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MODEL QUESTION PAPER
MID TERM EXAMINATION
SESSION : 2022 - 2023
SUBJECT : MATHEMATICS
CLASS - X

Time Allowed : 3 Hours **Maximum Marks : 80**

General Instructions

- The first 15 minutes is the Reading Time. You are not allowed to write during this time.
- Section – A – Contains 10 MCQ type question carrying 1 mark each.
- Section – B – Contains 6 question carrying 2 marks each.
- Section – C – Contains 6 question carrying 3 marks each.
- Section – D – Contains 5 question carrying 4 marks each.
- Section – E – Contains 4 question carrying 5 marks each. These are case study based questions
- Try not to change the order of the questions.
- No extra paper will be provided for rough work. It should be done in the same answer scripts.
- Marks may be deducted for untidy presentation and incorrect spellings.
- Use of correction ink/whitener is not allowed.

SECTION – A **(1 × 10 = 10)**

Choose the correct option :

1. The zeros of the polynomial $x^2 + 2x - 195$ are

a) - 15, 13 b) 15, 13

c) 15, -13 d) -15, -13

2. The solution for the following pair of equations are

$2x + y = 7$, $4x - 3y + 1 = 0$

a) (2, 3) b) (1, 3)

c) (0, 2) d) (-1, 3)

3. The system of equations $4x + 6y = 7$ and $12x + 18y = 21$ has

a) infinitely many solutions. b) no solution

c) unique solution d) none of the above

4. The nature of roots for the equation $x^2 + ax - 4 = 0$ for all real values of a is

a) no real roots b) real and equal

c) real and distinct d) none of the above

5. Which term of the AP 21, 18, 15,…..is zero?

a) 6^{th} b) 7^{th}

c) 8^{th} d) 5^{th}

6. If $(2p+1)$, 13, $(5p - 3)$ are in AP, then the value of p is

a) 3 b) 4

c) -4 d) 5

7. If A (–6 , 7) and B (– 1, –5) are two given points then the value of 2AB is

a) 12 units b) 24 units

c) 13 units d) 26 units

8. If $2\cos 3\theta = 1$ then the value of θ is

a) 20^o b) 30^o

c) 60^o d) 90^o

9. For some integer m, every odd integer is of the form

a) m + 1 b) m

c) 2m d) 2m + 1

10. If $\Delta ABC \sim \Delta DEF$ such that 2AB = DE and BC = 6 cm, find EF.

a) 12 cm b) 13 cm

c) 10 cm d) 14 cm

SECTION – B **(2 × 6 = 12)**

11. Find two rational numbers between $\sqrt{2}$ and $\sqrt{3}$.

12. Find the largest number that divides 245 and 1029 leaving remainder 5 in each case.

13. If α, β are the zeros of the polynomial $f(x) = x^2 - x - 2$, find the polynomial whose zeros are $(1+ 2\alpha)$ and $(1 + 2\beta)$.

14. For what value of k the system of equations $3x+y = 1$ and $kx + 2y = 5$ has a unique solution?

15. Find the solution of the quadratic equation $3\sqrt{3}\,x^2 + 10x + \sqrt{3} = 0$

16. If one root of the equation $3x^2 - 10x + k = 0$ is reciprocal of the other, find the value of k.

SECTION – C **(3 × 6 = 18)**

17. The 7^{th} term of an AP is 4 and its common difference is – 4. Find the first term.

18. Prove that $\frac{5}{\sqrt{2}}$ is an irrational number given that $\sqrt{2}$ is irrational.

19. Write the denominator of the rational number $\frac{257}{5000}$ in the form $2^m \times 5^n$, where m and n are non negative integers. Hence write its decimal expansion without actual division.

20. A rational number in its decimal expansion is 1.7112. What can you say about the prime factors of q, when this number is expressed in the form of $\frac{p}{q}$.

21. The first and the last terms of an AP are 10 and 361 respectively. If its common difference is 9 then find the number of terms and their total sum.

22. A line intersects the y-axis and x-axis at the points P and Q respectively. If (2, –5) is the mid point of PQ, then find the coordinates of P and Q.

SECTION – D **(4 × 5 = 20)**

23. If A(4, 3), B(–1, y) and C(3, 4) are the vertices of a right triangle ABC, right-angled at A, then find the value of y.

24. Find the value of k for which the equation $x^2 + k(2x + k – 1) + 2 = 0$ has real and equal roots.

25. Devi wants to make a rectangular pond on the road side for the purpose of providing drinking water for street animals. The area of the pond will be decreased by 3 square feet if its length is decreased by 2 ft. and breadth is increased by 1 ft. Its area will be increased by 4 square feet if the length is increased by 1 ft. and breadth remains same. Find the dimensions of the pond.

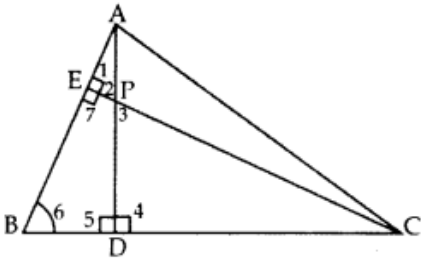
26. In ΔABC, altitudes AD and CE intersect each other at the point P. Prove that:

i) ΔAPE ~ ΔCPD

ii) AP × PD = CP × PE

iii) ΔADB ~ ΔCEB

iv) AB × CE = BC × AD



27. Prove that $(1 + \tan^2 A / 1 + \cot^2 A) = (1 – \tan A / 1 – \cot A)^2 = \tan^2 A$

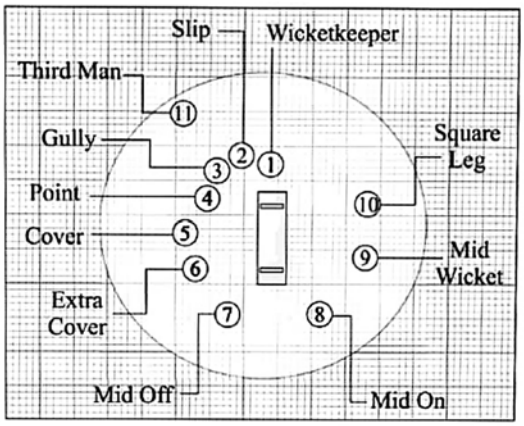
SECTION – E **(5 × 4 = 20)**

28. Case Study Based – 1

Cricket Fielding Positions and Players

In the sport of cricket the Captain sets the field according to a plan. He instructs the players to take a position at a particular place. There are two reasons to set a cricket field-to take wickets and to stop runs being scored.

The following graph shows the position of players during a cricket match.



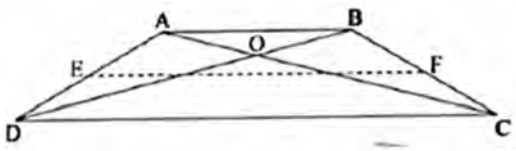
a) If the distance between the points showing the players at Gully A(1, 0) and as wicketkeeper B(4, p) is 5m, then find the value of p **(2½)**

b) Suppose the length of a line segment joining the players of Mid-off and Mid-on be 10 units. If the coordinates of its one end are (2, –3) and the abscissa of the other end is 10 units, then find its ordinate **(2½)**

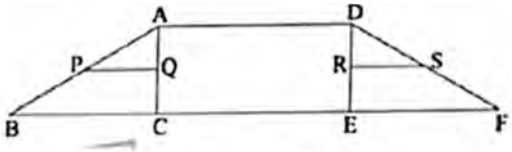
29. Case Study Based – 2

Kerala

Kerala is a state in Southern India. The state is known as a tropical paradise of waving palms and wide sandy beaches. This map of the Indian province of Kerala shows its arca can be approximated using a simple straight-sided shape. The shape has two parallel sides 561 km and 216 km long. The other sides are 180 km and 211 km long. Its parallel sides are 100 km apart. Shreya observed the shape formed by four straight lines and explored it on her notebook in different ways shown below.



Shape I



Shape II

Refer to Shape I

a) Let ABCD is a trapezium with AB || DC, E and F are points on non-parallel sides AD and BC respectively such that EF is parallel to AB. Then $\frac{AE}{ED}$ =

(i) $\frac{BF}{CD}$ (ii) $\frac{AB}{CD}$

(iii) $\frac{BF}{FC}$ (iv) None of these

b) Here, AB || CD. If DO = $3x - 19$, OB = $x - 5$, CO = $x - 3$ and AO = 3, the value of x is

(i) 5 or 8 (ii) 8 or 9

(iii) 10 or 12 (iv) None of these

c) Again AB || CD. If DO = $3x - 1$, OB = $5x - 3$, AO = $6x - 5$ and OC = $2x + 1$, then the value of x is

(i) 0 (ii) 1

(iii) 2 (iv) 3

Refer to Shape II

d) *In ΔABC, PQ || BC. If AP = 2.4 cm, AQ = 2 cm, QC = 3 cm and BC = 6 cm, AB* and PQ are respectively

(i) AB = 6 cm, PQ = 2.4 cm (ii) AB = 4.8 cm, PQ = 8.2 cm

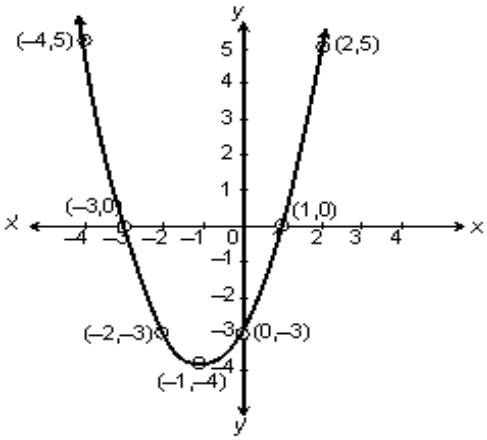
(iii) AB = 4 cm, PQ = 5.3 cm (iv) AB = 8.4 cm, PQ = 2.8 cm

e) In ΔDEF, if RS || EF, DR = $4x - 3$, DS = $8x - 7$, ER = $3x - 1$ and FS = $5x - 3$, then the value of x is

(i) 1 (ii) 2

(iii) 8 (iv) 10

30. Case Study Based – 3



In a game to entertain themselves the students of class X have drawn the figure with chalk on the ground. They have certain questions in their mind which they want to solve. Please answer and solve their questions.

i) Find the number of zeroes of the polynomial p(x). **(1)**

ii) Find the zeroes of the polynomial p(x). **(2)**

iii) Write the expression of the polynomial represented in the graph. **(2)**

31. **Case Study Based – 4**

If sin(A + B) = 1 and tan (A – B) = 1/√3, find the value of:

a) tan A + cot B **(1½)**

b) sec A – cosec B **(1½)**

c) If 15 sin A – 8 cos A = 0, find $\frac{\sin A - \cos A}{\sin A + \cos A}$ **(2)**

ANSWER

1. a) – 15, 13

2. a) 2, 3

3. a) infinitely many solutions

4. c) real and distinct

5. c) 8^{th}

6. b) $p = 4$

7. d) 26 units.

8. a) θ = 20°

9. d) 2m+1

10. a) EF = 12 cm

11. $\frac{3}{2}$, 1.62 can be a rational numbers between $\sqrt{2}$ and $\sqrt{3}$

12. 245 – 5 = 240

1029 – 5 = 1024

$$
\begin{array}{l}
240\overline{)1024}\,(4 \\
\quad -960 \\
\quad \overline{64}\overline{)240}\,(3 \\
\quad\quad -192 \\
\quad\quad \overline{48}\overline{)64}\,(1 \\
\quad\quad\quad -48 \\
\quad\quad\quad \overline{16}\overline{)48}\,(3 \\
\quad\quad\quad\quad -48 \\
\quad\quad\quad\quad \overline{\times}
\end{array}
$$

HCF = 16

13. $p(x) = x^2 - x - 2$

P(x) = (x – 2)(x +1)

Let α = 2, β = -1

Then, $(1+2\alpha) = 5, (1+2\beta) = -1$

Sum of roots $= 4$, product of roots $= -5$

The required polynomial is $k(x^2 - 4x - 5)$

14. $3x + y = 1$ and $kx + 2y = 5$

For unique solution $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\frac{3}{k} \neq \frac{1}{2}$ $\qquad k \neq 6$

15. $3\sqrt{3}x^2 + 10x + \sqrt{3} = 0$

$3\sqrt{3}x^2 + 9x + x + \sqrt{3} = 0$

$\left(x + \sqrt{3}\right)\left(3\sqrt{3}x + 1\right) = 0$

$x = -\sqrt{3}$ $\qquad$ or $\qquad x = \frac{-1}{3\sqrt{3}}$ Ans.

16. $3x^2 - 10x + k = 0$

$\alpha \times \frac{1}{\alpha} = \frac{k}{3}$ $\qquad \boxed{k = 3}$ Ans.

17. $a + 6d = a_7$

$a + (-24) = 4$

$a = 4 + 24 = 28$ Ans.

18. Let $\frac{5}{\sqrt{2}}$ be rational number. Then it can be expressed as $\frac{5}{\sqrt{2}} = \frac{a}{b}$ where a, b are coprime integers and $b \neq 0$

$\frac{5\sqrt{2}}{2} = \frac{a}{b}$

$\sqrt{2} = \frac{2a}{5b}$

R.H.S. is rational. Hence L.H.S. is also rational. But we know $\sqrt{2}$ is irrational, which contradicts our assumption.

Hence $\frac{5}{\sqrt{2}}$ is irrational.

19. $\frac{257}{5000} = \frac{257 \times 2}{2^3 \times 5^4 \times 2} = \frac{514}{2^4 \times 5^4}$

Decimal expansion 0.0514

20. $1.7112 = \frac{17112}{10000}$

q has factors of the form $2^n \times 5^m$ for whole numbers n and m.

21. $a = 10$ $a_n = 361$ $d = 9$

$361 = 10 + (n - 1)9$

$351 = (n - 1)9$

$n = 40$

Total number of terms = 40

$$S_{40} = \frac{\cancel{40}^{20}}{\cancel{2}}(10 + 361)$$

$= 20 \times 371$

$S_{40} = 7420$

22. Let the coordinates be P(0, y) and Q(x, 0)

$\frac{0 + x}{2} = 2$ $\frac{y + 0}{2} = -5$

$x = 4,$ $y = -10$

P(0, –10), Q(4, 0)

23. A (4, 3), B(–1, y) and C(3, 4)

$BC^2 = AB^2 + AC^2$

$(-1 - 3)^2 + (y - 4)^2 = (4 + 1)^2 + (3 - y)^2 + (4 - 3)^2 + (3 - 4)^2$

$y^2 - 8y + 32 = y^2 - 6y + 36$

$y = -2$ Ans.

24. $x^2 + k(2x + k - 1) + 2 = 0$

For real and equal roots

D = 0

$x^2 + 2kx + (k^2 - k + 2) = 0$

$4k^2 - 4(k^2 - k + 2) = 0$

$k - 2 = 0$

$k = 2$ Ans.

25. Let the length be x and breadth be y

Area = xy

$(x - 2)(y + 1) = xy - 3$ ..... (i)

$(x + 1)\, y = xy + 4$ ..... (ii)

From (i) $x - 2y = -1$

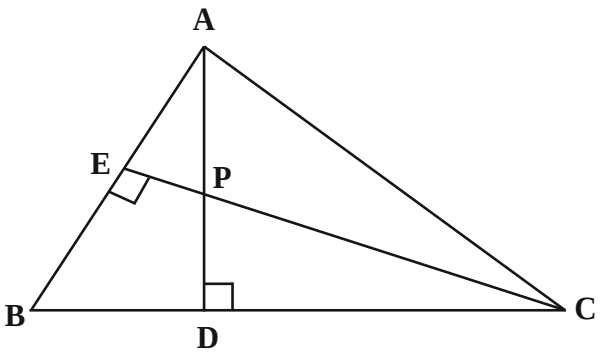
From (ii) $y = 4$

$\therefore\ x = 7$

Length of rectangular pond = 7 ft

Breadth of rectangular pond = 4 ft

26. Given: AD ⊥ BC and CE ⊥ AB



i) To prove ΔAPE ~ ΔCPD

Proof: ∠AEP = ∠CDP = 90°

∠AEP = ∠CPD (vertically opposite angles)

∴ ΔAPE ~ ΔCPD by AA similarity proved.

ii) AP × PD = CP × PE

Proof: ∵ ΔAPE ~ ΔCPD ∴ $\frac{AP}{CP} = \frac{PE}{PD}$ (Corresponding sides are proportional)

∴ AP × PD = CP × PE proved

iii) ΔADB ~ ΔCEB

proof : ∠ADB = ∠CEB = 90°

∠ABD = ∠CBE Common angle

∴ ΔADB ~ ΔCEB by AA similarity proved

iv) AB × CE = BC × AD

proof: ∵ ΔADB ~ ΔCEB

$\therefore \frac{AB}{CB} = \frac{AD}{CE} \Rightarrow$ AB × CE = CB × AD

27. $\frac{1+\tan^2 A}{1+\cot^2 A} = \frac{\sec^2 A}{\cos ec^2 A} = \tan^2 A$

$$\frac{(1-\tan A)^2}{(1-\cot A)^2} = \frac{(\cos A - \sin A)^2}{\cos^2 A} \times \frac{\sin^2 A}{(\sin A - \cos A)^2}$$

$= \tan^2 A$ proved

28. a) $(1-4)^2 + p^2 = 25$

$p^2 = 16$, p = 4

b) $100 = (2-10)^2 + (-3-y)^2$

$100 = 64 + 9 + 6y + y^2$

$y^2 + 6y - 27 = 0$

$(y-3)(y+9) = 0$

y = 3 or y = – 9

29. a) iii) BF/FC

b) iii) 8 or 9

c) iii) 2

d) i) AB = 6 cm PQ = 2.4 cm

e) i) 1

30. i) 2

ii) –3, 1

iii) $x^2 - 2x - 3$

31. $\sin(A + B) = 1 \Rightarrow A + B = 90°$

$\tan(A - B) = \frac{1}{\sqrt{3}}$ $A - B = 30°$

$2A = 120°$

$A = 60°$ $B = 30°$

a) $\tan A + \cot B = \tan 60° + \cot 30°$

$= \sqrt{3} + \sqrt{3} = 2\sqrt{3}$

b) $\sec A - \operatorname{cosec} B = \sec 60° - \operatorname{cosec} 30°$

$= 2 - 2 = 0$

c) $\tan A = \frac{8}{15}$

$\frac{\tan A - 1}{\tan A + 1} = \frac{8-15}{8+15} = -\frac{7}{23}$

▽ ▽ ▽