then the potential difference $V = El$.
- $El = j\rho l \quad \therefore \vec{E} = \vec{j}\rho$

$$= \vec{j} = \frac{\vec{E}}{\rho} = \vec{E}\sigma$$

σ is called the conductivity. $\sigma = 1/\rho$.

* Drift of electrons and the origin of resistivity.

- If we consider N ~~atom~~ number of electrons moving then their average velocity will be zero since their directions are random.
- If V_i is the velocity of the i^{th} electron, then

$$\frac{1}{N}\sum_{i=1}^{N} V_i = 0$$

- In the presence of electric field, electrons will be accelerated by

$$a = -\frac{eE}{m}$$

$-e \rightarrow$ charge of electron.
$m \rightarrow$ mass of electron.

$$V_i = v_i + \frac{-eE}{m}t$$

$V_i \rightarrow$ velocity of i^{th} electron
$v_i \rightarrow$ velocity after last collision.

- The collisions of the electrons do not occur at regular intervals but at random times.
- Let τ be the average time between successive collisions.
- * $V_d \equiv (V_i)_{average} = (V_i)_{average} - \frac{eE}{m}(t_i)_{average}$.
- For a small time interval Δt, all electrons to the left of the area at distances upto $|V_d|\Delta t$ would have crossed the area.
- * If n is the ~~nu~~

* $= 0 - \frac{eE}{m}\tau = -\frac{eE\tau}{m}$

* If n is the number of free electrons per unit volume in the metal then there are $n\Delta t |V_d| A$ such electrons.
- For a charge $-e$ then the total charge transported across area A to the right in time Δt is $-neA|V_d|\Delta t$.
- Direction of electric field E is in the opposite direction.
- The amount of charge crossing the area A in time Δt is given as $I\Delta t$, 'I' is magnitude of current.
- $I\Delta t = +neA|V_d|\Delta t$
- Substituting the value of $|V_d|$.
- $I\Delta t = neA \frac{eE}{m}\tau |E|\Delta t = \frac{e^2A}{m}\tau n \Delta t |E|$
- $I = |j| A$.

- $|j|A = \frac{e^2 A}{m} \tau n |E|$
- $|j| = \frac{ne^2}{m} \tau |E|$
- $\therefore \vec{j} = \frac{ne^2}{m} \tau \vec{E}$
- $\therefore \sigma = \frac{ne^2}{m} \tau$. τ & n are constants and independent of E.

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* Mobility -
- Conductivity arises from mobile charge carriers.
- An important quantity is the mobility μ defined as the magnitude of the drift velocity per unit electric field.
- $\mu = \frac{|V_d|}{E}$
- The SI unit of mobility is m^2/Vs and is 10^4 of the mobility. In CGS unit is cm^2/Vs.
- Mobility is positive.
- $V_d = \frac{e \tau E}{m}$ $\therefore$ $\mu = \frac{e \tau \cancel{E}}{m \times \cancel{E}} = \frac{e \tau}{m}$

where τ is the average collision time for electrons.

* Temperature dependence of Resistivity.
- The resistivity of a material is found to be dependent on the temperature.
- The resistivity of a metallic conductor is given by $\rho_T = \rho_0 [1 + \alpha (T - T_0)]$.

α is called the temperature co-efficient of resistivity.

- $\rho = \frac{1}{\sigma} = \frac{m}{ne^2\tau}$.

  ρ thus depends inversely both on the number of free electrons per unit volume and on the average time τ between collisions.
- As temperature increases the frequent collisions also increases, hence the average time of collisions τ decreases with temperature.
- n is not dependent on temperature.
- For insulators and semiconductors, 'n' increases with temperature.
- ρ decreases with temperature.

* Electrical Energy, Power.

- Consider a conductor with end points A & B
- Current flows from A to B.
- Electric potentials are V(A) & V(B) such that V(A) > V(B).
- In time interval Δt, an amount of charge $\Delta Q = I \Delta t$ travels from A to B
- $\Delta Q\, V(A)$ potential energy at A, $\Delta Q\, V(B)$ potential energy at B.
- Change in potential energy

  ΔU_{pot} = Final potential energy − Initial potential energy

  $= \Delta Q [V(B) - V(A)] = -\Delta Q V$

  $\Delta U_{pot} = -I V \Delta t < 0$.
- If charges moved without collision then their kinetic energy would also change according to conservation of total energy

$\therefore \Delta K = -\Delta U_{pot} = -(-IV\Delta t)$

$\Delta K = IV\Delta t > 0.$

- Due to collisions, the free charges move with drift velocity.
- During collisions the energy gained by the charges is shared with the atoms which vibrate more vigorously due to which conductor heats up.
- The amount of energy dissipated as heat in the conductor during the time interval Δt is $\Delta W = IV\Delta t.$
- The energy dissipated per unit time is the power dissipated.
- $P = \frac{\Delta W}{\Delta t} = \frac{IV\Delta t}{\Delta t}$
- $P = VI.$
- Using Ohm's law $V = IR$
- $P = IRI = I^2R$
- $P = V.\frac{V}{R} = \frac{V^2}{R}$
- The power loss is also known as ohmic loss.

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* Consider a device having resistance R to which a power P is to be delivered by transmission cables having resistance R_c.
- If V is the voltage across conductor R and I the current through it then $P = VI.$
- The power dissipated in the connecting wires, which is wasted as P_c, $P_c = I^2R_c$

- $P_c = \frac{P^2 R_c}{V^2}$ $\quad [I = \frac{P}{V}]$.
- The power wasted in the connecting wires is inversely proportional to V^2.
- To reduce P_c, these wires carry current at enormous values of V and this is the reason for the high voltage danger signs on transmission lines.

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* Electromotive Force Emf - (ε)

- Definition - It is defined as the amount of work done in transferring a unit positive charge round the circuit in which the cell is connected.
- Definition - Emf of a cell is defined as the potential difference between the two electrodes of a cell when the cell is in open circuit.
- The term potential difference of a cell is the potential difference between the two electrodes of a cell in a closed circuit.
- The relation between potential difference and emf of a cell is given as:

$V = \varepsilon - Ir$. $\quad r \rightarrow$ internal resistance of the cell.

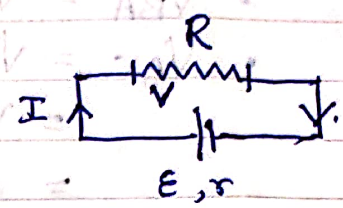
* Internal resistance of a cell - (r).

- Definition - It is the opposition offered by the material of electrolyte of the cell to the flow of the current through it.
- The internal resistance depends on

a) temperature of the electrolyte

b) concentration of electrolyte.

c) area of electrodes in electrolyte.
d) nature of electrolytes.



* Consider an external resistor R connected to a cell of emf ε and internal resistance 'r'.
- If I is the current in circuit then.

$$V = \varepsilon - Ir$$
$$IR = \varepsilon - Ir$$
$$\varepsilon = IR + Ir$$
$$\varepsilon = I(R+r)$$
$$\boxed{I = \frac{\varepsilon}{R+r}}$$

The maximum current that can be drawn from the cell is for $R=0$ then $I_{max} = \frac{\varepsilon}{r}$.

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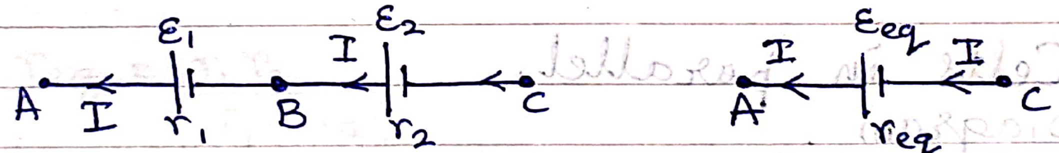
<u>Cells in series and parallel</u>

Diagram for Cells in series.



- Consider two cells having emfs ε_1 & ε_2 & internal resistances r_1 & r_2.
- $V(A)$, $V(B)$ & $V(C)$ are the potentials at points A, B & C.
- Then $V(A) - V(B)$ is the potential difference between the positive and negative terminals of the first cell.

- $V_{AB} = V(A) - V(B) = \varepsilon_1 - Ir_1$
- $V_{BA} = V(B) - V(C) = \varepsilon_2 - Ir_2$
- $V_{AC} = V(A) - V(C)$

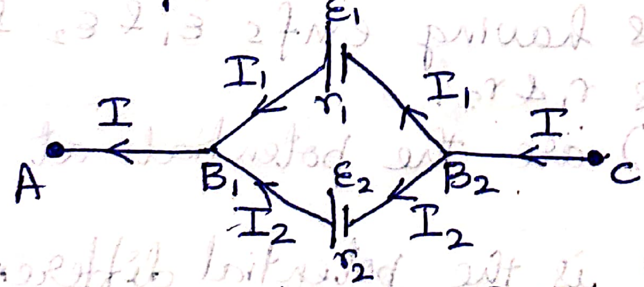
$$= V(A) - V(B) + V(B) - V(C)$$
$$= \varepsilon_1 - Ir_1 + \varepsilon_2 - Ir_2$$
$$= (\varepsilon_1 + \varepsilon_2) - I(r_1 + r_2)$$

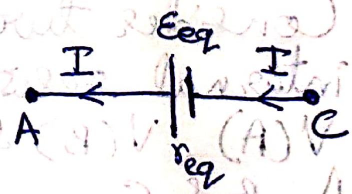
- $V_{AC} = \varepsilon_{eq} - Ir_{eq}$
- $\therefore \varepsilon_{eq} = \varepsilon_1 + \varepsilon_2$
- $r_{eq} = r_1 + r_2$

- If we connect two negatives then
  $V_{BC} = -\varepsilon_2 - Ir_2$
  then $\varepsilon_{eq} = \varepsilon_1 - \varepsilon_2$. $\boxed{(\varepsilon_1 > \varepsilon_2)}$.

- The equivalent emf of a series combination of n cells is just the sum of their individual emfs.
- The equivalent internal resistance of a series combination of n cells is just the sum of their internal resistances.

★ Cells in parallel.
Diagram.





- At points B_1 & B_2 $I = I_1 + I_2$.

- $V = V(B_1) - V(B_2) = \varepsilon_1 - I_1 r_1$ for first cell.
- $V = V(B_1) - V(B_2) = \varepsilon_2 - I_2 r_2$ for second cell.

- $V = \varepsilon_1 - I_1 r_1$ $\quad\quad$ $V = \varepsilon_2 - I_2 r_2$

  $I_1 = \dfrac{\varepsilon_1 - V}{r_1}$ $\quad\quad$ $I_2 = \dfrac{\varepsilon_2 - V}{r_2}$

- $I = \dfrac{\varepsilon_1 - V}{r_1} + \dfrac{\varepsilon_2 - V}{r_2}$.

- $I = \dfrac{\varepsilon_1}{r_1} - \dfrac{V}{r_1} + \dfrac{\varepsilon_2}{r_2} - \dfrac{V}{r_2} = \left(\dfrac{\varepsilon_1}{r_1} + \dfrac{\varepsilon_2}{r_2}\right) - V\left(\dfrac{1}{r_1} + \dfrac{1}{r_2}\right)$

- $V = \left(\dfrac{\varepsilon_1}{r_1} + \dfrac{\varepsilon_2}{r_2}\right) - I \dfrac{r_1 r_2}{r_1 + r_2}$.

- $V = \left(\dfrac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 + r_2}\right) - I \dfrac{r_1 r_2}{r_1 + r_2}$.

- Replace the combination by a single cell. between B_1 & B_2 of emf ε_{eq} & internal resistance r_{eq}.
- then $V = \varepsilon_{eq} - I r_{eq}$.
- $\varepsilon_{eq} = \dfrac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 + r_2}$

- $r_{eq} = \dfrac{r_1 r_2}{r_1 + r_2}$.

- $\dfrac{1}{r_{eq}} = \dfrac{1}{r_1} + \dfrac{1}{r_2}$

- $\dfrac{\varepsilon_{eq}}{r_{eq}} = \dfrac{\varepsilon_1}{r_1} + \dfrac{\varepsilon_2}{r_2}$.

- For n number of cells
  $$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} + \cdots + \frac{1}{r_n}.$$

- $$\frac{\varepsilon_{eq}}{r_{eq}} = \frac{\varepsilon_1}{r_1} + \cdots\cdots + \frac{\varepsilon_n}{r_n}.$$

Kirchoff's Rules.

a) Junction Rule - At any junction, the sum of the currents entering the junction is equal to the sum of currents leaving the junction.

b) Loop Rule - The algebraic sum of changes in potential around any closed loop involving resistors and cells in the loop is zero.

- 'I' is determined to be positive, the actual current in the resistor is in the direction of the arrow.
- If 'I' turns out to be negative, the current actually flows in a direction opposite to the arrow.

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Wheatstone's Bridge

- It is an application of Kirchoff's rules.
- The bridge has four resistors R_1, R_2, R_3 & R_4
- Between the points A & C a source is connected.
- Between the points B & D a galvanometer is connected.
- Special case - current I_g through G is equal to zero
- $I_1 = I_3$ & $I_2 = I_4$
- Consider the loop ADBA $(I_g = 0)$

$I_1 R_1 + 0 - I_2 R_2 = 0.$

- $I_1 R_1 = I_2 R_2.$

$$\frac{I_1}{I_2} = \frac{R_2}{R_1}$$

- Consider the loop CBDC

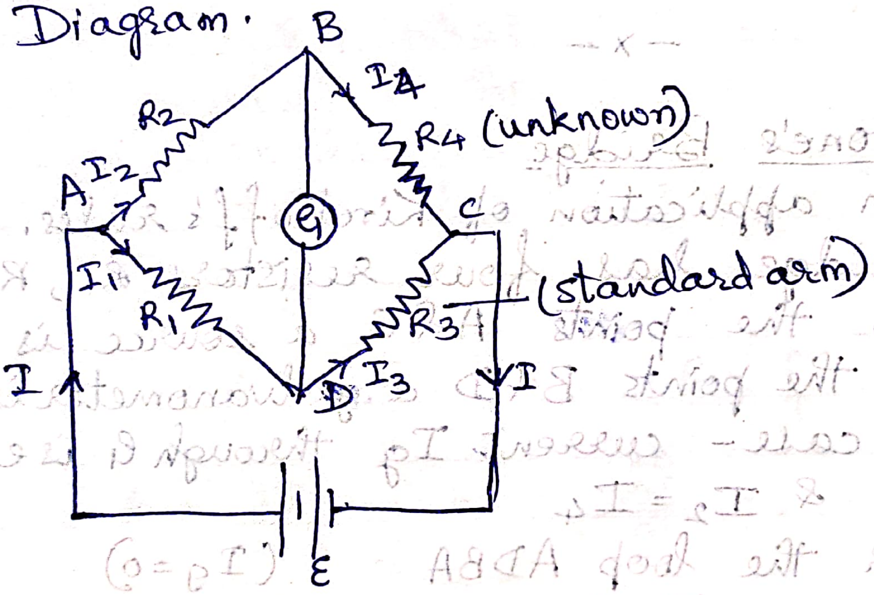
$-I_4 R_4 + 0 + I_3 R_3 = 0.$

$I_3 R_3 = I_4 R_4$ so $I_1 R_3 = I_2 R_4$

$$\frac{I_1}{I_2} = \frac{R_4}{R_3} \quad \therefore \boxed{\frac{R_2}{R_1} = \frac{R_4}{R_3}}$$

- The four resistors is called the balance condition for the galvanometer to give zero or null deflection.
- If R_4 is not known, then R_1 & R_2 are kept as known resistances, R_3 is varied till the galvanometer shows a null deflection then $R_4 = R_3 \frac{R_2}{R_1}$
- A practical device using this principle is called the metre bridge.

- Diagram.



Exercises

1) The storage battery of a car has an emf of 12V. If the internal resistance of the battery is 0.4Ω, what is the maximum current that can be drawn from the battery?

- Solution data $V' = 12V$ $r = 0.4\Omega$

$V' = Ir$.

$$I = \frac{V'}{r} = \frac{12}{0.4} = \frac{\overset{30}{\cancel{120}}}{\underset{1}{\cancel{4}}} = 30A.$$

$I = 30A.$

2) A battery of emf 10V and internal resistance 3Ω is connected to a resistor. If the current in the circuit is 0.5A, what is the resistance of the resistor? What is the terminal voltage of the battery when the circuit is closed?

- Solution data $E = 10V$; $r = 3\Omega$ $I = 0.5A$

$$I = \frac{E}{R+r}$$

$$R + r = \frac{E}{I} \quad \therefore \quad R = \frac{E}{I} - r$$

$$R = \frac{10}{0.5} - 3 = \frac{\overset{20}{\cancel{100}}}{\underset{1}{\cancel{5}}} - 3 = 17\Omega$$

$V' = IR$

$V' = 0.5 \times 17 = 8.5V$

3) At room temperature 27°c the resistance of a heating element is 100 Ω. What is the temperature of the element if the resistance is found to be 117 Ω, given that the temperature co-efficient of the material of the resistor is $1.7 \times 10^{-4}/°c$.

• Solution

data $R_{27} = 100\Omega$ $R_t = 117\Omega$

$\alpha = 1.7\times10^{-4}/°c$ $t = ?$

$$R_t = R_{27}\left[1 + \alpha(t-27)\right].$$

$$\frac{R_t}{R_{27}} = 1 + \alpha t - \alpha 27$$

$$\frac{117}{100} = 1 + \alpha t - 1.7\times10^{-4}\times 27.$$

$$1.17 - 1 = \alpha(t-27)$$

$$0.17 = 1.7\times10^{-4}(t-27)$$

$$\frac{0.17}{1.7\times10^{-4}} = t-27.$$

$$\frac{\cancel{1.7}\times10^{-1}}{\cancel{1.7}\times10^{-4}} = t-27.$$

$$t = 10^{-1+4} + 27 = 10^3 + 27$$

$$t = 1000 + 27 = 1027°c.$$

4) A negligibly small current is passed through a wire of length 15m and uniform cross section 6×10^{-7} m^2 and its resistance is measured to be 5.0Ω. What is the resistivity of the material at the temperature of the experiment?

• Solution data $l = 15m$ $A = 6\times10^{-7}m^2$ $R = 5\Omega$

$\rho = ?$

$R = \frac{\rho l}{A}$ ∴ $\rho = \frac{RA}{l}$

$\rho = \frac{\cancel{5}^{1}\times\cancel{6}^{2}\times10^{-7}}{\cancel{15}_{3}} = 2\times10^{-7}\Omega m.$

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5) A silver wire has a resistance of 2·1Ω at 27·5°c, and a resistance of 2·7Ω at 100°c. Determine the temperature co-efficient of resistivity of silver.

• Solution data $R_1 = 2{\cdot}1\Omega$ $R_2 = 2{\cdot}7\Omega$

$t_1 = 27{\cdot}5°c$ $t_2 = 100°c.$

$\alpha = ?$

$$\alpha = \frac{R_2 - R_1}{R_1t_2 - R_2t_1}$$

$$= \frac{2{\cdot}7 - 2{\cdot}1}{(2{\cdot}1)(100)-(2{\cdot}7)(27{\cdot}5)} = \frac{0{\cdot}6}{210 - 74{\cdot}25}$$

$$\alpha = \frac{0{\cdot}6}{135{\cdot}75} = 4{\cdot}4\times10^{-3}/°c$$

$$R_2 = R_0(1+\alpha t_2)$$

$$R_0 = \frac{R_2}{1+\alpha t_2} = \frac{2{\cdot}7}{1+4{\cdot}4\times10^{-3}\times100}$$

$$R_0 = \frac{2{\cdot}7}{1{\cdot}44} = 1{\cdot}875\Omega.$$

6) A heating element using nichrome connected to a 230V supply draws an initial current of 3.2A which settles after a few seconds to a steady value of 2.8 A. What is the temperature of the heating element if the room temperature is 27°c? Temperature co-efficient of resistance of nichrome averaged over the temperature range involved is $1.7 \times 10^{-4} /°c$.

• Solution data $V = 230V$ $t = 27°c$ $I = 3.2A$

at $t°c$ $I = 2.8A$. $\alpha = 1.7 \times 10^{-4} /°c$

$$R_{27} = \frac{V}{I} = \frac{230}{3.2} = 71.875\,\Omega$$

$$R_t = \frac{V}{I} = \frac{230}{2.8} = 82.143\,\Omega$$

$$R_t = R_{27}\left[1 + \alpha(t - 27)\right]$$

$$82.143 = 71.875\left[1 + \alpha(t - 27)\right]$$

$$\cancel{\frac{82.143}{71.875}} = \cancel{\left[1 + \alpha(t - 27)\right]}$$

$$\cancel{10.268 - 1} = \cancel{\alpha(t - 27)}$$

$$\cancel{\frac{9.268}{\alpha}} = \cancel{t - 27}$$

$$\cancel{t} = \cancel{\frac{9.268}{1.7 \times 10^{-4}}} + \cancel{27}$$

$$t = \cancel{5.4517 \times 10^{4} + 27}$$

$$\frac{82.143}{71.875} - 1 = \alpha(t - 27)$$

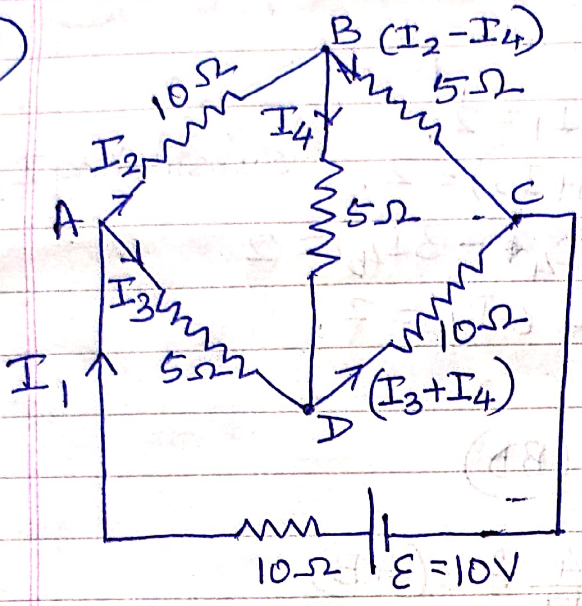
$$\frac{1.142 - 1}{1.7 \times 10^{-4}} = t - 27$$

$$\therefore t = \frac{0.142}{1.7 \times 10^{-4}} + 27$$

$$t = 835.29 + 27$$

$$t = 862.29°c$$

7) Determine the current in each branch of the network shown in the figure.



Solution – (1) Consider the mesh ABDA

$10I_2 + 5I_4 - 5I_3 = 0$ Dividing by 5

$2I_2 + I_4 - I_3 = 0$

$\therefore I_3 = 2I_2 + I_4$ — (1)

(2) Consider the mesh BCDB

$5(I_2 - I_4) - 10(I_3 + I_4) - 5I_4 = 0$ Dividing by 5

$I_2 - I_4 - 2(I_3 + I_4) - I_4 = 0$

$I_2 - I_4 - 2I_3 - 2I_4 - I_4 = 0$

$I_2 - 2I_3 - 4I_4 = 0$ Substituting value of I_3.

$I_2 - 2(2I_2 + I_4) - 4I_4 = 0$

$I_2 - 4I_2 - 2I_4 - 4I_4 = 0$

$-3I_2 - 6I_4 = 0$

$-3I_2 = 6I_4$

$I_2 = \frac{-6I_4}{3} = -2I_4$

$\boxed{I_2 = -2I_4}$ — (2)

(3) Consider the mesh ABCEA.

$10I_2 + 5(I_2 - I_4) + 10I_1 = 10$ Dividing by 5

$2I_2 + I_2 - I_4 + 2I_1 = 2$.

$3I_2 - I_4 + 2I_1 = 2$

$3(-2I_4) - I_4 + 2I_1 = 2$

$-6I_4 - I_4 + 2(I_2 + I_3) = 2$ Substituting I_2 & I_3

$-6I_4 - I_4 + 2(-2I_4 - 3I_4) = 2$

$-6I_4 - I_4 - 4I_4 - 6I_4 = 2$

$-17I_4 = 2$

* $I_4 = -\frac{2}{17}$ A. (BD)

* $I_2 = -2 \times \left(-\frac{2}{17}\right) = \frac{4}{17}$ A (AB)

* $I_3 = -3\left(-\frac{2}{17}\right) = \frac{6}{17}$ A (AD)

$I_1 = I_2 + I_3 = \frac{4}{17} + \frac{6}{17} = \frac{10}{17}$ A = total current

$BC = I_2 - I_4 = \frac{4}{17} - \left(-\frac{2}{17}\right) = \frac{4}{17} + \frac{2}{17} = \frac{6}{17}$ A

$DC = I_3 + I_4 = \frac{6}{17} + \left(-\frac{2}{17}\right) = \frac{6}{17} - \frac{2}{17} = \frac{4}{17}$ A.

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8) A storage battery of emf 8.0 V and internal resistance 0.5 Ω is being charged by a 120 V dc supply using series resistor of 15.5 Ω. What is the terminal voltage of the battery during charging? What is the purpose of having a series resistor in the charging circuit.

• Solution - data. emf = 8 V, r = 0.5 Ω, V = 120 V, R = 15.5 Ω

Charging current $I = \frac{V - \varepsilon}{R + r} = \frac{120 - 8}{15.5 + 0.5}$

$I = \frac{112}{16} = 7$ A.

Pg 21.

Terminal voltage $= E + Ir = 8 + 7 \times 0.5$
$= 8 + 3.5$
$= 11.5V.$

- The series resistor is used in the charging circuit to reduce the current from the external source as high current may damage the battery.

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9) The number ~~of~~ density of free electrons in a copper conductor is $8.5 \times 10^{28}/m^3$. How long does an electron take to drift from one end of a wire 3.0 m long to its other end? The area of cross-section of the wire is $2.0 \times 10^{-6} m^2$ and it is carrying a current of 3.0 A.

- Solution - data - $n = 8.5 \times 10^{28}/m^3$ $l = 3.0 m$. $A = 2.0 \times 10^{-6} m^2$
$I = 3A$. $e = 1.6 \times 10^{-19} C$

$I = neAv_d$. $\therefore v_d = \frac{I}{neA}$.

$$v_d = \frac{3}{8.5 \times 10^{28} \times 1.6 \times 10^{-19} \times 2.0 \times 10^{-6}}.$$

$v_d = 1.103 \times 10^{-4} m/s$.

$v = \frac{l}{t}$ $\therefore t = \frac{l}{v} = \frac{3}{1.103 \times 10^{-4}} = 27,200 s$.

$t = \frac{27,20\cancel{0}\cancel{0}s}{36\cancel{0}\cancel{0}} = 7.5h.$

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