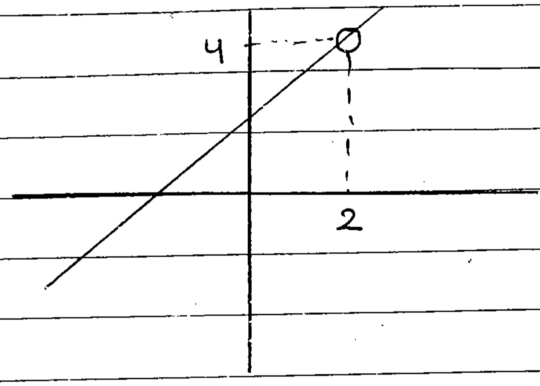
#1. $y = f(x) = \frac{x^2-4}{x-2} = x+2 \;;\; x \neq 2$

$f(2) = $ n.d.
but as $x \to 2$
$f(x) \to 4$



\# If x is sufficiently close to 2 f is arbitarily close to 4.

#2. Deleted neighbourhood of a point a $(a-\delta, a+\delta) - \{a\}$
δ is small +ve.

\# $(a-\delta, a)$ is left neighbourhood
\# $(a, a+\delta)$ is right neighbourhood.
Meaning of small +ve is something like
.000000000000000000000……000……000……00000001

#3. Left & right hand limit

$\lim_{x \to a^-} f(x) = l$ (Left hand limit)

$\lim_{x \to a^+} f(x) = M$ (Right hand limit)

#4. Existence of limit.
★ Very Important

$$\boxed{\begin{array}{l} L.H.L. = R.H.L. = \text{finite} \\ f(a^-) = f(a^+) = \text{finite} \end{array}}$$

7 Indeterminate Form

#1. $\frac{0}{0}$ #2. $\frac{\infty}{\infty}$ #3. $\infty - \infty$ #4. $0 \times \infty$

#5 ∞^0 #6. 0^0 #7. 1^∞ (5, 6, 7 → log)

Determinate Form

#1. $0^\infty = 0$ #2. $\infty + \infty = \infty$

★ V. Imp

Note: $[x]$ & $\{x\}$ have no limits at all Integers.

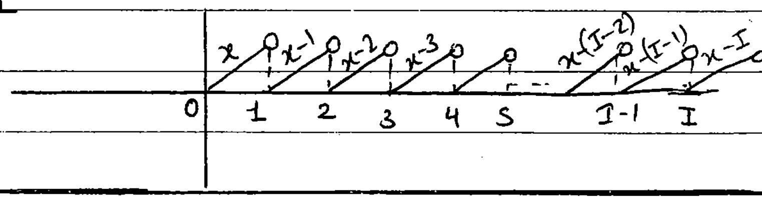
$\lim\limits_{x \to I} [x]$

$$\boxed{\begin{array}{lll} x \to I^- & I-1 & (\text{Exact}) \\ x \to I^+ & I & (\text{Exact}) \end{array}}$$

$\lim\limits_{x \to I} \{x\}$ $\lim\limits_{x \to I^+} \{x\} = \lim\limits_{x \to I^+} x - I = 0^+$ in limit language

$= 0$

& $\lim\limits_{x \to I^-} \{x\} = 1$



Question: i) $\lim\limits_{x \to \pi/2} e^{\tan x}$

⇒ $f\left(\frac{\pi}{2}^-\right) = e^\infty = \infty$ & $f\left(\frac{\pi}{2}^+\right) = e^{-\infty} = 0$ Limit D.N.E.

ii) $\lim\limits_{x \to 0} \frac{|x|}{x}$

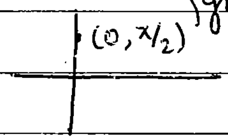
⇒ $f(0^+) = 1$ & $f(0^-) = -1$ ∴ Limit D.N.E.

iii) $\lim\limits_{x\to 0} \tan^{-1}\left(\frac{1}{x}\right)$

$\Rightarrow$ $f(0^+) = \tan^{-1}(\infty) = \frac{\pi}{2}$ & $f(0^-) = \tan^{-1}(-\infty) = -\frac{\pi}{2}$ Limit D.N.E.

★ Value of limit may or may not be in range of function.

iv) $\lim\limits_{x\to 0} \sin^{-1}(\sec x)$

$\Rightarrow$ 

graph of fu^n

NONSENSE

$\sin^{-1}x$ is not well defined.

v) $\lim\limits_{x\to 1} [x] + \sqrt{\{x\}}$

$\Rightarrow$ $f(1^+) = \text{Exact } 1 + \sqrt{0^+}$ & $f(1^-) = \text{Exact }(0) + \sqrt{1^-}$

$= 1^+ = 1$ $\quad$ $= \text{Exact } 0 + 1^-$

$= 1^- = 1$

Limit exist.

vi) $\lim\limits_{x\to 1} \left[[x] + \sqrt{\{x\}}\right]$

$\Rightarrow$ $f(1^+) = [1^+] = 1$ & $f(1^-) = [1^-] = 0$

Limit D.N.E.

vii) $\lim\limits_{x\to \pi/2} [\sin x]$

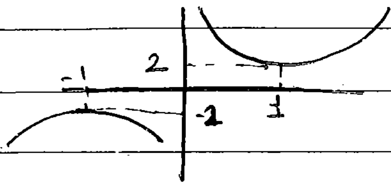
$\Rightarrow$ $f\left(\frac{\pi^-}{2}\right) = [1^-] = 0$ & $f\left(\frac{\pi^+}{2}\right) = [1^-] = 0$ Limit Exists.

viii) $\lim\limits_{x\to \pi/2} [\cos x]$

$\Rightarrow$ $f\left(\frac{\pi^-}{2}\right) = [0^+] = 0$ & $f\left(\frac{\pi^-}{2}\right) = [0^-] = -1$ Limit D.N.E.

ix) $\lim_{x \to -1} \left\{x + \frac{1}{x}\right\}$

$\left\{x + \frac{1}{x}\right\} = 2$ (min & max)



$f(-1^+) = \{-2^+\} = 1$

& $f(-1^-) = \{-2^-\} = 1$

Limit exist

x) $\lim_{x \to 1} \frac{x}{[x]}$

Domain $= R - [0,1)$

$f(1^-) = \frac{1^-}{[1^-]} = \frac{1}{\text{Exact } 0} = \text{n.d.}$ not in domain

$f(1^+) = \frac{1^+}{1} = 1$ Limit exist.

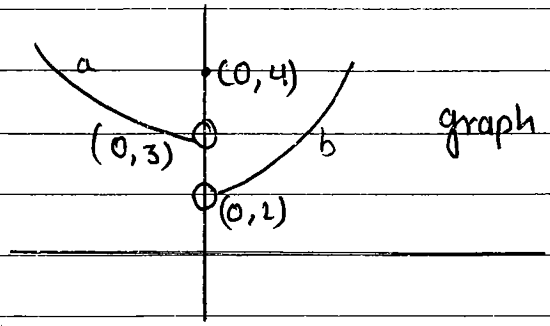
xi) $\lim_{x \to 1} \frac{[x]}{x}$

$f(1^+) = \frac{1}{1} = 1$ & $f(1^-) = \frac{[1^-]}{1} = 0$ Limit D.N.E.

Concept of one sided limit

Note: If one neighbourhood not in domain then definition of existence of limit is $\boxed{\lim_{x \to a^+} f(x) = \text{finite}}$ a^- not in domain.

Question: $\cancel{\lim} f(x^3-x^2) = \lambda \lim\limits_{x\to 0} f(2x^4-x^5)$ $\lambda = ?$



graph of f

Solution: $x^2(x-1)$ & $x^4(2-x)$

$+ve\,(-ve) = -ve \Rightarrow a$ $\quad$ $(+ve)(+ve) = +ve \Rightarrow b$

$f(0^-) = 3^+ = 3$ $\quad$ $f(0^+) = 2^+ = 2$

$3 = \lambda(2)$

$\Rightarrow \boxed{\lambda = \frac{3}{2}}$

Five fundamental Theorem:

If $\lim\limits_{x\to c} f(x) = L$ & $\lim\limits_{x\to c} g(x) = M$

Then i) sum rule

$\lim\limits_{x\to c} f(x) + g(x) = L+M$

ii) difference rule

$\lim\limits_{x\to c} f(x) - g(x) = L-M$

iii) Product rule

$\lim\limits_{x\to c} fg(x) = LM$

iv) $\lim\limits_{x\to c} \frac{f}{g} = \frac{L}{M}$ $\quad M \neq 0$

v) Constant multiple rule

$\lim\limits_{x\to c} k f(x) = KL$

Note:

#1. If $\lim\limits_{x\to c} f$, $\lim\limits_{x\to c} g$ exists then $\lim\limits_{x\to c} f \pm g$, $\lim\limits_{x\to c} fg$ & $\lim\limits_{x\to c} \frac{f}{g}$ exists

#2. If f exists & g D.N.E

Then $f \pm g$ DNE $fg, \frac{f}{g}$ can't say.

#3. f, g DNE Then

$f \pm g, fg, f/g$ can't say

Example

(i) g exist f DNE

→ $\lim_{x\to 1} x[x]$ $f(1^+)=1$ $f(1^-)=0$ DNE

→ $\lim_{x\to 0} x[x]$ $f(0^-)=0$ $f(0^+)=0$ Exist.

→ $\lim_{x\to 0} \frac{x}{\operatorname{sgn}(x)}$ $f(0^+)=0$ $f(0^-)=0$ Exist.

→ $\lim_{x\to 2} \frac{x}{[x]}$ $f(2^-)=1$ $f(2^+)=2$ DNE

Example

(ii) f, g DNE

→ $\lim_{x\to 1} [x]+\{x\}$ $=1$ Exist

→ $\lim_{x\to 0} [x]+\operatorname{Sgn}(x)$ $f(0^-)=-2$ & $f(0^+)=1$ DNE

→ $\lim_{x\to 1} \{x\}[x]$ $=0$ Exist

→ $\lim_{x\to 0} \{x\}[x]$ $f(0^-)=-1$ & $f(0^+)=0$ DNE

Important Identities/Formula

#1. $a^4 + b^2a^2 + b^4 = (a^2 - ab + b^2)(a^2 + ab + b^2)$

#2. $x^4 + x^2 + 1 = (x^2 - x + 1)(x^2 + x + 1)$

#3. Binomial Theorem $n \in N$ (in this expansion has $n+1$ terms)

$$(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + {}^nC_3 x^3 + \ldots + {}^nC_n x^n$$

* For any index (mostly Q)

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{\underline{|2}} x^2 + \frac{n(n-1)(n-2)}{\underline{|3}} x^3 \ldots \infty$$

Condition of convergence of sequence $|x| < 1$ (IIT me yahi sequence aata convergence mtlb jo kisi ek no. may be zero ko tend ∞ jaane par.)

#4. S_n G.P. is $\dfrac{a(1-r^n)}{(1-r)}$

#5. S_∞ $\lim_{n \to \infty}$ $r \in (-1, 1)$ $\dfrac{a}{1-r}$

An Important logic

		Coeff	~~Highest degree~~
#6	$\sum n = \dfrac{n(n+1)}{2}$	$\dfrac{1}{2}$	n^2
#7	$\sum r^2 = \dfrac{n(n+1)(2n+1)}{6}$	$\dfrac{1}{3}$	n^3
#8	$\sum n^3 = \left(\dfrac{n(n+1)}{2}\right)^2$	$\dfrac{1}{4}$	n^4
#9	$\sum n^7 = \dfrac{n^8}{8} + \cdots$	$\dfrac{1}{8}$	n^8

Various Strategies to Solve Limit

- # Factorise
- # Rationalise
- # Double rationalise
- # Binomial Theorem
- # Law of Love $\lim\limits_{x\to\infty} \frac{1}{x^n} = 0$ ($\infty \Rightarrow$ in denominator)

Question: $\lim\limits_{x\to 2} \dfrac{2^x + 2^{3-x} - 6}{\sqrt{2^{-x}} - 2^{1-x}}$

Solution: $\lim\limits_{x\to 2} \dfrac{2^x}{2^x}\left(\dfrac{2^x+2^{3-x}-6}{\sqrt{2^{-x}} - 2^{1-x}}\right) = \lim\limits_{x\to 2} \dfrac{2^{2x}+8-6.2^x}{2^{x/2}-2}$

Substitute $2^x = y$

$$\lim_{y\to 4} \frac{(y-2)(y-4)}{\sqrt{y}-2} = (y-2)(\sqrt{y}+2) = 2(4) = \boxed{8}$$

Question: $\lim\limits_{x\to \pi/4} \dfrac{-1+\cot^3 x}{2+\cot x+\cot^3 x}$

Solution: $\lim\limits_{y\to 1} \dfrac{(y-1)(y^2+y+1)}{(y-1)(y^2+y+2)} = \dfrac{3}{4}$

Question: $\lim\limits_{x\to 9} \dfrac{3-\sqrt{x}}{4-\sqrt{2x-2}}$

Solution: $\lim\limits_{x\to 9} \dfrac{(3-\sqrt{x})}{(4-\sqrt{2x-2})} \times \dfrac{(3+\sqrt{x})}{(3+\sqrt{x})} \times \dfrac{(4+\sqrt{2x-2})}{(4+\sqrt{2x-2})}$

$= \lim\limits_{x\to 9} \dfrac{\cancel{9-x}}{\cancel{18}-2x}\,\dfrac{1}{2} \times \dfrac{4+\sqrt{2x-2}}{3+\sqrt{x}} = \dfrac{1}{2}\times\dfrac{8}{6} = \dfrac{2}{3}$

Question: $\lim\limits_{x \to 1} \cos\left(\frac{x}{x+1}\right) \cos\left(\frac{x}{x-1}\right)$

Solution: $\cos \frac{x}{2} = 0$ & $\cos \frac{x}{0} \approx \cos \infty \simeq [-1, 1] \to$ finite

note: hum 1 ni rakh the h.

$$\lim_{x \to 1} \cos\left(\frac{x}{x+1}\right) \cos\left(\frac{x}{x-1}\right) = 0 \times [-1,1] = 0$$

Question: $\lim\limits_{x \to 0} \frac{\sqrt{ax+b} - 2}{x} = 3$ find a, b.

by limit, $\frac{0}{0}$ = finite $\quad \frac{1}{\to 0} = \infty$

So, $\sqrt{b} - 2 = 0$

$\Rightarrow \boxed{b = 4}$

now,

$$\lim_{x \to 0} \frac{\sqrt{ax+b} - 2}{x} \frac{(\sqrt{ax+b} + 2)}{\sqrt{ax+b} + 2} = 3$$

$$\lim_{x \to 0} \frac{a\cancel{x} + \cancel{4} - \cancel{4}}{\cancel{x}(\sqrt{ax+4} + 2)} = 3$$

$$\lim_{x \to 0} \frac{a}{4} = 3 \Rightarrow \boxed{a = 12}$$

Question: $\lim\limits_{n \to \infty} \frac{(3n+3)!}{(n+1)^3 (3n)!}$

Solution: Trick: Divide by highest order of ∞

$n^3, n^2, n, 1$

$$\lim_{n \to \infty} \frac{(3n+3)(3n+2)(3n+1)(3n)!}{(n+1)(n+1)(n+1)(3n)!}$$

$$\lim_{n \to \infty} \frac{\left(3 + \frac{3}{n}\right)\left(3 + \frac{2}{n}\right)\left(3 + \frac{1}{n}\right)}{\left(1 + \frac{1}{n}\right)\left(1 + \frac{1}{n}\right)\left(1 + \frac{1}{n}\right)} = \frac{(3)^3}{(1)^3} = 27$$

OR

Get rid of lower order ∞

$$\frac{\text{Coeff of } n^3 \text{ in Nr}}{\text{Coeff of } n^3 \text{ in Dr}}$$

$$\lim_{n\to\infty} \frac{(3n+\cancel{3})(3n+\cancel{2})(3n+\cancel{1})}{(n+\cancel{1})(n+\cancel{1})(n+\cancel{1})} = \frac{3\times3\times3}{1\times1\times1} = \frac{27}{1}$$

Question: $\lim_{x\to\infty} \frac{\sqrt{2x^2+3}}{3x-7}$

Solution: $\lim_{x\to\infty} \frac{\sqrt{(x^2)\left(2+\frac{3}{x^2}\right)}}{x\left(3-\frac{7}{x}\right)} = \lim_{x\to\infty} \frac{|x|\sqrt{2+\frac{3}{x^2}}}{x\left(3-\frac{7}{x}\right)} = -\frac{\sqrt{2}}{3}$ //

OR.

$$\lim_{x\to\infty} \frac{\sqrt{2x^2+\cancel{3}}^{\text{ Gone}}}{3x-\cancel{7}_{\text{ Gone}}} = \lim_{x\to\infty} \frac{|x|}{x}\frac{\sqrt{2}}{3} = -\frac{\sqrt{2}}{3}$$

Question: $\lim_{x\to\infty} \frac{\left\{\sum_{r=1}^{n}(2r^2+r-1)\right\}\left\{\sum_{r=1}^{n}(3r^5+4r)\right\}}{\left\{\sum_{r=1}^{n}(4r^3+4r^2+7)\right\}\left\{\sum_{n=1}^{n}(7r^4+3)\right\}}$

Solution: $\cancel{2\frac{n^3}{3}+\cdots}$ $\frac{\left(\frac{2n^3}{3}+\cdots\right)\left(\frac{3n^6}{6}+\cdots\right)}{\left(\frac{4n^4}{4}+\cdots\right)\left(\frac{7n^5}{5}+\cdots\right)}$ or $\frac{\sum_{n=1}^{n} 2r^2 \sum_{n=1}^{n} 3r^5}{\sum_{r=1}^{n} 4r^3 \sum_{r=1}^{n} 7r^4}$

$$= \frac{\frac{2n^3}{3}\times\frac{3n^6}{6}}{\frac{4n^4}{4}\times\frac{7n^5}{5}} = \frac{\frac{2}{3}\times\frac{1}{2}}{7/5} = \frac{5}{21}$$

Question: $\lim\limits_{n\to\infty} \dfrac{(n^3-2n^2+1)^{1/2} + (n^4+1)^{1/3}}{(3n^6+6n^5+2)^{1/4} - (3n^7+4n^3+1)^{1/5}}$

Solution:

$\frac{3}{2} = 1.5 \qquad \frac{4}{3} = 1.33$

$\frac{6}{4} = \frac{3}{2} = 1.5 \qquad \frac{7}{5} \approx 1.4$

$$\frac{1^{3/2}}{3^{1/4}} = \frac{1}{3^{1/4}} \text{ Ans}$$

OR

Divide in n^r & d^r by $n^{3/2}$

$$\lim_{n\to\infty} \frac{\left(\frac{n^3 - \cancel{2n^2} + \cancel{1}}{n^3}\right)^{1/2} + \left(\frac{n^8 + \cancel{2n^4+1}}{\cancel{n^9}}\right)^{1/6}}{\left(\frac{3n^6 + \cancel{6n^5+2}}{n^6}\right)^{1/4} - \left[\frac{(3n^7 + \cancel{4n^3+1})^2}{\cancel{n^{15}}}\right]^{1/10}} = \frac{1^{1/2}}{3^{1/4}}$$

Question: $\lim\limits_{n\to\infty} \dfrac{(n+1)^5 - (n-1)^5}{(n+1)^4 + (2n+1)^4 + (3n+1)^4 - (4n+1)^4}$

Solution: By Binomial Expansion,

$$\frac{{}^5C_4 n^4 + {}^5C_4 n^4}{n^4 + (2n)^4 + (3n)^4 - (4n)^4}$$

$$= \frac{{}^5C_4 + {}^5C_4}{1+16+81-256} = \frac{2\times 5}{-138} = \frac{-10}{138}$$

Question: $\prod\limits_{x=2}^{\infty} \left(\frac{x^3+1}{x^3-1}\right) = ?$

Solution: $\dfrac{(x+1)(x^2-x+1)}{(x-1)(x^2+x+1)}$ $\rightarrow \dfrac{x(x-1)+1}{x(x+1)+1}$ $\rightarrow x = x+0$ $\downarrow$ 1 term will be left.

$x: x+2 \rightarrow$ 2 terms will be left

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$$\lim_{n\to\infty}\left(\frac{\cancel{3}}{1}\times\frac{\cancel{4}}{2}\times\frac{5}{\cancel{3}}\times\cdots\frac{n+1}{n-1}\right)\left(\frac{\cancel{3}}{\cancel{7}}\times\frac{\cancel{7}}{13}\times\cdots\frac{n^2-n+1}{n^2+n+1}\right)$$

$$\lim_{n\to\infty}\frac{n(n+1)}{2}\times\frac{3}{(n^2+n+1)}$$

$$=\frac{3n^2}{2n^2}=\frac{3}{2}$$

Question: $\lim_{x\to\infty}\sqrt{4x^2+x}-\sqrt{\frac{4x^3}{x+2}}$

Solution:

$$\lim_{x\to\infty}\frac{\left(\sqrt{4x^2+x}-\sqrt{\frac{4x^3}{x+2}}\right)\left(\sqrt{4x^2+x}+\sqrt{\frac{4x^3}{x+2}}\right)}{\left(\sqrt{4x^2+x}+\sqrt{\frac{4x^3}{x+2}}\right)}$$

$$\lim_{x\to\infty}\frac{4x^2+x-\frac{4x^3}{x+2}}{\left(\sqrt{4x^2+x}+\sqrt{\frac{4x^3}{x+2}}\right)}$$

$$\lim_{x\to\infty}\frac{\cancel{4x^3}+x^2+8x^2+\cancel{2x}-\cancel{4x^3}}{(x+\cancel{2})\left(\sqrt{4x^2+\cancel{x}}+\sqrt{\frac{4x^3}{x+\cancel{2}}}\right)}=\frac{9}{4}$$

Question: $\lim_{x\to\infty}\sqrt{x^2-2x-1}-\sqrt{x^2-7x-3}$

Solution: $$\lim_{x\to\infty}\frac{\left(\sqrt{x^2-2x-1}-\sqrt{x^2-7x-3}\right)\left(\sqrt{x^2-2x-1}+\sqrt{x^2-7x-3}\right)}{\left(\sqrt{x^2-2x-1}+\sqrt{x^2-7x-3}\right)}$$

$$\lim_{x\to\infty}\frac{\cancel{x^2}-2x-\cancel{1}-\cancel{x^2}+7x+\cancel{3}}{\sqrt{x^2-\cancel{2x}-\cancel{1}}+\sqrt{x^2-\cancel{7x}-\cancel{3}}}=\frac{5}{2}$$

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Question: $\lim_{x\to-\infty} \sqrt{x^2-2x-1} - \sqrt{x^2-7x-3}$

Solution: $\lim_{x\to-\infty} \dfrac{5x+2}{|x|\sqrt{1-\frac{2}{x}-\frac{1}{x^2}} + |x|\sqrt{1-\frac{7}{x}-\frac{3}{x^2}}}$

$\lim_{x\to-\infty} \dfrac{5}{-1-1} = \dfrac{5}{-2}$

Question: $\lim_{x\to-2} \dfrac{1}{x+2} - \dfrac{12}{x^3+8}$ ($\infty-\infty$ form)

Solution: $\lim_{x\to-2} \dfrac{x^2-2x+4-12}{(x+2)(x^2-2x+4)}$ (LCM)

$\lim_{x\to-2} \dfrac{(x-4)\cancel{(x+2)}}{\cancel{(x+2)}(x^2-2x+4)}$

$= \dfrac{-2-4}{(-2)^2-2(-2)+4} = \dfrac{-6}{12} = -\frac{1}{2}$

Question: $\lim_{x\to\pi/2} \tan^2 x\left(\sqrt{2\sin^2 x+3\sin x+4} - \sqrt{\sin^2 x+6\sin x+2}\right)$

Solution: $\lim_{x\to\pi/2} \dfrac{\sin^2 x}{\cos^2 x}\left(\dfrac{2\sin^2 x+3\sin x+4-\sin^2 x-6\sin x-2}{\sqrt{2\sin^2 x+3\sin x+4}+\sqrt{\sin^2 x+6\sin x+2}}\right)$

$\lim_{x\to\pi/2} \dfrac{\sin^2 x}{\cos^2 x} \dfrac{(\sin^2 x-3\sin x+2)}{(\sqrt{2\sin^2 x+3\sin x+4}+\sqrt{\sin^2 x+6\sin x+2})}$

$\lim_{x\to\pi/2} \dfrac{\sin^2 x}{(1-\sin^2 x)} \dfrac{(\sin x-1)(\sin x-2)}{(\sqrt{} + \sqrt{})}$

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$$\lim_{x\to\pi/2} \frac{-\sin^2 x(\sin x-2)}{(1+\sin x)(\sqrt{\ } + \sqrt{\ })}$$

$$\frac{-1(1-2)}{(1+1)(3+3)} = \frac{1}{12}$$

OR

$$\lim_{x\to\pi/2} \frac{\sin^2 x}{\cos^2 x}\left\{\frac{\sin^2 x-3\sin x+2}{\sqrt{\ }+\sqrt{\ }}\right\}$$

$$\lim_{x\to\pi/2} \frac{1}{\cos^2 x}\frac{(\sin^2 x-3\sin x+2)}{6}$$

Putting limit will not affect

Note: If an expression is non-zero finite appearing in multiplication or division put value as soon as possible.

* Limit applied only once.

Learn

Standard Limit

$\lim_{x\to a} \frac{x^n-a^n}{x-a} = na^{n-1}$

$\lim_{x\to 1} \frac{x^n-1}{x-1} = n$

Proof: $x=a+h$

$$\lim_{h\to 0} \frac{(a+h)^n-a^n}{h}$$

$$\lim_{h\to 0} \frac{{}^nC_0\cancel{a^n} + {}^nC_1 a^{n-1}\cancel{h} + \overset{0}{{}^nC_2 a^{n-2} h^{\cancel{2}}} + \overset{0}{\ldots} + \overset{0}{{}^nC_n h^{\cancel{n}\,n-1}} - \cancel{a^n}}{\cancel{h}} = na^{n-1}$$

Question: $\lim_{x\to 2} \frac{x^{n+1}-2^{n+1}}{x-2} = 80 \quad n\in N \quad n=?$

Solution: $(n+1)2^n = 80$

$= 5\times 16$

$\boxed{n=4}$

Question: $\lim_{x\to 0} \frac{(\cos x)^{1/3}-(\cos x)^{1/2}}{\sin^2 x} = ?$

Solution: $\frac{1}{3}$ & $\frac{1}{2}$ L.C.M of 2 & 3 = 6.

Put $\cos x = y^6$. $x\to 0 \Rightarrow y\to 1$

$\lim_{y\to 1} \frac{y^2-y^3}{1-y^{12}} = \frac{y^2(1-y)}{(1-y^{12})} = \frac{\cancel{y^2}(1)^2}{12(1)^{11}} = \frac{1}{12}$

Question: $\lim_{x\to 1} \frac{(7+x^3)^{1/3}-\sqrt{3+x^2}}{x-1}$

Solution: For B.E we need $(1+x)^n$

Put $x=1+h$

$\lim_{h\to 0} \frac{(7+1+h^3+3h^2+3h)^{1/3}-\sqrt{3+1+2h+h^2}}{h}$

$\lim_{h\to 0} \frac{(8+h^3+3h^2+3h)^{1/3}-(4+2h+h^2)^{1/2}}{h}$

$\lim_{h\to 0} \frac{2\left(1+\frac{h^3}{8}+\frac{3h^2}{8}+\frac{3h}{8}\right)^{1/3}-2\left(1+\frac{h}{2}+\frac{h^2}{4}\right)^{1/2}}{}$

(h) → Tells us how far in exponsion

$\lim_{h\to 0} \cancel{2}\left[1+\frac{1}{3}\left(\frac{3\cancel{h}}{8}+\frac{3h^2}{8}+\frac{h^3}{8}\right)+\frac{1}{3}\left(\frac{-2}{3}\right)\left(\frac{3h}{8}+\cancel{\frac{3h^2}{8}}+\frac{h^3}{8}\right)^2+\dots\right]$

$-\cancel{2}\left[1+\frac{1}{2}\left[\frac{2\cancel{h}}{2}+\frac{h^2}{4}\right]+\frac{1}{2}\left(\frac{-1}{2}\right)\left[\frac{h}{2}+\frac{h^2}{4}\right]^2\dots\right]$

$$= \frac{2}{8} - \frac{1}{2} = \frac{1}{2}$$

Question: $\lim_{x\to\infty} \sqrt[3]{x^3+3x^2} - \sqrt{x^2-2x}$

Solution: $\lim_{x\to\infty} x\left(1+\frac{3}{x}\right)^{1/3} - x\left(1-\frac{2}{x}\right)^{1/2}$

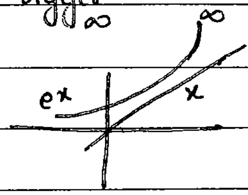
$$\lim_{x\to\infty} x\left[\cancel{1} + \frac{1}{3}\left(\frac{3}{x}\right) + \frac{\frac{1}{3}\left(\frac{-2}{3}\right)}{L2}\left(\frac{3}{x}\right)^2 + \cdots\right]$$

$$- \cancel{x}\left[\cancel{1} + \frac{1}{2}\left(\frac{-2}{x}\right) + \frac{\frac{1}{2}\left(\frac{-1}{2}\right)}{L2}\left(\frac{-2}{x}\right)^2 \cdots\right]$$

$$\lim_{x\to\infty} 1+1 = 2$$

Question: $\lim_{x\to\infty} [(x+1)(x+2)(x+3)\ldots(x+100)]^{1/n} - x = \text{finite} \quad n\in N \; n=?\; l=?$

Solution: $e^x - x = \infty$ (bigger ∞ − smaller ∞ $= \infty$) $\qquad x^2 - x = \infty$ (bigger ∞ − smaller ∞ $= \infty$)



for finite

$Kx^n - x^n = \text{finite}$

$\Rightarrow n = 100$

$$\lim_{x\to\infty} \left[x^{100} + x^{99}(1+2+3\ldots+100) + x^{98}(\ldots) + \ldots\right]^{\frac{1}{100}} - x$$

$$\lim_{x\to\infty} x\left[1 + \frac{1}{x}(5050) + \cdots\right]^{\frac{1}{100}} - x$$

$$\lim_{x\to\infty} \cancel{x}\left[\cancel{1} + \frac{1}{100}\,\frac{1}{x}(5050) + \left(\frac{1}{x}(\ldots)\right)^2 \frac{\frac{1}{100}\left(\frac{1}{100}-1\right)}{L2} \cdots\right] - \cancel{x}$$

$$= \frac{5050}{100} = 50.50$$

Question: $\lim_{n\to\infty} (2^n+3^n+4^n)^{1/n}$

Solution: $\lim_{n\to\infty} 4\left(\cancel{\frac{2^n}{4^n}}^{\to 0}+\cancel{\frac{3^n}{4^n}}^{\to 0}+1\right)^{1/n}$

$4\times1=4$

Question: $\lim_{x\to 0} \left(1^{\operatorname{cosec}^2x}+2^{\operatorname{cosec}^2x}+3^{\operatorname{cosec}^2x}+\dots 100^{\operatorname{cosec}^2x}\right)^{\sin^2x}$

Solution: $\lim_{x\to 0} 100\left(\cancel{\left(\frac{1}{100}\right)^{\operatorname{cosec}^2x}}^{\to 0}+\cancel{\left(\frac{2}{100}\right)^{\operatorname{cosec}^2x}}^{\to 0}+\cancel{\left(\frac{3}{100}\right)^{\operatorname{cosec}^2x}}^{\to 0}+\dots 1\right)^{\sin^2x}$

$\lim_{x\to 0} 100\times1=100$

Question: $\lim_{x\to\infty} \frac{x^2+1}{x+1}-ax-b=0$ find a,b

$=3$

$=\infty$

Solution: $\lim_{x\to\infty} \frac{x^2+1-a(x^2+x)-b(x+1)}{x+1}=0$

$\lim_{x\to\infty} \frac{x^2(1-a)+x(-b-a)+1-b}{x+1}=0$

Exact 0 $x\to\infty=0$

i) $\Rightarrow a=1$

$\frac{-b-a}{1}=0 \Rightarrow \boxed{b=-1}$ $c\in R$

ii) $a=1$

$\frac{-b-a}{1}=3$ $\boxed{b=-4}$ $c\in R$

iii) $+\infty$

for $+\infty$

$1-a>0 \Rightarrow \boxed{a<1}$ $b,c\in R$.

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Q $\lim_{x \to 2} \frac{ax^2+bx+c}{x-2} = 3$ find a, b, c $\quad a, b, c \in R$

Solution $\quad 4a + 2b + c = 0$ for making $\frac{0}{0}$ form else it will ~~become linear.~~ → by substituting $x = 2$

$x = 2 + h$

$$\lim_{h \to 0} \frac{a(2+h)^2 + b(2+h) + c}{h} = 3$$

$$\frac{ah^2 + h(4a+b) + \overbrace{(4a+2b+c)}^{=0}}{h} = 3$$

$\Rightarrow \quad 4a + b = 3$

So, we have,

$4a + 2b + c = 0$ & $4a + b = 3$.

$(a, b, c) = (a, 3-4a, 4a-6)$ & $a \in R$

Sandwich Theorem

$f(x) \leq g(x) \leq h(x) \quad \forall x$

& $\lim_{x \to c} f(x) = \lim_{x \to c} h(x) = L$

$\Rightarrow \lim_{x \to c} g(x) = L$

* $\boxed{x - 1 < [x] \leq x}$

Question: $\lim_{x\to -\infty} \frac{5x^2 - \sin 3x}{x^2+10}$

Solution: $\lim_{x\to-\infty}\left(\frac{5x^2-1}{x^2+10} \le \frac{5x^2-\sin 3x}{x^2+10} \le \frac{5x^2+1}{x^2+10} \right)$

$$5 \le \lim_{x\to -\infty} \frac{5x^2-\sin 3x}{x^2+10} \le 5$$

$$\Rightarrow \lim_{x\to -\infty} \frac{5x^2-\sin 3x}{x^2+10} = 5$$

Question: $\lim_{n\to\infty} \frac{[1^2x]+[2^2x]+[3^2x]+\dots+[n^2x]}{1^2+2^2+3^2\dots+n^2}$

Solution:

$$\frac{(1^2x-1)+(2^2x-1)+(3^2x-1)-(n^2x-1)}{\frac{n(n+1)(2n+1)}{6}} < f(x) \le \frac{1^2x+2x^2\dots n^2x}{\frac{n(n+1)(2n+1)}{6}}$$

$$\lim_{n\to\infty}\left(x - \frac{1}{\frac{n(n+1)(2n+1)}{6}} < f(x) < x \right)$$

$$x < \lim_{n\to\infty} f(x) < x$$

$$\lim_{n\to\infty} f(x) = x$$

Question: $\lim_{n\to\infty} \frac{n}{n^2+1} + \frac{n}{n^2+2} + \frac{n}{n^2+3} + \dots + \frac{n}{n^2+n}$ H.P.

Solution:

$$\underbrace{\frac{n}{n^2+n}}_{=\text{Smallest}} + \frac{n}{n^2+n} \dots n \text{ times} < f(x) < \underbrace{\frac{n}{n^2+1}}_{=\text{Biggest}} + \frac{n}{n^2+1} \dots n \text{ times}$$

$$\lim_{n\to\infty}\left(n\left(\frac{n}{n^2+n}\right) < f(x) < n\left(\frac{n}{n^2+1}\right)\right)$$

$$1 < \lim_{n\to\infty} f(x) < 1$$

$$\lim_{n\to\infty} f(x) = 1$$

Question: $\lim_{x\to 0} \frac{x}{a}\left[\frac{b}{x}\right]$

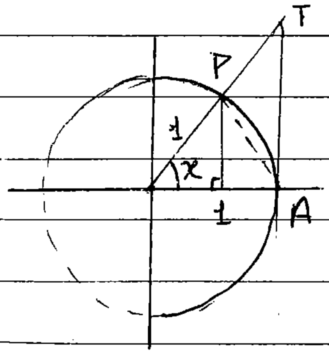
Solution: $\lim_{x\to 0} \frac{x}{a}\left(\frac{b}{x} - \underbrace{\left\{\frac{b}{x}\right\}}_{[0,1)}\right)$

$$\lim_{x\to 0} \frac{b}{a} - \frac{x}{a}[0,1) = \frac{b}{a}$$

Theorem (Limit of Trig. fns.)

x is in radians

$$\lim_{x\to 0}\frac{\sin x}{x} = 1 = \lim_{x\to 0}\frac{\tan x}{x} = \lim_{x\to 0}\frac{\tan^{-1}x}{x} = \lim_{x\to 0}\frac{\sin^{-1}x}{x} = \lim_{x\to 0}\frac{x}{\tan x} = \lim_{x\to 0}\frac{x}{\sin x}$$



area of ΔOAP < area of sector $\widehat{OAP}$ < area of ΔOAT

$$\frac{1}{2}\times 1\times 1\times \sin x < \frac{x}{2\pi}\pi r^2 < \frac{1}{2}\times 1\times \tan x$$

$$\frac{\sin x}{x} < 1 < \frac{\tan x}{x} \qquad \lim_{x\to 0}\left(\underset{1}{1} < \underset{1}{\frac{x}{\sin x}} < \underset{1}{\sec x}\right)$$

Sandwich theorem

& for $\tan^{-1}x$ = $x = \tan\theta$

$$\lim_{\theta \to 0} \frac{\theta}{\tan\theta} = 1$$

√Imp
Note: $\frac{\sin x}{x} \to 1$ from left side (1^-) $\boxed{\frac{\sin x}{x} < 1}$

$\frac{\tan x}{x} \to 1$ from right side (1^+)

Question: $\lim_{x\to 0} \left[\frac{\sin x}{x}\right] = \lim_{x \to 0} [1^-] = 0$

Question: $\lim_{x\to 0} \left[\frac{\tan x}{x}\right] = \lim_{x\to 0} [1^+] = 1$

Question: $\left[\lim_{x\to 0} \frac{x}{\tan^{-1}x}\right] = [1] = 1$

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Notes

Question: $\lim_{x\to n}\left[\frac{\sin x}{x} + \frac{\sin(2^2 x)}{x/2} + \frac{\sin(3^2 x)}{x/3} \dots n \text{ terms}\right] = ?$

Solution $\lim_{x\to n}\left[\frac{\sin x}{x} + \left(\frac{\sin 4x}{4x}\right)8 + \left(\frac{\sin 9x}{9x}\right)27 \dots n \text{ terms}\right]$

$\lim_{x\to n}\left[1^- + 8^- + 27^- + \dots + n^-\right]$

$\lim_{x\to n}\left[\left(\left[\frac{n(n+1)}{2}\right]^2\right)^-\right] = \left[\frac{n(n+1)}{2}\right]^2 - 1$

* If $\tan x$ is given in place of $\sin x$.
Answer will be $\left[\frac{n(n+1)}{2}\right]^2$

Question: Show that $\lim_{x\to 0} \frac{\sin x - x}{x^3} = \frac{-1}{6}$

Solution: $\lim_{x\to 0} \frac{\sin x - x}{x^3} = \frac{-1}{6}$ $\quad x \to 3x \quad 3x \to 0$ if $x \to 0$

Let $\lim_{x\to 0} \frac{\sin x - x}{x^3} = L$

then $L = \lim_{3x\to 0} \frac{\sin 3x - 3x}{(3x)^3}$

$= \lim_{3x\to 0} \frac{3\sin x - 4\sin^3 x - 3x}{27x^3}$

lim defined lim defined

$= \lim_{x\to 0} \frac{3\sin x - 3x}{27x^3} - \lim_{x\to 0} \frac{4\sin^3 x}{27x^3}$

$L = \frac{L}{9} - \frac{4}{27}$

$\Rightarrow \frac{8L}{9} = \frac{-4}{27} \Rightarrow L = \frac{-4\times 9}{8\times 27} = \frac{-1}{6}$

Question: $\lim_{x \to 1} (1-x) \tan\left(\frac{\pi}{2}x\right)$

Solution: $x = 1+h \quad x \to 1 \Rightarrow h \to 0$

$$\lim_{h \to 0} -h \tan\left(\frac{\pi}{2}(1+h)\right)$$

$$\lim_{h \to 0} -h \cot\left(\frac{\pi h}{2}\right) = \lim_{h \to 0} -h \cot\left(\frac{h\pi}{2}\right)$$ $\cot\left(\frac{\pi}{2}+\theta\right) = -\cot\theta$

$$\lim_{h \to 0} \frac{-h}{-\tan\left(\frac{\pi}{2}h\right)} \times \frac{\pi}{2} \times \frac{1}{\frac{\pi}{2}}$$

$$\lim_{h \to 0} \frac{h\frac{\pi}{2}}{\tan\left(\frac{\pi}{2}h\right)} \times \frac{2}{\pi} = \frac{2}{\pi}$$

Formula learn

$$\boxed{\lim_{x \to 0} \frac{1-\cos x}{x^2} = \frac{1}{2}}$$

Question: $\lim_{x \to 0} \frac{\sqrt{1+\tan x} - \sqrt{1+\sin x}}{x^3}$

Solution: $\lim_{x \to 0} \frac{(1+\tan x - 1 - \sin x)}{x^3(\sqrt{1+\tan x} + \sqrt{1+\sin x})}$

$$= \lim_{x \to 0} \frac{\tan x - \sin x}{x^3(2)}$$

$$= \lim_{x \to 0} \frac{\frac{\sin x}{\cos x} - \sin x}{2x^3}$$

$$= \lim_{x \to 0} \frac{\sin x}{x}\left(\frac{1-\cos x}{2x^2\cos x}\right)$$

$$= \lim_{x \to 0} \frac{\sin x}{x} \times \left(\frac{1-\cos x}{x^2}\right) \times \frac{1}{2\cos x}$$

$$= \lim_{x \to 0} 1 \times \frac{1}{2} \times \frac{1}{2 \times 1} = \frac{1}{4}$$

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Question: $\lim_{x\to 0} \frac{2-\sqrt{3}\cos x - \sin x}{(6x-\pi)^2}$

Solution: $\frac{2-\sqrt{3}}{\pi^2}$

Question: $\lim_{x\to \pi/6} \frac{2-\sqrt{3}\cos x - \sin x}{(6x-\pi)^2}$

$\lim_{x\to \pi/6} \frac{2-(\sqrt{3}\cos x + \sin x)}{36\left(x-\frac{\pi}{6}\right)^2}$

$\lim_{x\to \pi/6} \frac{2\left[1-\cos\left(x-\frac{\pi}{6}\right)\right]}{36\left(x-\frac{\pi}{6}\right)^2} = \frac{2}{36}\times\frac{1}{2} = \frac{1}{36}$

Important
Question: $\lim_{x\to 0} \frac{1-(\cos x)\sqrt{\cos 2x}}{\tan^2 x}$

To check your answer:
* If after applying limit there is calculation of + or − it means you have applied partial limit.

Solution: $\lim_{x\to 0} \frac{1-\cos^2 x(\cos 2x)}{\tan^2 x(1+\cos x\sqrt{\cos 2x})}$

$\lim_{x\to 0} \frac{1-(1-\sin^2 x)(1-2\sin^2 x)}{\tan^2 x(1+\cos x\sqrt{\cos x})}$

$\lim_{x\to 0} \frac{3\sin^2 x - 2\sin^4 x}{\tan^2 x(1+1)}$

$\lim_{x\to 0} \frac{\frac{3\sin^2 x - 2\sin^4 x}{x^2}}{\frac{\tan^2 x}{x^2}\times 2} = \frac{3+0}{2} = 3/2$

Question: $\lim_{x \to 0} \dfrac{\cos\left(\frac{\pi}{2\cos x}\right)}{\sin(\sin x^2)}$

Solution: $\lim_{x \to 0} \dfrac{\cos\left(\frac{\pi}{2\cos x}\right)}{\dfrac{\sin(\sin x^2)}{\sin x^2} \times \dfrac{\sin x^2 \times x^2}{x^2}}$

$$\lim_{x \to 0} \frac{\cos\left(\frac{\pi}{2\cos x}\right)}{x^2} = \lim_{x \to 0} \frac{\sin\left(\frac{\pi}{2} - \frac{\pi}{2\cos x}\right) \times \left(\frac{\pi}{2} - \frac{\pi}{2\cos x}\right)}{\left(\frac{\pi}{2} - \frac{\pi}{2\cos x}\right) \quad x^2}$$

$$= \lim_{x \to 0} \frac{\pi}{2}\left[\frac{\cos x - 1}{x^2 \cos x}\right]$$

$$= \frac{\pi}{2} \times \frac{-1}{2} \times \frac{1}{1} = -\frac{\pi}{4}$$

Question: $\lim_{x \to 0} \dfrac{\cos^{-1}(1-x)}{\sqrt{x}}$

Solution: $x > 0 \Rightarrow$ one sided limit $x \to 0^+$

Substitute $1 - x = \cos\theta$.

$$\lim_{x \to 0^+} \frac{\cos^{-1}(\cos\theta)}{\sqrt{1-\cos\theta}} = \lim_{x \to 0^+} \frac{\sqrt{\theta^2}}{\sqrt{1-\cos\theta}} = \frac{1}{\sqrt{2}}$$

Question: $\lim_{x \to 8} \dfrac{\sin\{x-10\}}{\{10-x\}}$

* Check L.H.L./R.H.L. when f^n is discontinous

Solution: $\lim_{x\to 8} \frac{\sin\{x\}}{\{-x\}}$

$\lim_{x\to 8^-} \frac{\sin 1}{0^+} = \infty$ $\qquad \{-x\} = 1-\{x\}$

$\lim_{x\to 8^+} \frac{\sin 0^+}{1} = 0$

D.N.E.

Question: $\lim_{n\to\infty} \cos\frac{x}{2}\cos\frac{x}{4}\dots\cos\frac{x}{2^n}$

Solution: $\lim_{n\to\infty} \frac{\sin x}{2^n\sin\left(\frac{x}{2^n}\right)} = \lim_{n\to\infty} \overbrace{\left(\frac{x}{2^n}\right)}^{\to 0} \frac{\sin x}{x} \cdot \frac{1}{\sin\left(\frac{x}{2^n}\right)}$

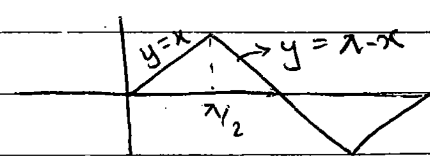
$= \frac{\sin x}{x}$ //

Question: $\lim_{x\to \frac{1}{\sqrt{2}}^+} \frac{\cos^{-1}(2x\sqrt{1-x^2})}{x-\frac{1}{\sqrt{2}}}$

Solution: $x = \sin\theta$

$\lim_{\theta\to\frac{\pi}{4}^+} \frac{\cos^{-1}(\sin 2\theta)}{\sin\theta-\frac{1}{\sqrt{2}}} = \lim_{\theta\to\frac{\pi}{4}^+} \frac{\pi/2 - \sin^{-1}(\underbrace{\sin 2\theta}_{\to\frac{\pi}{2}^+})}{\sin\theta-\frac{1}{\sqrt{2}}}$

$= \lim \frac{\frac{\pi}{2} - \pi + 2\theta}{\sin\theta - 1/\sqrt{2}}$



$= \lim \frac{2\left(\theta - \frac{\pi}{4}\right)}{\sin\theta - \sin\pi/4}$

$= \lim \frac{2\cancel{\left(\theta - \frac{\pi}{4}\right)}}{2\sin\left(\frac{\theta-\pi/4}{2}\right)\cos\left(\frac{\theta+\pi/4}{2}\right)} = \lim_{\theta\to\frac{\pi}{4}^+} 2$

Question: $\lim_{x\to 0} \prod_{k=1}^{\infty} \left[\frac{1+2\cos\left(\frac{2x}{3^k}\right)}{3}\right]$

Solution: $\left(1+2\cos\frac{2x}{3}\right)\left(1+2\cos\frac{2x}{9}\right)\cdots\left(1+2\cos\frac{2x}{3^{n-1}}\right)\left(1+2\cos\frac{2x}{3^n}\right) \times \frac{\sin\frac{x}{3^n}}{\sin\frac{x}{3^n}}$

$\left(\quad " \quad\right)\left(\quad " \quad\right)\cdots\left(\quad " \quad\right)\frac{\left(\sin\frac{x}{3^{n-1}}\right)}{\sin\left(\frac{x}{3^n}\right)}$

$= \frac{\sin x}{\sin\left(\frac{x}{3^n}\right)}$

$\Rightarrow \lim_{x\to 0} \frac{1}{3^n}\frac{\sin x}{\sin\left(\frac{x}{3^n}\right)} = \lim_{x\to 0} \left(\frac{x}{3^n}\right)\frac{\sin x}{x} \times \frac{1}{\sin\left(\frac{x}{3^n}\right)}$

$= 1$

Explaination:

Every angle is 1/3rd of its consecutive angle.

Let. → starting from smallest angle.

$(1+2\cos\theta)\ (1+2\cos 3\theta)\ (1+2\cos 9\theta)\ldots$ — A, B

$(1-2\sin^2\theta/_2)$

$(1+2-4\sin^2\theta/_2)$

$(3-4\sin^2\theta/_2)\frac{\sin\theta/_2}{\sin\theta/_2}$ → half angle of A

$\frac{\sin 3\theta/_2}{\sin\theta/_2}$ → We have got half angle of B

Exponential function

$\lim_{x\to 0} \frac{a^x - 1}{x} = \ln a \quad (a>0)$

$\lim_{x\to 0} \frac{e^x - 1}{x} = 1$

$\lim_{x\to \infty} x(e^{1/x} - 1) = 1$

$\lim_{x\to 0} \frac{\ln(1+x)}{x} = 1$

Expansion of function by (Maclawin Series)

(Not in iit)

$$f(x) = f(0) + f'(0)x + \frac{f''(0)x^2}{\lfloor 2} + \frac{f'''(0)x^3}{\lfloor 3} + \dots \infty$$

$e^x = 1 + x + \frac{x^2}{\lfloor 2} + \frac{x^3}{\lfloor 3} + \dots\infty \qquad x \in R$

$a^x = e^{\ln a} = 1 + \cancel{x\ln a}\; x\ln a + \frac{\cancel{x}(\ln a)^2}{\lfloor 2} + \frac{\cancel{x}(\ln a)^3}{\lfloor 3} + \dots$

$$a^x = 1 + \ln a + \frac{(\ln a)^2}{\lfloor 2} + \frac{(\ln a)^3}{\lfloor 3} + \dots$$

$\sin x = 0 + x\,1 - 0 - \frac{x^3}{\lfloor 3} + 0 + \frac{x^5}{\lfloor 5} \dots$

$$= x - \frac{x^3}{\lfloor 3} + \frac{x^5}{\lfloor 5} - \frac{x^7}{\lfloor 7} \dots$$

$\cos x = 1 - 0 \cancel{+} - \frac{x^2}{\lfloor 2} + 0 + \frac{x^4}{\lfloor 4}$

$$= 1 - \frac{x^2}{\lfloor 2} + \frac{x^4}{\lfloor 4}$$

$\tan x = x + \frac{x^3}{3} + \frac{2}{15}x^5 + \dots$

$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} \dots \qquad -1 \le x \le 1$

$\tan^{-1} x = x - \frac{x^3}{3} + \dots$

Question: $\lim\limits_{x\to 0} \dfrac{e^{\tan x} - e^x}{\tan x - x}$

Solution: $\lim\limits_{x\to 0} \dfrac{e^x\left(e^{\tan x - x} - 1\right)}{\tan x - x} = e^0 \times 1 = 1$

Question: $\lim\limits_{x\to 0} \dfrac{e^{x^2} - \cos x}{x^2}$

Solution: $\lim\limits_{x\to 0} \left[\left(\dfrac{e^{x^2}-1}{x^2}\right) + \left(\dfrac{1-\cos x}{x^2}\right)\right]$

$= 1 + \frac{1}{2} = \frac{3}{2}$

Question: $\lim\limits_{x\to 0} \dfrac{e^{\sin(2x)} - e^{\sin x}}{x}$

Solution: $\lim\limits_{x\to 0} \dfrac{\left(e^{\sin 2x - \sin x} - 1\right)}{x(\sin 2x - \sin x)} (\sin 2x - \sin x) e^{(\sin x)}$

$\lim\limits_{x\to 0} \dfrac{\sin 2x - \sin x}{x} = 2 - 1 = 1$

Question: $\lim\limits_{x\to 0} \dfrac{1 - 7^x - 8^x + 56^x}{\sqrt{2\cos x + 7 - 9}}$

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Question: $\lim_{x\to 0} \frac{1-7^x-8^x+56^x}{\sqrt{2\cos x+7}-3}$

Solution $1-a-b+ab$.

$$\lim_{x\to 0} \frac{x^2(7^x-1)(8^x-1)}{x \times x} \times \frac{(\sqrt{2\cos x+7}+3)}{(2\cos x+7-9)}$$

$$\lim_{x\to 0} \frac{x^2\,(6)(\ln 7)(\ln 8)}{2(\cos x-1)}$$

$$\Rightarrow -\frac{6\ln 7\ln 8}{4}$$

Question: $\lim_{x\to 0} \frac{\cos(xe^x)-\cos(xe^{-x})}{x^3}$

Solution: $\lim_{x\to 0} \frac{-2\sin\left(x\left(\frac{e^x-e^{-x}}{2}\right)\right)\sin\left(x\left(\frac{e^x+e^{-x}}{2}\right)\right)}{x^3}$

$$\lim_{x\to 0} \frac{-2\cancel{x}\left(\frac{e^x-e^{-x}}{2}\right)\cancel{x}\left(\frac{e^x+e^{-x}}{2}\right)}{x^{\cancel{3}}}$$

$$\lim_{x\to 0} \frac{-2}{\cancel{e^x}^{\,1}}\left(\cancel{\frac{e^{2x}-1}{2x}}^{\,1}\right)\left(\cancel{\frac{e^x+e^{-x}}{2}}\right) \quad \frac{2}{2}=1$$

$$\Rightarrow -2$$

Question: $\lim_{x\to\infty} \frac{e^{1/x^2}-1}{2\tan^{-1}(x^2)-\pi}$

Solution $\lim_{x\to\infty} \frac{\frac{1}{x^2}}{-2\cot^{-1}(x^2)} = -\frac{1}{2}$

Question: $\lim_{x \to 1} (1-x) \log_x 2$

Solution: $x = 1+h$ $\xrightarrow{}$ base change

$$\lim_{h \to 0} \frac{-h \ln 2}{\ln(1+h)} = -\ln 2$$

Question: $\lim_{x \to a} \frac{x^x - a^a}{x-a}$

Solution: $\lim_{x \to a} \frac{(x^x - x^a) + (x^a - a^a)}{x-a}$ $\left[\frac{x^n - a^n}{x-a} = n x^{n-1} \right]$

$$\lim_{x \to a} \left(\frac{x^x - x^a}{x-a} \right) + a^a$$

$$\lim_{x \to a} x^a \frac{(x^{x-a} - 1)}{x-a} + a^a$$

$$\cancel{\lim_{x \to a} x^a \left(e^{\ln x(x-a)} - 1 \right)}$$

$$\lim_{x \to a} x^a \left(\frac{e^{(x-a)\ln x} - 1}{(x-a)\ln x} \right) \ln x + a^a$$

$$a^a \ln a + a^a = a^a (\ln a + 1)$$

Question: $\lim_{x \to 1} \frac{x^2 + ax + b}{\ln^2(2-x)}$ exist find a, b, l.

Solution: $x = 1+h$

$$\lim_{h \to 0} \frac{1 + 2h + h^2 + a + ah + b}{\left(\frac{\ln^2(1-h)}{(-h)^2} \right) \times (-h)^2}$$

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$$\lim_{x\to 0} \frac{h^2 + h(2+a) + a+b+1}{h^2}$$

$\infty + \infty$ - D.N.E

$a=-2 \quad b=1 \quad l=1$

* 1^∞

$(\to 1)^{\to\infty}$ $(\text{exact } 1)^\infty = 1$

$$\lim_{x\to 0} (1+x)^{1/x} = \lim_{x\to\infty} \left(1+\frac{1}{x}\right)^x = e$$

$$= e^{1/x \ln(1+x)}$$

Question: $\lim_{x\to 0} (1+2x)^{5/x}$

Solution: $\lim_{x\to 0} \left[(1+2x)^{\frac{1}{2x}}\right]^{10} = e^{10}$

* Expansion of $(1+x)^{1/x}$

$$e^{\frac{1}{x}\ln(1+x)} = e^{1 - x/2 + x^2/3 - x^3/4 + \dots}$$

$$= e^1 \left\{1 + \left(-\frac{x}{2} + \frac{x^2}{3} - \frac{x^3}{4} + \dots\right) + \frac{\left(-\frac{x}{2} + \frac{x^2}{3} + \dots\right)^2}{\lfloor 2} + \frac{(\quad)^3}{\lfloor 3}\right\}$$

$$= e^1 \left(1 - \frac{x}{2} + x^2\left(\frac{11}{24}\right) + x^3(\quad)\dots\right)$$

Question: $\lim_{x\to 0} \frac{(1+x)^{1/x} - e}{x}$

Solution: $\lim_{x\to 0} \frac{e - \frac{ex}{2} + \frac{11ex^2}{24} - e}{x} = -\frac{e}{2}$

Question: $\lim_{x\to 0} \dfrac{(1+x)^{1/x} - e + ex/2}{x^2}$

Solution: $\dfrac{e - \frac{ex}{2} + ex^2\left(\frac{11}{24}\right) + \frac{ex}{2}}{x^2} = \dfrac{11e}{24}$

Question: $\lim_{n\to\infty} \lim_{x\to 0} \dfrac{(1+x)^{1/x} + (1+2x)^{1/2x} + (1+3x)^{1/3x} + \dots (1+nx)^{1/nx} - ne}{n^2 x} = ?$

Solution: $\dfrac{1}{n^2}\left[\dfrac{(1+x)^{1/x} - e}{x} + \left[\dfrac{(1+2x)^{1/2x} - e}{2x}\right] \times 2 + \dots\right]$

$= \dfrac{-e}{2} \dfrac{n(n+1)}{2} = \dfrac{-e}{4}$

* Generalised formula of 1^∞

Let $\lim_{x\to a} f(x) = 1$ & $\lim_{x\to a} g(x) \to \infty$

Then $\boxed{\lim_{x\to a} (f(x))^{g(x)} = e^{\lim_{x\to a} g(x)(f(x)-1)}}$

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Proof: $e^{g \ln f} = e^{g \frac{\ln(1+f-1)}{f-1}(f-1)}$

$= e^{g(f-1)}$

Question: $\lim_{n\to\infty} (3^{1/n} + 5^{1/n} + 7^{1/n} - 2)^n$

Solution: $e^{\ln(3^{1/n} + 5^{1/n} + 7^{1/n} - 2 - 1)}$

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$$e^{\frac{(3^{\frac{1}{n}}-1)+(5^{1/n}-1)+(7^{1/n}-1)}{\frac{1}{n}}}$$

$$= e^{\ln 3+\ln 5+\ln 7} = e^{\ln 105} = 105$$

Question: $\lim_{x\to\infty}\left(\sin\frac{1}{x}+\cos\frac{1}{x}\right)^x$

Solution: $e^{x(\sin\frac{1}{x}+\cos\frac{1}{x}-1)} = e^{\left(\frac{\sin\frac{1}{x}}{\frac{1}{x}}+\frac{\cos\frac{1}{x}-1}{\frac{1}{x^2}}\times\cancel{\frac{1}{x}}^{\,0}\right)}$

$$= e^{1+0} = e$$

Question: $\lim_{x\to 0}\left(\frac{5}{2+\sqrt{9+x}}\right)^{\operatorname{cosec}x}$

Solution: $e^{\operatorname{cosec}x\left(\frac{5-2-\sqrt{9+x}}{2+\sqrt{9+x}}\right)}$

$$= e^{\frac{1}{\sin x}\frac{x}{x}\left[\frac{3-\sqrt{9+x}}{5}\right]}$$

$$= e^{\frac{3-\sqrt{9+x}}{5x}} = e^{\frac{-x}{5x}\frac{1}{3+\sqrt{9+x}}} = e^{\frac{-1}{30}}$$

Question: $\lim_{n\to\infty}\left(\frac{\sqrt{n^2+n}-1}{n}\right)^{2(\sqrt{n^2+n}-1)}$

Solution: $\lim_{n\to\infty}\left(\cancel{\frac{\sqrt{n^2+n}-1}{n}}^{\,1}\right)^{2(\sqrt{n^2+n}-1)}$

$$= e^{\lim (2\sqrt{n^2+n}-1)\left(\frac{\sqrt{n^2+n}-1-n}{n}\right)}$$

$$= (2\sqrt{n^2+n}-1)\left(\frac{\cancel{n^2}+n-\cancel{n^2}-1-2n}{n(\sqrt{n^2+n}+1+n)}\right)$$

$$= \frac{2(\sqrt{n^2+\cancel{n}}-\cancel{1})(-n-\cancel{1})}{n(\sqrt{n^2+\cancel{n}}+\cancel{1}+n)}$$

$$= \frac{-2}{2} = -1$$

$$\Rightarrow \quad e^{-1} = \frac{1}{e}$$

Qustion: $\lim\limits_{x\to 2}\left(1+5^{(x-2)^{-1}}\right)^{-1} + [x-1]$

Solution:- $\lim\limits_{x\to 2^-} [1+5^{-\infty}]^{-1} + 0$

$$[1+0]^{-1} + 0 = 1$$

$$\lim_{x\to 2^+} [1+5^{\infty}]^{-1} + 1$$

$$\frac{1}{\infty} + 1 = 0+1 = 1$$

Qustion: $\lim\limits_{x\to\infty}\left(\frac{1^{1/x}+2^{\frac{1}{x}}+3^{\frac{1}{x}}+\dots+n^{\frac{1}{x}}}{n}\right)^{nx} \quad n\in N$

Solution: $e^{\lim\limits_{x\to\infty} nx\left[\frac{1^{1/x}+2^{1/x}+3^{1/x}+\dots n^{1/x}}{n}-1\right]}$

$$e^{\lim\limits_{x\to\infty} nx\left[\frac{1^{1/x}+2^{1/x}+3^{1/x}+\dots n^{1/x}-n}{n}\right]}$$

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$$e^{\lim_{x\to\infty} \frac{(1^{1/x}-1)+(2^{1/x}-1)\dots+(n^{1/x}-1)}{1/x}}$$

$$= e^{\ln(1\times 2\times 3\times\dots n)} = e^{\ln \lfloor n} = \lfloor n$$

Question: $f(n) = \lim_{x\to 0}\left[\left(1+\sin\left(\frac{x}{2}\right)\right)\left(1+\sin\left(\frac{x}{2^2}\right)\right)\left(1+\sin\left(\frac{x}{2^3}\right)\dots\left(1+\sin\left(\frac{x}{2^n}\right)\right)\right]^{1/x}$

$\lim_{n\to\infty} f(n) = ?$

Solution: $\lim_{x\to 0}\left[1+\sin\left(\frac{x}{2}\right)\right]^{1/x} = e^{\lim_{x\to 0}\frac{1}{x}\left[1+\sin\left(\frac{x}{2}\right)-1\right]}$

$$= e^{\lim_{x\to 0} \frac{\sin x/2}{x/2}\times 2} = e^{\frac{1}{2}}$$

$$\Rightarrow f(n) = e^{1/2}.e^{1/4}.e^{1/8}.e^{1/16}\dots e^{\frac{1}{2^n}}$$

$$= e^{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}\dots\frac{1}{2^n}} \leftarrow \text{Infinite G.P.}$$

$$\lim_{n\to\infty} f(n) = e^{\frac{1/2}{1-1/2}} = e^{\frac{1/2}{1-1/2}} = e$$

Question: $\lim_{x\to 0}\left(\sin^2\left(\frac{\pi}{2-ax}\right)\right)^{\sec^2\left(\frac{\pi}{2-bx}\right)}$

Solution: $e^{\lim_{x\to 0} \sec^2\left(\frac{\pi}{2-bx}\right)\left[\sin^2\left(\frac{\pi}{2-ax}\right)-1\right]}$

$$e^{\lim_{x\to 0} \frac{-\cos^2\left(\frac{\pi}{2-ax}\right)}{\cos^2\left(\frac{\pi}{2-bx}\right)}} \rightarrow \text{tending to } 90^\circ$$

$$e^{\lim_{x\to 0} \frac{-\sin^2\left(\frac{\pi}{2}-\frac{\pi}{2-ax}\right)}{\sin^2\left(\frac{\pi}{2}-\frac{\pi}{2-ax}\right)}} \rightarrow \overset{\text{tends to}}{0^\circ}$$

$$e^{\lim\limits_{x\to 0} \frac{-\left[\frac{\pi}{2} - \frac{\pi}{2-ax}\right]^2}{\left[\frac{\pi}{2} - \frac{\pi}{2-bx}\right]^2}} = e^{\lim\limits_{x\to 0} \frac{-\left[\frac{\pi(\cancel{2}-ax-\cancel{2})}{2(2-ax)}\right]^2}{\left[\frac{\pi(\cancel{2}-bx-\cancel{2})}{2(2-bx)}\right]^2}}$$

$$= e^{-\frac{a^2}{b^2}} \text{ Ans}$$

Question: $\lim\limits_{x\to\infty} x^2 \sin\left(\ln\sqrt{\cos\left(\frac{\pi}{x}\right)}\right)$

Solution: $\lim\limits_{x\to\infty} x^2 \sin\left(\ln\sqrt{\cos\left(\frac{\pi}{x}\right)}\right) \to 0$

$$\lim_{x\to\infty} x^2\left(\ln\sqrt{\cos\left(\frac{\pi}{x}\right)}\right) = \lim_{x\to\infty} \frac{x^2}{2}\ln\left(\cos\frac{\pi}{x}\right)$$

$$= \lim_{x\to 0} \frac{x^2}{2}\left(\ln\left(1 + \left(\cos\frac{\pi}{x} - 1\right)^{\to 0}\right)\right)$$

$$= \lim_{x\to 0} \frac{x^2}{2} \frac{\left(\cos\frac{\pi}{x} - 1\right)}{\left(\frac{\pi}{x}\right)^2} \times \left(\frac{\pi}{x}\right)^2$$

$$= \lim_{x\to 0} \frac{\cancel{x^2}}{2} \times \left(\frac{-1}{2}\right) \times \frac{\pi^2}{\cancel{x^2}} = -\frac{\pi^2}{4}$$

Built in limit

Question: $f(x) = \lim\limits_{n\to\infty} \frac{\tan(\pi x^2) + (x+1)^n \sin x}{x^2 + (x+1)^n}$

$\lim\limits_{x\to 0} f(x) = ?$

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Solution: Define f in neighbourhood of $x=0$.

$(1^-)^\infty \to 0 \quad (1^+)^\infty \to \infty$

$$f(x) = \begin{cases} \dfrac{\tan(\lambda x^2)}{x^2} & x<0 \\ \sin x & x>0 \end{cases} \longrightarrow \lim_{n\to\infty} \frac{\cancel{\dfrac{\tan(\lambda x^2)}{(x+1)^n}}^{\to 0} + \sin x}{\cancel{\dfrac{x^2}{(x+1)^n}}^{\to 0} + 1}$$

$$f(x) = \begin{cases} \lambda & x<0 \\ 0 & x>0 \end{cases}$$

$f(0^+) = 0 \qquad f(0^-) = \lambda \qquad$ limit DNE

Question: $f(x) = \lim_{n\to\infty} \dfrac{\cos(\pi x) - (x^{2n})\sin(x-1)}{1 + x^{2n+1} - x^{2n}}$

$\lim_{x\to 1} f(x) = ?$

Solution: $f(x) = \begin{cases} \cos(\pi x) & 1^- \\ -\dfrac{\sin(x-1)}{x-1} & 1^+ \end{cases}$

$f(1^-) = -1 \qquad f(1^+) = -1 \qquad f(x) = -1$

Important limit related to Expansion

Learn

#1. $\lim_{x\to 0} \dfrac{e^x - 1 - x}{x^2} = \dfrac{1}{2}$ #2. $\lim_{x\to 0} \dfrac{x - \sin x}{x^3} = \dfrac{1}{6}$

#3. $\lim_{x\to 0} \dfrac{\tan x - x}{x^3} = \dfrac{1}{3}$ #4. $\lim_{x\to 0} \dfrac{e^x - e^{-x} - 2x}{x^3} = \dfrac{1}{3}$

Question: $\lim_{x\to 0} \dfrac{1-\sqrt{\cos 2x}\,\sqrt[3]{\cos 3x}\,\sqrt[4]{\cos 4x}\cdots\sqrt[n]{\cos nx}}{x^2} = 10$

$n = ?$

Solution: $\lim_{x\to 0} \dfrac{1-\left(1-\frac{(2x)^2}{L2}\cdots\right)^{1/2}\left(1-\frac{(3x)^2}{L2}\cdots\right)\left(1-\frac{(4x)^2}{L2}+\cdots\right)^{\frac{1}{4}}(\,)(\,)(\,)\cdots\left(1-\frac{n^2x^2}{L2}\right)^{\frac{1}{n}}}{x^2}$

$= \lim_{x\to 0} \dfrac{\cancel{1}-\left(\cancel{1}+\frac{1}{2}\left(\frac{-(2x)^2}{L2}\right)\right)\left(\cancel{1}+\frac{1}{3}\left(\frac{-(3x)^2}{L2}\right)\cdots\right]\left(\cancel{1}+\frac{1}{4}\left(\frac{(-4x)^2}{L2}\right)\cdots\right]\cdot\left[\cancel{1}+\frac{1}{n}\left(\frac{-n^2x^2}{L2}\right)\cdots\right.}{x^2}$

$= \frac{1}{2}\left(\frac{+4}{2}\right) + \frac{1}{3}\left(\frac{+9}{L2}\right) + \frac{1}{4}\left(\frac{16}{L2}\right)\cdots \frac{1}{n}\left(\frac{n^2}{L2}\right)$

$= \frac{2}{2}+\frac{3}{2}+\frac{4}{2}\cdots\frac{n}{2} = 10$

$= 2+3+4+\cdots n = 20$

$= \frac{n(n+1)}{2} - 1 = 20$

$= n(n+1) - 2 = 40$

$= n(n+1) = 42$

$\boxed{n=6}$

Important Question: $\lim_{x\to 0}\left(\dfrac{\sqrt{\dfrac{\cos 2x + (1+3x)^{1/3}}{2}} - \sqrt[3]{\dfrac{4\cos^3 x - \ln(1+x)^4}{4}}}{x}\right)$

Solution: $\dfrac{\left(\dfrac{1+\left(1+\frac{1}{3}(3x)\cdots\right)}{2}\right)^{1/2} - \left(\dfrac{4-4x}{4}\right)^{1/3}}{x}$

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$$\frac{\left(1+\frac{x}{2}\right)^{1/2}-(1-x)^{1/3}}{x}$$

$$\frac{\left(1+\frac{1}{2}\times\frac{x}{2}\right)-\left(1+\frac{1}{3}(-x)\right)}{x}$$

$$=\frac{1}{4}+\frac{1}{3}=\frac{7}{12}$$

Question: $\lim_{x\to 0} \frac{1}{(\sin^{-1}x)^2}-\frac{1}{x^2}=?$

Solution: $\frac{1}{x^2}-\frac{1}{x^2}=0$ $\times$ (Partial limit)

$$\lim_{x\to 0}\frac{x^2-(\sin^{-1}x)^2}{x^4}=\frac{x-\sin^{-1}x}{x^3}\times\underbrace{\left(\frac{x+\sin^{-1}x}{x}\right)}_{=2}$$

$$=2\times\left(\frac{x-\sin^{-1}x}{x^3}\right) \qquad x=\sin\theta$$

$$=2\ \frac{(\sin\theta-\theta)}{\frac{\sin^3\theta}{\theta^3}\times\theta^3}$$

$$=2\times-\frac{1}{6}=-\frac{1}{3}\ \text{Ans}$$

Question: $\lim_{x\to 0}\frac{(1-\cos 2x)^3+2\sin 3x\tan^5(2x)-\ln(1+\tan^{10}x+\sin^6x)+x^6}{(e^{2x^6}-1)+(1-\cos x)^2\sin^2(2x)+\ln(1+x^6)+x^8-\sin^7x}$

Solution: Divide by x^6

$$\lim_{x\to 0}\left(\frac{1-\cos 2x}{x^2}\right)^3+2\frac{\sin 3x\tan^5(2x)}{x^6}-\underbrace{\ln\left(\frac{1+\tan^{10}+\sin^6x}{\tan^{10}+\sin^6x}\right)}_{+1}\times\frac{(T^{10}+S^6)}{x^6}$$

$$2\quad+\frac{1}{4}+1+0+0$$

or.

$$\frac{(1-\cos 2x)^3 + 2\sin 3x \tan^5(2x) - \ln(1+\tan^{10}x\} + \sin^6 x) + x^6}{(e^{2x^6}-1) + (1-\cos x)^2 \sin^2(2x) + \ln(1+x^6) + \cancel{x^8} - \sin^7 \cancel{x}}$$

* $x \to 0$ divide by $x^?$?= smalles power of x which standard limit exist

$$= \frac{8 \times \frac{1}{8} + 2\times 3\times 2^5 - 1 + 1}{2 + \frac{1}{4} + 1} = \frac{4(1+192)}{10} = \frac{772}{10} = \frac{386}{5}$$

Question: $\lim\limits_{x\to 0} \dfrac{\sin x - x^2 - \{x\}\{-x\}}{x\cos x - x^2 - \{x\}\{-x\}}$

Solution: RHL

$$\frac{\sin x - x^2 - x(1-x)}{x\cos x - x^2 - x(1-x)} = \frac{\sin x - \cancel{x^2} - x + \cancel{x^2}}{x\cos x - \cancel{x^2} - x + \cancel{x^2}}$$

$$= \frac{\sin x - x}{x(\cos x - 1)} = \frac{-1/6}{-1/2} = \frac{1}{3}$$

LHL $\dfrac{\sin x - x^2 - (1+x)(-x)}{x\cos x - x^2 + (x^2+x)} = \dfrac{\sin x + x}{x(\cos x + 1)} = \dfrac{2}{2} = 1$

Question: $\lim\limits_{x\to 0} \dfrac{A\cos x + B\sin x - 5}{x^4}$ exists & is finite find A, B & l

Solution: $A = 5$

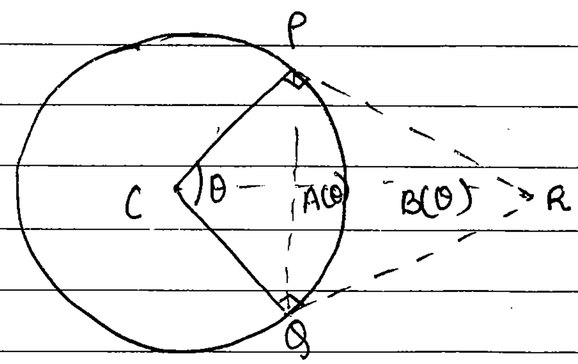
$$\lim_{x\to 0} \frac{5\left(1-\frac{x^2}{2}+\frac{x^4}{24}\right)+Bx\left(x-\frac{x^3}{6}\right)-5}{x^4}$$

$$-\frac{5}{2}+B=0 \Rightarrow B=\frac{5}{2}$$

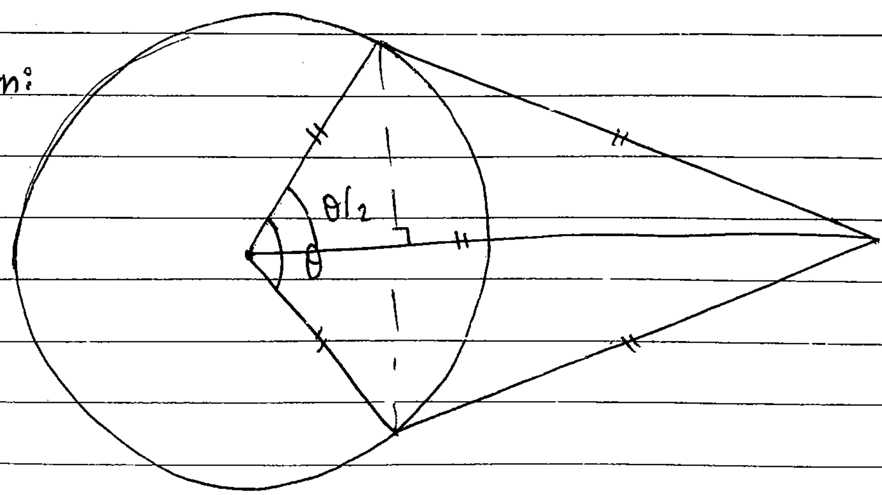
for l $\quad \frac{5}{24}-\frac{B}{6}=l \Rightarrow \frac{5}{24}-\frac{5}{12}=-\frac{5}{24}=l$

Geometrical limit

Question: An arc PQ of a circle subtends a central angle θ as shown. Let $A(\theta)$ be the area between chord PQ and arc PQ. $B(\theta)$ be the area between tangents lines PR & QR & arc PQ. Find $\lim_{\theta\to 0} \frac{A(\theta)}{B(\theta)}$



Solution:



$A(\theta) = \text{Sector} - \Delta CPQ$

$= \frac{\theta}{2\pi}(\pi r^2) - \frac{1}{2} r^2 \sin\theta$

$B(\theta) = \text{quad } PCQR - \text{Sector}$

$= 2 \cdot \frac{1}{2} r r \tan\frac{\theta}{2} - \frac{\theta r^2}{2}$

$$\frac{A(\theta)}{B(\theta)} = \frac{\theta/2 - \frac{\sin\theta}{2}}{\tan\theta/2 - \theta/2} = \frac{\frac{1}{2}\left(\frac{\theta - \sin\theta}{\theta^3}\right) \times \theta^3}{\frac{(\tan\theta/2 - \theta/2)}{(\theta/2)^3} \times (\theta/2)^3} = \frac{\frac{1}{6} \times \frac{1}{2} \times 8}{1/3}$$

$= 2$