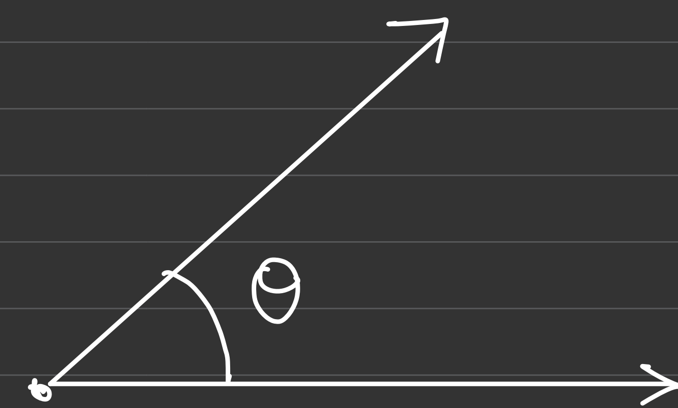
Basic Mathematics

(Nurture Batch)

Basic Mathematics

- **Trigonometry :-**

Angle :- (θ)



$0 \leq \theta < 90^\circ \Rightarrow$ Acute Angle

$\theta = 90^\circ \Rightarrow$ Right Angle

$90^\circ < \theta < 180^\circ \Rightarrow$ obtuse Angle

$180^\circ < \theta < 360^\circ \Rightarrow$ Reflex Angle

$\theta = 180^\circ \Rightarrow$ St. line

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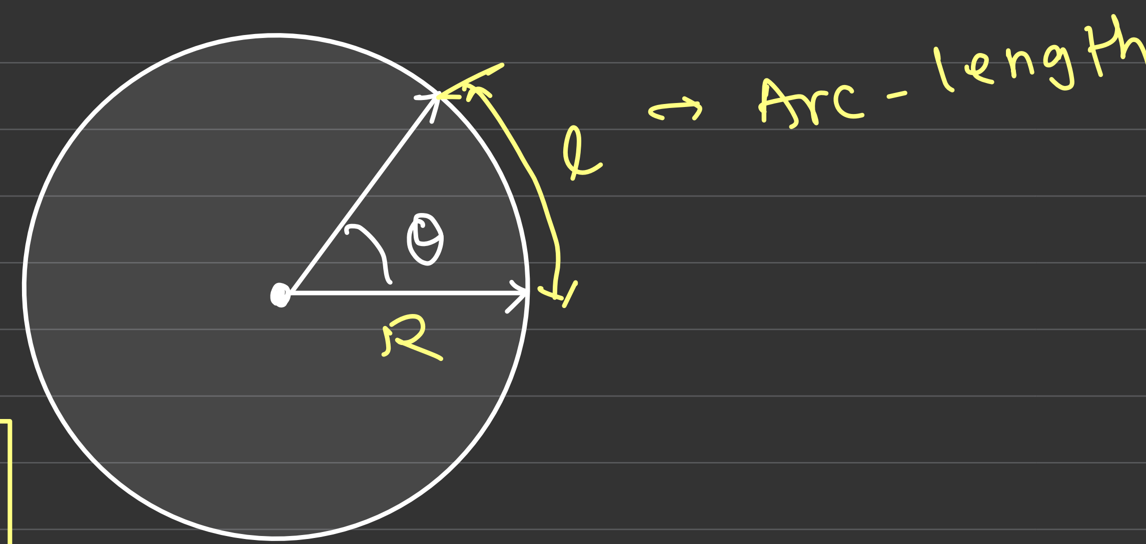


* $$\boxed{\theta = \frac{\text{Arc-length}}{\text{Radius}}}$$

- θ → is always taken in anticlockwise (+ve)

  θ → clockwise (−ve)

- Unit of Angle
  - → degree
  - → Radian
  - → Minutes
  - → Second

Conversion

- $180^\circ = \pi$ radian

or $\boxed{1^\circ = \left(\frac{\pi}{180}\right) \text{ radian}}$

- $\boxed{1 \text{ Radian} = \left(\frac{180}{\pi}\right)^\circ}$

$1^\circ = 60'$ (minute)

- $\boxed{1' = \left(\frac{1}{60}\right)^\circ}$

- $1' = 60''$ (Second)

- $1'' = \left(\frac{1}{60}\right)'$

Qu:- Convert in to Radian:-

(a) $45^\circ = \left(\frac{\pi}{4}\right)$. (b) $30^\circ = \pi/6$ (c) $60^\circ = \frac{\pi}{3}$ (d) $90^\circ = \pi/2$

(e) $120^\circ = \frac{2\pi}{3}$ (f) $135^\circ = \frac{3\pi}{4}$ (g) $150^\circ = \left(\frac{5\pi}{6}\right)$ (h) $180^\circ = \pi$

$$1^\circ = \left(\frac{\pi}{180}\right) \text{ radian}$$

$$45^\circ = 45 \frac{\pi}{180}$$

$$= \left(\frac{\pi}{4}\right)$$

$\pi = 3.14$

Qu:- Convert in degree

(a) $\frac{\pi}{15} = 12^\circ$

(b) $\frac{\pi}{12} = 15^\circ$

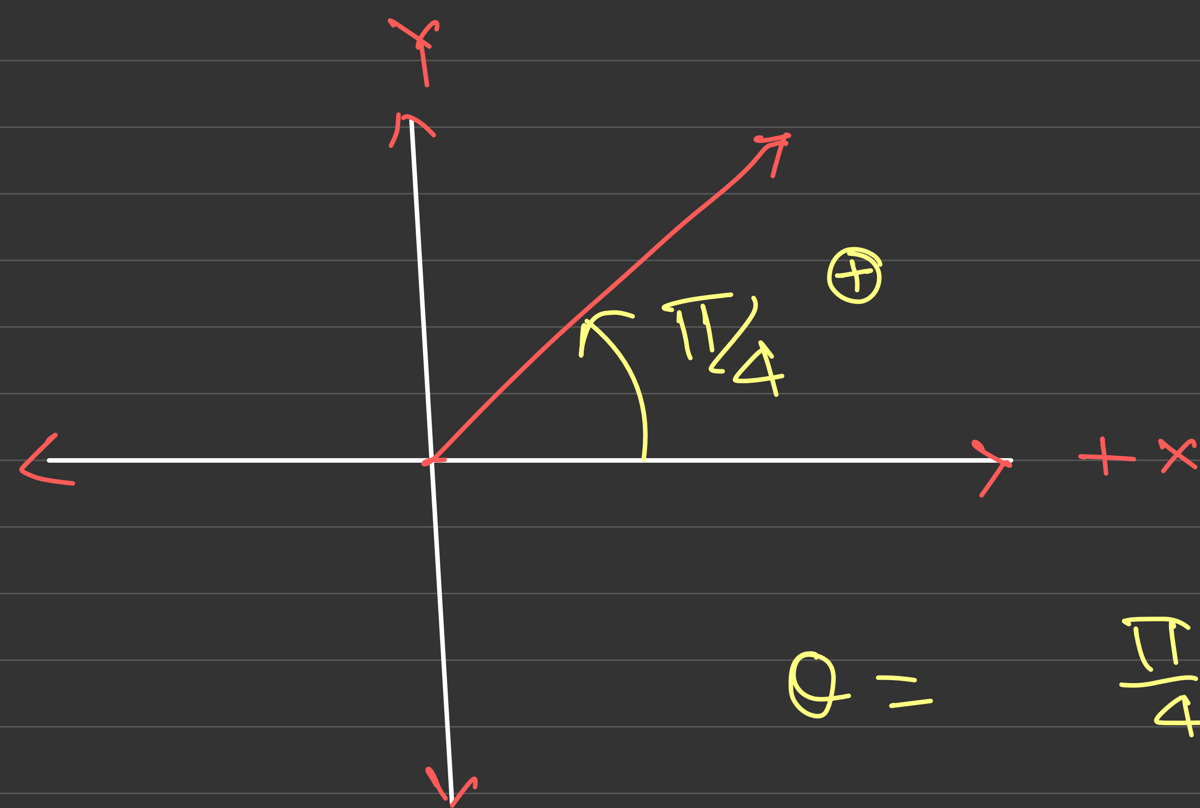
(c) $\frac{\pi}{8} = 22.5^\circ$

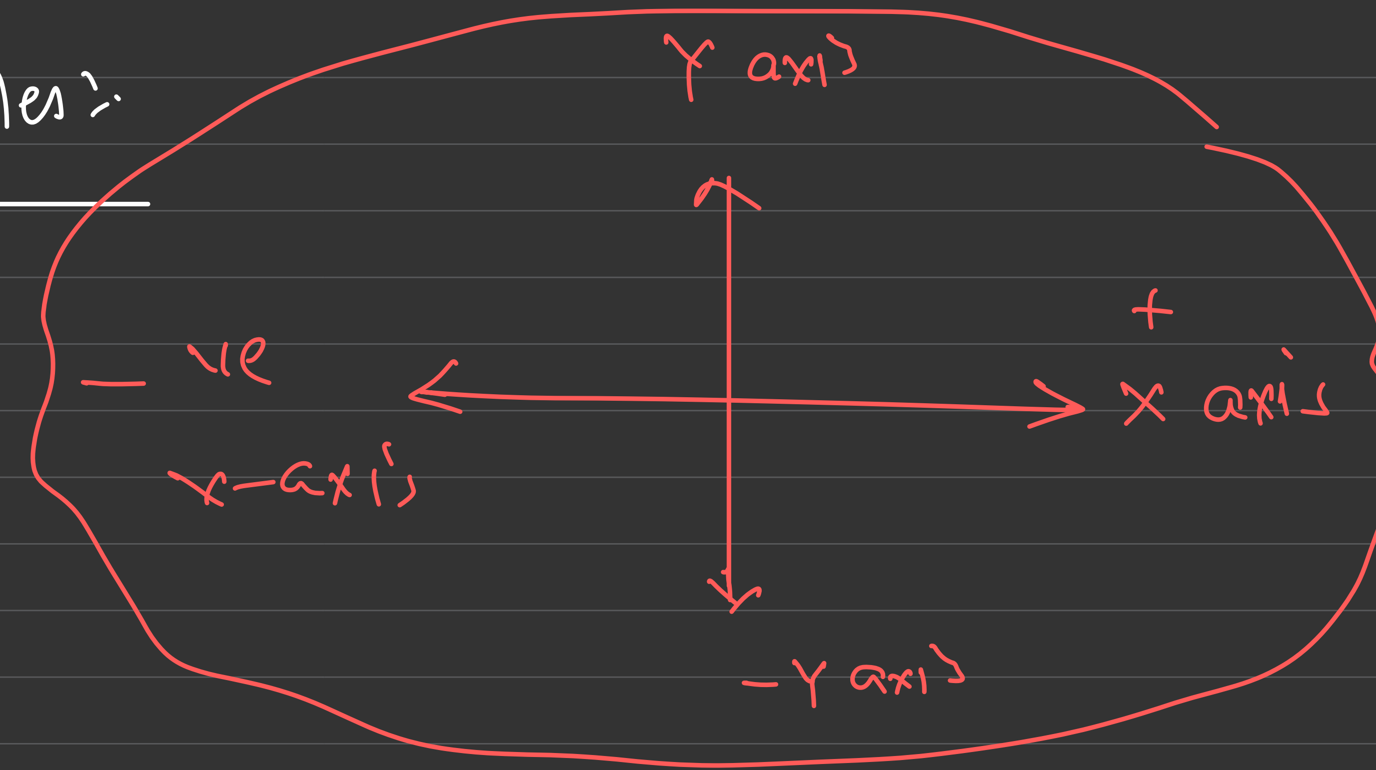
(d) $\frac{5\pi}{4} = 225^\circ$

(e) $\frac{8\pi}{3} = 480^\circ$

$$\frac{\pi}{15} = \left(\frac{180}{15}\right)^\circ$$

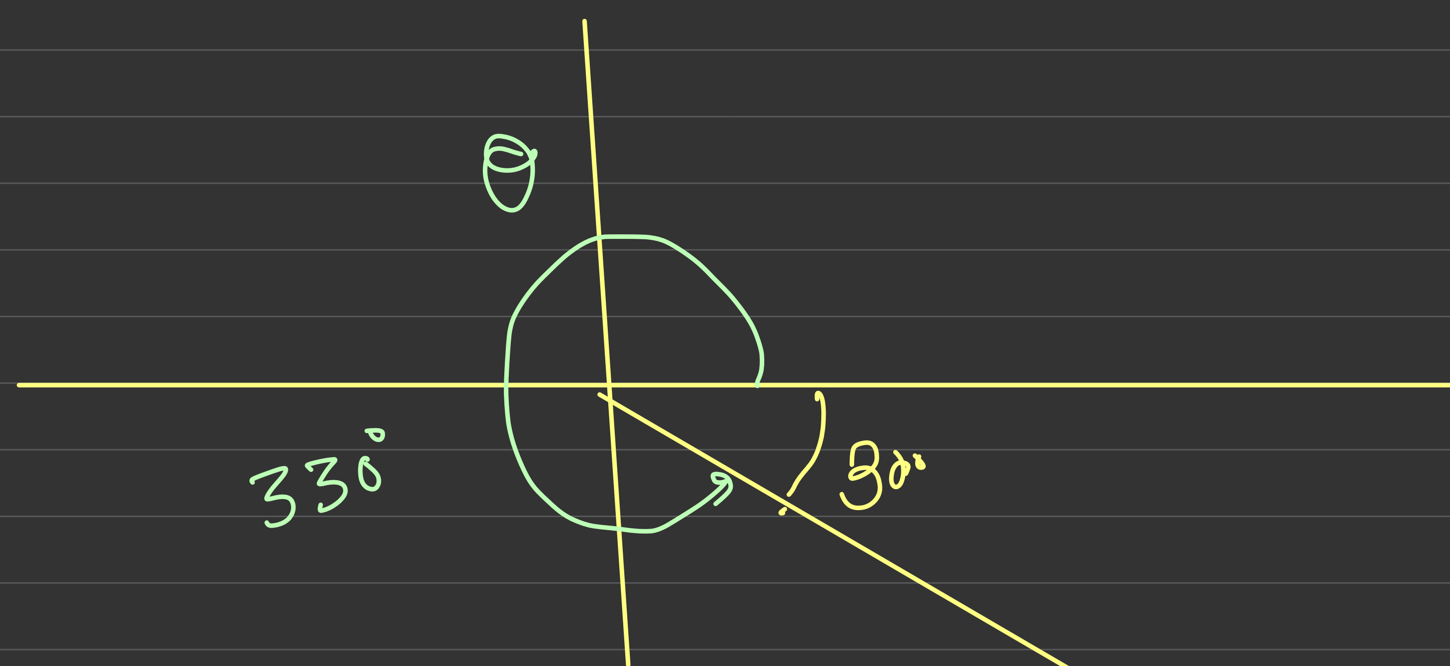
Positive and Negative Angles:





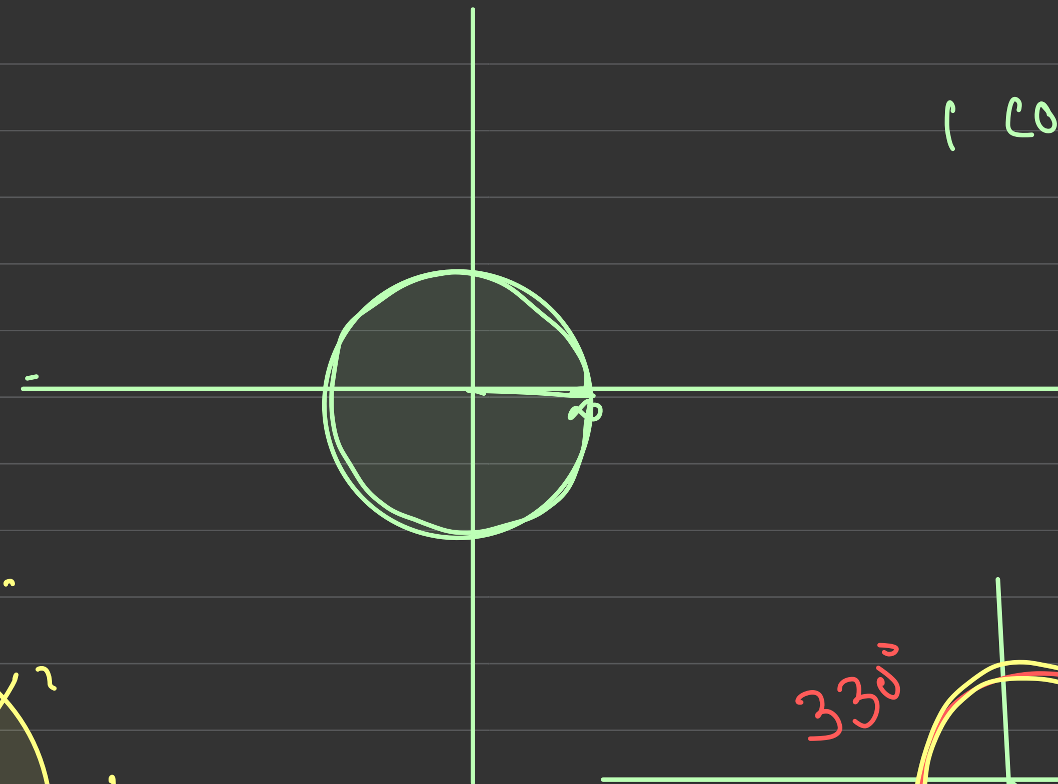
$$\theta = \frac{\pi}{4} = 45^\circ$$

$$= -(360^\circ - 45^\circ) = \underline{-315^\circ}$$

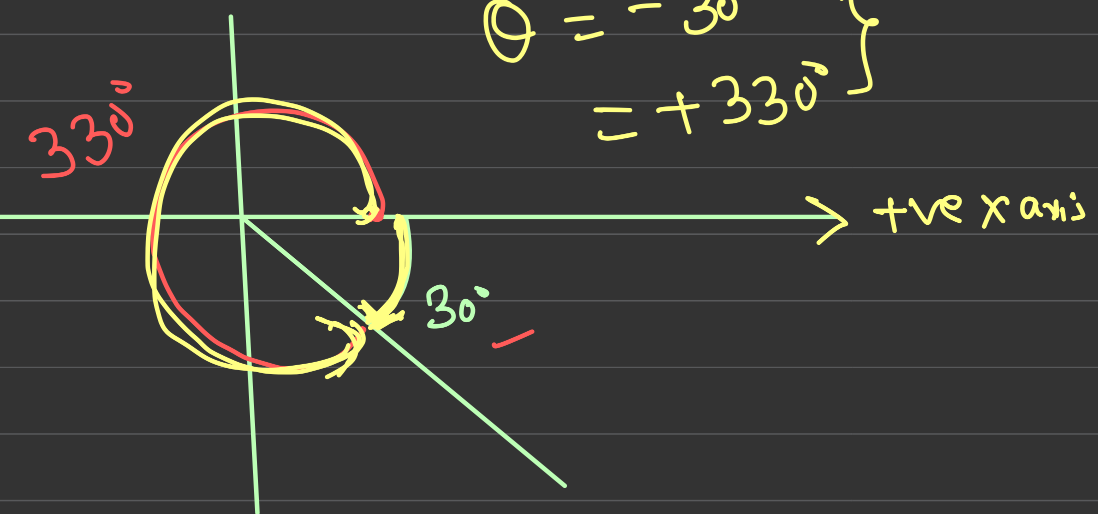


$$\left.\begin{aligned}\theta &= -30^\circ \\ &= 330^\circ\end{aligned}\right\}$$

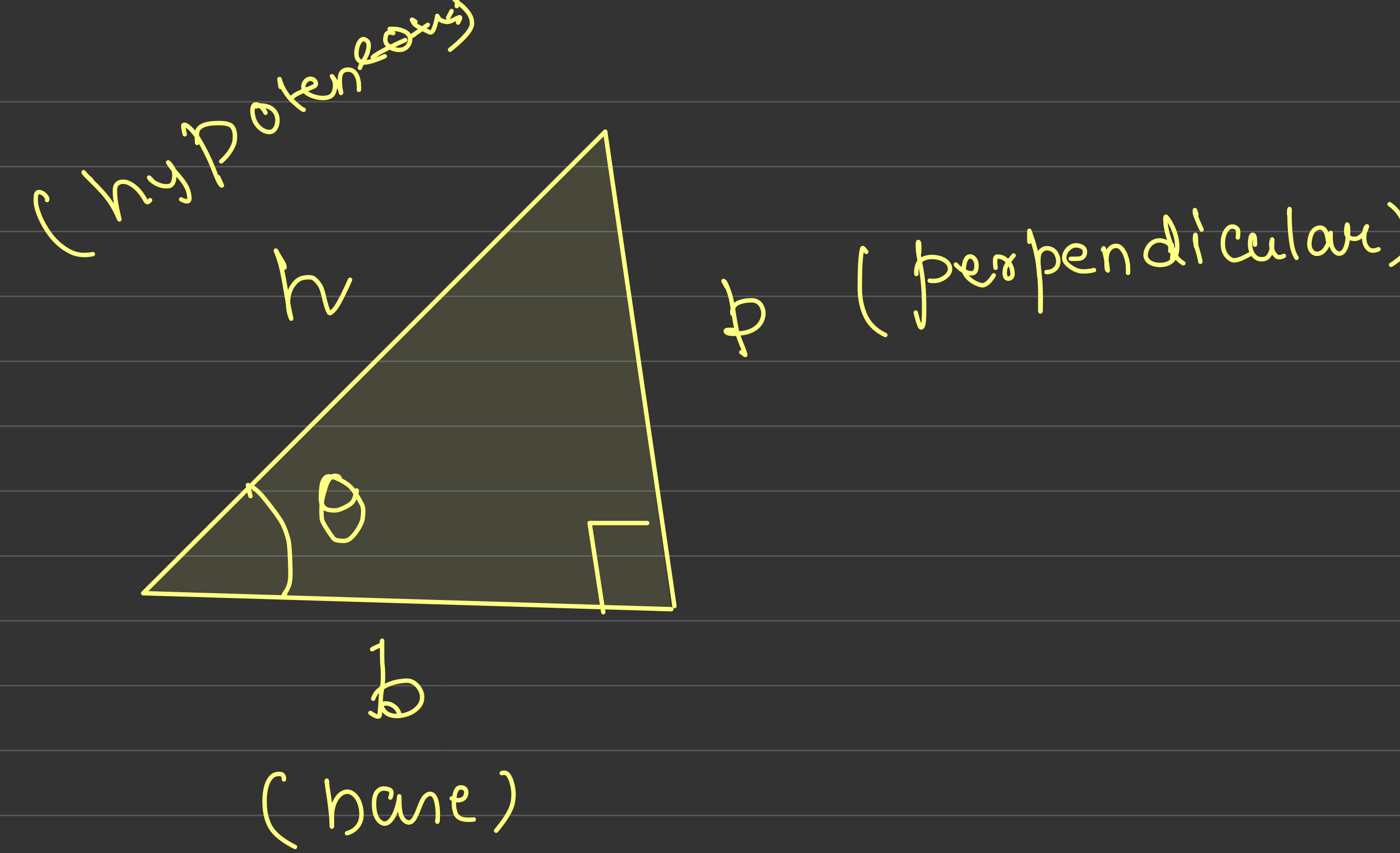
1 comple rotation, $\theta = 360^\circ$



$$\left.\begin{aligned}\theta &= -30^\circ \\ &= +330^\circ\end{aligned}\right\}$$



Trigonometric Ratios:



Pythogorus theorm

$$\boxed{h^2 = p^2 + b^2}$$

** Pythogorus triplets

- ⇒ 3, 4, 5 (6, 8, 10) (1.5, 2, 2.5)
- ⇒ 9, 40, 41

- $\sin\theta = \dfrac{p}{h}$

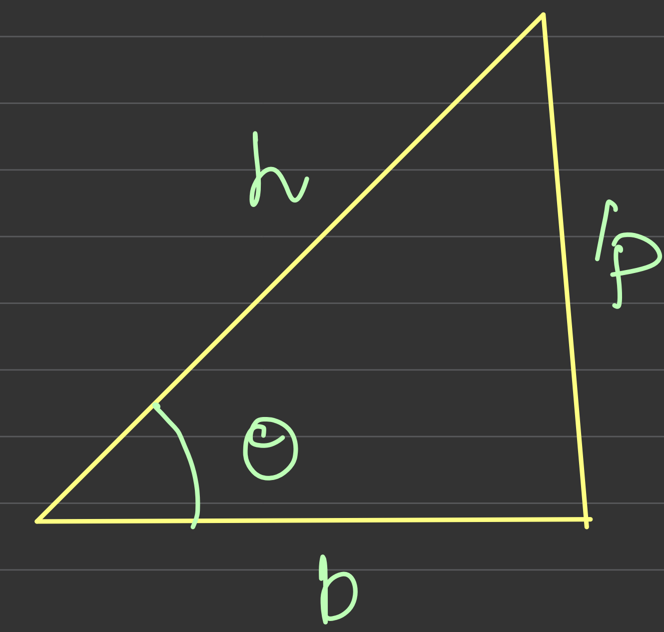
- $\cos\theta = \dfrac{b}{h}$

- $\tan\theta = \dfrac{p}{b}$

- $\cot\theta = b/p$

- $\sec\theta = \dfrac{h}{b}$

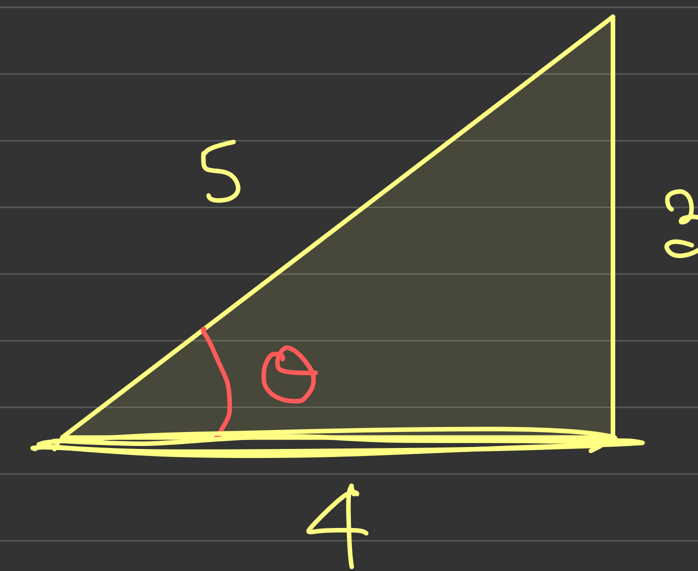
- $\mathrm{cosec}\,\theta = \dfrac{h}{p}$



$$\left.\begin{aligned}
\mathrm{cosec}\,\theta &= \frac{1}{\sin\theta}\\
\sec\theta &= \frac{1}{\cos\theta}\\
\cot\theta &= \frac{1}{\tan\theta}
\end{aligned}\right\}$$

$$\left.\begin{aligned}
\sin^2\theta + \cos^2\theta &= 1\\
\sec^2\theta - \tan^2\theta &= 1\\
\mathrm{cosec}^2\theta - \cot^2\theta &= 1
\end{aligned}\right\}$$

Qu: If $\sin\theta = 3/5$, find all other trigonometric Ratios,



$\cos\theta = \frac{4}{5}$

$\tan\theta = \frac{3}{4}$

$\sec\theta = \frac{5}{4}$

$\csc\theta = \frac{5}{3}$

$\cot\theta = \frac{4}{3}$

$\sin\theta = \frac{3}{5} = \frac{p}{h}$

$h^2 = p^2 + b^2$

$\Rightarrow b^2 = h^2 - p^2$

$b = \pm\sqrt{h^2 - p^2}$

$b = \pm\sqrt{h^2 - p^2}$

$= \pm\sqrt{25 - 9}$

$= \pm 4$

$= 4$

#

- $\sqrt{16} = 4$

- $x^2 = 16$

  $x = \pm 4$

- $\sqrt{x^2} = |x|$ → (modulus)

  $=$ Mod of x

  $=$ Absolute value of x

- $|-4| = 4$, $|10| = 10$

~~$\sqrt{(+4)^2} = 4$~~

~~$\sqrt{(-4)^2} = -4$~~

Wrong

- $x^2 = 16$

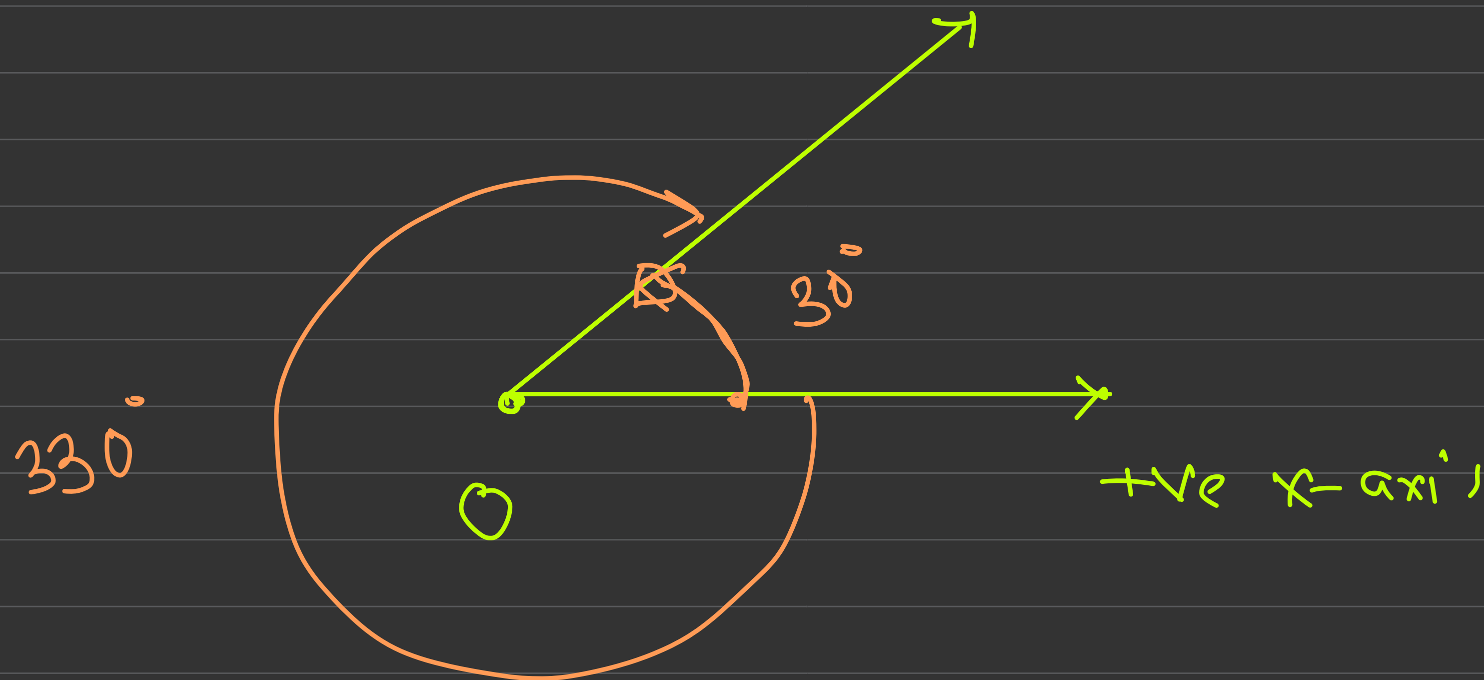
  $x = \pm\sqrt{16}$

  $\boxed{x = \pm 4}$

- $\sqrt{}$ ⇒ is symbol of +ve square root o $ax^2 + bx + c = 0$

$$\alpha, \beta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$\sqrt{}$

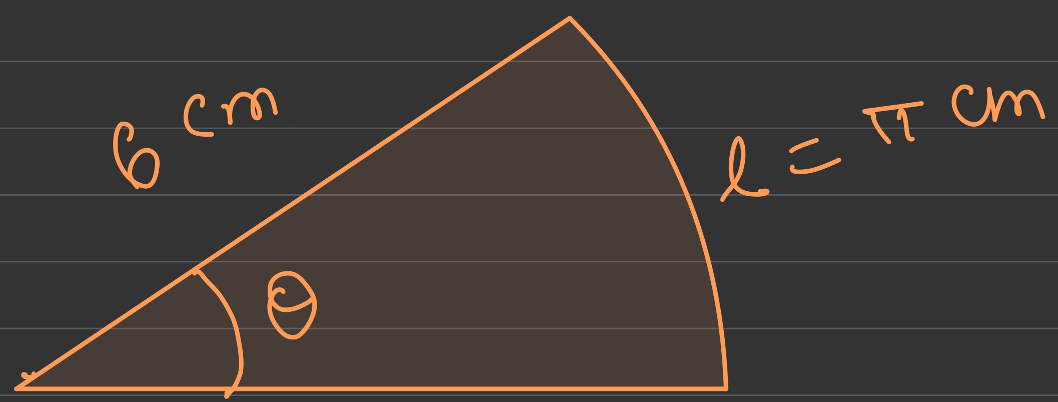


$\theta = 30°$ → Anticlockwise

$= -330°$ → Clockwise

Qu: A circular arc of arc-length π cm. Find the angle subtended by it at center if Radius = 6 cm

(Degree & Radian)

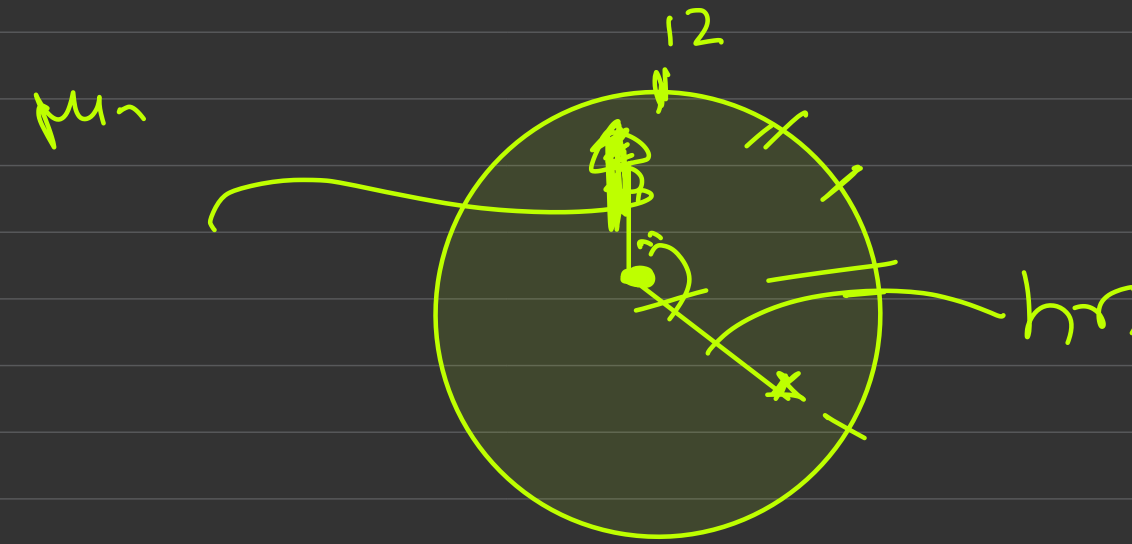


Soln:

$$\theta = \frac{l}{r} = \frac{\pi}{6}$$

$$= 30^\circ$$

Qu: When clock shows 4 o'clock; how much angle do its minute and hour hand makes?



$12 \rightarrow 1 \qquad \frac{360^\circ}{12} = 30^\circ$

$12 \rightarrow 2$

$12 \rightarrow 4 \quad \rightarrow \quad 30 \times 4 = 120^\circ = \frac{2\pi}{3}$ radian

Small Angle Approximation

$\theta \rightarrow$ very - very small (up to 5°)

- $\sin\theta \sim \theta$ (θ must be radian)
- $\tan\theta \sim \theta$ (" ")
- $\cos\theta \sim 1$

ex!.

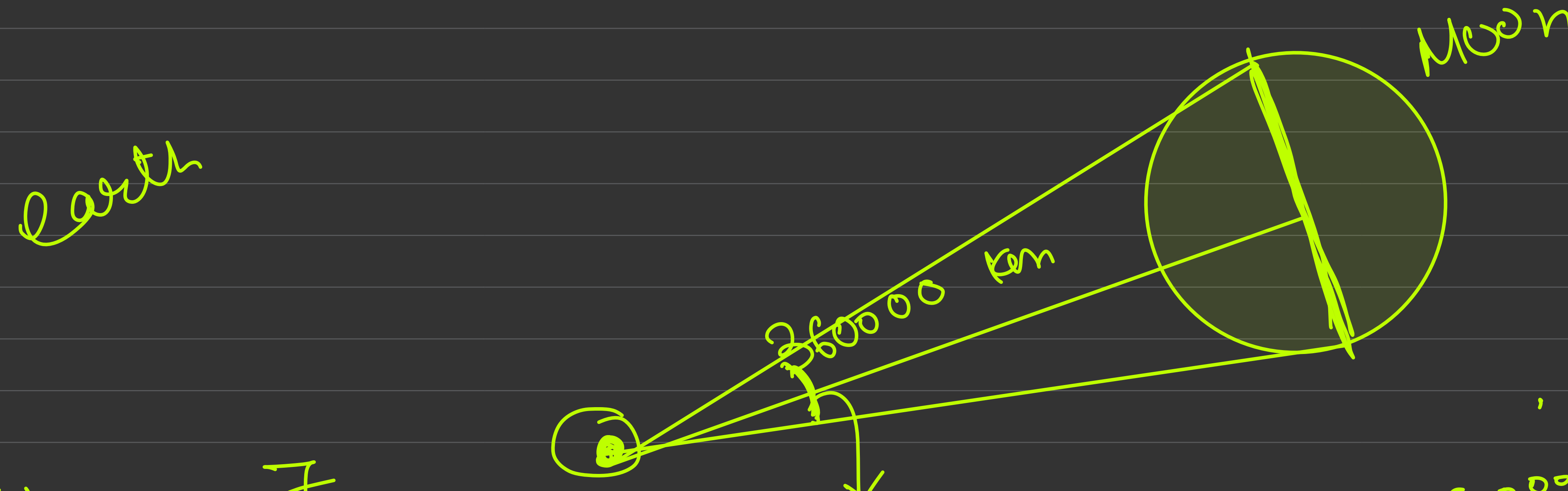
- $\sin 1^\circ = \left(\frac{\pi}{180}\right) = \left(\frac{3.14}{180}\right)$

$\sin(3.5^\circ) = \frac{3.5\pi}{180}$

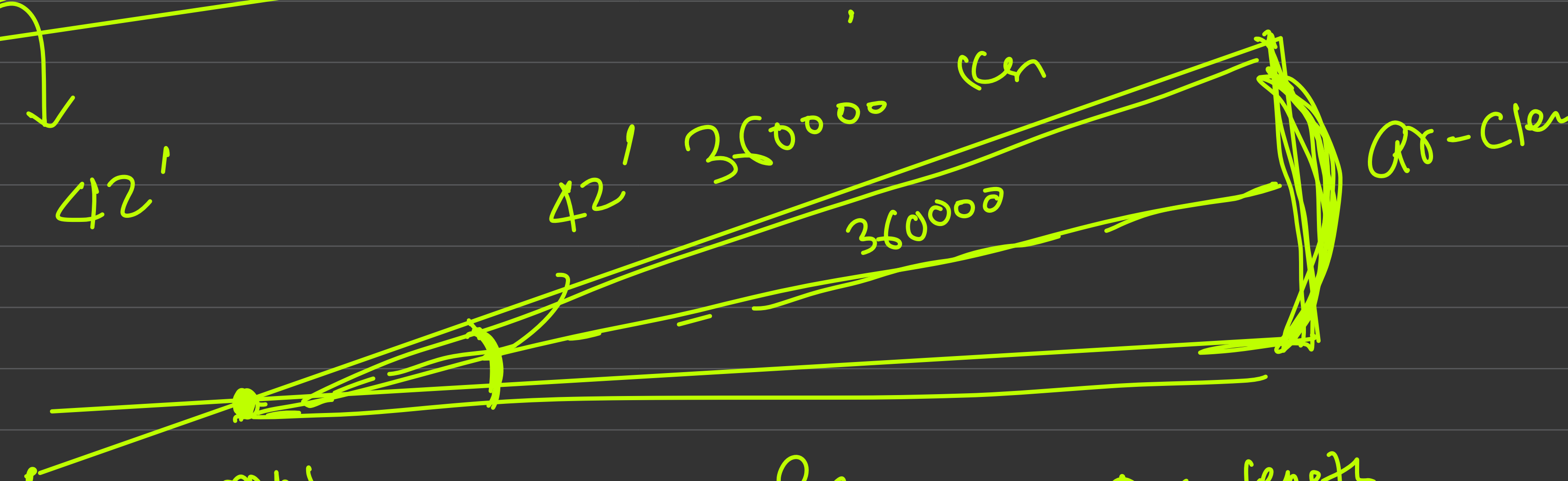
- $\tan 3^\circ = \frac{3\pi}{180} = \frac{\pi}{60} = \left(\frac{3.14}{60}\right)$

Qu: The moon's distance from earth is 360000 km. and its diameter subtends an angle of 42′ at the eye of an observer. The diameter of moon in km is

(a) 4400 (b) 1000 (c) 3600 (d) 8800



$$\left(\frac{42}{60}\right)^\circ$$



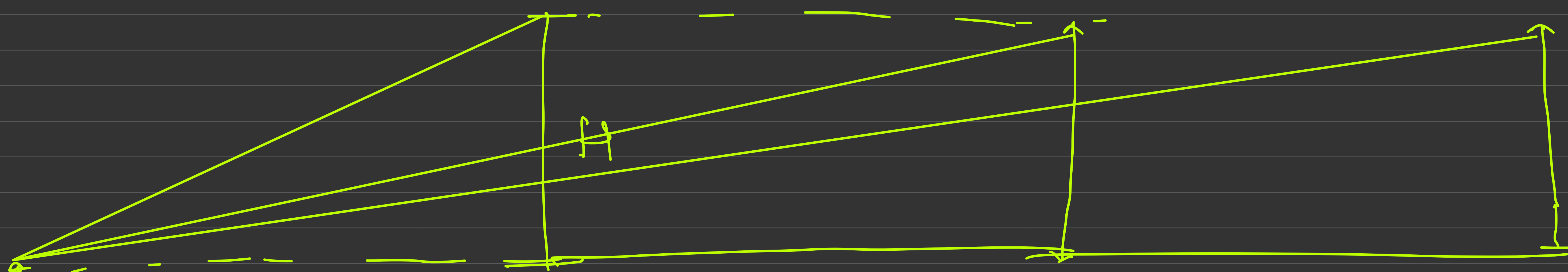
Arc-length

$$= \cancel{360000}^{\,200} \times \frac{\cancel{42}^{\,7}}{\cancel{60}} \times \frac{\pi}{\cancel{180}}$$

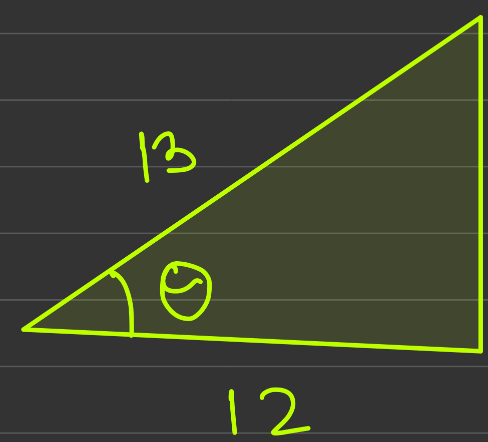
$$= 200 \times \cancel{7} \times \frac{22}{\cancel{7}} = 4400$$

$$\theta = \frac{l}{R} \Rightarrow \text{arc-length} = R \times \theta$$

⊙ → very small ⇒ $\boxed{\text{arc. length} \sim \text{diameter}}$



Qu: $\cos\theta = \frac{12}{13} = \frac{b}{h}$ find $\sin\theta$ & $\tan\theta$



$p = 5$

$p = \sqrt{(13)^2 - (12)^2}$

$= \sqrt{25}$

$= 5$

$a^2 - b^2$

$= (a+b)(a-b)$

- $\sin\theta = \frac{5}{13}$
- $\tan\theta = 5/12$

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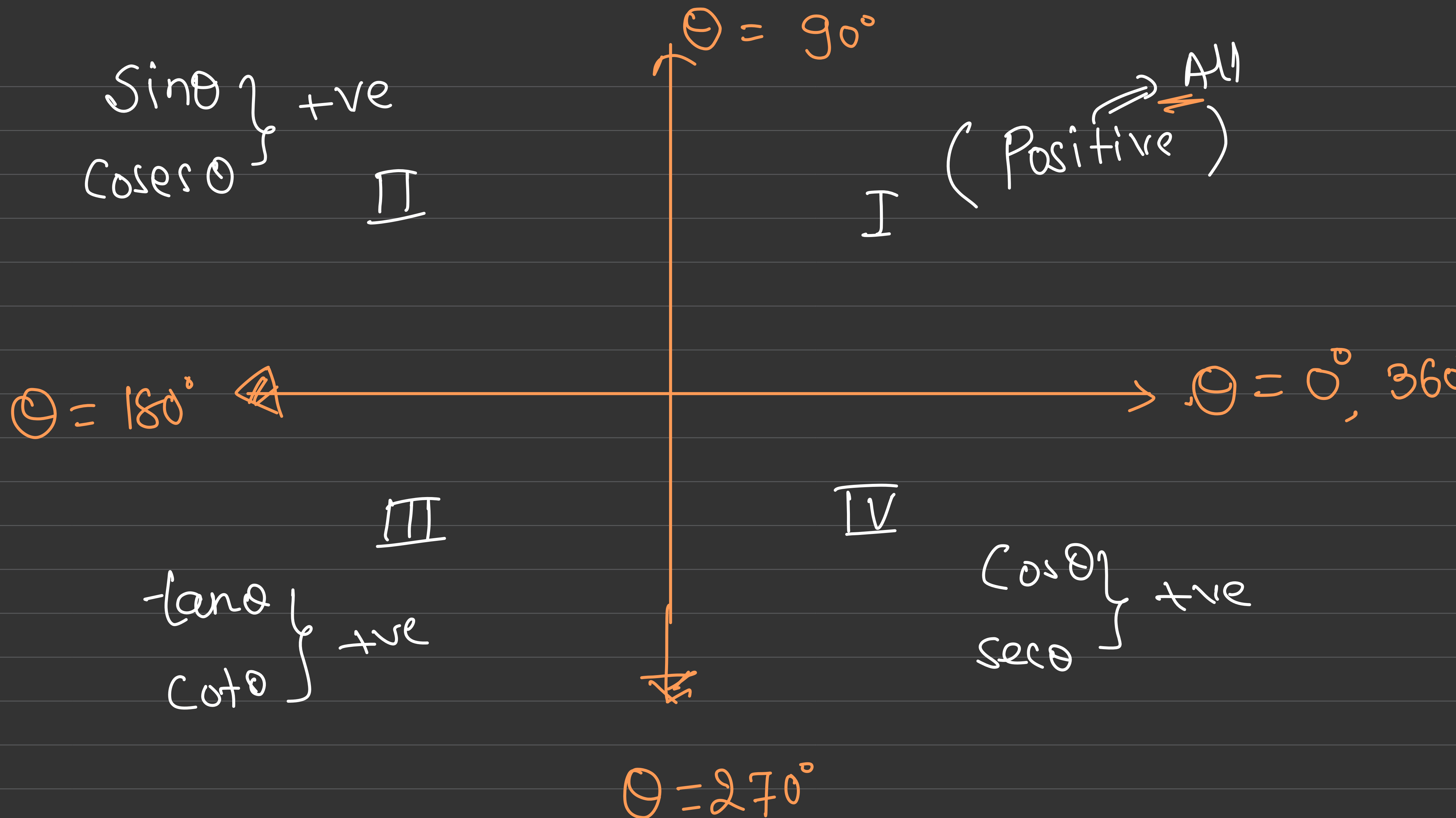
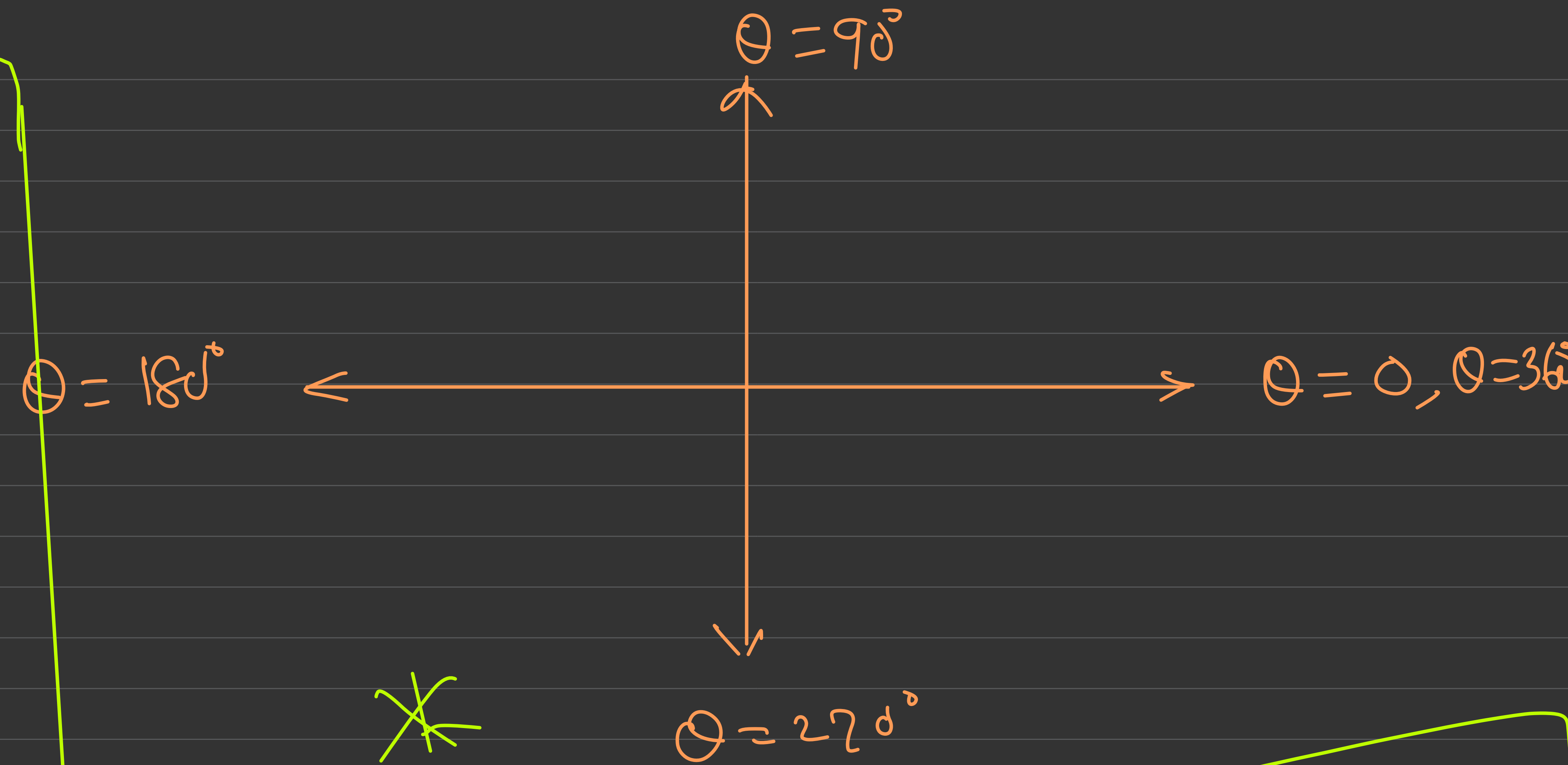


After School To College

All — Sin — Tan / cot — Cos / sec

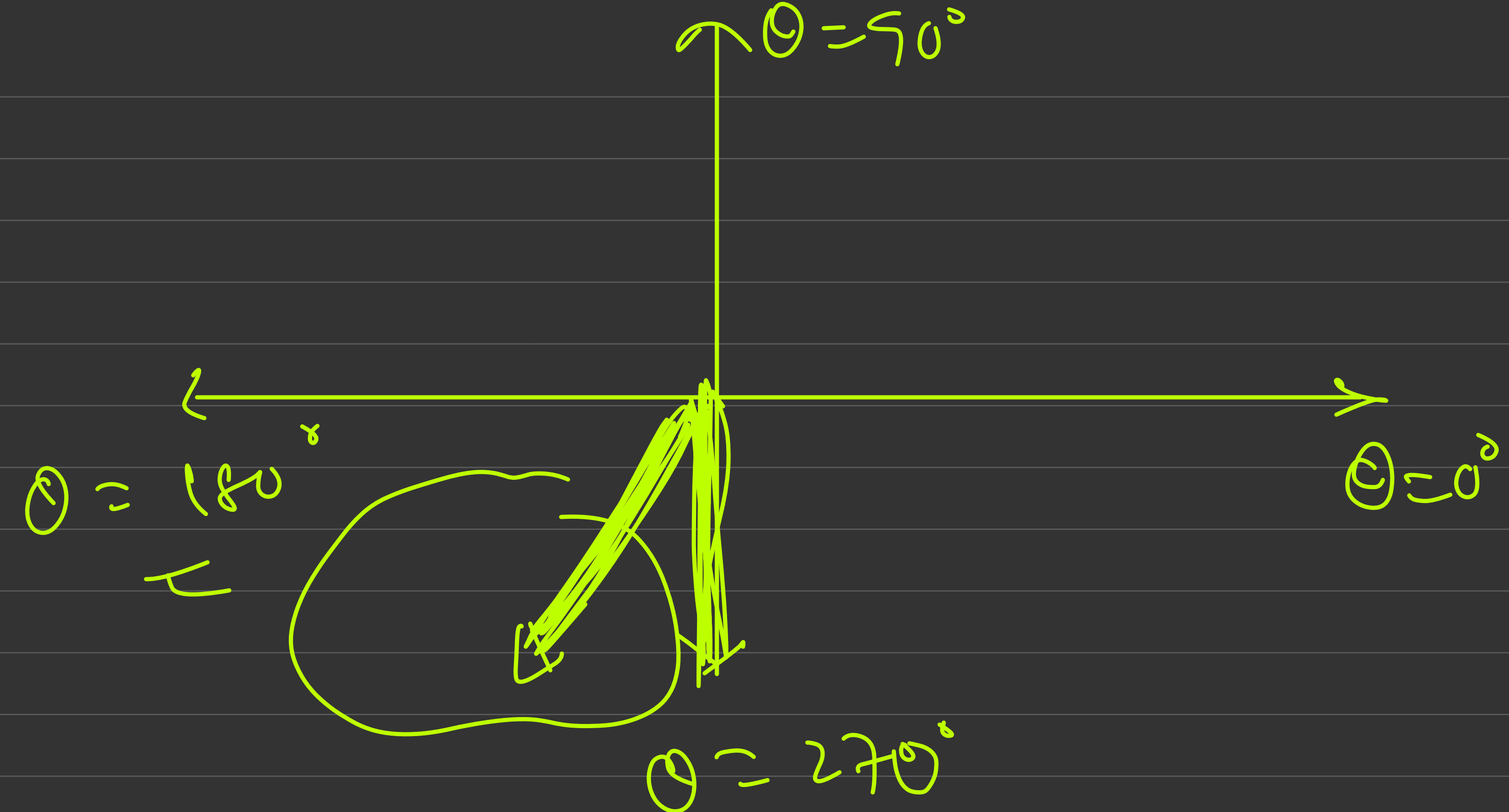
- $\sin(90^\circ - \theta) = \cos\theta$
- $\cos(90^\circ - \theta) = \sin\theta$
- $\tan(90^\circ - \theta) = \cot\theta$
- $\operatorname{cosec}(90^\circ - \theta) = \sec\theta$
- $\sec(90^\circ - \theta) = \operatorname{cosec}\theta$
- $\cot(90^\circ - \theta) = \tan\theta$



$$\left.\begin{array}{l} 90 \pm \theta \\ 270 \pm \theta \\ 450 \pm \theta \\ 630 \pm \theta \end{array}\right\} \qquad \left.\begin{array}{l} \sin \longleftrightarrow \cos \\ \tan \longleftrightarrow \cot \\ \sec \longleftrightarrow \operatorname{cosec} \end{array}\right\}$$

$\sin(90 - \theta) = +\cos\theta$

1st Quadrant



$\sin(270 - \theta) = -\cos\theta$

$0^\circ < \theta < 90^\circ$

$\tan(90^\circ + \theta) = -\cot\theta$

$\sin(\pi + \theta) = -\sin\theta$

$\sin(180^\circ + \theta) =$

(1) $\sin(\underbrace{90^\circ + \theta}) = \cos\theta$

(2) $\cos(\underbrace{180^\circ - \theta}) = -\cos\theta$

(3) $\tan(\underbrace{180^\circ - \theta}) = -\tan\theta$

(4) $\sin(\underbrace{180 - \theta}) = \sin\theta$

(5) $\operatorname{cosec}(\underline{270 - \theta}) = -\sec\theta$

(6) $\sec(\underline{270^\circ} + \theta) = +\operatorname{cosec}\theta$

(7) $\underline{\cos}(\underline{270} + \theta) = \sin\theta$

(8) $\underline{\sin}(360^\circ - \theta) = -\sin\theta$

(9) $\operatorname{cosec}(360^\circ - \theta) = -\operatorname{cosec}\theta$

(10) $\cot(\underline{270^\circ} - \theta) =$

$\underline{\tan\theta}$

Hint:

① $\sin(90^\circ - \theta)$

$\cos(90^\circ - \theta)$

$\tan(90^\circ - \theta)$

$\cot(90^\circ - \theta)$

$\sec(90^\circ - \theta)$

$\mathrm{cosec}(90^\circ - \theta)$

③ $\sin(180^\circ - \theta)$

② $\sin(90 + \theta)$

$\cos(90 + \theta)$

④ $\sin(180^\circ + \theta)$

⑤ $\sin(270 - \theta)$

⑥ $\sin(270 + \theta)$

⑦ $\sin(360 - \theta)$

⑧ $\sin(-\theta) = -\sin\theta$

Solution

Standard Values :-

Degree →	0°	30°	45°	60°	90°
Radian →	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
Sin	$\sqrt{\frac{0}{4}} = 0$	$\sqrt{\frac{1}{4}} = \frac{1}{2}$	$\sqrt{\frac{2}{4}} = \frac{1}{\sqrt{2}}$	$\sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$	$\sqrt{\frac{4}{4}} = 1$
Cos	1	$\sqrt{3}/2$	$\frac{1}{\sqrt{2}}$	$1/2$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞ (Not defined)
Cot	ND (∞)	$\sqrt{3}$	1	$1/\sqrt{3}$	0
Sec	1	$2/\sqrt{3}$	$\sqrt{2}$	2	ND (∞)
Cosec	ND (∞)	2	$\sqrt{2}$	$2/\sqrt{3}$	1

$\sin(90^\circ - \theta) = \cos\theta$

$\cos(90^\circ - \theta) = \sin\theta$

$\tan(90^\circ - \theta) = \cot\theta$

$\cot(90^\circ - \theta) = \tan\theta$

$\sec(90^\circ - \theta) = \operatorname{cosec}\theta$

$\operatorname{cosec}(90^\circ - \theta) = \sec\theta$

$\sin(90^\circ + \theta) = \cos\theta$

$\cos(90^\circ + \theta) = -\sin\theta$

$\tan(90^\circ + \theta) = -\cot\theta$

$\cot(90^\circ + \theta) = -\tan\theta$

$\sec(90^\circ + \theta) = -\operatorname{cosec}\theta$

$\operatorname{cosec}(90^\circ + \theta) = \sec\theta$

$\sin(180^\circ - \theta) = \sin\theta$

$\cos(180 - \theta) = -\cos\theta$

$\tan(180^\circ - \theta) = -\tan\theta$

$\cot(180^\circ - \theta) = -\cot\theta$

$\sec(180^\circ - \theta) = -\sec\theta$

$\operatorname{cosec}(180^\circ - \theta) = \operatorname{cosec}\theta$

$\sin(180^\circ + \theta) = -\sin\theta$

$\cos(180 + \theta) = -\cos\theta$

$\tan(180^\circ + \theta) = \tan\theta$

$\cot(180^\circ + \theta) = \cot\theta$

$\sec(180^\circ + \theta) = -\sec\theta$

$\operatorname{cosec}(180^\circ + \theta) = -\operatorname{cosec}\theta$

$$\sin(270^\circ - \theta) = -\cos\theta$$
$$\cos(270^\circ - \theta) = -\sin\theta$$
$$\tan(270^\circ - \theta) = \cot\theta$$
$$\cot(270^\circ - \theta) = \tan\theta$$
$$\sec(270^\circ - \theta) = -\operatorname{cosec}\theta$$
$$\operatorname{cosec}(270^\circ - \theta) = -\sec\theta$$

$$\sin(270^\circ + \theta) = -\cos\theta$$
$$\cos(270^\circ + \theta) = \sin\theta$$
$$\tan(270^\circ + \theta) = -\cot\theta$$
$$\cot(270^\circ + \theta) = -\tan\theta$$
$$\sec(270^\circ + \theta) = -\operatorname{cosec}\theta$$
$$\operatorname{cosec}(270^\circ + \theta) = \sec\theta$$

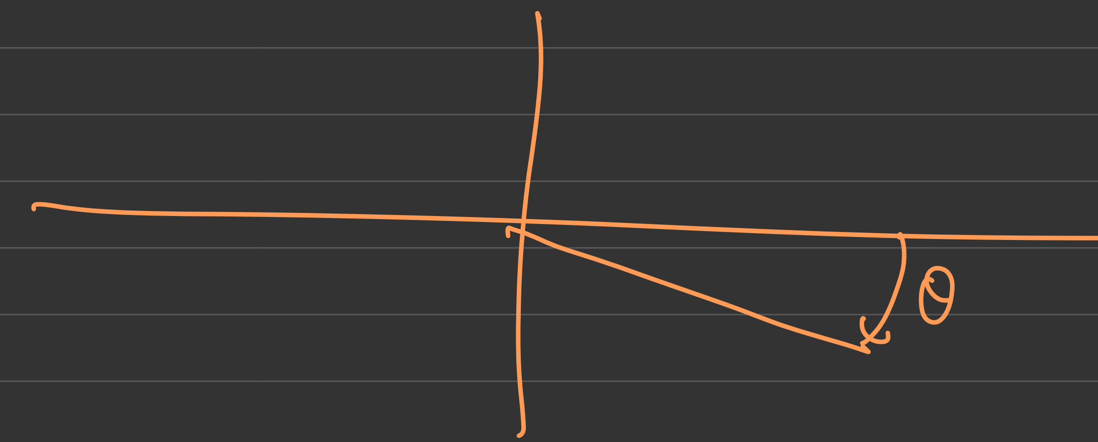
$$\sin(360^\circ - \theta) = -\sin\theta$$
$$\cos(360^\circ - \theta) = \cos\theta$$
$$\tan(360^\circ - \theta) = -\tan\theta$$
$$\cot(360^\circ - \theta) = -\cot\theta$$
$$\sec(360^\circ - \theta) = \sec\theta$$
$$\operatorname{cosec}(360^\circ - \theta) = -\operatorname{cosec}\theta$$

$$\sin(\quad -\theta) = -\sin\theta$$
$$\cos(\quad -\theta) = \cos\theta$$
$$\tan(\quad -\theta) = -\tan\theta$$
$$\cot(\quad -\theta) = -\cot\theta$$
$$\sec(\quad -\theta) = \sec\theta$$
$$\operatorname{cosec}(\quad -\theta) = -\operatorname{cosec}\theta$$

4th Qn

$$\boxed{\operatorname{cosec}(\underbrace{360^\circ - \theta}) = -\operatorname{cosec}\theta}$$

$$\boxed{\sin(-\theta) = -\sin\theta}$$



$\sin 120^\circ = \sin(90^\circ + 30^\circ) = \cos 30^\circ = \sqrt{3}/2$

$\longrightarrow \sin(180^\circ - 60^\circ) = \sin 60^\circ = \sqrt{3}/2$

Solve:

① $\cos 120^\circ = \cos(90^\circ + 30^\circ) = -\sin 30^\circ$

$= -1/2$

② $\sin 150^\circ = 1/2$

③ $\cos 150^\circ = -\dfrac{\sqrt{3}}{2}$

④ $\sin 135^\circ = \dfrac{1}{\sqrt{2}}$

⑤ $\cos 135^\circ = -\dfrac{1}{\sqrt{2}}$

⑥ $\sin 240^\circ = -\sqrt{3}/2$

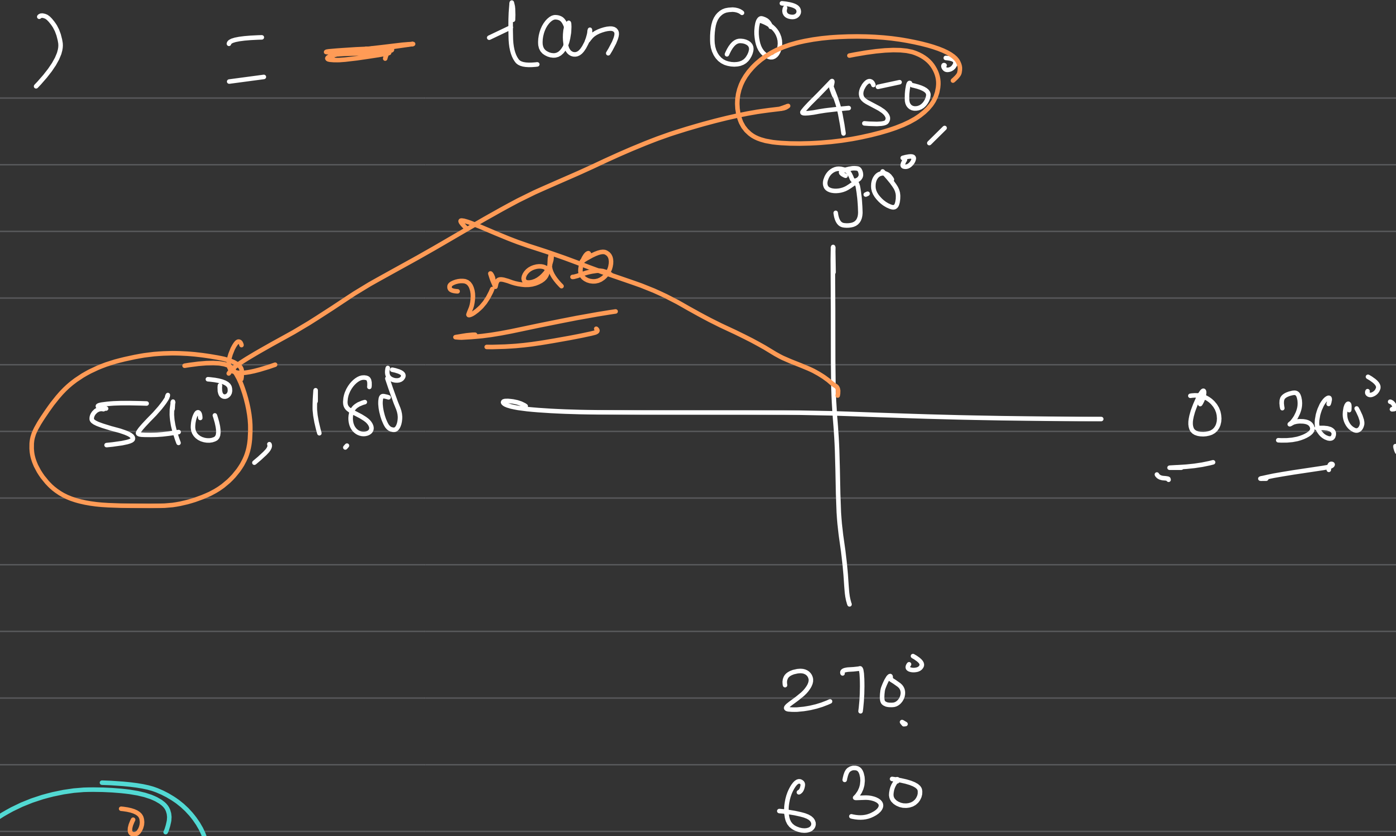
⑦ $\cos 240^\circ = -1/2$

⑧ $\cos 210^\circ = -\dfrac{\sqrt{3}}{2}$

⑨ $\sin 210^\circ = -1/2$

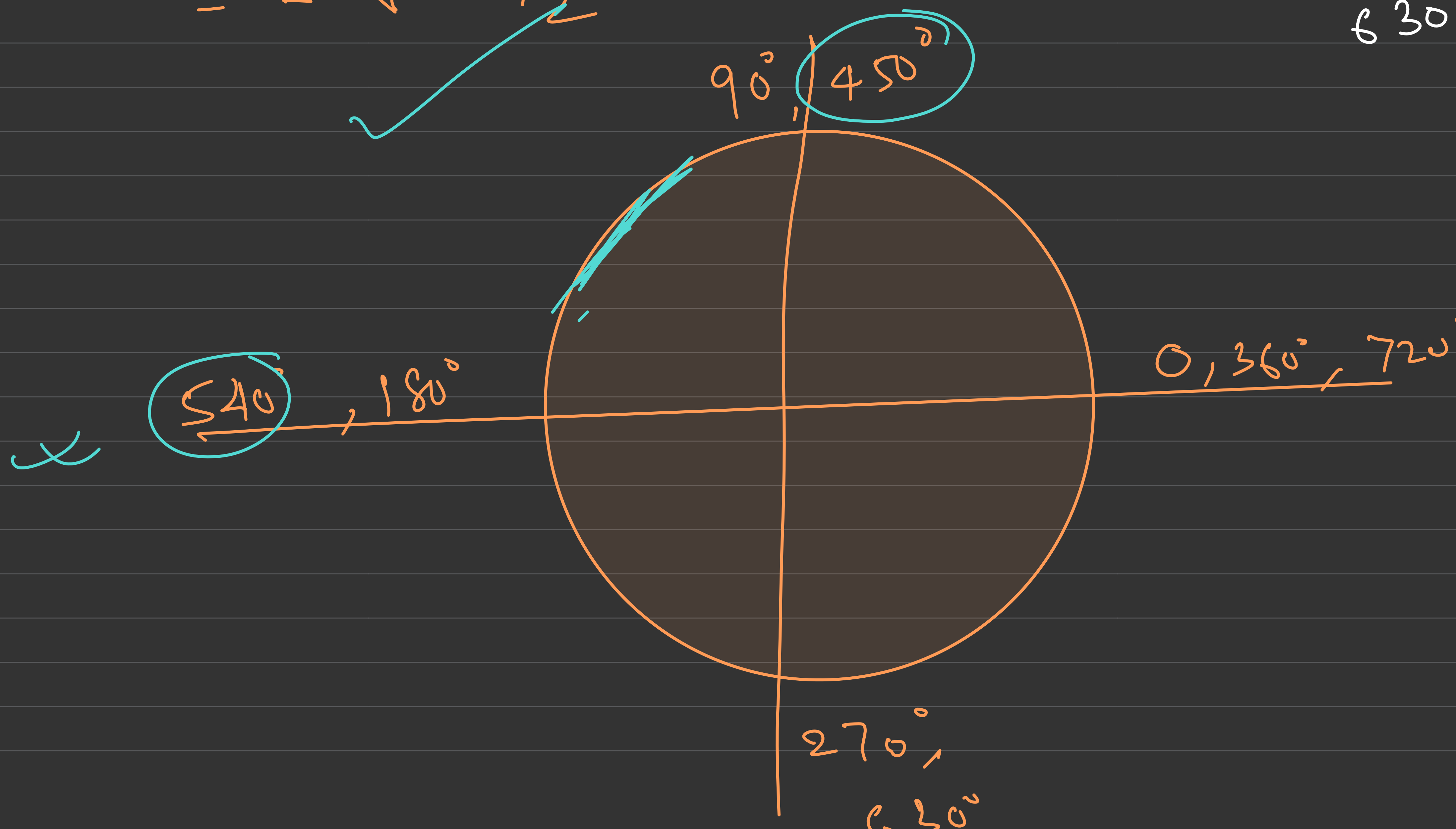
⑩ $\tan 480^\circ =$

$$\tan 480^\circ = \tan(540^\circ - 60^\circ) = -\tan 60^\circ$$



$$= -\tan 60^\circ$$

$$= -\sqrt{3} \quad \text{Ans}$$



$$\sin 240^\circ = \sin(270^\circ - 30^\circ) = -\cos 30^\circ = -\sqrt{3}/2$$

$$= \sin(180^\circ + 60^\circ) = -\sin 60^\circ = -\sqrt{3}/2$$

Compound Angle Formula:-

- $\sin(A+B) = \sin A \cos B + \cos A \sin B$
- $\sin(A-B) = \sin A \cos B - \cos A \sin B$
- $\cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B$
- $\cos(A-B) = \cos A \cos B + \sin A \cdot \sin B$
- $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$
- $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \cdot \tan B}$

$$\tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)} = \frac{\sin A\cos B + \cos A \sin B}{\cos A \cos B - \sin A \sin B}$$

$$= \frac{\left(\frac{\sin A \cancel{\cos B}}{\cos A \cancel{\cos B}} + \frac{\cancel{\cos A} \sin A}{\cancel{\cos A} \cdot \cos B}\right) \times \cancel{\cos A \cos B}}{\left(\frac{\cancel{\cos A} \cdot \cancel{\cos B}}{\cancel{\cos A}\ \cancel{\cos A}} - \frac{\sin A \quad \sin B}{\cos A \quad \cos B}\right) \times \cancel{\cos A \cos B}}$$

$$\boxed{\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}}$$

$$\tan(A-B) = \frac{\sin(A-B)}{\cos(A-B)} = \frac{\sin A\cos B - \cos A\sin B}{\cos A\cos B + \sin A\sin B}$$

$$= \frac{\left(\frac{\sin A\cos B}{\cos A\cos B} - \frac{\cos A\sin B}{\cos A\cos B}\right)\cancel{\cos A\cos B}}{\left(\frac{\cos A\cos B}{\cos A\cos B} + \frac{\sin A\sin B}{\cos A\cdot\cos B}\right)\cancel{\cos A\cos B}}$$

$$\boxed{\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\cdot\tan B}}$$

Qu:

$\sin 15^\circ = \sin(45^\circ - 30^\circ)$ $\quad 0^\circ / 30^\circ / 45^\circ / 60^\circ / 90^\circ$

$= \sin 45^\circ \cdot \cos 30^\circ - \cos 45^\circ \cdot \sin 30^\circ$

$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$

$= \left(\frac{\sqrt{3} - 1}{2\sqrt{2}}\right)$ ✓ ★★

$\Rightarrow \sin(60^\circ - 45^\circ) = \sin 60^\circ \cdot \cos 45^\circ - \cos 60^\circ \cdot \sin 45^\circ$

$= \left(\frac{\sqrt{3} - 1}{2\sqrt{2}}\right)$ ★★★

Qu! $\underline{\cos 15^\circ} = \cos(45^\circ - 30^\circ)$

$$= \cos 45^\circ \cdot \cos 30^\circ + \sin 45^\circ \cdot \sin 30^\circ$$

$$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$$

$$= \frac{\sqrt{3}+1}{2\sqrt{2}}$$

Qu! $\tan 15^\circ = \dfrac{\sin 15^\circ}{\cos 15^\circ} = \dfrac{\sqrt{3}-1}{\sqrt{3}+1}$

Qu! $\sin 75^\circ = \sin(90^\circ - 15^\circ) = \cos 15^\circ = \dfrac{\sqrt{3}+1}{2\sqrt{2}}$

Qu! $\cos 75^\circ = \cos(90^\circ - 15^\circ) = \sin 15^\circ = \dfrac{\sqrt{3}-1}{2\sqrt{2}}$

- Complementary Angle $\alpha + \beta = 90^\circ$

- $\sin 30^\circ = \cos 60^\circ$

- $\sin 15^\circ = \cos 75^\circ$

- $\sin 105^\circ = \sin(90^\circ + 15^\circ) = \cos 15^\circ = \dfrac{\sqrt{3}+1}{2\sqrt{2}}$.

Reduction Formula:

- $\sin(2A) = \sin(A+A) = \sin A \cos A + \cos A \sin A$

$$\boxed{\sin 2A = 2 \sin A \cos A}$$

- $\cos(2A) = \cos(A+A) = \cos A \cdot \cos A - \sin A \cdot \sin A$

$$\boxed{\cos 2A = \cos^2 A - \sin^2 A}$$

$\sin^2 A + \cos^2 A = 1 \Rightarrow \cos^2 A = 1 - \sin^2 A$

$$\boxed{\cos 2A = 1 - 2\sin^2 A}$$

$\sin^2 A = 1 - \cos^2 A$

$$\boxed{\cos 2A = 2\cos^2 A - 1}$$

$\boxed{\sin^2 A = \dfrac{1-\cos 2A}{2}}$ $\Rightarrow$ $1-\cos 2A = 2\sin^2 A$

$\boxed{\cos^2 A = \dfrac{1+\cos 2A}{2}}$ $\Rightarrow$ $1+\cos 2A = 2\cos^2 A$

$$\boxed{\left.\begin{aligned} \#\quad & 1-\cos A = 2\sin^2 A/2 \\ \#\quad & 1+\cos A = 2\cos^2 A/2 \end{aligned}\right\}}$$

Qu!: $\sin 22.5^\circ = \sin\left(\frac{45^\circ}{2}\right)$

$$1 - \cos A = 2\sin^2 A/2$$

Assume $A = 45^\circ$ $\quad A/2 = 22.5^\circ$

$$1 - \cos 45^\circ = 2 \sin^2 22.5^\circ$$

$$\Rightarrow \left(1 - \frac{1}{\sqrt{2}}\right) = 2 \sin^2 22.5^\circ$$

$$\Rightarrow \sin^2 22.5^\circ = \left[\frac{\sqrt{2}-1}{2\sqrt{2}}\right]$$

$$\sin 22.5^\circ = \pm \left(\frac{\sqrt{2}-1}{2\sqrt{2}}\right)^{1/2}$$

$$\sin 22.5^\circ = + \left(\frac{\sqrt{2}-1}{2\sqrt{2}}\right)^{1/2}$$

$\left(\because 22.5^\circ \text{ is in } 1^{st} \text{ Quadrant} \Downarrow \sin \Rightarrow +ve\right)$

Note:

1) $-1 \le \sin\theta \le 1$

$(\sin\theta)_{min} = -1$

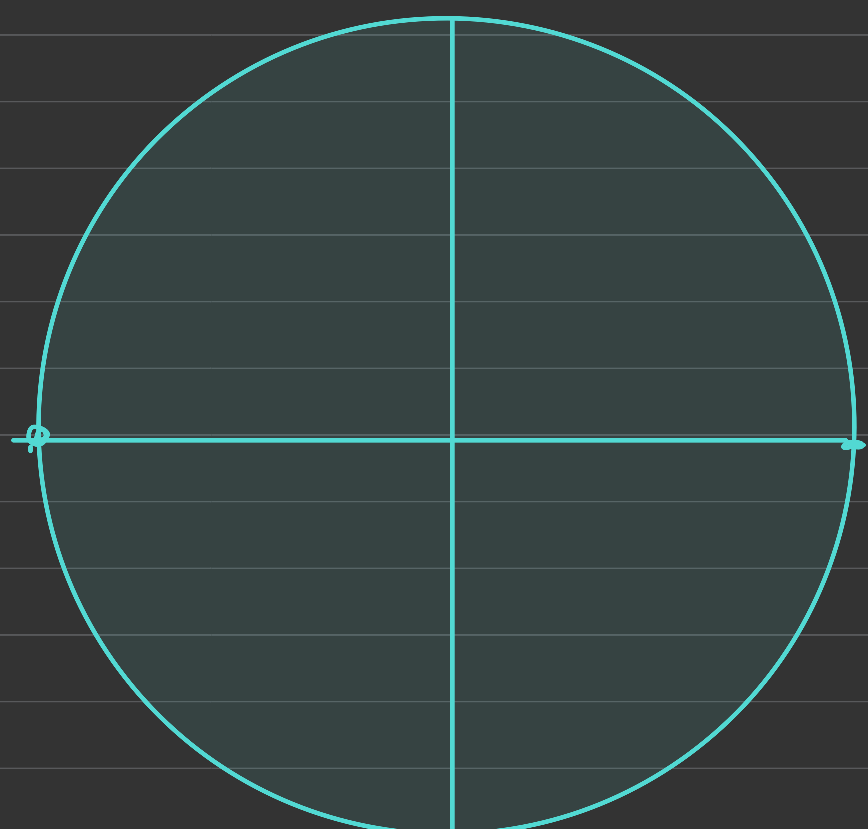
$(\sin\theta)_{max} = 1$

2) $-1 \le \cos\theta \le 1$ → $(\cos\theta)_{max} = 1$

$(\cos\theta)_{min} = -1$

$\sin 90^\circ = 1$

$\sin 180^\circ = 0$



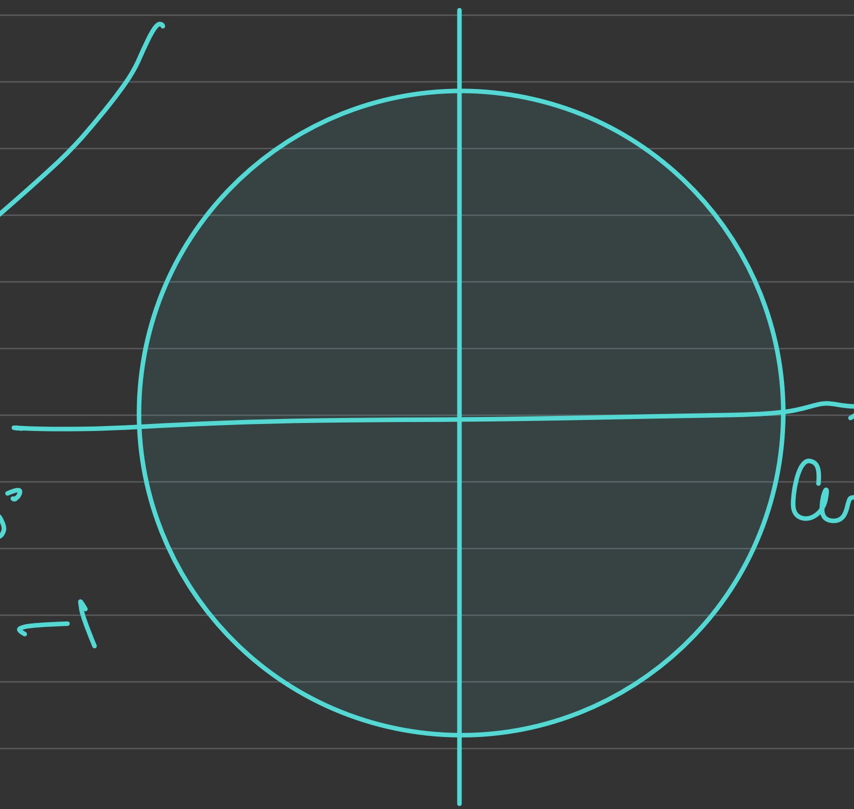
$0^\circ = 0$

$\sin 270^\circ = -1$

Sin

$\cos 90^\circ = 0$

$\cos 180^\circ = -1$



$\cos 0 = 1$

$\cos 270^\circ = 0$

RACE → Basic Math – 1 ⇒ Try to solve

→ DPP – 1

→ **Module (1) Basic Mathematics** ⇒ B.B – 1

#

Phase – 1 students ⇒ 2 : PM –

Google Meet

Inverse f

	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\cot\theta$	$\sec\theta$	$\operatorname{cosec}\theta$
$\sin\theta$	$\sin\theta$	$\sqrt{1-\cos^2\theta}$	$\frac{\tan\theta}{\sqrt{1+\tan^2\theta}}$	$\frac{1}{\sqrt{1+\cot^2\theta}}$	$\frac{\sqrt{\sec^2\theta-1}}{\sec\theta}$	$\frac{1}{\operatorname{cosec}\theta}$
$\cos\theta$	$\sqrt{1-\sin^2\theta}$	$\cos\theta$	$\frac{1}{\sqrt{1+\tan^2\theta}}$	$\frac{\cot\theta}{\sqrt{1+\cot^2\theta}}$	$\frac{1}{\sec\theta}$	$\frac{\sqrt{\operatorname{cosec}^2\theta-1}}{\operatorname{cosec}\theta}$
$\tan\theta$	$\frac{\sin\theta}{\sqrt{1-\sin^2\theta}}$	$\frac{\sqrt{1-\cos^2\theta}}{\cos\theta}$	$\tan\theta$	$\frac{1}{\cot\theta}$	$\sqrt{\sec^2\theta-1}$	$\frac{1}{\sqrt{\operatorname{cosec}^2\theta-1}}$
$\cot\theta$	$\frac{\sqrt{1-\sin^2\theta}}{\sin\theta}$	$\frac{\cos\theta}{\sqrt{1-\cos^2\theta}}$	$\frac{1}{\tan\theta}$	$\cot\theta$	$\frac{1}{\sqrt{\sec^2\theta-1}}$	$\sqrt{\operatorname{cosec}^2\theta-1}$
$\sec\theta$	$\frac{1}{\sqrt{1-\sin^2\theta}}$	$\frac{1}{\cos\theta}$	$\sqrt{1+\tan^2\theta}$	$\frac{\sqrt{1+\cot^2\theta}}{\cot\theta}$	$\sec\theta$	$\frac{\operatorname{cosec}\theta}{\sqrt{\operatorname{cosec}^2\theta-1}}$
$\operatorname{cosec}\theta$	$\frac{1}{\sin\theta}$	$\frac{1}{\sqrt{1-\cos^2\theta}}$	$\frac{\sqrt{1+\tan^2\theta}}{\tan\theta}$	$\sqrt{1+\cot^2\theta}$	$\frac{\sec\theta}{\sqrt{\sec^2\theta-1}}$	$\operatorname{cosec}\theta$

$$\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$$

$$\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$$

$$\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$$

$$\cos C - \cos D = 2\sin\frac{C+D}{2}\sin\frac{D-C}{2}$$

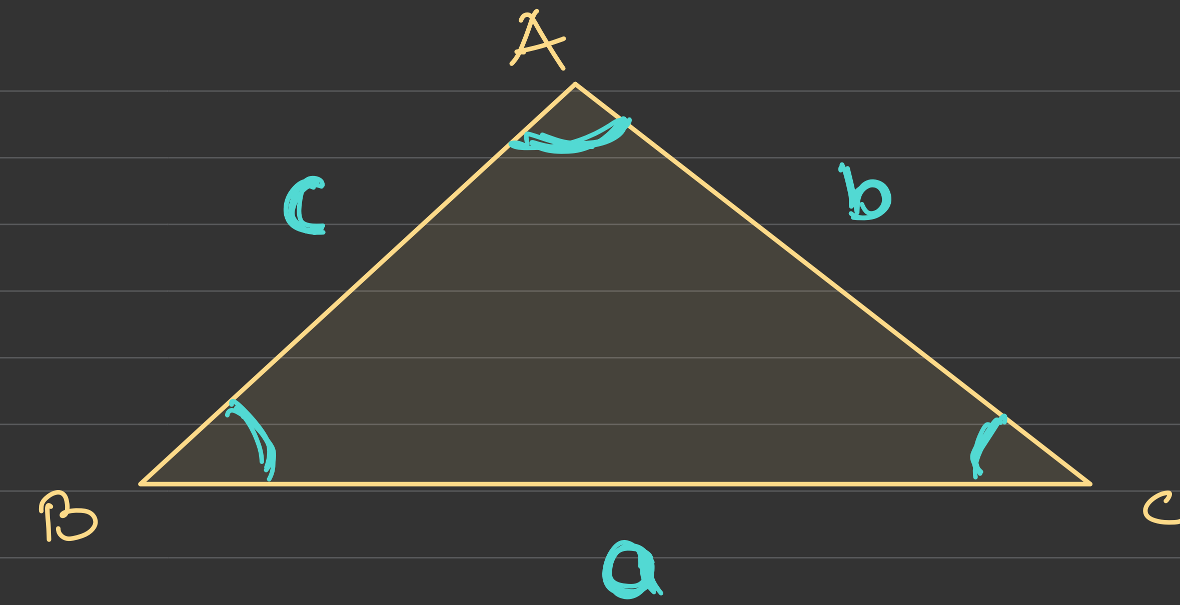
* $1 + \sin A = \left(\sin\frac{A}{2} + \cos\frac{A}{2}\right)^2$

* $1 - \sin A = \left(\sin\frac{A}{2} - \cos\frac{A}{2}\right)^2$

- $\sin 3A = 3\sin A - 4\sin^3 A$
- $\cos 3A = 4\cos^3 A - 3\cos A$
- $\tan 3A = \dfrac{3\tan A - 4\tan^3 A}{1 - 3\tan^2 A}$
- $1 + \sin 2A = \sin^2 A + \cos^2 A + 2\sin A \cdot \cos A = (\sin A + \cos A)^2$

163. Theorem : *In any triangle ABC,*

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

lar manner it may be shown that

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

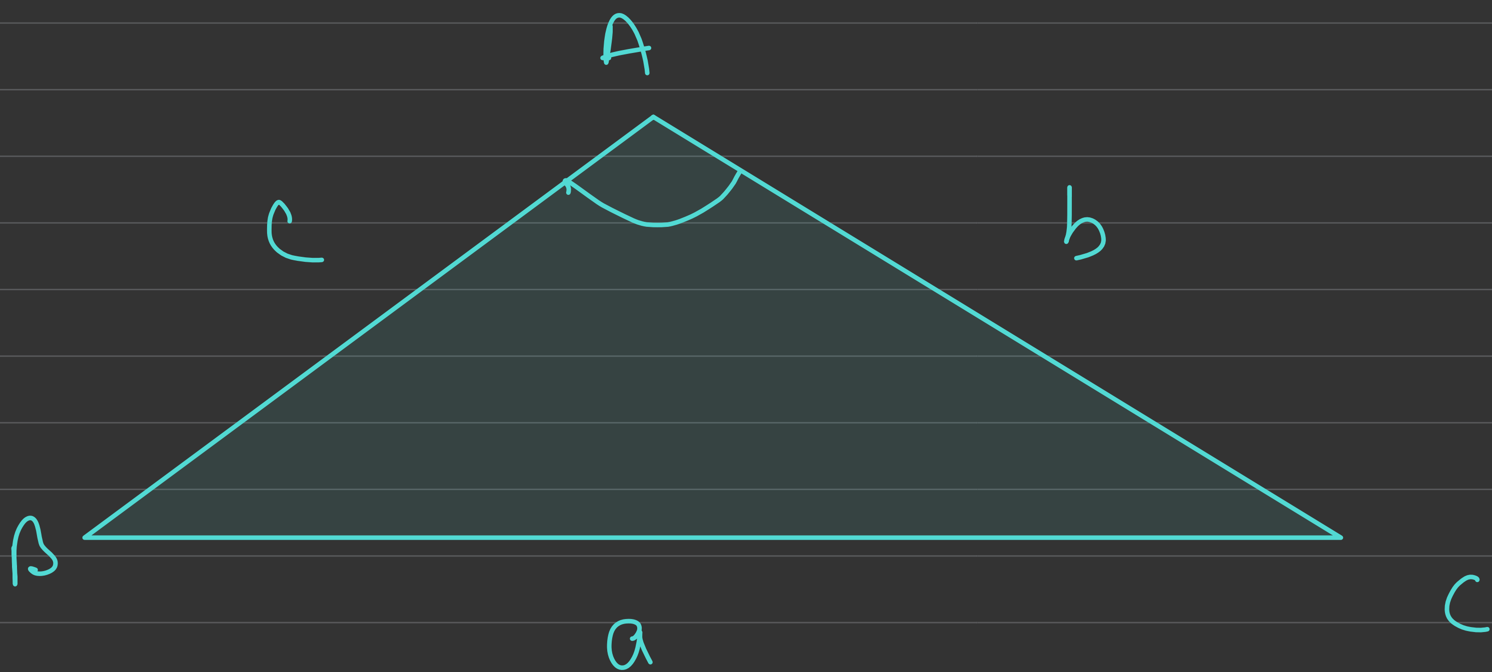
$$\cos B = \frac{c^2 + a^2 - b^2}{2ca}$$

$$\boxed{\cos A = \frac{b^2 + c^2 - a^2}{2bC}}$$

$$\boxed{\cos B = \frac{a^2 + c^2 - b^2}{2ac}}$$

$$\boxed{\cos C = \frac{a^2 + b^2 - c^2}{2ab}}$$

vector



$a = b\cos C + c\cos A$

$b = a\cos C + c\cos A$

$c = a\cos B + b\cos C$

** Projection of Vector

• Find the maximum and minimum value of

(a) $C = 10 - 4\cos\theta$

(b) $d = 3\cos\theta - 4$

(c) $e = 3\cos^2\theta - 4$

(d) $f = 4 - 3\sin^2\theta$

Soln:

(a) $(\cos\theta)_{max} = 1$, $(\cos\theta)_{min} = -1$

$C_{max} = 10 + 4 = 14$

$C_{min} = 10 - 4 = 10$

(b) $d_{max} = -1$

$d_{min} = -7$

(c) $0 \le \cos^2\theta \le 1$

$e_{max} = 3\times1 - 4 = -1$

$e_{min} = 3\times0 - 4 = -4$

(d) $f_{max} = 4$, $f_{min} = 1$

$$x^2 \geq 0$$

$$(x^2)_{min} = 0$$

$$-1 \leq \cos\theta \leq 1 \qquad 0 \leq \cos^2\theta \leq 1$$

$$-1 \leq \sin\theta \leq +1$$

$$0 \leq \sin^2\theta \leq 1$$

ex!

$$f = 4 - 3\sin^2\theta$$

$$\left.\begin{aligned}(\sin^2\theta)_{max} &= 1\\ (\sin^2\theta)_{min} &= 0\end{aligned}\right\}$$

$$f_{max} = 4 - 3\times 0 = 4$$

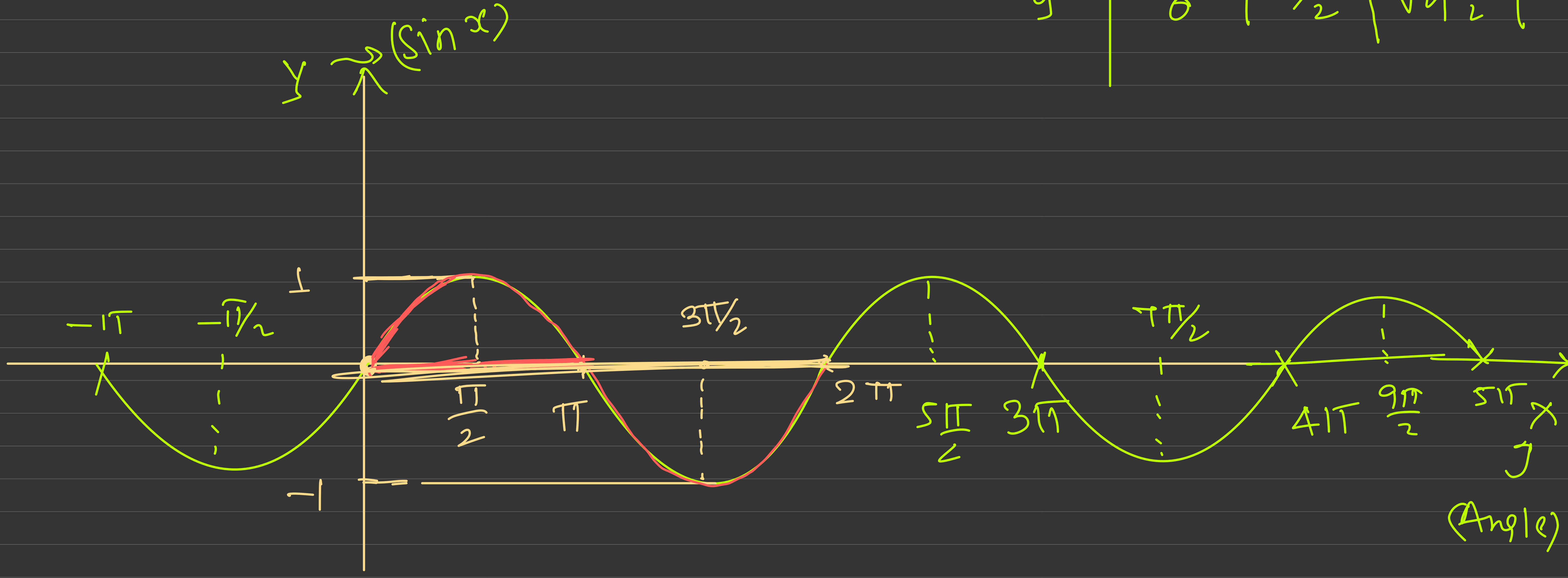
$$f_{min} = 4 - 3\times 1$$

$$= 1$$

Graph of Sinx & Cosx

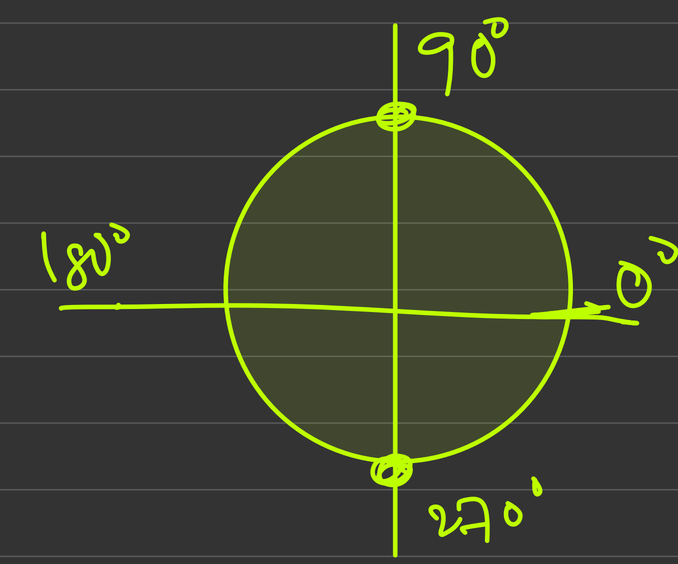
1) $y = \sin x$

x	0	30°	45°	60°	90°
		$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
y	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1



- $-1 \le \sin x \le 1$

- $\boxed{\sin x = 1}$ At $x = \frac{\pi}{2}$



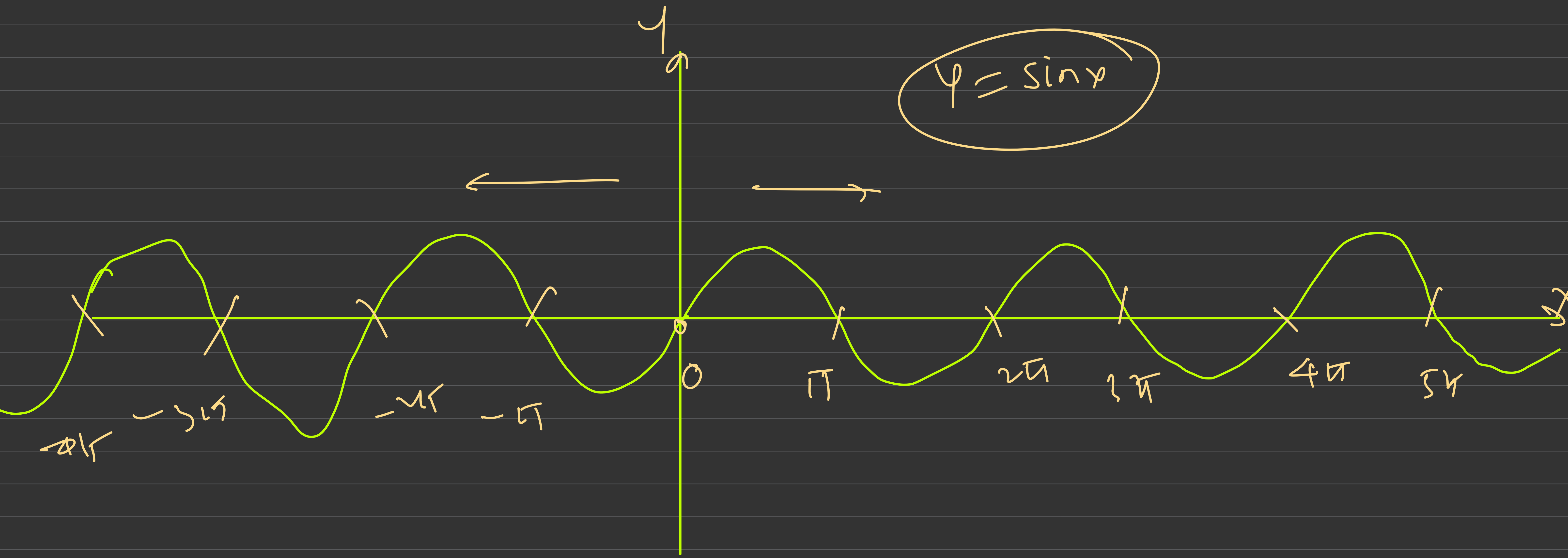
- $\sin x = -1$ At $x = 3\pi/2 = 270°$
- $\sin x = 0$ At $x = 0$ and π

- $\sin x$ is $\underbrace{\text{periodic function}}$

$\Downarrow$

The function repeats itself after a certain period

- Period of $\sin x = \underline{\underline{2\pi}}$
- $\sin x$ is increasing from $\cancel{x} = 0°$ to $\cancel{x} = 90°$.



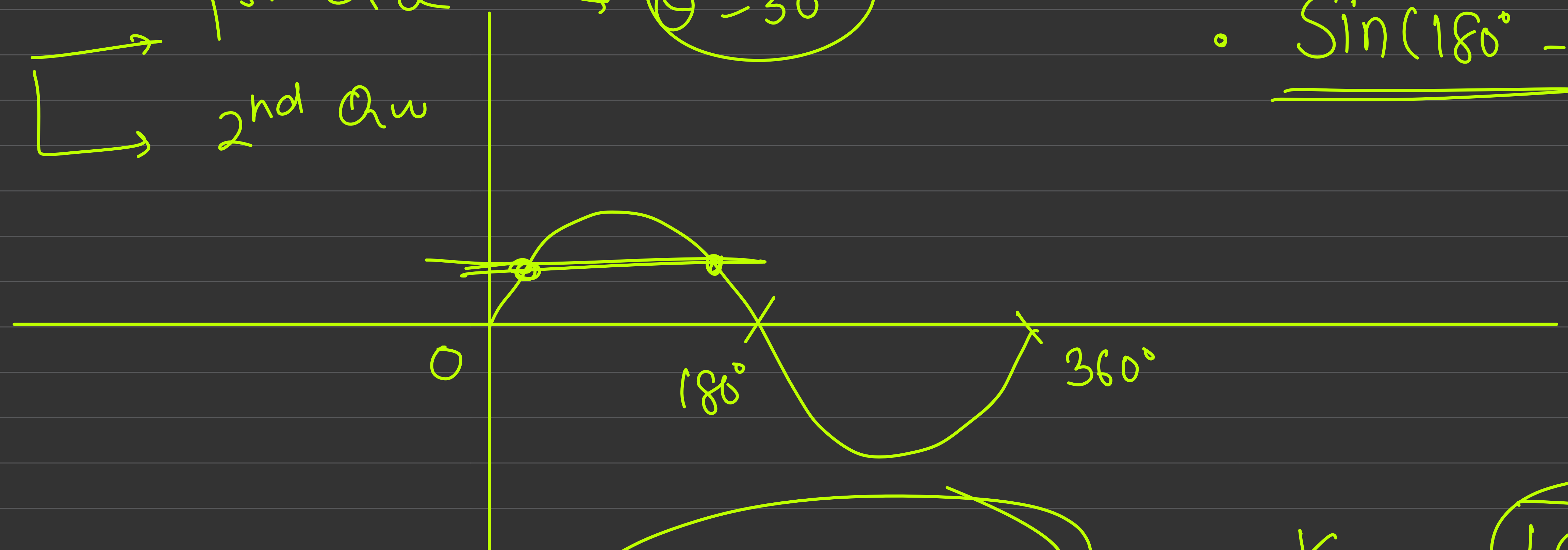
y = sinx
y
x
0
π
2π
3π
4π
5π
-π
-2π
-3π
-4π

Qu:

$\sin\theta = \frac{1}{2} = \sin 30^\circ$ find the the value of θ between 0 to 360°.

$\theta \to$ 1st Qa $\to$ $\theta = 30^\circ$

$\theta \to$ 2nd Qu



- $\sin(180^\circ - \theta) = \sin\theta$

$\boxed{\sin\theta} = \sin(180^\circ - \theta) = \frac{1}{2} = \boxed{\sin 30^\circ}$

$\Rightarrow \sin\theta = \sin 30^\circ \Rightarrow \theta = 30^\circ;$

$\Rightarrow \sin(180^\circ - \theta) = \sin 30^\circ \Rightarrow 180^\circ - \theta = 30^\circ \Rightarrow \boxed{\theta = 150^\circ}$

- $\sin\theta = \frac{1}{2}$

  $\theta = \underline{30^\circ}$, $\boxed{180^\circ - 30^\circ = 150^\circ}$

- $\sin\theta = \frac{\sqrt{3}}{2}$

  $\theta = 60^\circ, \quad 120^\circ$

- $\sin\theta = \frac{1}{\sqrt{2}}$

  $\theta = 45^\circ, \; 135^\circ$

$$\sin\theta = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

$$\theta = 15^\circ, \quad 165^\circ$$

V.V.I

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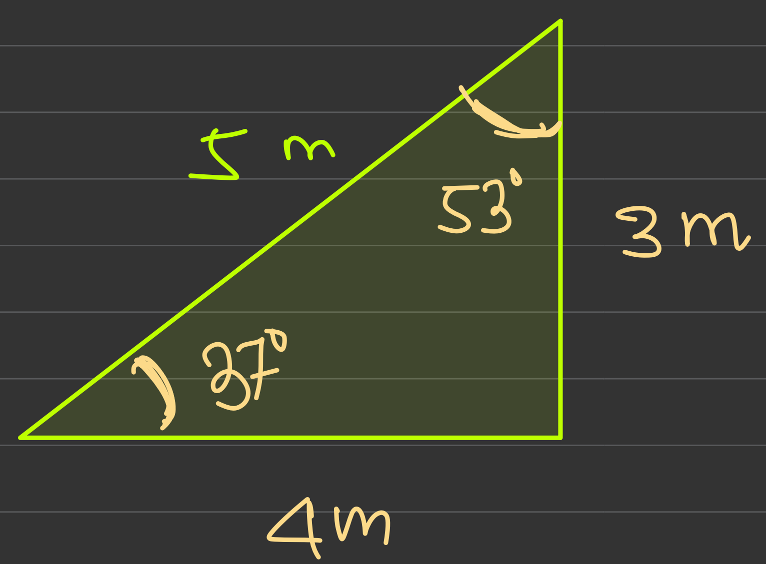


$\sin 37^\circ = 3/5$

$\cos 37^\circ = 4/5$

$\tan 37^\circ = 3/4$

$\sin 53^\circ = \frac{4}{5}$

$\cos 53^\circ = 3/5$

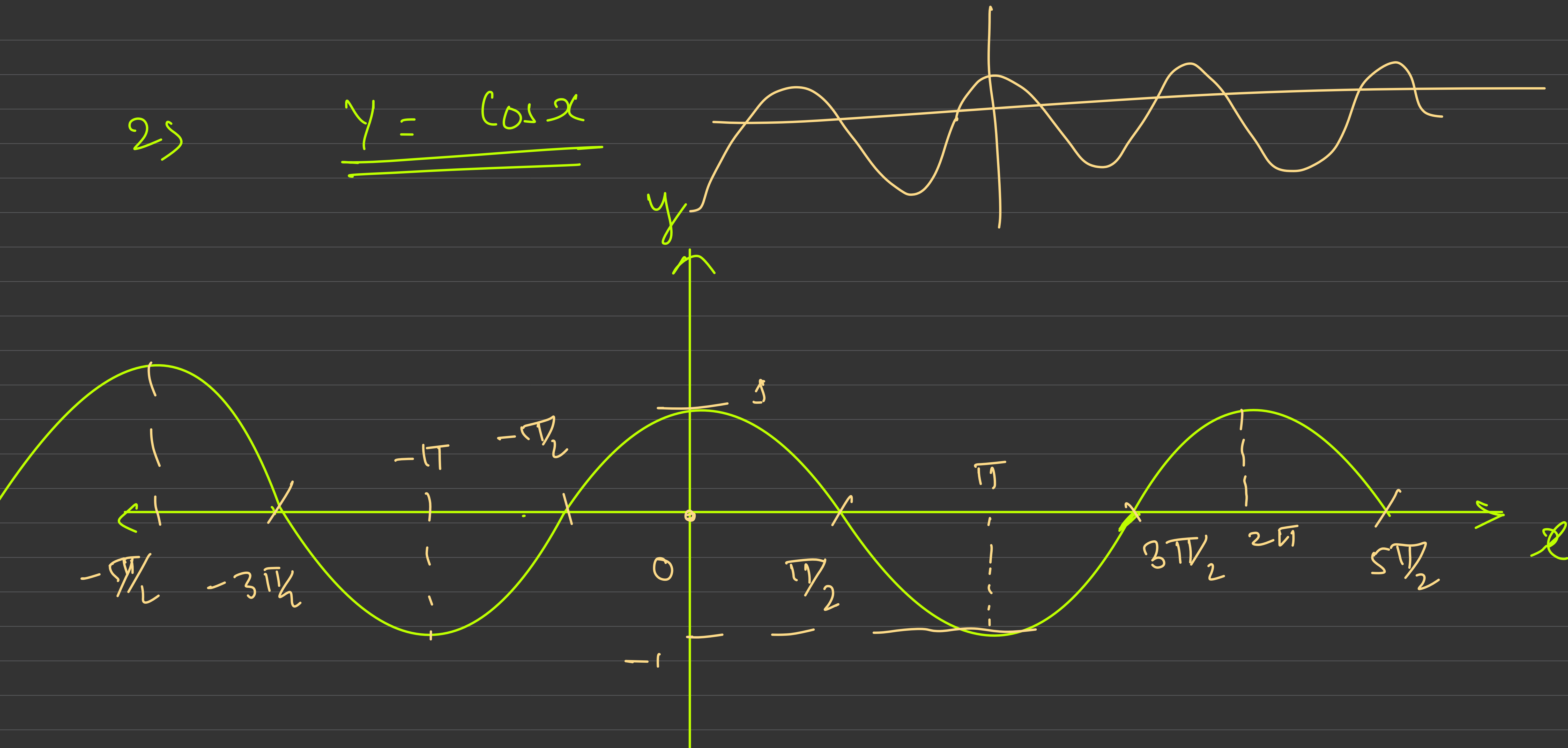
$\tan 53^\circ = 4/3$

$$\sin\theta = 3/5$$

$$\theta = 37^\circ, 143^\circ$$

2)

$y = \cos x$



- $-1 \leq \cos x \leq 1$

- $\cos x = 1$ at $x = 0$

- $\cos x = -1$ at $x = \pi$

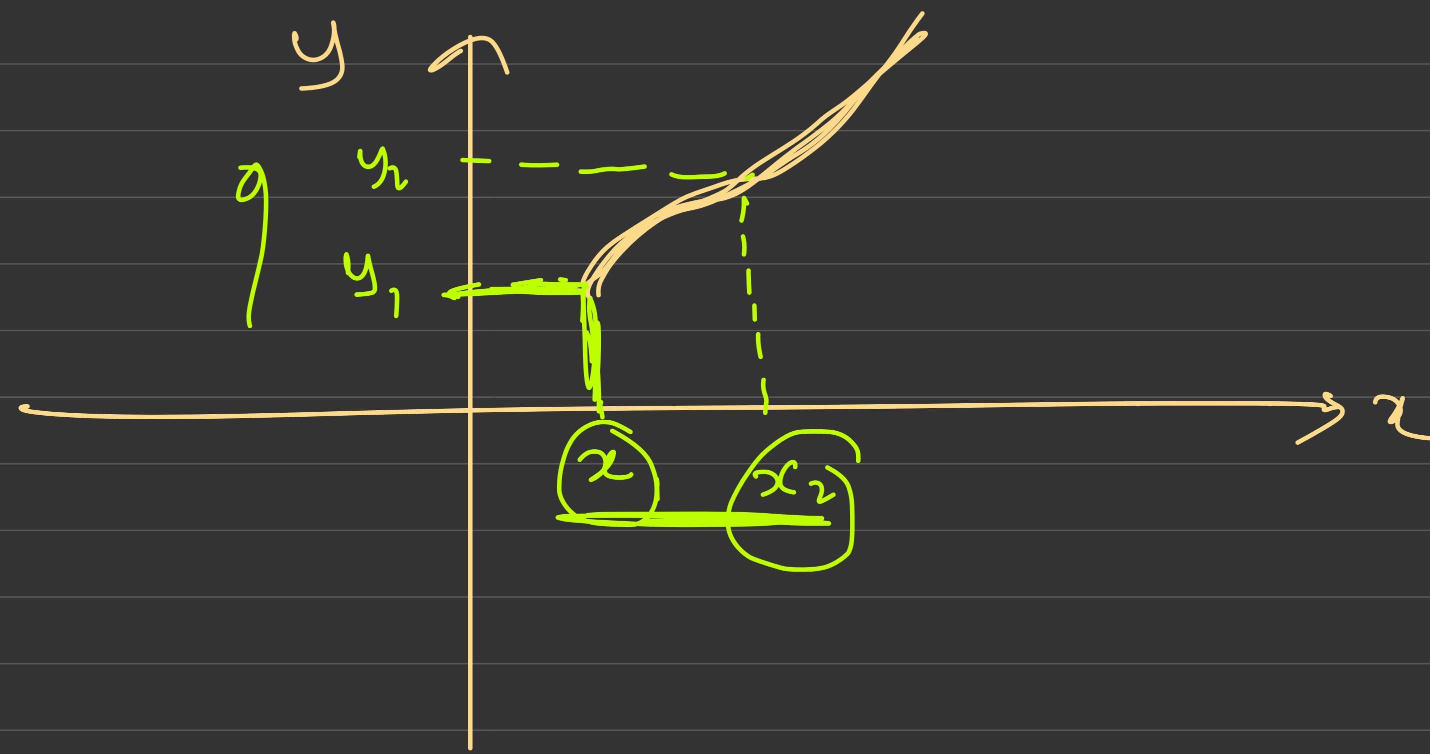
- $\cos x = 0$ at $x = \frac{\pi}{2}, \frac{3\pi}{2}$

- $\cos x$ is a periodic function.

and period $= \underline{\underline{2\pi}}$

- $\cos x$ is decreasing from $\underline{x = 0^\circ \text{ to } x = 90^\circ}$

1)



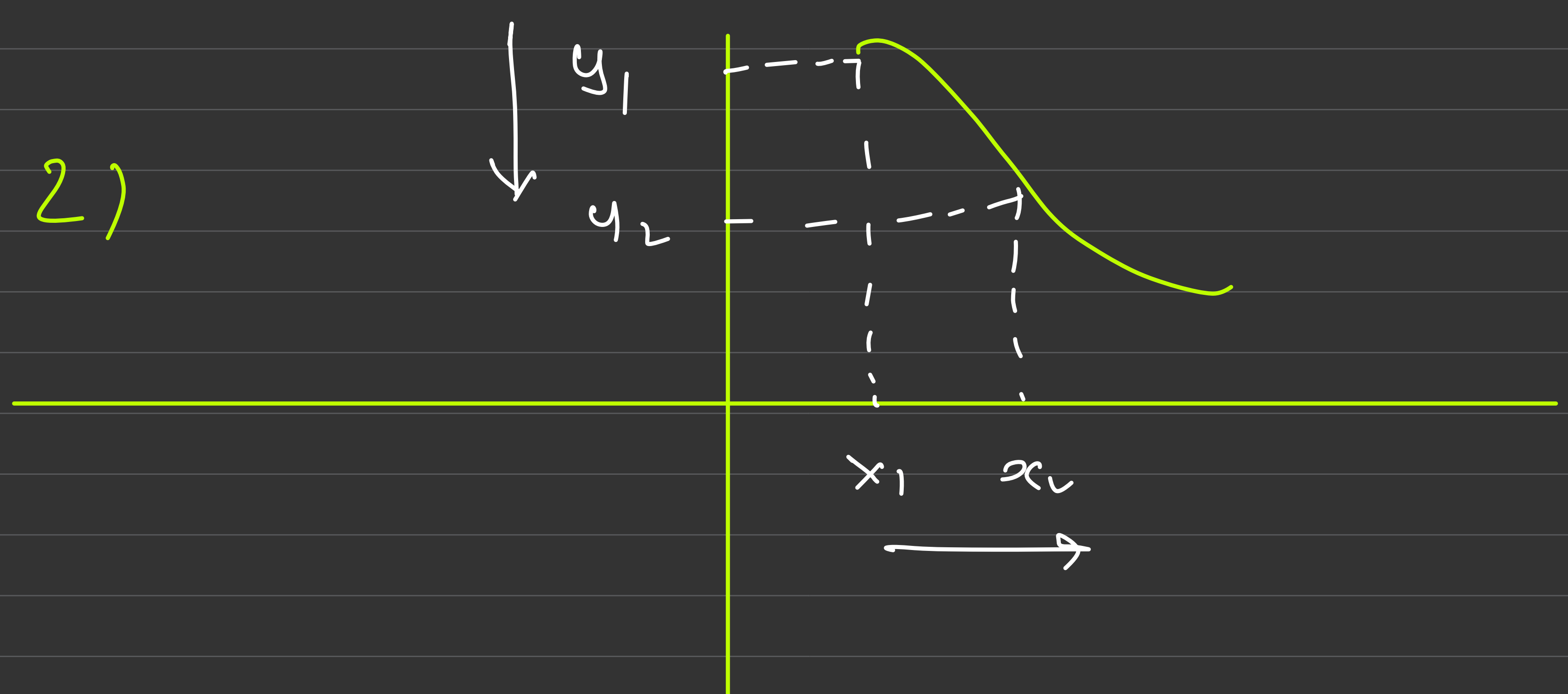
Increasing function

$x \uparrow \quad y \uparrow$

$x_2 > x_1$

$y_2 > y_1$

2)



$x \uparrow \quad y \downarrow$ (Decreasing function)

$$\left.\begin{aligned} x_2 &> x_1 \\ y_2 &< y_1 \end{aligned}\right\}$$

- $\sin x$ is increasing between 0 to 90°

$\theta_1 > \theta_2$ $\quad [\theta_1, \theta_2 \in (0, 90^\circ)]$

- $\sin\theta_1 > \sin\theta_2$

- $\cos\theta_1 < \cos\theta_2$ $\quad$ ($\cos\theta \rightarrow$ decreasing function)

- $\theta_2 = 30^\circ$, $\theta_1 = 60^\circ$

$\sin\theta_2 = \frac{1}{2} = 0.5$ $\quad$ $\sin\theta_1 = \sqrt{3}/2 = \frac{1.732}{2} = 0.86$

$\cos\theta_2 = 0.86$ $\quad$ $\cos\theta_1 = 0.5$

$\theta_2 > \theta_1$

$\Rightarrow \sin\theta_2 > \sin\theta_1$

$\boxed{\cos\theta_2 < \cos\theta_1}$

$\cos\theta = 1/2$

$$\theta \rightarrow \begin{cases} \text{1st Quadrant} = 60^\circ \\ \text{4th Quadrant} = (360^\circ - 60^\circ) = 300^\circ \end{cases}$$

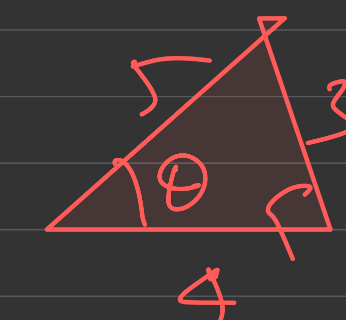
$\cos\theta = 1/2 = \cos 60^\circ \Rightarrow \boxed{\theta = 60^\circ}$

$\cos(360^\circ - \theta) = \frac{1}{2} = \cos 60^\circ \Rightarrow 360^\circ - \theta = 60^\circ$

$\Rightarrow \boxed{\theta = 300^\circ}$

Qu! 1. $\cos\theta = \sqrt{3}/2$ $\qquad \theta = 30^\circ, 330^\circ$

Qu: 2 $\cos\theta = 1/\sqrt{2}$ $\qquad \theta = 45^\circ, 315^\circ$



Qu: 3 $\cos\theta = 4/5$ $\qquad \theta = 37^\circ, 323^\circ$

Qu! 4 $\cos\theta = \dfrac{\sqrt{3}+1}{2\sqrt{2}}$ $\qquad \theta = 15^\circ, 345^\circ$

Qu! 5 $\cos\theta = \dfrac{\sqrt{3}-1}{2\sqrt{2}}$ $\qquad \theta = 75^\circ, 285$

$\sin 15^\circ = \cos 75^\circ$

H.W.

Ex:- 1 $\longrightarrow$ 1 to 11

Ex 1. 2

Ex - 3 $\longrightarrow$ 1 , 2

$\sin\theta = -\frac{1}{2}$

θ → 3rd Quadrant

→ 4th Quadrant

$\theta = -30^\circ, -150^\circ$

$360^\circ - 150^\circ$

$360^\circ - 30^\circ$

$= 330^\circ, 210^\circ$

$\sin\theta = \frac{1}{2}$

$\Downarrow$

$\theta = 30^\circ, 150^\circ$

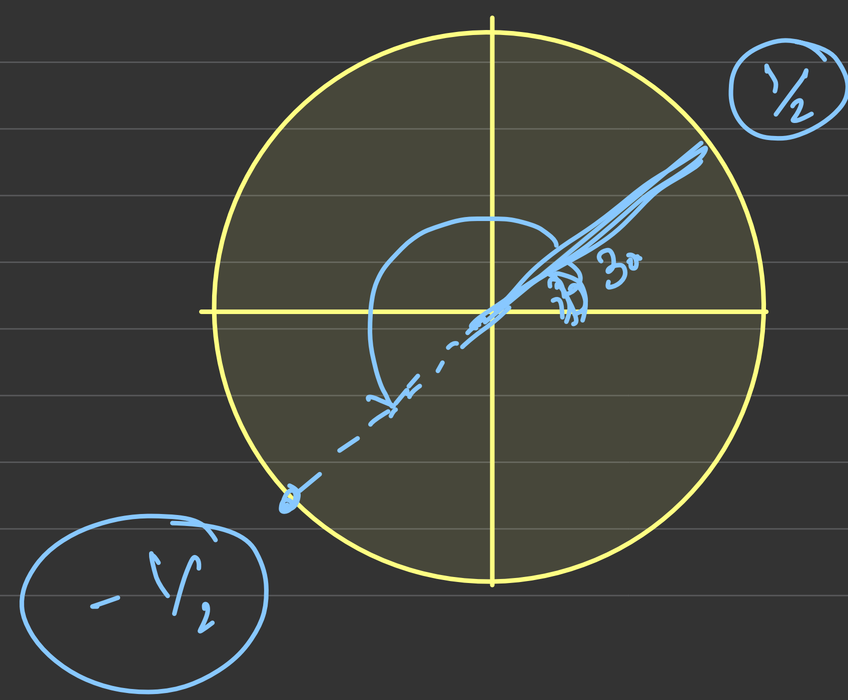
$\sin\theta = \frac{1}{2}$

$\theta = 30^\circ$

150°

$\sin\theta = -\frac{1}{2} \Rightarrow \theta = 30^\circ + 180^\circ$

$= 150 + 180^\circ$



① $\sin\theta = -\frac{1}{\sqrt{2}} \Rightarrow \theta = 360^\circ - 45^\circ = 315^\circ$

$= 360^\circ - 135^\circ = 225^\circ$

② $\sin\theta = -\sqrt{3}/2$

$\sin\theta = \frac{\sqrt{3}}{2}$

$\theta = 60^\circ, 120^\circ$

$\theta = 60 + 180^\circ, 120 + 180^\circ$

$= 240^\circ, 300^\circ$

③ $\sin\theta = -\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)$

$\boxed{\sin\theta = \frac{\sqrt{3}-1}{2\sqrt{2}} \quad \theta = 15^\circ, 165^\circ}$

$\theta = 15 + 180^\circ, 165 + 180^\circ$

$= 195^\circ, 345^\circ$

Qu. $\sin\theta = -3/5$

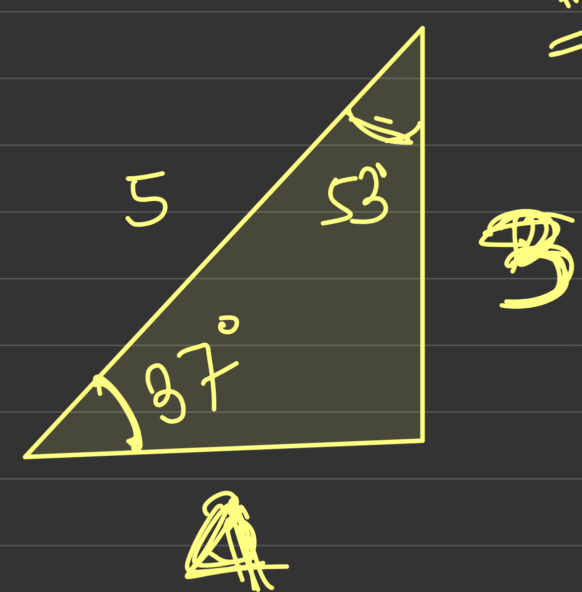
$\Downarrow$

$\theta = 180 + 37^\circ, \quad 180^\circ + (180^\circ - 37^\circ)$

$\sin\theta = 3/5$

$\Downarrow$

$\theta = 37^\circ, \; 180^\circ - 37^\circ$



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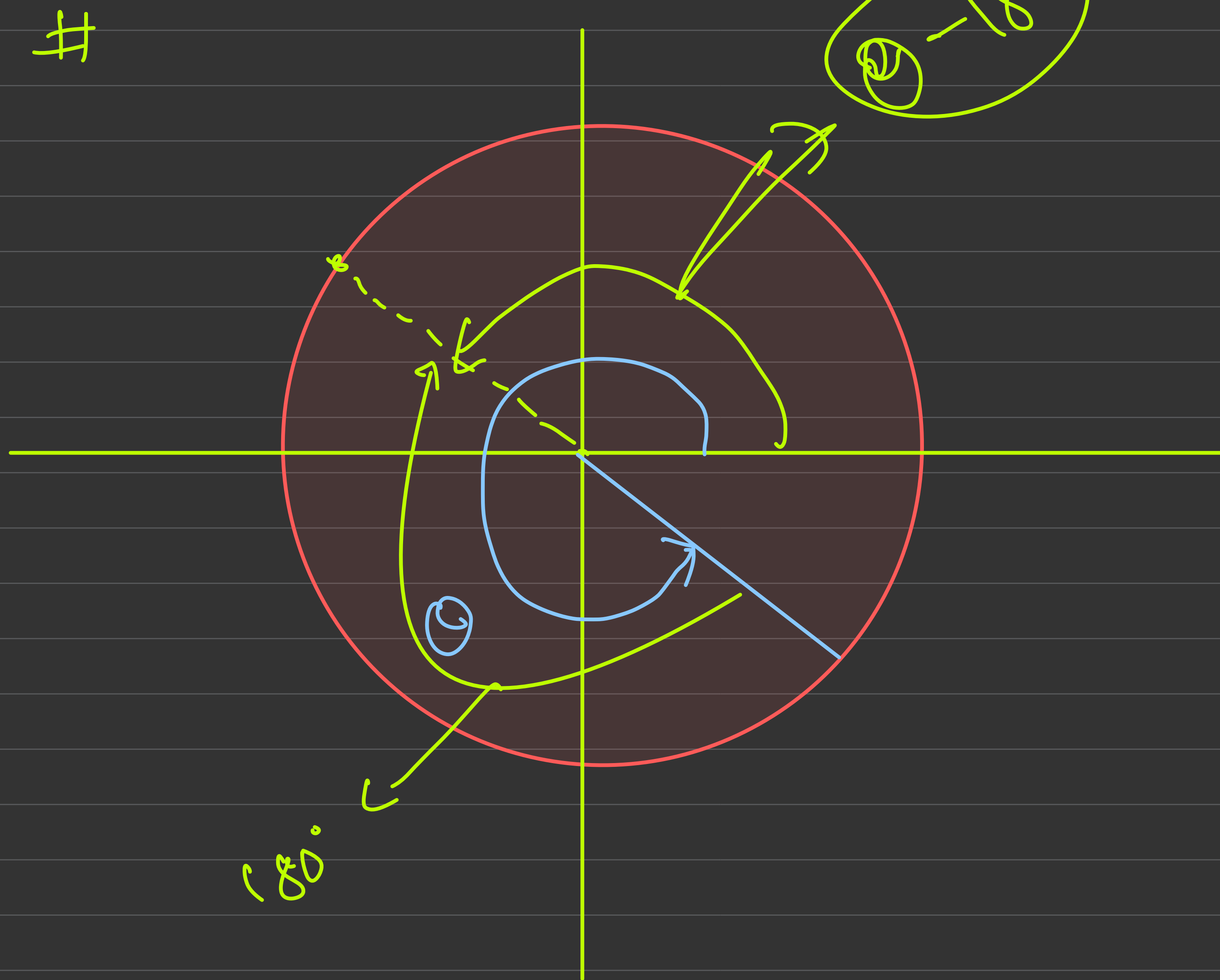
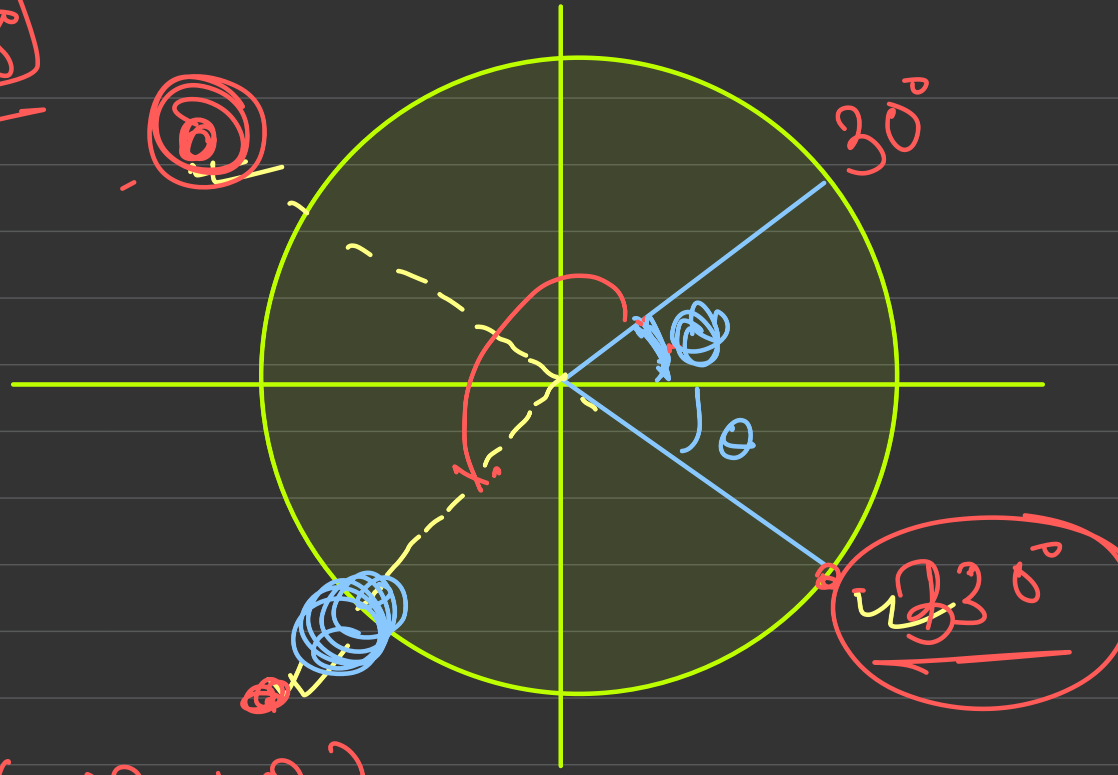


$\boxed{330° - 180°}$



$(180 + \theta)$

$= \underline{210°}$

① $\cos\theta = -\frac{1}{2}$

$\Downarrow$

$\theta = \underline{240^\circ}, \quad 360^\circ - 180^\circ = \underline{120^\circ}$

$\cos\theta = \frac{1}{2}$

$\Downarrow$

$\theta = 60^\circ, \quad \underline{300^\circ}$

② $\cos\theta = -\frac{\sqrt{3}}{2}$

$\Downarrow$

$\theta = 210^\circ, \quad 150^\circ$

$\cos\theta = \frac{\sqrt{3}}{2}$

$\theta = \underline{30^\circ, \quad 330^\circ}$

③ $\cos\theta = -\frac{3}{5}$

$\Downarrow$

$\theta = 180 + 53^\circ, \quad \left((360^\circ - 53) - 180^\circ \right)$

$\cos\theta = \frac{3}{5}$

$\Downarrow$

$\theta = 53^\circ, \quad (360^\circ - 53^\circ)$

④ $\cos\theta = -\frac{1}{\sqrt{2}}$

$\Downarrow$

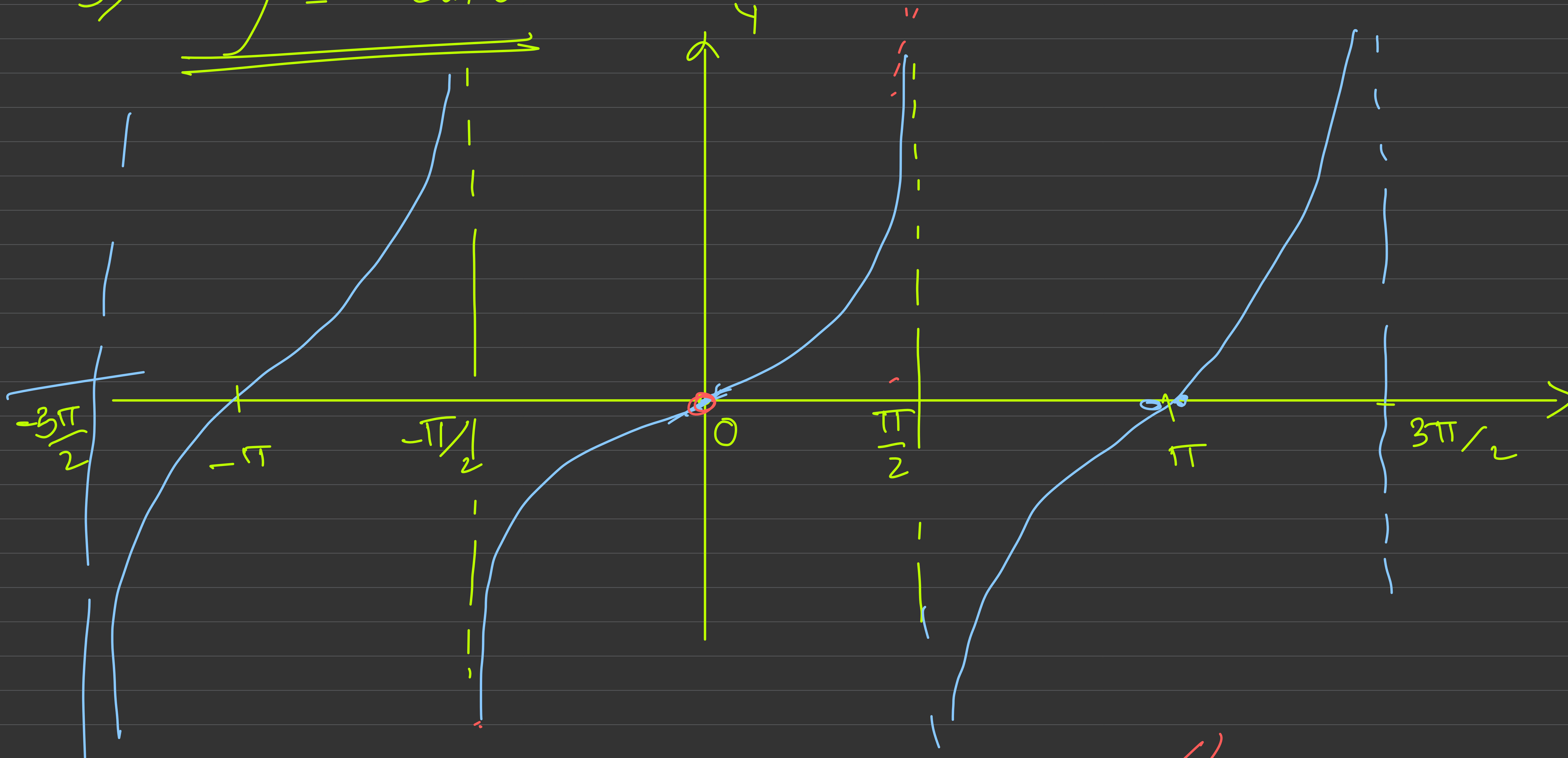
$\theta = 45^\circ + 180^\circ, \quad 315^\circ - 180^\circ$

$\cos\theta = \frac{1}{\sqrt{2}}$

$\Downarrow$

$\theta = 45^\circ, \quad 315^\circ$

3> $y = \tan\theta$



• $\tan\theta \Rightarrow$ Periodic function

↳ of period π

4)

$$y = a\sin\theta + b\cos\theta \rightarrow y_{max} = ?$$

$$y_{min} = ?$$

$\sin\theta = 1 \Rightarrow \theta = 90^\circ$

$\cos\theta = 1 \Rightarrow \theta = 0^\circ$

Put $\theta = 90^\circ$, $y = a \times 1 + b \times 0$

$= a$

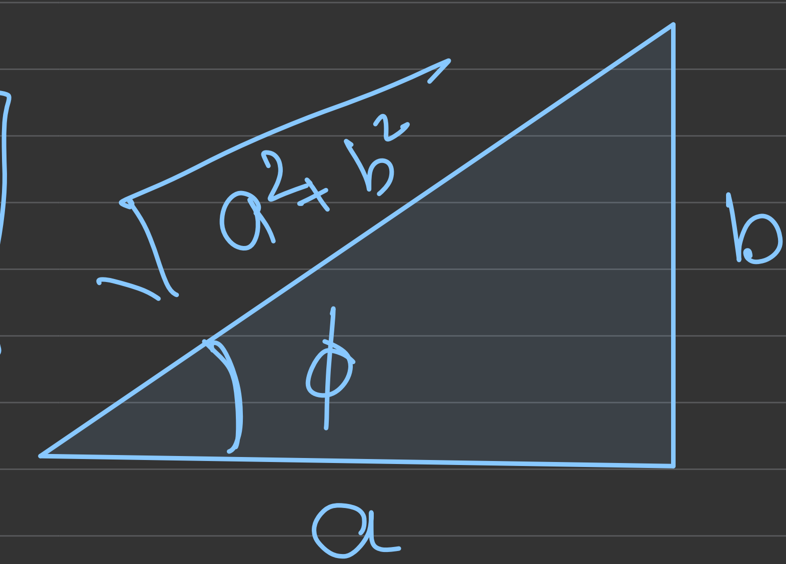
$\theta = 0^\circ$, $y = a \times 0 + b \times 1$

$= b$

$$y = \left[\frac{\overset{\swarrow \cos\phi}{a}}{\sqrt{a^2+b^2}} \sin\theta + \frac{\overset{\swarrow \sin\phi}{b}}{\sqrt{a^2+b^2}} \cos\theta \right] \sqrt{a^2+b^2}$$

**

$$y = \sqrt{a^2+b^2} \left[\sin\theta . \cos\phi + \cos\theta . \sin\phi \right]$$



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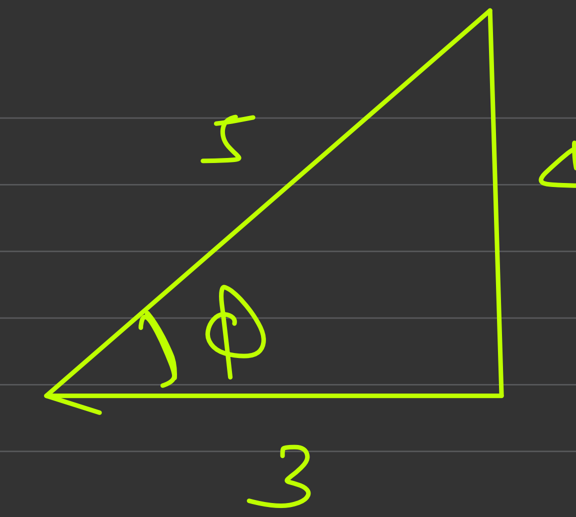
$$= \sqrt{a^2+b^2} \, \sin(\theta + \phi)$$

***

$$\left\{ \begin{aligned} y_{max} &= \sqrt{a^2+b^2} \\ y_{min} &= -\sqrt{a^2+b^2} \end{aligned} \right\}$$

$$\left\{ \begin{aligned} \cos\phi &= \frac{a}{\sqrt{a^2+b^2}} \\ \sin\phi &= \frac{b}{\sqrt{a^2+b^2}} \\ \tan\phi &= b/a \end{aligned} \right.$$

$$y = 3\sin\theta + 4\cos\theta$$



$$y_{max} = \sqrt{3^2+4^2} = 5$$

$$y_{min} = -\sqrt{3^2+4^2} = -5$$

$$\tan\phi = 4/3$$

$$\phi = 53^\circ$$

find A and ϕ.

$$y = A\sin(\theta+\phi)$$

$$= 5\sin(\theta + 53^\circ)$$

$$y = a\sin\theta + b\cos\theta = \sqrt{a^2+b^2}\,\sin(\theta+\phi)$$

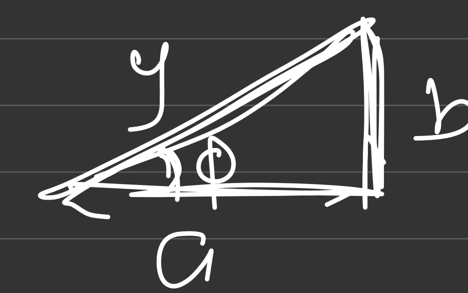
$y = a\sin\theta + b\cos\theta$

$$y_{max} = \sqrt{a^2+b^2}$$

$$y_{min} = -\sqrt{a^2+b^2}$$

$$\cos\theta = \sin(90^\circ+\theta)$$

$$y = \sqrt{a^2+b^2}\,\sin(\theta+\phi)$$



$$y = a\sin\theta + b\sin(\theta+90^\circ)$$

$$\sqrt{a^2+b^2}\,\sin(\theta+\phi)$$

$$\tan\phi = \left(\frac{b}{a}\right)$$

$$y = \sqrt{a^2+b^2}\,\sin(\theta+\phi)$$

Qu:-

$$y = \sqrt{3}\sin\theta + \cos\theta$$

phi

ψ φ

$$y_{max} = \sqrt{(\sqrt{3})^2 + (1)^2} = 2$$

$$y_{min} = -\sqrt{(\sqrt{3})^2 + (1)^2} = -2$$

$$y = A\sin(\theta + \phi)$$

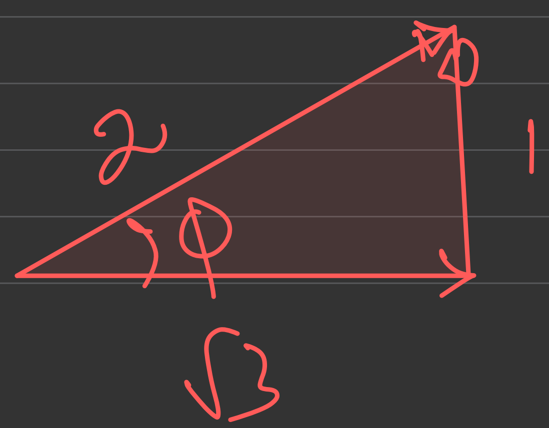
$$\tan\phi = \frac{b}{a}$$

$$= 2\sin(\theta + \phi)$$

$$= \frac{1}{\sqrt{3}}$$

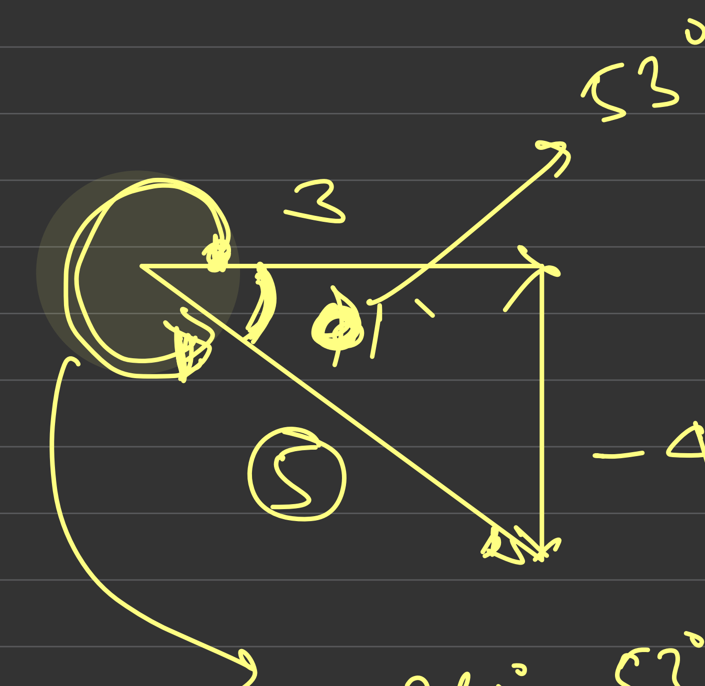
$$= 2\sin(\theta + 30^\circ)$$

$$\phi = 30^\circ$$



Qu: $y = 3\sin\theta - 4\cos\theta$

$= 3\sin\theta + (-4)\cos\theta$



$y_{max} = 5$

$y_{min} = -5$

$360° - 53°$

$= 307°$

$y = 5\sin(\theta + \phi)$

$= 5\sin(\theta + 307°)$

$5\sin(\theta - 53°)$

$$\sin\left(\frac{701\pi}{2}\right) = \sin\left(\frac{700\pi + \pi}{2}\right)$$

$$= \sin\left(350\pi + \pi/2\right)$$

Odd Number

**

$$\sin\{(2n+1)\pi + \theta\} = -\sin\theta$$

$\pi, 3\pi, 5\pi$

odd multiples of π

$$\sin\{2n\pi + \theta\} = \sin\theta$$

even numbers

even multiples of π

$0, 2\pi, 4\pi$

$\sin\{(2n+1)\pi + \theta\} = -\sin\theta$, $n \in I$

*** $\boxed{\sin(2n\pi + \theta) = \sin\theta , \quad n \in I}$

$\boxed{\pi, 3\pi, 5\pi}$

odd multiples of π

$(2n+1)\pi$

$(2n\pi)$

$0, 2\pi, 4\pi, \ldots$

even multiple of π

*** $\sin(2n\pi + \theta) = \sin\theta$

$\cos(2n\pi + \theta) = \cos\theta$

$\tan(2n\pi + \theta) = \tan\theta$

2, 4, 6, ... ⇒ even

1, 3, 5, 7 ... ⇒ odd

Qu!

$$\sin\left(\frac{701\pi}{2}\right) = \sin\left(\underline{350\pi} + \frac{\pi}{2}\right)$$

$$= \sin\frac{\pi}{2} = 1$$

Qu:

$$\sin\left(\frac{699\pi}{2}\right) = \sin\left(\frac{700\pi - \pi}{2}\right)$$

$$= \sin\left(\underbrace{350\pi}_{2n\pi} - \frac{\pi}{2}\right) = -\sin\frac{\pi}{2}$$

$$= -1$$

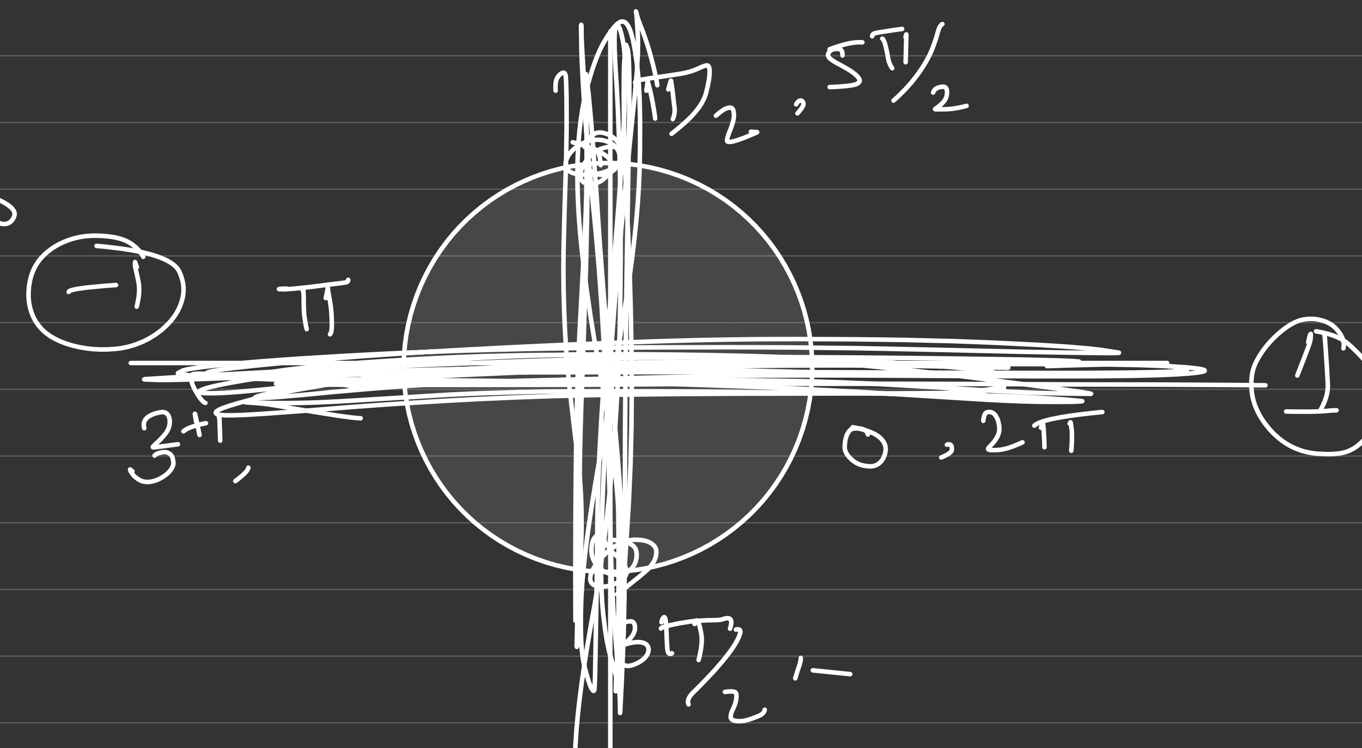
Qu:. $\cos\left(\frac{1391\pi}{2}\right) = \cos\left(\frac{1390+1\pi}{2}\right) = \cos\left(695\pi + \pi/2\right)$

$= -\cos \pi/2 = 0$

$\parallel$

0

$\cos\left(\text{odd multiples of } \pi/2\right) = 0$



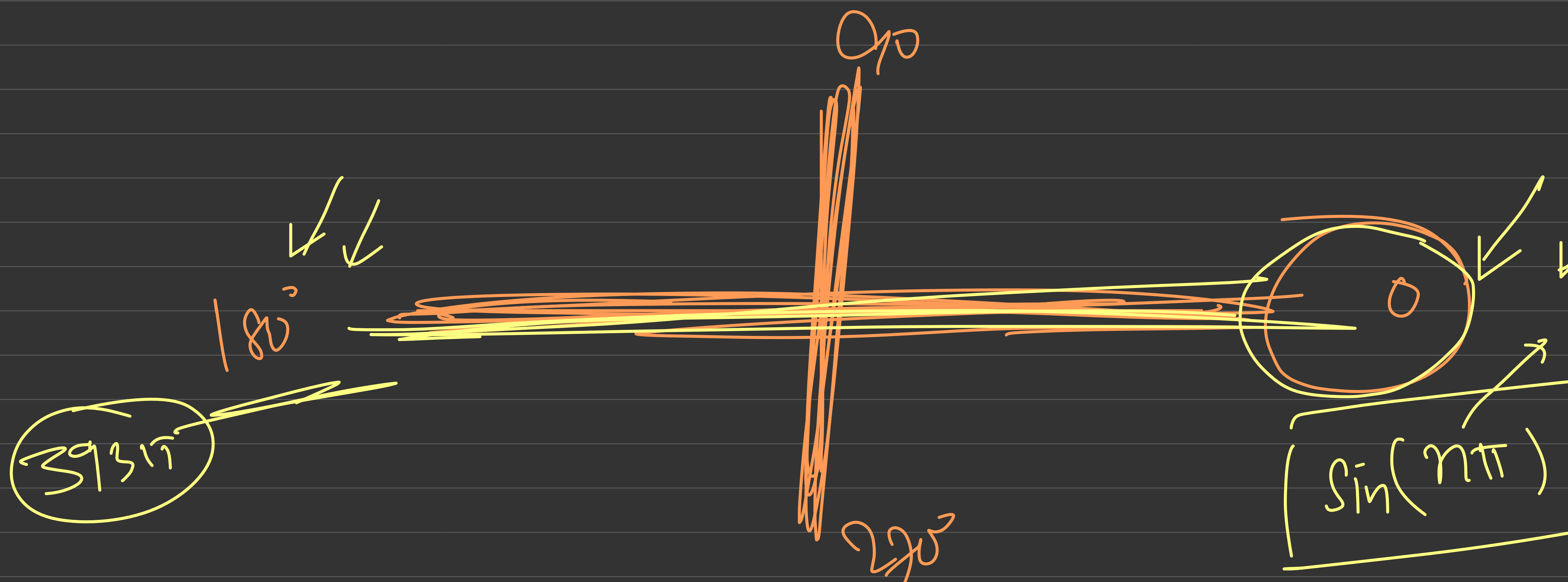
Qu:. $\cos(693\pi) = -1$

odd multiples of π → -1 ———— $+1$ ← even multiples of π

Qu:- $\cos(392\pi) = 1$

Qu:- $\sin(393\pi)$

(A) 1 (B) -1 (C) 0 (D) None of these



$\boxed{\sin(n\pi) = 0}$

Qu!.

$\sin\left(\frac{393\pi}{2}\right) = \sin\left(\frac{392\pi + \pi}{2}\right) = \sin\left(196\pi + \pi/2\right)$

(a) 0 (b) 1 (c) -1 (d) None of these

$$\boxed{\sin \pi/2 = 1}$$

$$* \left\{ \begin{aligned} \sin(2n\pi + \theta) &= \sin\theta \\ \sin(2n\pi - \theta) &= -\sin\theta \end{aligned} \right\}$$

25-05-2021

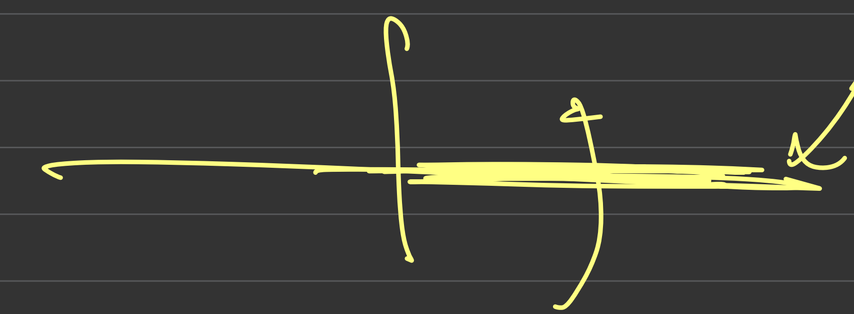
$\sin\left(701 \times \frac{\pi}{2}\right)$

odd no.

even no: -

$\sin(630\ 90)$

$\sin(\underline{63000} + 90^\circ)$



$\sin\left(\frac{(700+1)\pi}{2}\right) =$

$= \sin(\underline{90^\circ} \times \boxed{700} + 90^\circ)$

$= \sin\left(350 \times \pi + \frac{\pi}{2}\right)$

$= \sin(\underline{350} \times \underline{180} + 90^\circ)$

$= \sin 90^\circ = 1$

$\sin\theta = 1$ $\Rightarrow$ $\theta = \sin^{-1}(1/2)$

$= \text{Inverse of sine } (1/2)$

* $(\sin\theta)^{-1} = \dfrac{1}{\sin\theta}$

* $\sin^{-1}(x) =$ Represents angle

$\theta = \sin^{-1}(1) \Rightarrow \boxed{\sin\theta = 1}$

$\theta = \sin^{-1}(-\sqrt{3}/2) \Rightarrow \sin\theta = -\sqrt{3}/2$

$(a+b)^2 = (a^2 + 2ab + b^2)$

$(a-b)^2 = (a^2 - 2ab + b^2)$

$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

$= a^3 + b^3 + 3a^2b + 3ab^2$

$(a-b)^3 = a^3 - b^3 - 3ab(a-b)$

$= a^3 - b^3 - 3a^2b + 3ab^2$

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$(a+b+c)^2 = a^2 + b^2 + c^2$

$+ 2ab + 2bc + 2ca$

$a^2 - b^2$

$= (a-b)(a+b)$

Quadratic Equation

$$ax^2 + bx + c = 0 \quad , \quad a \neq 0$$

$\Downarrow$

$$\alpha, \beta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$D = b^2 - 4c$

$\hookrightarrow$ discriminant

$$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad , \quad \beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

$$\alpha + \beta = \frac{-b}{a} = -\frac{(\text{Coefficient of } x)}{(\text{Coefficient of } x^2)}$$

$$\alpha\beta = c/a = \frac{\text{Const term}}{\text{Coeff of } x^2}$$

Qu:

$$x^2 - 5x + 6 = 0$$

$$\Rightarrow x^2 - 3x - 2x + 6 = 0$$

$$\Rightarrow x(x-3) - 2(x-3) = 0$$

$$(x-3)(x-2) = 0 \rightarrow \left.\begin{array}{l} x = 3 \\ x = 2 \end{array}\right\}$$

$$\#\quad D > 0 \longrightarrow 2 \text{ distinct Roots}$$

$$D = 0 \longrightarrow \text{equal roots}$$

$$D < 0 \longrightarrow \text{No real roots}$$

Logarithm

$$a^x = N \qquad (a = \text{base}) \; , \; (x = \text{Power})$$

$$\Rightarrow \boxed{x = \log_a N} \quad (\text{argument})$$

$$\left.\begin{matrix} a > 0 \\ N > 0 \end{matrix}\right\} \quad (a \neq 1)$$

Common Base

→ $10 \Rightarrow \log N$ ← $\log_{10} N$

→ $e \Rightarrow \ln N$ (Natural logarithim)

$= \log_e N$

$$a^x = N \quad \Rightarrow \quad \boxed{x = \log_a N}$$

ex:-

$$4^x = 5 \quad \Rightarrow \quad x = \log_4 5 \quad (\log 5 \text{ base } 4)$$

\# $$\log x = 2 \quad \Rightarrow \quad x = (10)^2 = 100$$

\# $$\log x^2 = 4^2 \quad \Rightarrow \quad x =$$

$$\hookrightarrow x^2 = (10)^{16} \quad \Rightarrow \boxed{x = \pm 10^8}$$

$$x^2 = 10^{16}$$

$$x = \pm(10^{16})^{1/2}$$

$$= \pm 10^8$$

ex:.

$$\log_{x-5} 4 = 2 \Rightarrow (x-5)^2 = 4 = (2)^2$$

$$\Rightarrow x-5 = 2 \Rightarrow \boxed{x=7}$$

#

$$\log_{10} x^2 = 4^2 = 16$$

$$\Rightarrow 10^{16} = x^2 \Rightarrow x = \pm (10^{16})^{1/2} = \pm 10^8$$

#

$$\log_2 4\sqrt{32} = \log_2 \left\{(2)^2 \cdot (2^5)^{1/2}\right\}$$

$$(2)^2 \cdot (2^5)^{1/2} = \log_2 (2)^{(2+5/2)} = \left(2+5/2\right) \log_2 2$$

$$= 9/2$$

$$a^m \cdot a^n = a^{m+n}$$

$$(2)^2 \cdot (2)^{5/2} = (2)^{2+5/2} = \underline{(2)^{9/2}}$$

$$\log_2 (2)^{9/2} = \frac{9}{2} \log_2 2 = \frac{9}{2} \quad \text{Ans}$$

<u>Ex:</u>

$$\log_3 27\sqrt[3]{81} = \log_3 \left\{(3)^3 \; 3^{4/3}\right\}$$

$$(3^4)^{1/3} = \log_3 (3)^{3+4/3}$$

$$= \frac{13}{3} \log_3 3 = \frac{13}{3} \quad \text{Ans}$$

Qu!.

$$\log_{25}\left(125\sqrt[4]{25}\right) = \log_{25} (5)^3\cdot(5^2)^{1/4}$$

$$= \log_{(5)^2}\left(5^{3+2/4}\right)$$

$$= \frac{14}{4}\times\frac{1}{2}\cdot\log_5 5$$

$$= \frac{7}{4}$$

H.W

los.

h.p

Identities of logarithm

1) $\log_a 1 = 0 \Rightarrow \boxed{a^0 = 1}$ $\quad \log 1 = 0$, $\ln 1 = 0$, $\log_{20} 1 = 0$

2) $\log_a a = 1$ $\quad \left(\dfrac{\log a}{\log a} = 1 \right)$

3) $\log_{(1/a)} a = -1$ $\quad \left(\log_{a^{-1}} a = \dfrac{1}{(-1)} \log_a a = -1 \right)$

** 4) $\log_a (mn) = \log_a m + \log_a n$

** 5) $\log_a \left(\frac{m}{n}\right) = \log_a m - \log_a n$

$\log_a 1 =$ $\quad a^0 = 1$

$0 = \log_a 1$

$\log_a x^2$

$= 2 \log_a x$

$a^m \cdot a^n = a^{m+n}$

$\log_a (mn) = \log_a (m) + \log_a (n)$

6 $\log_a m^\alpha = \alpha \log_a m$

7. $\log_{(a^\beta)} m = \frac{1}{\beta} \log_a m$

8.

$$\log_a b = \frac{\log_n b}{\log_n a} = \frac{\log_e b}{\log_e a}$$

- $\log_6 1 = 0 \Rightarrow 6^0 = 1$

$$\log_a b = \frac{\log b}{\log a}$$

- $\log 1000 = \log_{10} (10)^3 = 3 \log_{10} 10 = 3$

- $\ln e^{100} = 100 \ln e = 100 \times 1 = 100$ $\qquad \log_e e = 1$

- $\log 63 = \log(9 \times 7) = \log 9 + \log 7$

  $= \log (3)^2 + \log 7$

  $= 2 \log 3 + \log 7$

- $\log \left(\frac{1}{1000} \right) = \log (10)^{-3}$

  $= -3$

Qu!

$$\log_{32} 64 = \log_{(2)^5} (2)^6 = 6 \log_{(2)^5} (2)$$

$$= \frac{\log 64}{\log 32} = \frac{6}{5} = \frac{6}{5} \log_2 2 \quad = 1$$

$$= 6/5 = 1.2$$

Qu!.

$$\log_9 81 = \log_9 (9)^2 = 2 \log_9 9 = 2$$

Qu!.

$\ln 2 = a$, $\ln 3 = b$

$$\ln(18) = \ln(2 \times 9) = \ln 2 + \ln 9 = \ln 2 + 2 \ln 3$$

$$= \underline{\underline{a + 2b}}$$

Q4 :- If $\ln 2 = a$, $\ln 3 = b$, $\ln 5 = c$, $\ln 7 = d$

Then calculate (in terms of a, b, c, d)

(a) $\ln 54 = \ln(2 \times 3^3) = \ln 2 + 3\ln 3$

$= a + 3b$

(b) $\ln 63 = \ln(3^2 \times 7) = 2\ln 3 + \ln 7 = 2b + d$

(c) $\ln 15 = \ln(5 \times 3) = \ln 5 + \ln 3 = b + c$

(d) $\ln\left(\dfrac{15}{63}\right) = \ln 15 - \ln 63$

$= (b + c) - (2b + d)$

$= -b + c - d$

Airthmatic Progression (A.P.)

$a, \quad \underbrace{a+d}_{d}, \quad \underbrace{a+2d}_{d}, \quad \underbrace{a+3d}_{d}, \; ---$

- d = Common diffrence

- $T_{nth} = a + (n-1)d$

- $\boxed{S_{nth} = \frac{n}{2}(a + T_n)}$

$= \frac{n}{2}[2a + (n-1)d]$

$\frac{n}{2}$ (1st term + last term)

$T_1 = a$

$T_2 = a + d$

$T_3 = a + 2d$

$T_4 = a + 3d$

Qu: $S_{50} = 1+2+3+ \cdots + 50$

$S_{50} = 25(1+50)$

$= 25 \times 51 = 1275$

Qu: $S = 7+8+9+ \cdots +30$

$30 = 7 + (n-1) \times 1 \Rightarrow n = 24$

$S_n = 12 \times (7+30)$

$= 12 \times 37$

$= 444$

Qu: $S = 1+3+5+7+ \cdots$ up to 10 terms

$T_n = 1 + 9 \times 2 = 19$

$\frac{10}{2}(1+19) = 100$

Qu. $a = 1$, $d = -2$, S_{10}

$n = 10$

$T_n = 1 + 9 \times (-2)$

$= -17$

$S_n = \frac{10}{2}(1 - 17) = \frac{10}{2} \times (-16) = -80$

###

$1+2+3+4+\cdots$ n term $= \dfrac{n(n+1)}{2}$

$(1)^2 + (2)^2 + (3)^2 + \cdots$ n terms $= \dfrac{n(n+1)(2n+1)}{6}$

Not AP / GP

$(1)^3 + (2)^3 + (3)^3 + \cdots + (n)^3 = \left[\dfrac{n(n+1)}{2}\right]^2$

Geometric Progression

$a, \quad ar, \quad ar^2, \quad ar^3, \quad ar^4, \quad \ldots$

$\frac{ar}{a} = r, \quad \frac{ar^2}{ar} = r$

$T_1 = a$

$T_2 = ar$

$T_3 = ar^2$

$T_4 = ar^3$

$T_n = ar^{n-1}$

- $\frac{T_n}{T_{n-1}} = r =$ Common Ratio $= \frac{T_{n+1}}{T_n}$

- $T_n = ar^{n-1}$ (nth term)

- $S_n = \frac{a(r^n - 1)}{(r-1)} = \frac{a(1-r^n)}{(1-r)}$

(Sum of n terms)

If $|r| < 1$

then S_∞ (Sum of Infinite no. of G.P.)

$$\boxed{S_\infty = \frac{a}{1-r}}$$ only if $|r| < 1$

$S_\infty = \dfrac{a(1-\cancel{r^n})}{(1-r)}$

$(1/2)^2 = 1/4$

$(1/2)^3 = 1/8$

$|r|^\infty \longrightarrow 0$

$(1/2)^\infty \longrightarrow 0$

Qu: $S_\infty = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$ up to ∞

$r = \frac{1}{2} < 1$

$$S_\infty = \frac{a}{1-r} = \frac{1}{1-\left(\frac{1}{2}\right)} = 2$$

Qu: $S = 1 + 2 + 4 + 8 + \cdots + 256$ $\rightarrow 2^8$

$$S = \frac{a(r^n - 1)}{(r-1)} = \frac{1\left((2)^9 - 1\right)}{(2-1)} = 256 \times 2 - 1 = 511$$

Qu: $S = 1 - \frac{1}{r} + \frac{1}{r^2} - \frac{1}{r^3} + \frac{1}{r^4} \cdots\cdots \infty$

where $r = 3$ $\quad$ C.R. $= -\frac{1}{r} = -\frac{1}{3}$

$$S_\infty = \frac{1}{1 + \frac{1}{3}} = \frac{3}{4} = 0.75.$$

Binomial Approximation

- $(a+b)^1 = a + b$
- $(a+b)^2 = a^2 + 2ab + b^2$
- $(a+b)^{3} = a^3 + 3a^2b + 3ab^2 + b^3$

$\longrightarrow$

Power of $a \downarrow$

Power of $b \uparrow$

- $(a+b)^4 =$

*

$$\boxed{S_n = \frac{a(r^n-1)}{r-1}}$$

$$\boxed{S_\infty = \frac{a}{r-1}, \quad |r| < 1}$$

			1		1			
		1		2		1		
	1		3		3		1	
1		4		6		4		1

Pascal's Triangle

1 1 $\longrightarrow (a+b)^1$

1 2 1 $\longrightarrow (a+b)^2$

1 3 3 1 $\longrightarrow (a+b)^3$

1 4 6 4 1 $\longrightarrow (a+b)^4$

1 5 10 10 5 1 $\longrightarrow (a+b)^5$

$$(a+b)^4 = 1a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + 1b^4$$

$$(a+b)^5 = 1a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + 1b^5$$

$$(1+x)^5 = 1 + 5x + 10x^2 + 10x^3 + 5x^4 + x^5$$

Approximation $(1+x)^n$ if $|x| << 1$ then

$$\boxed{(1+x)^n \approx 1 + nx}$$

$$\boxed{(1-x)^n \approx 1 - nx}$$

$$x = 0.01 \qquad x^2 = (0.01)^2 = 0.0001$$

$$\frac{1}{(1+x)}$$ if $x << 1$

$$(1+x)^{-1} = 1 - x$$

Qu:. $(0.98)^{1/2} = \{1 - 0.02\}^{1/2}$ $\boxed{0.02 \ll 1}$

$$= 1 - \cancel{\frac{1}{2}} \times \overset{0.01}{\cancel{0.02}}$$

$$\approx \underline{\underline{0.99}}$$

Qu:. $(1.05)^5 = (1 + 0.05)^5 = 1 + 5 \times 0.05$

$$= \underline{\underline{1.25}}$$

Qu:. $\dfrac{1}{(0.99)^3} = \dfrac{1}{(1 - 0.01)^3} = (1 - 0.01)^{-3}$

$$= 1 - (-3) \times 0.01$$

$$= \underline{\underline{1.03}}$$

Qu:. If the value of acceleration due to gravity with height is shown below

$$g(h) = \frac{g(\text{surface})}{\left(1 + h/R\right)^2}$$ where h = height

from earth surface and R = Radius of earth = 6400 km.

Find the value of 'g' at $h = 30$ km from earth surface.

(g surface $= 10\ m/s^2$)

Solve.

$$g(h) = \frac{g_{\text{surface}}}{\left(1 + h/R\right)^2} = g_{\text{sur}} \left(1 + \frac{h}{R}\right)^{-2}$$

Since $\underline{\underline{h << R}}$, we can apply Binomial approximation

$$g(h) = g_{surface}\left(1 - \frac{2h}{R}\right)$$

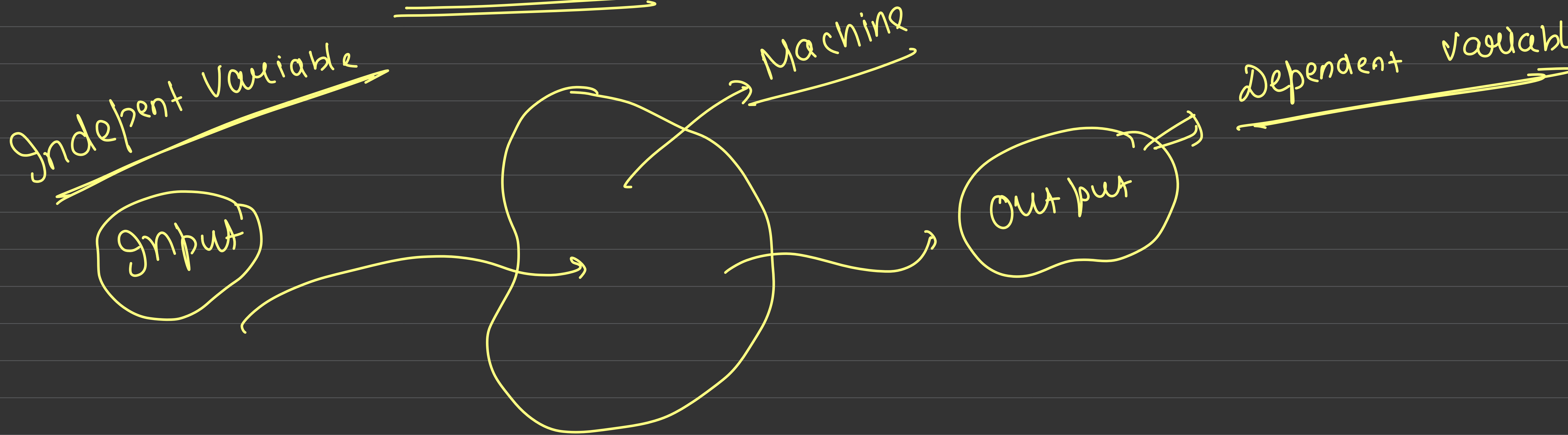
$$= 10\left(1 - \frac{30 \times 2}{6400}\right)$$

$$= 10\left(1 - \frac{6\cancel{0}}{640\cancel{0}}\right)$$

$$= \cancel{10}\left(\frac{640 - 6}{6\cancel{40}}\right) = \frac{63\cancel{4}}{64}$$

$$\approx 9.9$$

Function



dependent variable

Independent variable

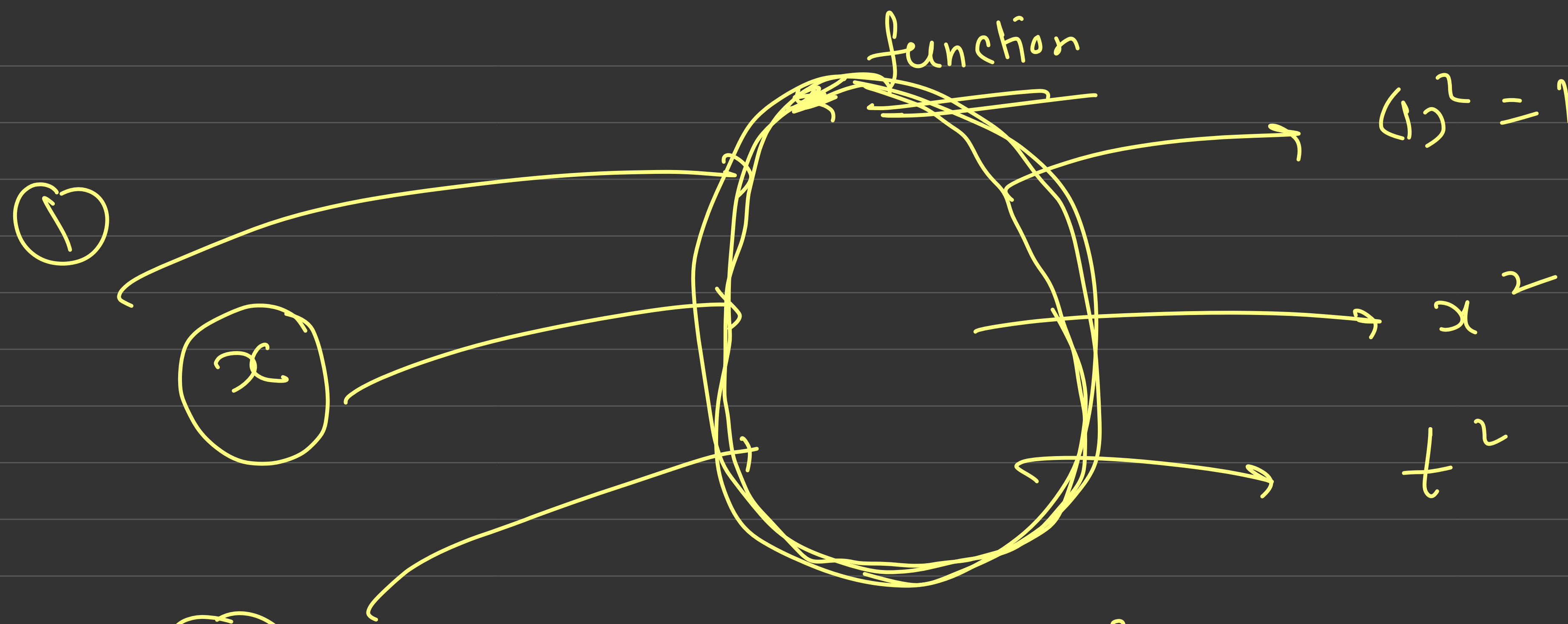
$$y = \frac{x^2}{4}$$

→ Dependent variable

→ Independent variable

1) 2

y	x
1	1
4	2



$$f = (x)^2$$

$$y = f(x) = x^2$$

This is a function which depends on 'x'

$$f(t) = t^2 \qquad f(\sin x) = \sin^2 x = (\sin x)^2$$

Qu:

$f(x) = \tan(x)$, $g(x) = x^2$

$f(\pi/4) = \tan(\pi/4)$

$= 1$

$g(\pi/4) = \left(\frac{\pi}{4}\right)^2$

$= \frac{\pi^2}{16} = \frac{10}{16}$

$f(g(x)) = \tan\{g(x)\}$

$= \tan(x^2)$

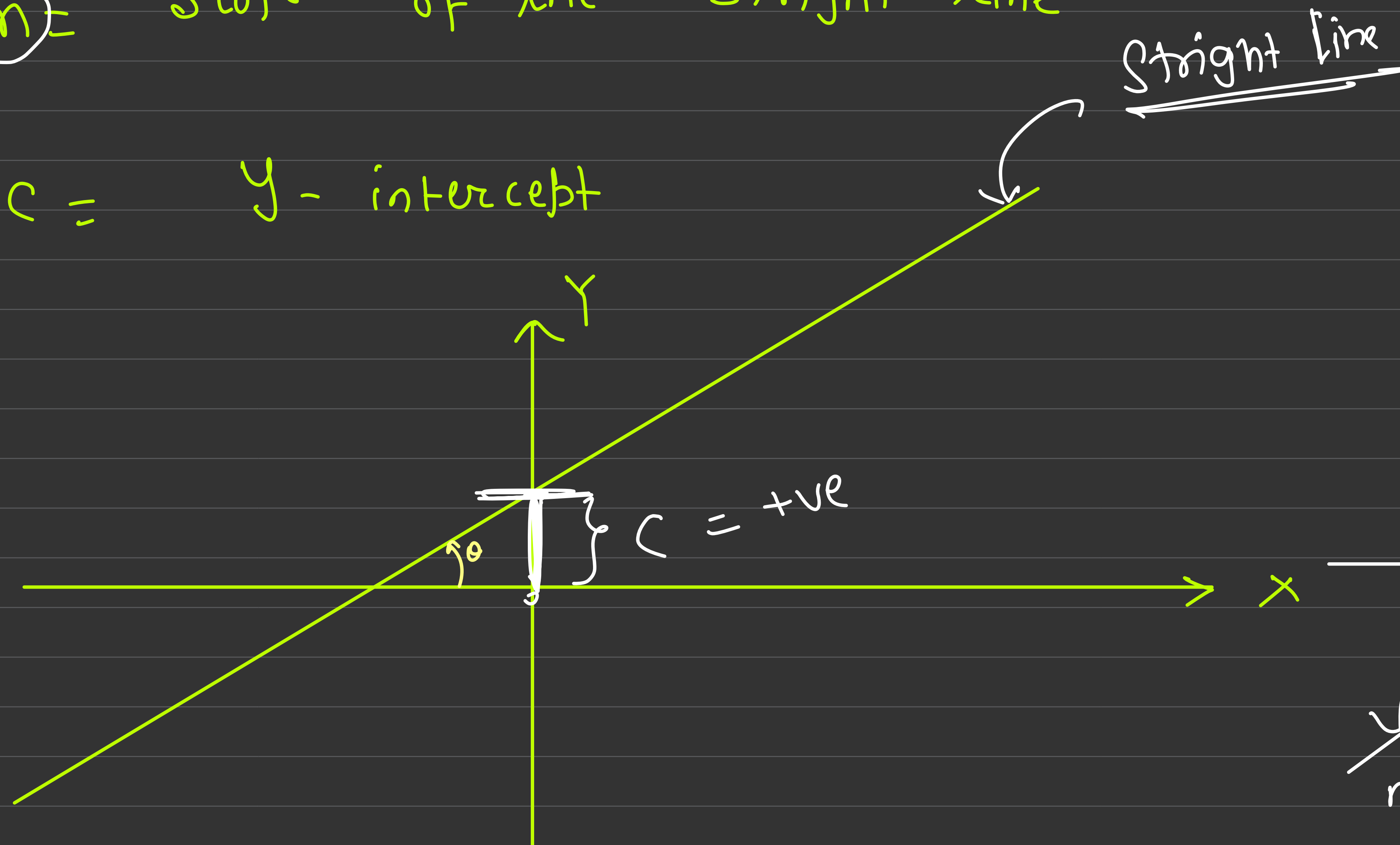
$g\{f(x)\} = [f(x)]^2 = [\tan x]^2 = \tan^2 x$

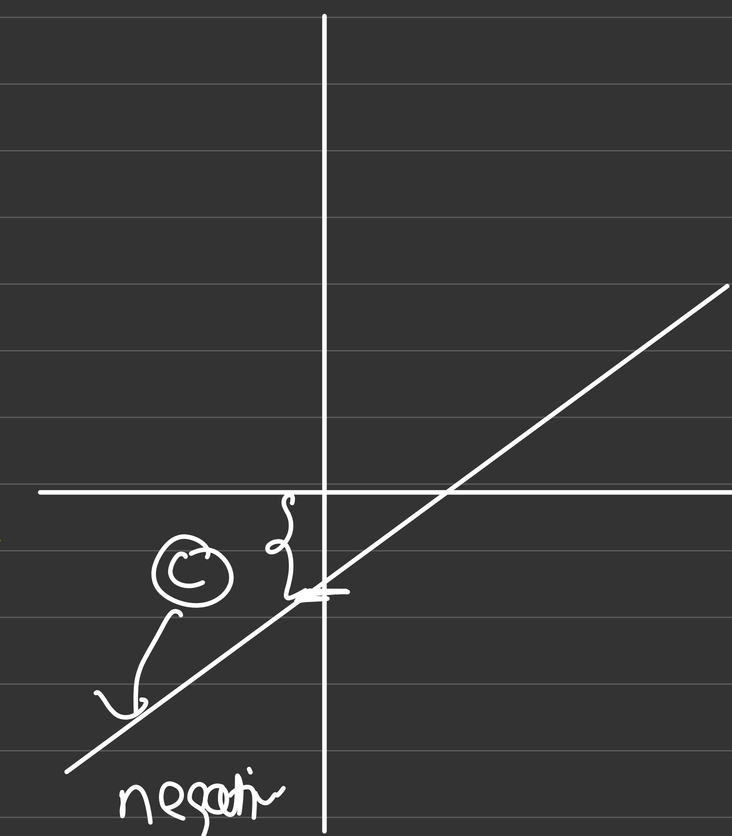
Stright - line

$\boxed{y = mx + c}$ ⟹ equation of stright line.

m = Slope of the stright line

c = y - intercept

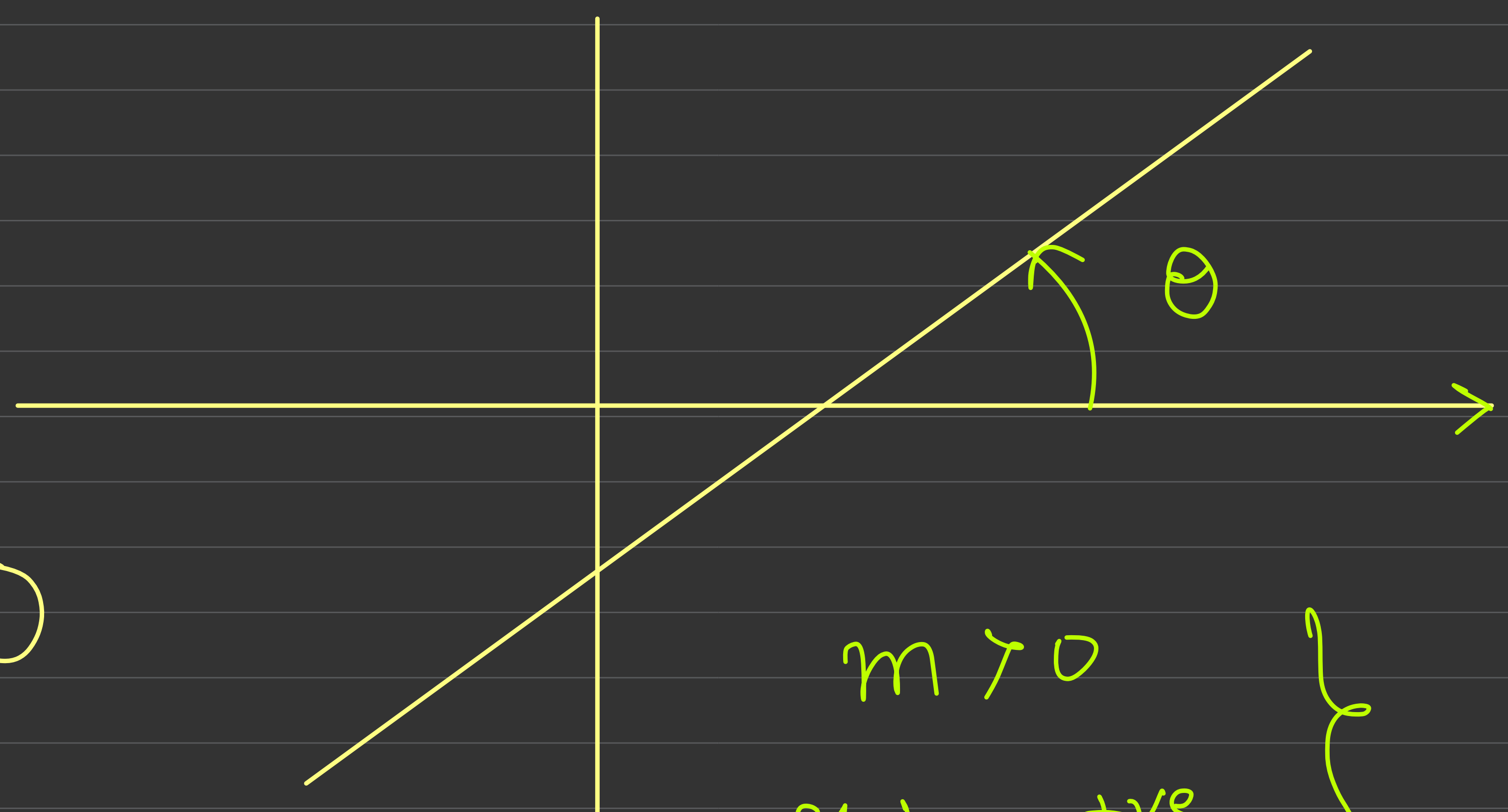




Slope = m = $\tan\theta$; where 'θ' ⟹ angle measured from +ve x - axis

$\tan\theta \Rightarrow +ve \;,\; 0 \le \theta < 90^\circ \Rightarrow$ Slope $= +ve$

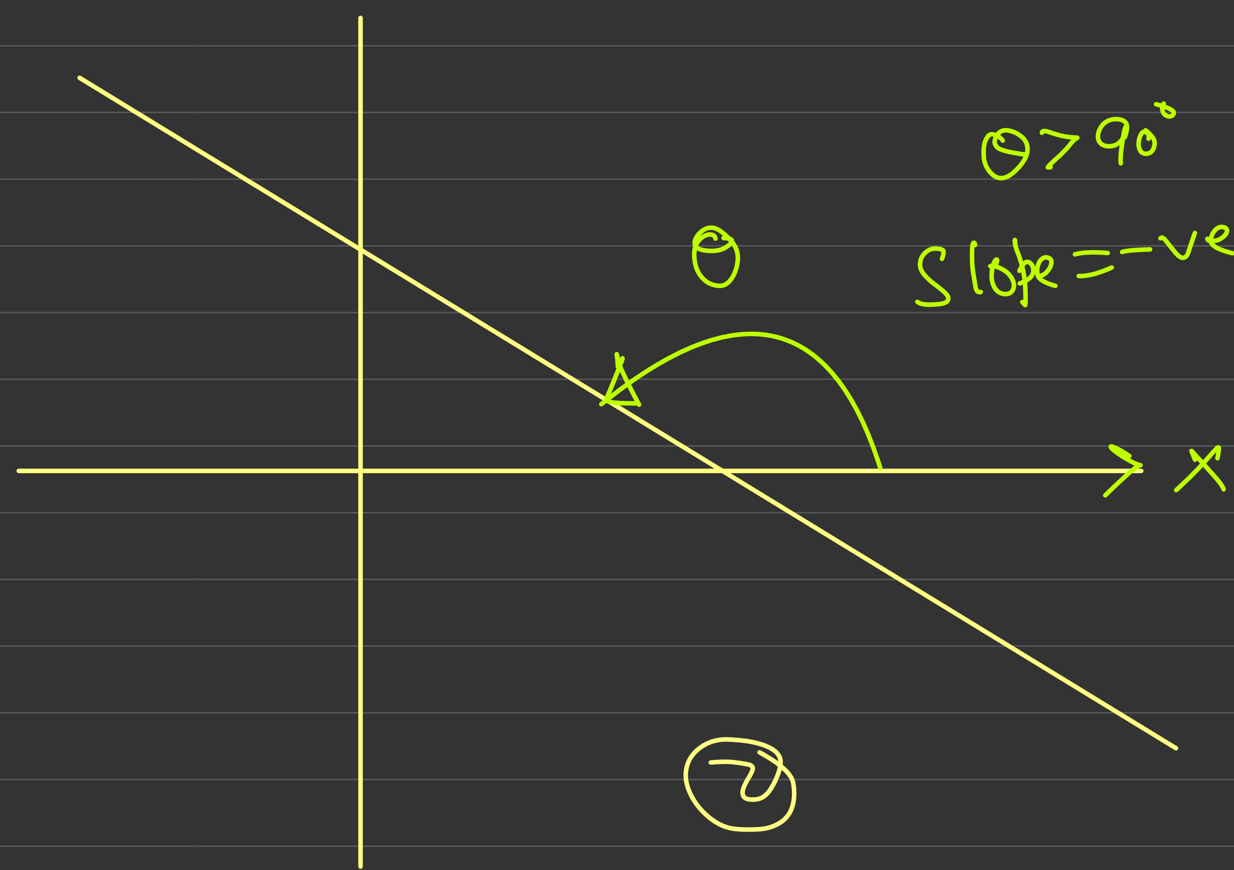
$\tan\theta \Rightarrow -ve \;,\; 90^\circ < \theta < 180^\circ \Rightarrow$ Slope $= -ve$



①

$m > 0$

Slope $= +ve$



$\theta > 90^\circ$

Slope $= -ve$

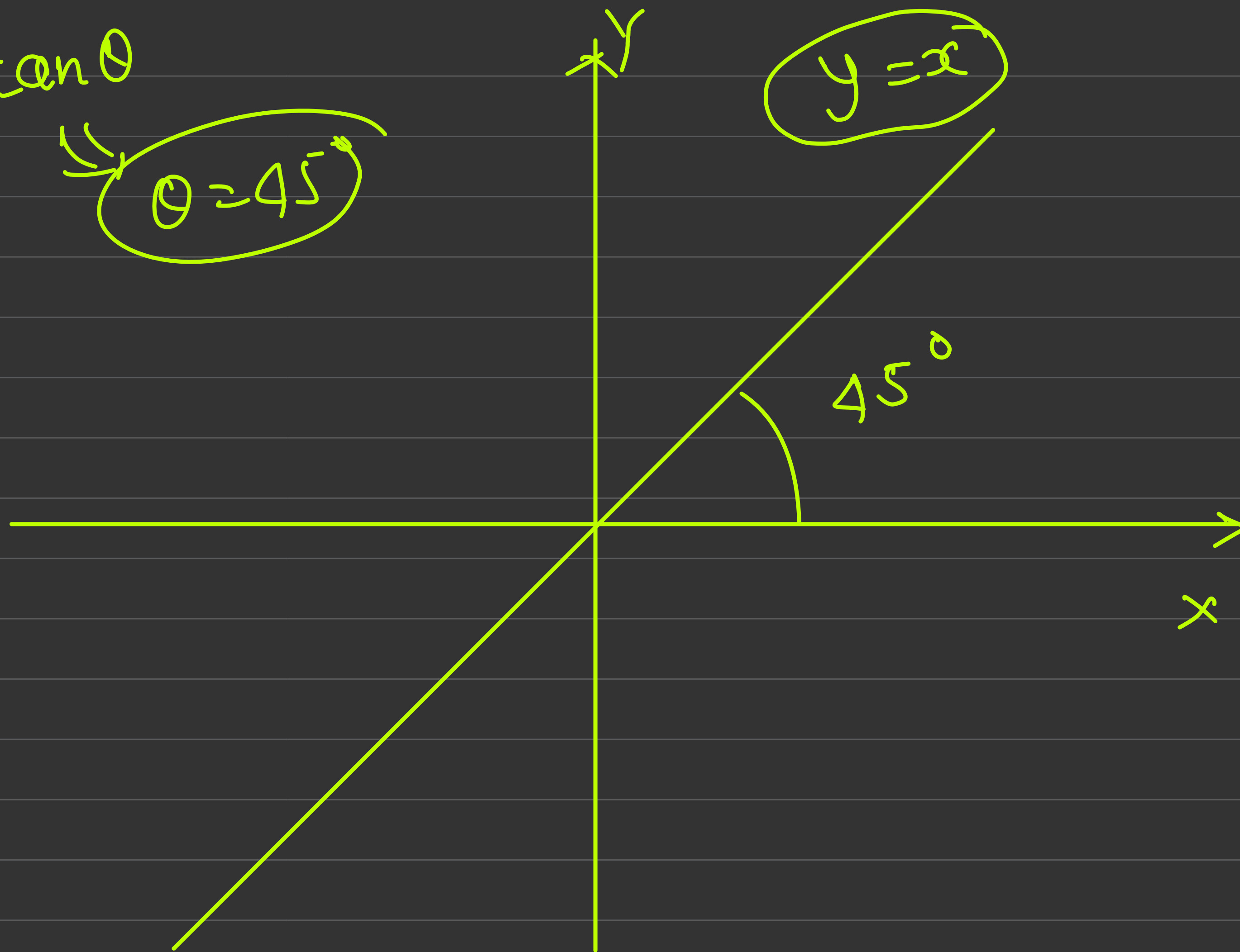
②

$y = x$

Slope $= 1 = \tan\theta$

$\theta = 45^\circ$

y - Intercept $= 0$



$y = -x$

Slope $= -1 = \tan\theta$

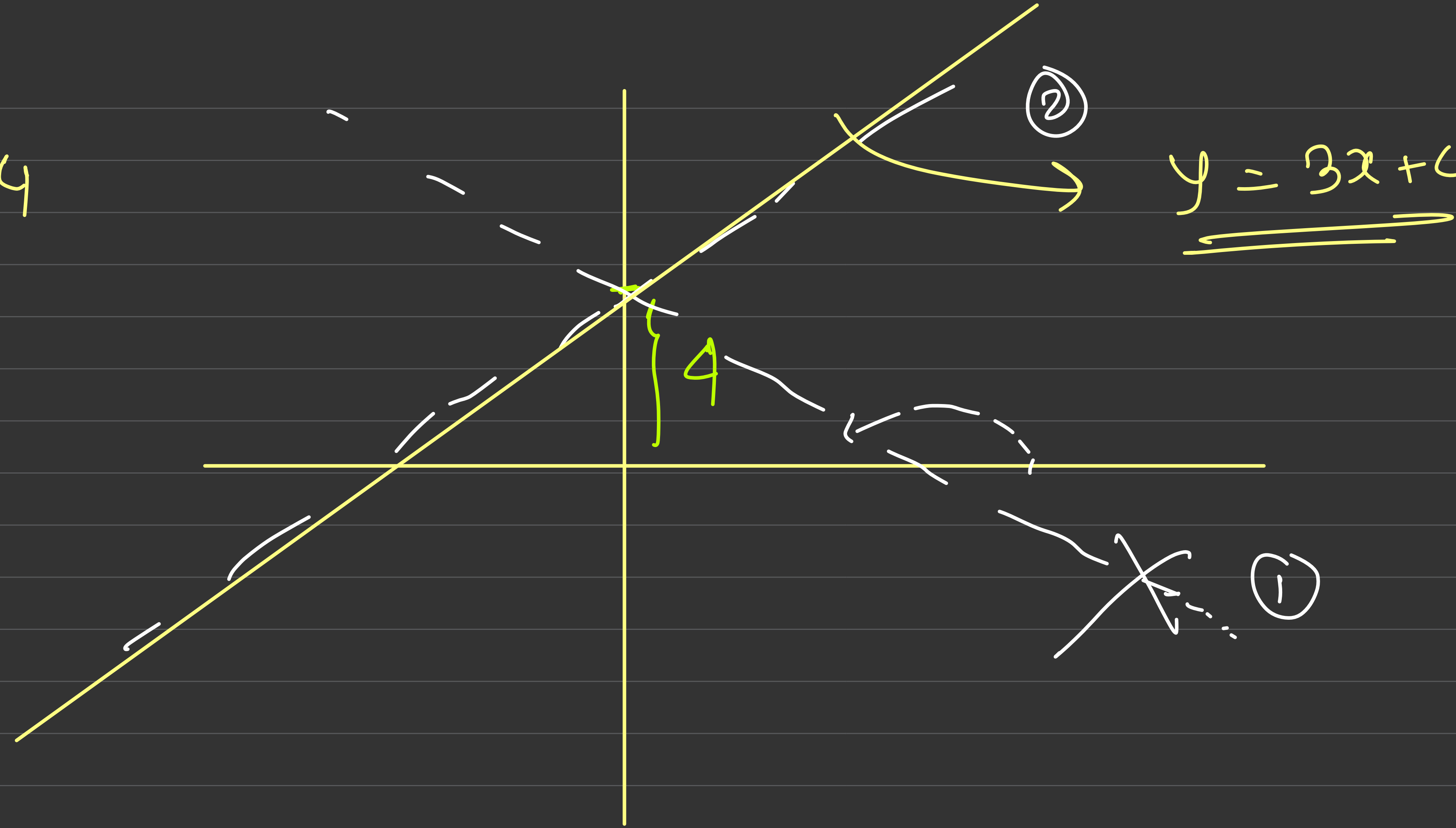
$\theta = 135^\circ$

Intercept $= 0$

$y = 3x + 4$

Slope = 3 > 0

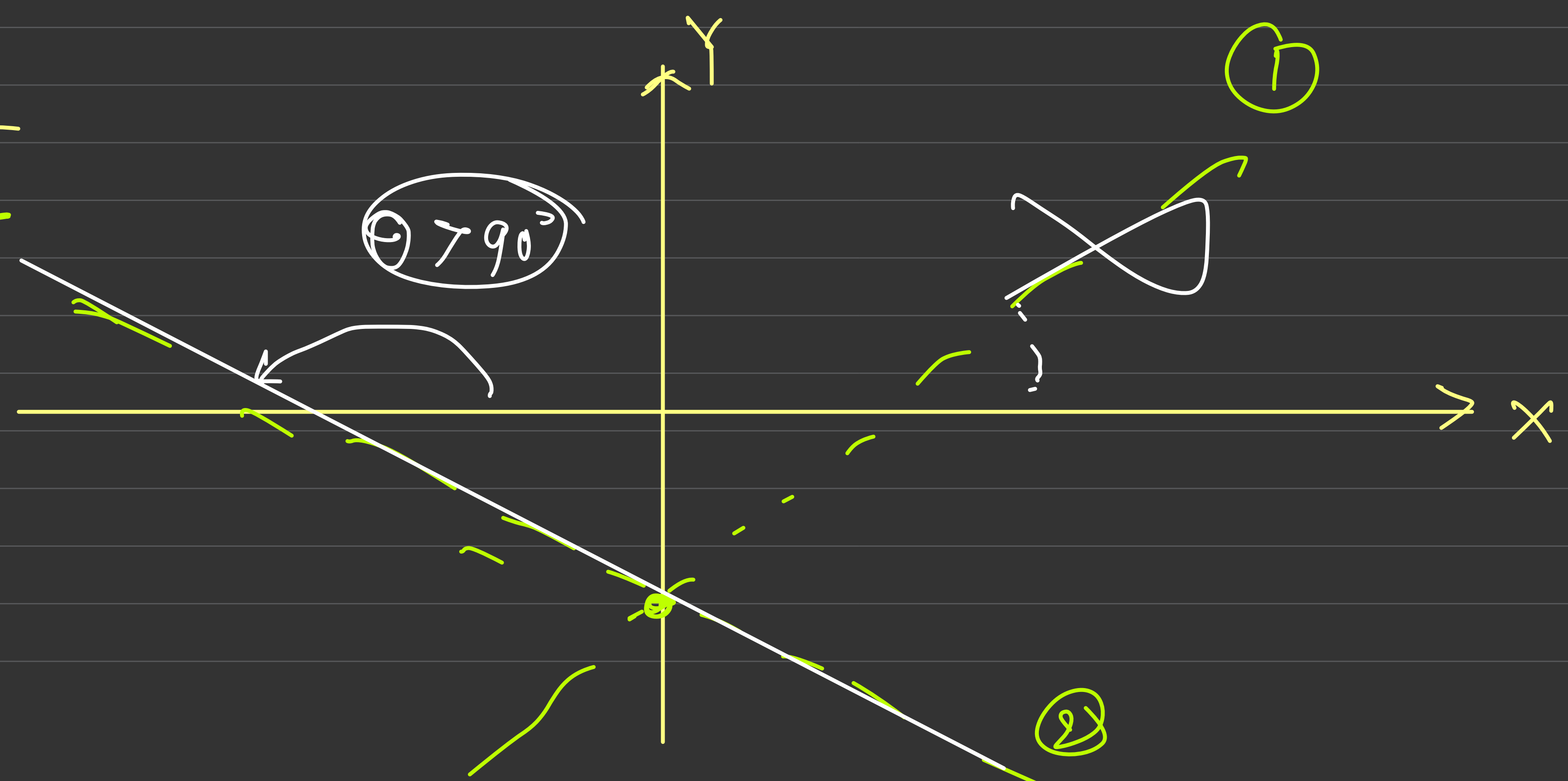
Intercept = +4 > 0



$y = -3x - 5$

Intercept = -5 < 0

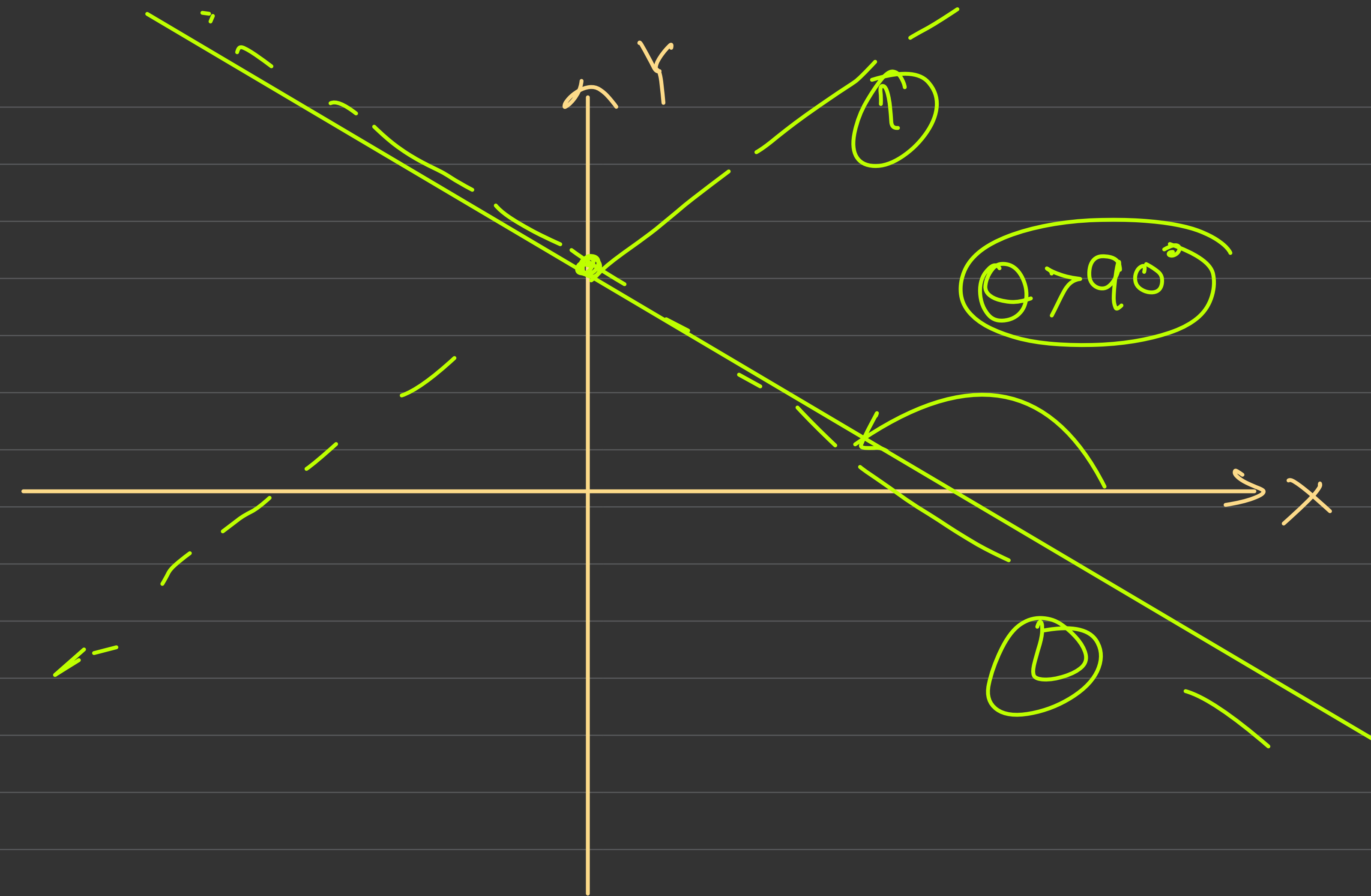
slope = -3 < 0



$y = -3x + 4$

Slope = -3

y-intercept = 4



$3x + 4y + 5 = 0 \Rightarrow$ $y = mx + c$

find slope and y- intercept

$\Rightarrow y = -\frac{3x}{4} - \frac{5}{4}$

Slope = $-3/4$

intercept = $-5/4$

Diffrentiation

$\frac{d}{dx}(\ )$ | Integration $\int$

$+ \quad - \quad \times \quad \div \quad |\ |$

↳ modulus

- $\Delta x = x_f - x_i$ → $\boxed{\text{diffrence in } x}$

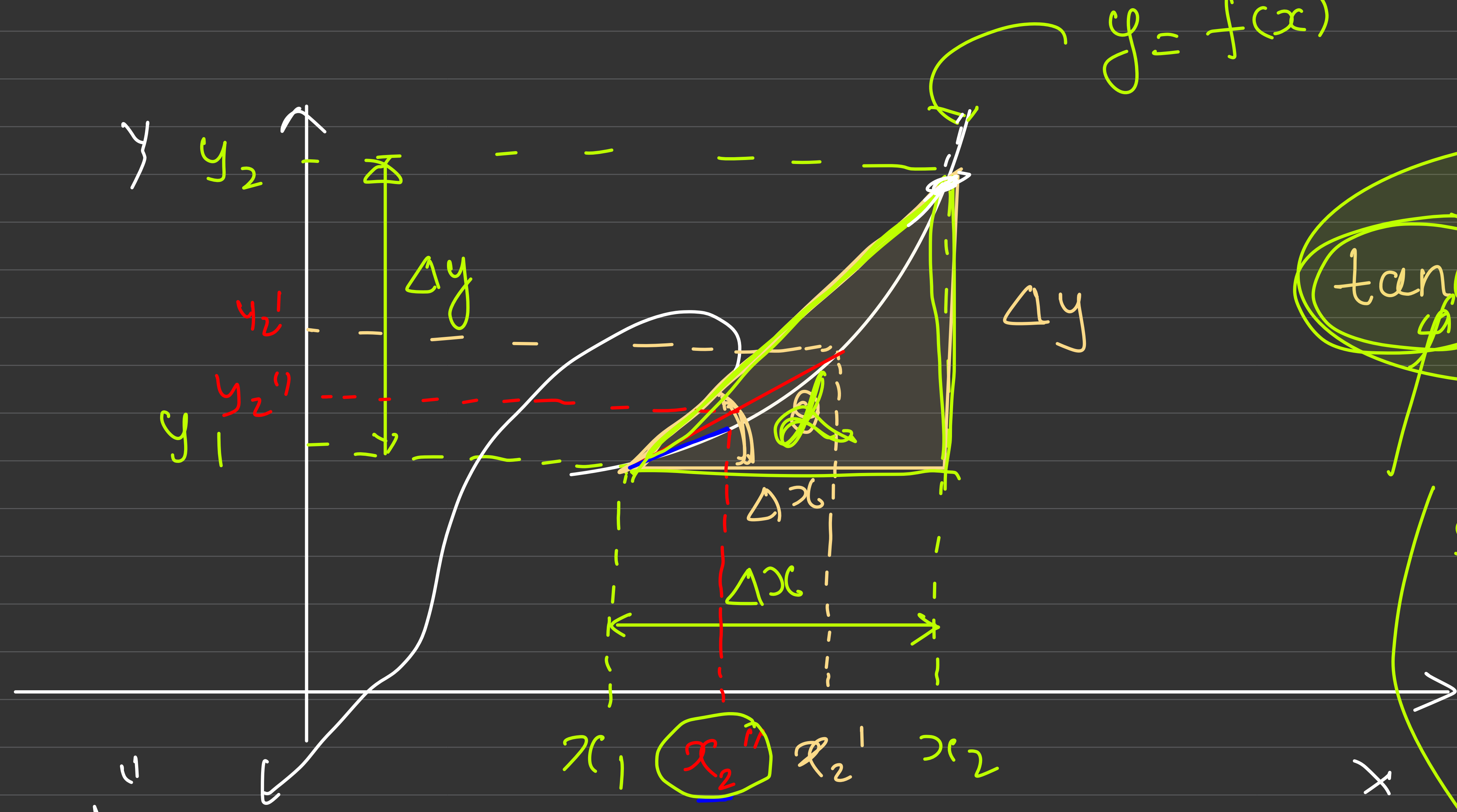
delta x

⇓

Significant value

- $dx \longrightarrow$ Very-very small change in x
- $dy \rightarrow$ " " " in y

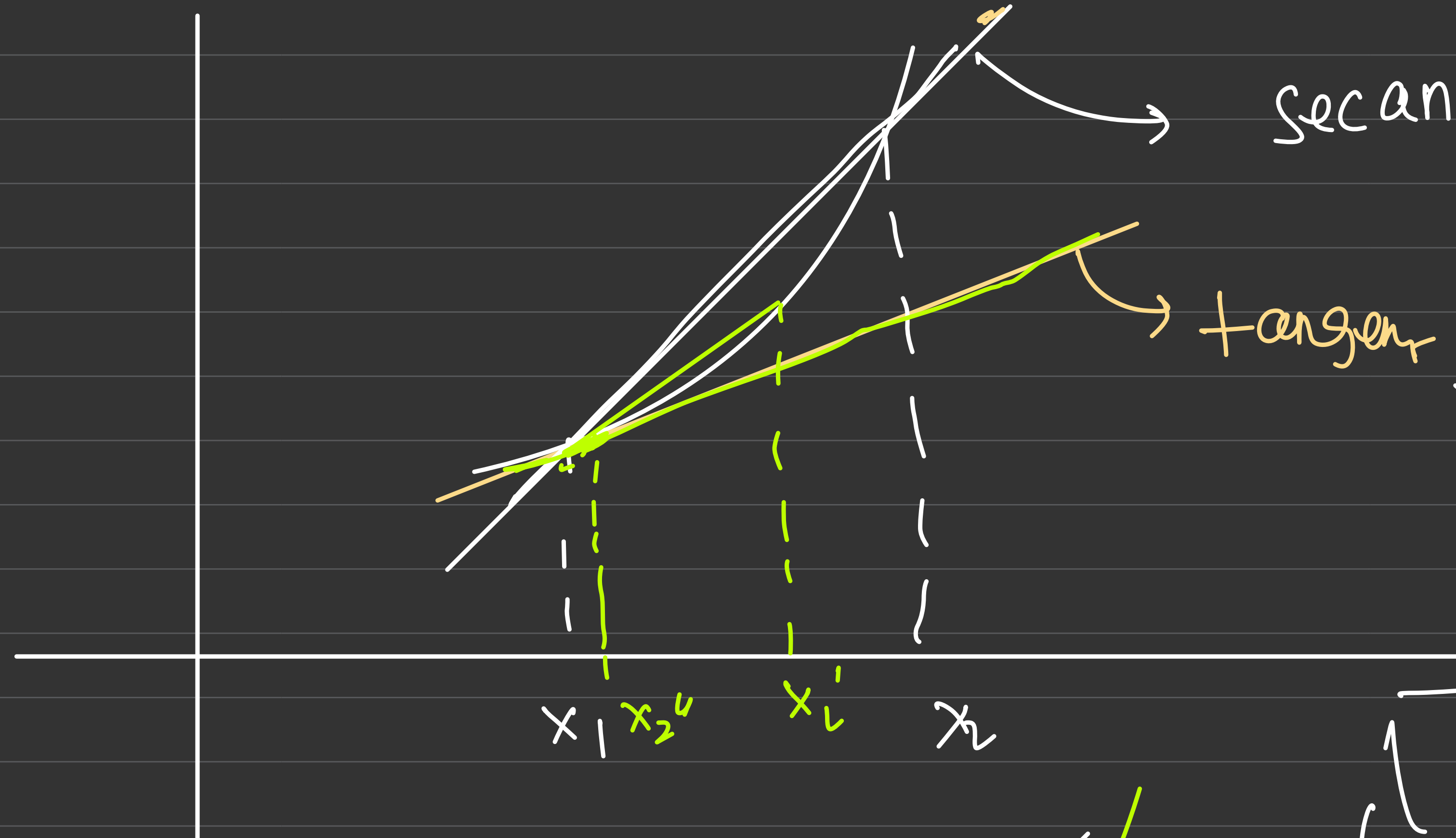
- $\infty \longrightarrow$ Infinity
- $h \longrightarrow$ "infinitesmal" (very-very small quantity)



"Secant"

$$\tan\alpha = \frac{\Delta y}{\Delta x}$$

(Secant line makes an angle "θ" with the horizontal)

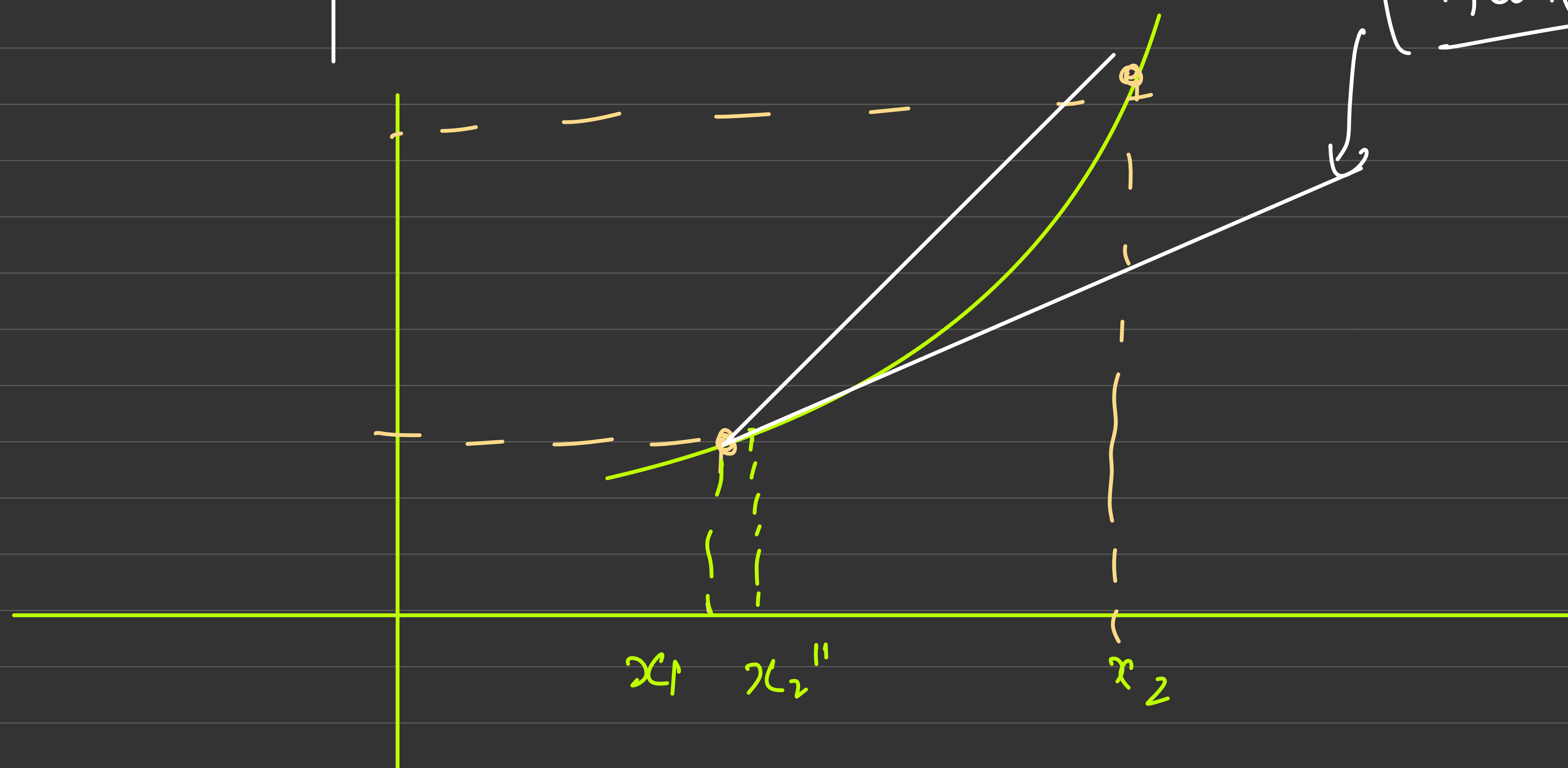


secant (It cuts the curve at two points)

tangent

(It touches the curve at one point)

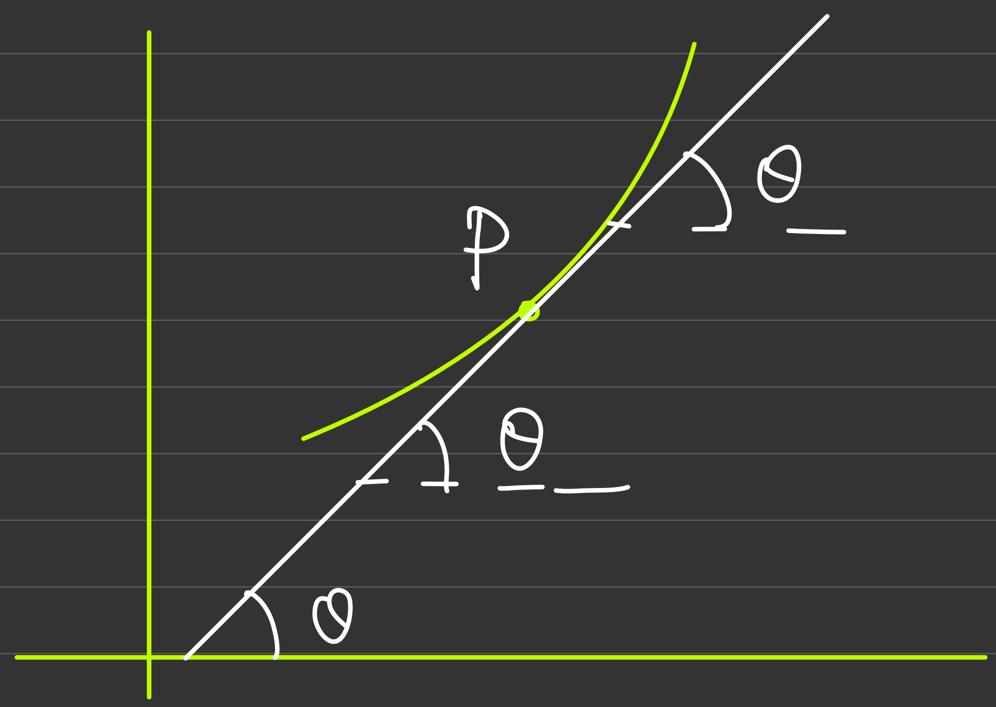
$\boxed{\text{tangent}}$



$\boxed{\text{Tangent}}$ ⇒ is the limiting value of Secant.

When $\Delta x \to 0$ (Δx tends to zero) then

Secant line will become tangent.



When $\Delta x \to 0 \Rightarrow dx$

$\Delta y \longrightarrow dy$

Slope of tangent

$\tan\theta = \dfrac{dy}{dx}$

θ = Angle made by tangent from +x axis axis

Physical Meaning of $\left(\frac{dy}{dx}\right)$

↳ It reeprresents the slope at a point.

$$\boxed{\frac{dy}{dx} \Rightarrow \text{Slope at any point} \Rightarrow \tan\theta}$$

$$\boxed{\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} \quad (\text{lim} - \text{limit})}$$

⇑

limit $\Delta x \to 0$ $\left\{ \begin{array}{l} \Delta x \Rightarrow dx \\ \Delta y \Rightarrow dy \end{array} \right\}$

$y = x^2$

$$x \longrightarrow x + \Delta x$$

$$y_f = (x+\Delta x)^2$$

$$y_i = (x)^2$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{y_f - y_i}{\Delta x} = \lim_{\Delta x \to 0} \frac{(x+\Delta x)^2 - x^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\{\cancel{x^2} + 2x\Delta x + \Delta x^2\} - \cancel{x^2}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2x\Delta x + (\Delta x)^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} [2x + \Delta x] \quad \Delta x \to 0$$

$$= \underline{2x}$$

Rule-1)

$$y = x^{n}$$

$$\boxed{\frac{dy}{dx} = n\,x^{n-1}}$$ ✓

Rule-2)

$$y = \text{Const}$$

$$\boxed{\frac{dy}{dx} = 0}$$

Rule-3)

$$y = c\,x^{2}$$

$$\frac{dy}{dx} = \frac{d}{dx}(cx^{2}) = c\,\frac{d}{dx}(x^{2}) = c\,(2x)$$

$$= 2cx$$

Rule-4.

$$\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$$

$$\frac{d}{dx}(u-v) = \frac{du}{dx} - \frac{dv}{dx}$$

Qu:- 1. $y = x^{9/2} = 9/2 \cdot x^{7/2}$

2. $y = \sqrt{x} = \frac{1}{2} x^{-1/2}$

$= \frac{1}{2\sqrt{x}}$

3. $y = x^2 + 4x$

$\frac{dy}{dx}$

$= \frac{d}{dx}(x^2 + 4x)$

$= \frac{d}{dx}(x^2) + \frac{d}{dx}(4x)$

$= 2x + 4$

4. $y = \frac{1}{x} = x^{-1}$

$\frac{dy}{dx} = \frac{-1}{x^2}$

5. $y = x^{-11/2}$

$\frac{dy}{dx} = \frac{-11}{2} \cdot x^{-13/2}$

$$y = x^2 + 4x$$

$$\frac{dy}{dx} = \frac{d}{dx}(x^2 + 4x) = \frac{d(x^2)}{dx} + \frac{d}{dx}(4x)$$

$$= 2x + 4\frac{d}{dx}(x)$$

$$= 2x + 4$$

$$y = x^2$$

$$\frac{dy}{dx} =$$

Qu :

$$y = \sqrt{5x^2} + \frac{1}{\sqrt{x}} = \sqrt{5}\,x + (x)^{-1/2}$$

$$\frac{dy}{dx} = \frac{d}{dx}\left(\sqrt{5}\,x + x^{-1/2}\right)$$

$$x^{-1/2}$$

$$= \frac{d}{dx}(\sqrt{5}\,x) + \frac{d}{dx}\left(x^{-1/2}\right)$$

$$-\frac{1}{2} \cdot x^{-1/2 - 1}$$

$$= \sqrt{5} \cdot \frac{dx}{dx} - \frac{1}{2} \cdot x^{-3/2}$$

$$\boxed{\frac{dy}{dx} = \sqrt{5} - \frac{x^{-3/2}}{2}}$$

Qu:
$$y = x^9 - \frac{1}{x^2} + 4 \qquad \frac{1}{x^2} = x^{-2}$$

$$\left.\frac{dy}{dx}\right|_{x=1} \qquad \frac{dy}{dx} = 9x^8 - \left[(-2)x^{-3}\right] + 0$$

$$\frac{dy}{dx} = 9x^8 + 2x^{-3}$$

$$\left.\frac{dy}{dx}\right|_{x=1} = 9(1)^8 + 2(1)^{-3}$$

$$= 9 + 2 = \underline{11 \text{ A}}$$

(Product Rule)

Rule-5

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

ex!:

$$\frac{d}{dx}\left((x+1)^2 \cdot x\right) = (x+1)^2 \cdot \frac{dx}{dx}$$

$$+ \; x \frac{d(x+1)^2}{dx}$$

$$= (x+1)^2 + x \; 2(x+1)$$

Rule-6 Quotient Rule

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{(v)^2}$$

Qu:

$$y = \frac{x^2+2}{x+2}$$

$$\frac{dy}{dx} = \frac{(x+2)\frac{d}{dx}(x^2+2) - (x^2+2)\frac{d}{dx}(x+2)}{(x+2)^2}$$

$$= \frac{(x+2)(2x) - (x^2+2)(1)}{(x+2)^2}$$

Rule-7.

$$\frac{d}{dx}(\sin x) = \cos x$$

Rule-8

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

ex:- $y = x \sin x$

$$\frac{dy}{dx} = x \cos x + \sin x (1)$$

$$= x\cos x + \sin x$$

Qu!: $y = \underset{u}{(x^2+2)}\ \underset{v}{\cos x}$

$*\ \frac{d}{dx}(\cos x) = -\sin x$

$$\frac{dy}{dx} = (x^2+2)(-\sin x) + \cos x\,(2x)$$

$$= -(x^2+2)\sin x + \underline{\underline{2x\cos x}}$$

① $\frac{d}{dx}(\text{Const}) = 0$

② $\frac{d}{dx}(x^n) = n x^{n-1}$

③ $\frac{d}{dx}(c\, x^n) = c \cdot n\, x^{n-1}$

④ $\frac{d}{dx}(u + v) = \frac{du}{dx} + \frac{dv}{dx}$

$\frac{d}{dx}(u - v) = \frac{du}{dx} - \frac{dv}{dx}$

(5) $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$ (Product Rule)

6) $$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\,du/dx - u\,dv/dx}{v^2}$$ (Quotient Rule)

7) $$\frac{d}{dx}(\sin x) = \cos x$$

8) $$\frac{d}{dx}(\cos x) = -\sin x$$

ex:. $y = \frac{\sin x}{x} = \frac{u}{v}$ $\qquad \left.\frac{dy}{dx}\right|_{x=\pi/2}$

$$u = \sin x$$
$$v = x$$

$$\frac{dy}{dx} = \frac{(x)\cos x - \sin x\,(1)}{(x)^2}$$

$$\left.\frac{dy}{dx}\right|_{x=\pi/2} = \frac{\left(\pi/2\right)\cdot \times(0) - 1\times 1}{\left(\frac{\pi^2}{4}\right)}$$

$$= \frac{4}{\pi^2} \approx \frac{4}{10}$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\cdot du/dx - u\,\frac{dv}{dx}}{v^2}$$

Qu!.

$$y = \tan x = \frac{\sin x}{\cos x}$$

$$\frac{dy}{dx} = \frac{(\cos x)(\cos x) - (\sin x)(-\sin x)}{(\cos^2 x)} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x}$$

$$= \sec^2 x$$

$$\boxed{\frac{d}{dx}(\tan x) = \sec^2 x}$$ **

Qu!

$$y = \cot x = \frac{\cos x}{\sin x}$$

$$\left.\frac{dy}{dx}\right|_{x=\pi/2} = \frac{(\sin x)(-\sin x) - (\cos x)(\cos x)}{(\sin x)^2}$$

$$= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$$

$$= \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x} = \frac{-1}{\sin^2 x}$$

$$= -\operatorname{cosec}^2 x$$

$$\boxed{\left.\frac{dy}{dx}\right|_{x=\pi/2} = -1}$$

$$\boxed{\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x} \; *$$

Qu:- $y = \sec x$ $\qquad \left.\frac{dy}{dx}\right|_{x=\pi/4}$

$$= \frac{1}{\cos x}$$

$$\frac{dy}{dx} = \frac{(\cos x)\times 0 - 1(-\sin x)}{\cos^2 x} = \frac{\sin x}{\cos x \cdot \cos x}$$

$$= \sec x \tan x.$$

$$\left.\frac{dy}{dx}\right|_{x=\pi/4} = \sec 45^\circ \cdot \tan 45^\circ$$

$$= \sqrt{2}$$

$$\boxed{\frac{d}{dx}(\sec x) = \sec x \cdot \tan x} \; *$$

Qu:

$$y = \operatorname{Cosec} x = \frac{1}{\sin x}$$

$$\frac{dy}{dx} = ?$$

$$\frac{dy}{dx} = \frac{(\sin x)\,(0) - (1).\cos x}{(\sin^2 x)} = -\frac{\cos x}{\sin x\,.\,\sin x}$$

$$= -\cot x \operatorname{cosec} x$$

$\frac{d}{dx}(\sin x) = \cos x$

$\frac{d}{dx}(\cos x) = -\sin x$

$\frac{d}{dx}(\tan x) = \sec^2 x$

$\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$

$\frac{d}{dx}(\sec x) = \sec x \cdot \tan x$

$\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cdot \cot x$

Rule-9. Diffrentiation of logarithimic function :-

$$\boxed{\frac{d}{dx}(\ln x) = \frac{1}{x}}$$

ex :-

$$\frac{d}{dx}(\log_a x) = \frac{d}{dx}\left(\frac{\ln x}{\ln a}\right) = \frac{1}{\ln a}\frac{d}{dx}(\ln x)$$

$$= \frac{1}{x \ln a}$$

$$\log_a x = \frac{\log_e x}{\log_e a} = \frac{\ln x}{\ln a}$$

$$\log_a x = \frac{\log_e x}{\log_e a} = \frac{\ln x}{\ln a}$$

$$\frac{d}{dx}\left(\log_a x\right) = \frac{d}{dx}\left(\frac{\ln x}{\ln a}\right) = \frac{1}{\ln a}\frac{d}{dx}(\ln x)$$

$$= \frac{1}{x \ln a}$$

Rule-10 Diffrentiation of exponential function.

$$\boxed{\frac{d}{dx}\left(e^{x}\right) = e^{x}}$$

Ex: ① $y = x \cdot e^{x}$

$$\frac{dy}{dx} = x \cdot e^{x} + e^{x}(1)$$

$$= e^{x}(x+1)$$

Ex: 2 - $y = \ln\left(e^{x}\right)$

$$= x \cdot (\ln e)$$

$$= x$$

$$\therefore \boxed{\frac{dy}{dx} = 1}$$

$$y = x \cdot e^x$$

$$u = x, \quad v = e^x$$

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$= (x) \cdot \frac{d}{dx}(e^x) + e^x \cdot \left(\frac{dx}{dx}\right)$$

$$= x \cdot e^x + e^x (1)$$

$$\boxed{y = e^x [x+1]}$$

Qu!.

$$y = \frac{\ln x}{e^x}$$

$$\because \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{(v)\frac{du}{dx} - (u)\frac{dv}{dx}}{(v)^2}$$

$$\frac{dy}{dx} = \frac{(e^x)\left(\frac{1}{x}\right) - \ln x \cdot (e^x)}{(e^x)^2}$$

$$= \frac{\cancel{e^x}\left[\,1/x - \ln x\,\right]}{(e^x)\,\cancel{(e^x)}}$$

$$= \frac{1}{e^x}\left(\frac{1}{x} - \ln x\right) = \frac{(1 - x\ln x)}{x \cdot e^x}$$

Qu!.

$$y = x^2 \cdot e^x \quad \text{(Product Rule)}$$

$$\frac{dy}{dx} = x^2 \cdot e^x + e^x(2x)$$

$$\frac{dy}{dx} = e^x\left[x^2 + 2x\right]$$ ✓

Rule-11. Chain Rule of Diffrentiation

[Outside - Inside Rule]

$$\frac{dy}{dx} = \frac{\text{Diffrentiation of } y}{\text{w.r.t. } x}$$

$$y = f(x)$$

$$\frac{d}{dx}(t^2) \Rightarrow \text{Can't be diffrentiated directly}$$

$$\frac{dy}{dx} = \frac{dy}{dx} \cdot \frac{dt}{dt} = \frac{dy}{\cancel{dt}} \cdot \frac{\cancel{dt}}{dx}$$

$$\boxed{\frac{dy}{dx} = \frac{dy}{\cancel{dw}} \cdot \frac{\cancel{dw}}{\cancel{dt}} \cdot \frac{\cancel{dt}}{dx}}$$

$$\frac{d}{dx}\left(t^2\right) = \frac{d}{dt}\left(t^2\right) \cdot \frac{dt}{dx}$$

$$\frac{d(t^2)}{\cancel{dt}} \cdot \frac{\cancel{dt}}{dx} = 2t \cdot \frac{dt}{dx}$$

$$y = z^2$$

$$\frac{dy}{dx} = \frac{d}{dx}\left(z^2\right) = \frac{d}{dz}\left(z^2\right)\left(\frac{dz}{dx}\right)$$

$$= 2z \cdot \left(\frac{dz}{dx}\right)$$

Q4:-

$y = e^{x^2}$

dy/dx

$\frac{d}{dy}(e^x) = e^x$

Let $x^2 = t$

$\boxed{y = e^t}$

$$\boxed{\frac{dy}{dx} = \left(\frac{dy}{dt}\right) \cdot \left(\frac{dt}{dx}\right)}$$

x^2

$$= \frac{d}{dt}(e^t) \quad \frac{d}{dx}(x^2)$$

$$= e^t \cdot 2x = e^{x^2} \cdot 2x$$

Race - 5

Practice-set-2

$\Rightarrow$

Name

→ Status - Race - C

Pr - C

→ Doubt Rac —

Pr - set -

#

$$\frac{d}{dx}(\text{const}) = 0$$

$$\frac{d}{dx}(x^n) = n\, x^{n-1}$$

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \cdot du/dx - u\, dv/dx}{v^2}$$

- $\frac{d}{dx}(\sin x) = \cos x$
- $\frac{d}{dx}(\cos x) = -\sin x$
- $\frac{d}{dx}(\tan x) = \sec^2 x$
- $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$
- $\frac{d}{dx}(\sec x) = \sec x \cdot \tan x$
- $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cdot \cot x$
- $\frac{d}{dx}(\ln x) = \frac{1}{x}$
- $\frac{d}{dx}(e^x) = e^x$

Qu:- $y = \sin(x^3)$ $\quad \frac{dy}{dx} = ?$

Let $z = x^3$ $\quad \left(\frac{dz}{dx}\right) = 3x^2$

$y = \sin(z)$ $\quad \left(\frac{dy}{dz}\right) = \cos z$

$$\frac{dy}{dx} = \left(\frac{dy}{dz}\right) \cdot \left(\frac{dz}{dx}\right)$$

$$= (\cos z) \cdot (3x^2)$$

$$= 3x^2 \cos(x^3)$$

Ans

Qu!. $y = e^{\sqrt{x}}$ find dy/dx

Soln!. $z = \sqrt{x} \Rightarrow \frac{dz}{dx} = \frac{1}{2} x^{-1/2} = \frac{1}{2\sqrt{x}}$

$y = e^{z} \Rightarrow \frac{dy}{dz} = e^{z}$

$\frac{d}{dx}(x^{1/2}) = \frac{1}{2} \cdot x^{-1/2}$

$= \frac{1}{2} \cdot \frac{1}{\sqrt{x}}$

$$\frac{dy}{dx} = \left(\frac{dy}{dz}\right)\left(\frac{dz}{dx}\right)$$

$$= e^{z} \cdot \frac{1}{2\sqrt{x}}$$

$$= \frac{1}{2\sqrt{x}} \cdot e^{\sqrt{x}}$$ Ans

**

$$\boxed{\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}}$$

Short-Cut Method $\Rightarrow$ outside $\rightarrow$ inside

$$y = e^{\left(\sqrt{x^2+1}\right)}$$

$$\frac{dy}{dx} = e^{\left(\sqrt{x^2+1}\right)} \times \frac{1}{\cancel{2}\sqrt{x^2+1}} \times (\cancel{2}x)$$

$$= \frac{x}{\sqrt{x^2+1}} \cdot e^{\sqrt{x^2+1}}$$

Qu: $y = \sin\sqrt{\left(x^2 + \frac{1}{x}\right)}$ find $\left(\frac{dy}{dx}\right)$

$$\frac{dy}{dx} = \left[\cos\sqrt{\left(x^2 + \frac{1}{x}\right)}\right] \times \frac{1}{2\sqrt{\left(x^2 + \frac{1}{x}\right)}} \times \left(2x - \frac{1}{x^2}\right)$$

Qu: $y = e^{-\sin x}$ find $\left(\frac{dy}{dx}\right)$ at $x = \pi/2$

$$\frac{dy}{dx} = \left(e^{-\sin x}\right) \cdot \left[-\cos x\right]$$

$$\left.\frac{dy}{dx}\right|_{\pi/2} = e^{-\sin \pi/2} \cdot \{-(0)\} = 0$$

Qu:

$$y = \ln\left(x + \sqrt{x^2+1}\right) \qquad \text{find } \left.\frac{dy}{dx}\right|_{x=1}$$

$$\frac{dy}{dx} = \left(\frac{1}{\left[x + \sqrt{x^2+1}\right]}\right) \times \left[1 + \frac{1}{\cancel{2}\sqrt{x^2+1}} \times (\cancel{2}x)\right]$$

$$= \frac{1}{\left(x + \sqrt{x^2+1}\right)} \cdot \left(1 + \frac{x}{\sqrt{x^2+1}}\right)$$

$$\left.\frac{dy}{dx}\right|_{x=1} = \frac{1}{1+\sqrt{2}} \left(1 + \frac{1}{\sqrt{2}}\right)$$

$$= \frac{1}{\sqrt{2}} \text{ Ans}$$

Qu: $y = \sin^3 x = (\sin x)^3$ $\quad \left.\dfrac{dy}{dx}\right|_{x=\pi/4}$.

$$\frac{dy}{dx} = 3\sin^2 x \times \cos x$$

$$\left.\frac{dy}{dx}\right|_{x=\pi/4} = 3 \times \left(\frac{1}{\sqrt{2}}\right)^2 \times \left(\frac{1}{\sqrt{2}}\right)$$

$$= \frac{3}{2\sqrt{2}} \text{ Ans}$$

Qu:

$$y = \underline{\sin}\left(\underline{\ln}\,\underline{\sqrt{x^2+1}}\right) \qquad \text{find } \left(\frac{dy}{dx}\right)$$

$$\frac{dy}{dx} = \cos\left(\ln\sqrt{x^2+1}\right) \cdot \frac{1}{\sqrt{x^2+1}} \times \frac{(\cancel{2}x)}{\cancel{2}\sqrt{x^2+1}}$$

$$= \frac{x}{(x^2+1)} \cdot \cos\left(\ln\left(\sqrt{x^2+1}\right)\right)$$

$y = \sin^{-1} x \Rightarrow \left(dy/dx \right)$

$\Rightarrow \quad \sin y = x$

$$\frac{dx}{dy} = \cos y \Rightarrow \frac{dy}{dx} = \frac{1}{\cos y} = \frac{1}{\sqrt{1 - \sin^2 y}}$$

$$= \frac{1}{\sqrt{1 - x^2}}$$

**

$$\therefore \boxed{\frac{d}{dx}\left(\sin^{-1} x\right) = \frac{1}{\sqrt{1 - x^2}}}$$

**

$$\boxed{\frac{d}{dx}\left(\cos^{-1} x\right) = -\frac{1}{\sqrt{1-x^2}}}$$

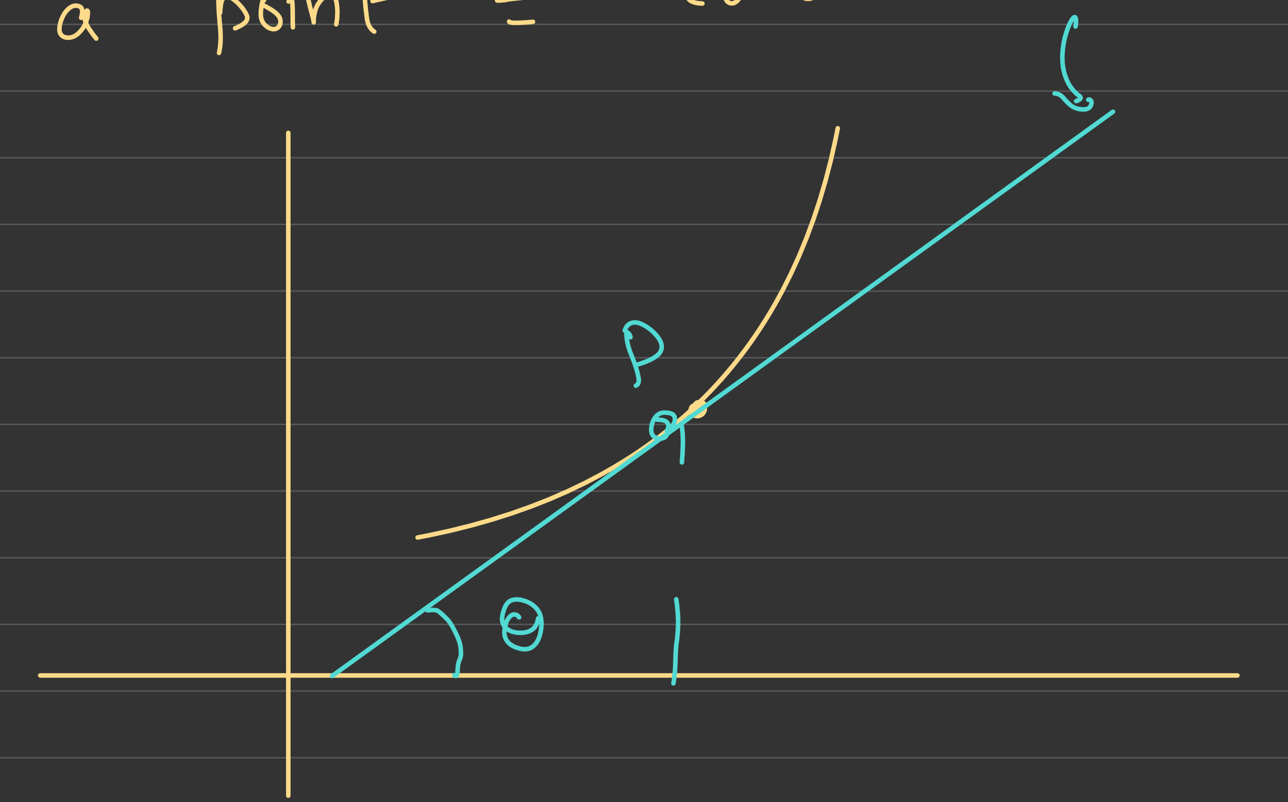
$$\boxed{\frac{d}{dx}\left(\tan^{-1} x\right) = \frac{1}{1+x^2}}$$

***

Physical Meaning of Diffrentiation

1) Slope of function at a point $= \tan\theta$ Slope

$$= \left(\frac{dy}{dx}\right)_P$$



2) $\frac{dy}{dx}$ = <u>change in y w.r.t. 'x'</u>

- Rate of change of $y = \left(\frac{dy}{dt}\right)$

"Time derivative"

- Rate of change of displacement $(x) = \frac{dx}{dt} =$ velocity

- Rate of change of velocity $(v) = \frac{dv}{dt} =$ acceleration

- Rate of change of momentum $(p) = \frac{dp}{dt} =$ Force

$$* \left[F = ma = m\frac{dV}{dt} = \frac{d}{dt}(mv) = \frac{dp}{dt} \right]$$

- Rate of flow of charge (q) = $\frac{dq}{dt}$ = Current

3) Gradient of any term means diffrentiation of that term w.r.t. distance/ displacement

- velocity gradient = $\frac{dv}{dx}$
  (v
- Temperature (T) gradient = $\frac{dT}{dx}$
- displacement gradient = $\frac{dx}{dx} = 1$

#

H. C. Verma — Part-1 (1 साल)

Concept of physics

IIT Kanpur

Qu! $y = \sin(4x+5)$ $\qquad dy/dx = 4\cos(4x+5)$

Qu! $y = e^{(3t+5)}$ $\qquad dy/dt = 3e^{3t+5}$

Qu: displacement of a particle $= x = (3t^2 + \sqrt{t})$

find (a) velocity of particle at $t = 1$ sec

(b) acceleration of particle at $t = 1$ sec

Soln:

$$v = \frac{dx}{dt} = 6t + \frac{1}{2\sqrt{t}}$$

$$v(t = 1\text{sec}) = 6\times 1 + \frac{1}{2\times\sqrt{1}} = \frac{13}{2} \text{ m/s}$$

$$a = \frac{dv}{dt} = 6 + \frac{1}{2}\left(\frac{-1}{2}\right) t^{-3/2}$$

$$\left[v = 6t + \frac{1}{2\sqrt{t}} \right]$$

$$\frac{1}{2} \; t^{-1/2}$$

$$\frac{1}{2}\left(-\frac{1}{2}\right) \cdot t^{-1/2 - 1}$$

$$a(t = 1\,sec) = 6 - \frac{1}{4}$$

$$= \frac{23}{4} \; m/s^2$$

Qu: Position of a particle is given by $x = 3\sin(4t + \pi/4)$

(1) find velocity and acceleration of particle at $t = \frac{\pi}{4}$ sec

(2) find maximum velocity and maximum acceleration of the Particle.

Sol^n:

$$x = 3\sin\left(4t + \frac{\pi}{4}\right) = 3\cos\left(4t+\frac{\pi}{4}\right) \times 4$$

$$v = \frac{dx}{dt} = 3\cos\left(4t+\frac{\pi}{4}\right) \cdot 4$$

$$= 12\cos\left(4t+\frac{\pi}{4}\right)$$

$$\bar{v}\left(t = \frac{\pi}{4}\text{ sec}\right) = 12\cos\left(\cancel{4}\times\frac{\pi}{\cancel{4}} + \frac{\pi}{4}\right)$$

$$= 12\cos\left(\pi + \frac{\pi}{4}\right)$$

$$= -12\cos(\pi/4) = \frac{-12}{\sqrt{2}}\text{ m/s}$$

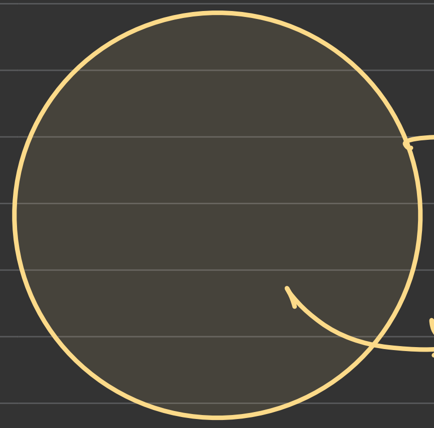
$$a = \frac{dv}{dt} = -12\sin\left(4t+\frac{\pi}{4}\right) \cdot 4 = -48\sin\left(4t+\frac{\pi}{4}\right)$$

$$a\left(t = \pi/4\text{ sec}\right) = -48\sin(\pi + \pi/4) = 48 \cdot \sin\left(\frac{\pi}{4}\right) = \frac{48}{\sqrt{2}}\text{ m/s}^2$$

$$V = 12\cos\left(4t + \frac{\pi}{4}\right) \Rightarrow V_{max} = 12 \text{ m/s}$$

$$a = -48\sin\left(4t + \frac{\pi}{4}\right) \Rightarrow a_{max} = -48(-1) = 48 \text{ m/s}^2$$

Qu: The radius of ballon is increasing at the rate 3 m/s. Calculate the rate of increase of surface area and volume of the ballon? (when $r = 10$ m)



$S.A. = 4\pi r^2$

$V = \frac{4}{3}\pi r^3$

Soln:

$$\frac{dr}{dt} = 3 \text{ m/s}$$

1) $$\frac{d}{dt}(SA) = \frac{d}{dt}(4\pi r^2) = \frac{d(4\pi r^2)}{dr} \cdot \frac{dr}{dt}$$

$$\frac{d(SA)}{dt} = \frac{d}{dr}(4\pi r^2) \cdot \frac{dr}{dt}$$

$$= 4\pi(2r) \cdot \frac{dr}{dt}$$

$$= 8\pi(10) \cdot (3) = 240\pi \ m^2/s$$

(ii)

$$\frac{dV}{dt} = \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right) = \frac{d}{dr}\left(\frac{4}{3}\pi r^3\right) \cdot \frac{dr}{dt}$$

$$= \frac{4}{\cancel{3}}\pi \cdot \cancel{3} r^2 \cdot \frac{dr}{dr}$$

$$= 4\pi(10)^2 \times 3 = \underline{\underline{1200\pi \ m^3/s}} \ A$$

Double Differentiation

differentiation of x w.r.t. time

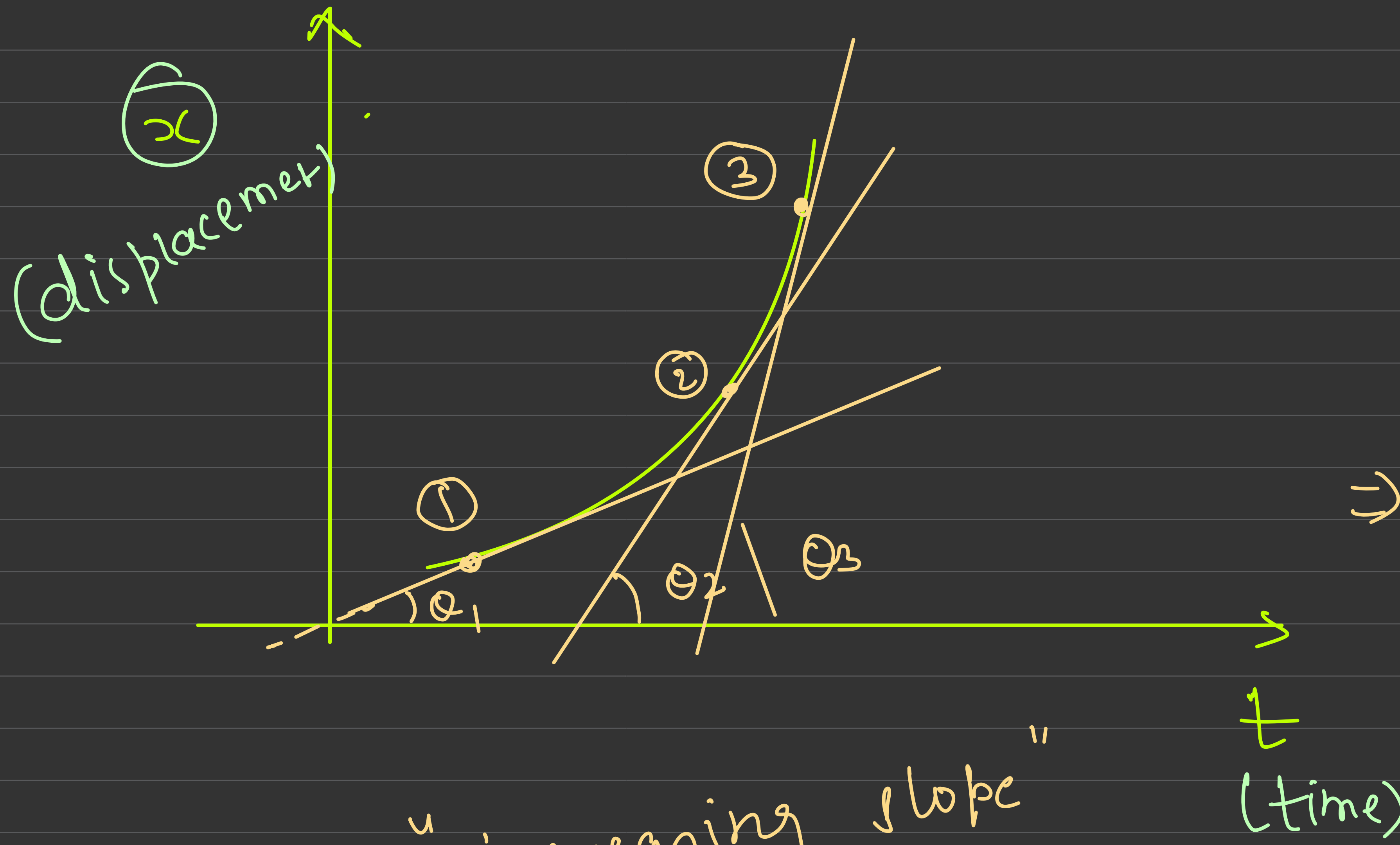
$$x \longrightarrow \frac{dx}{dt} \longrightarrow \frac{d^2x}{dt^2}$$

Double differentiation of 'x' w.r.t. time

- $\dfrac{dy}{dx} \rightarrow$ slope at any point

- $\dfrac{d^2y}{dx^2} \rightarrow$ Slope of (Slope of y)

①



"increasing slope"

Slope = $\tan\theta$

$\theta_3 > \theta_2 > \theta_1$

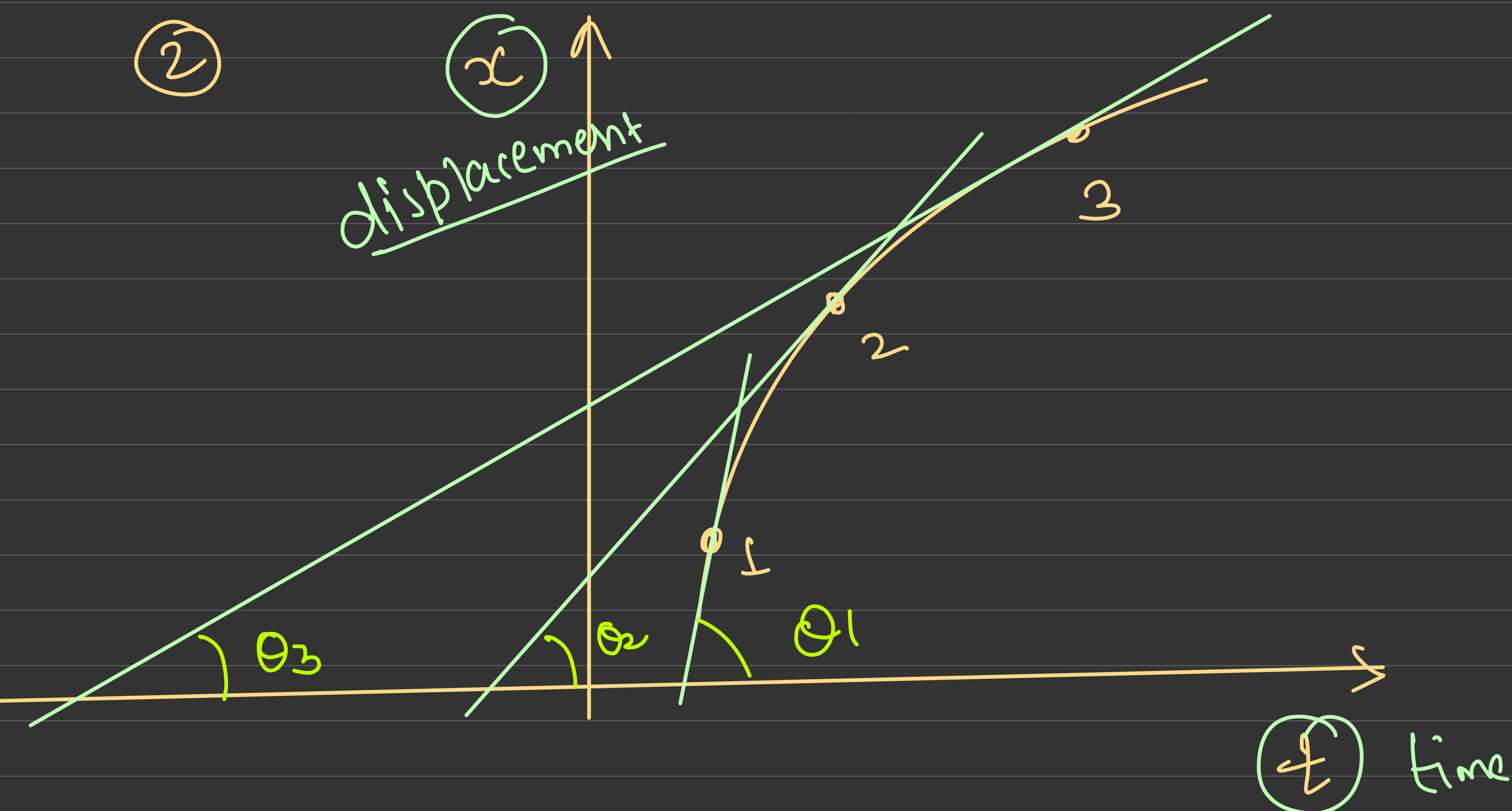
$\Rightarrow \tan\theta_3 > \tan\theta_2 > \tan\theta_1$

$\Rightarrow m_3 > m_2 > m_1$

$$\text{Slope} = \frac{dy}{dx}$$

- Slope of $x-t$ Curve $= \dfrac{dx}{dt}$ = velocity
- Particle velocity is increasing with time.
- Acceleration ✓

(2)



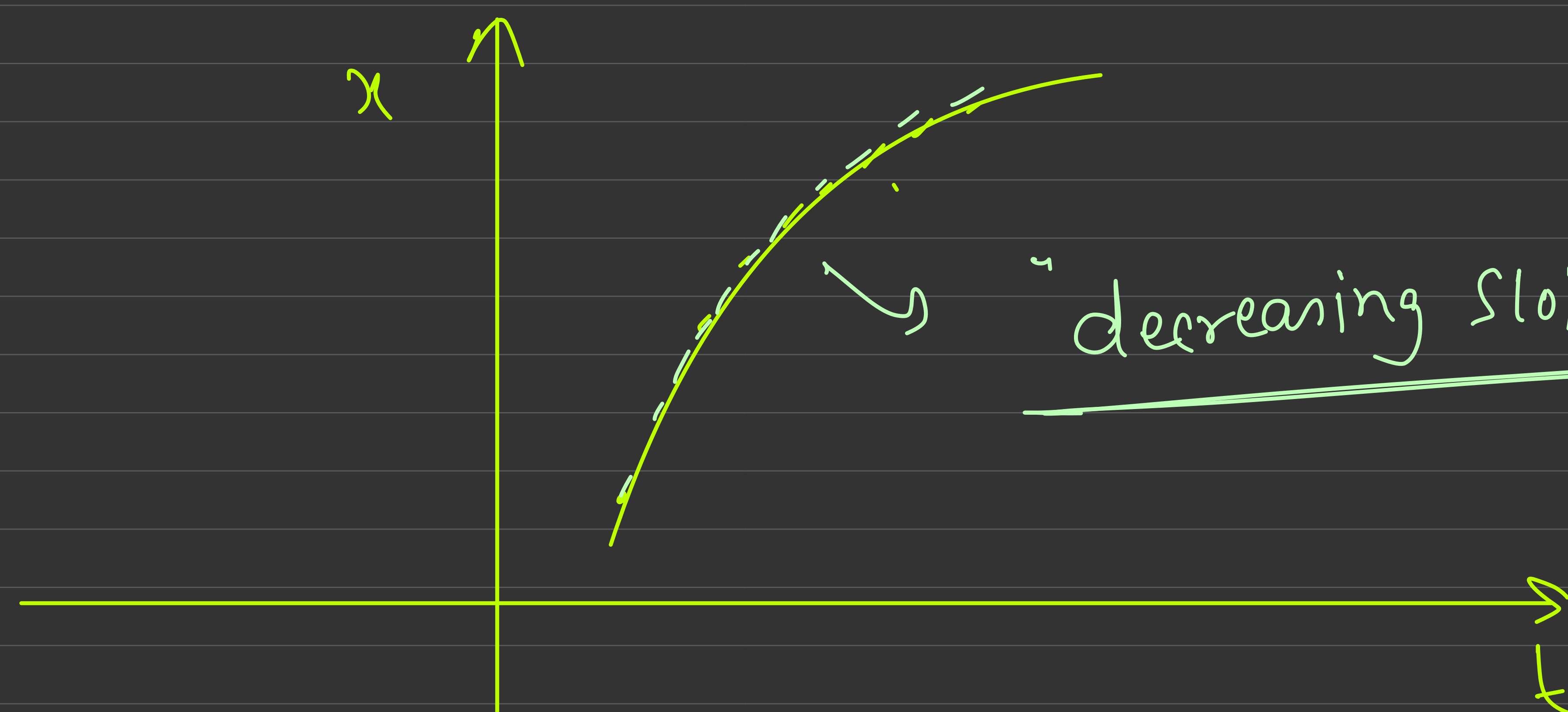
$$\theta_3 < \theta_2 < \theta_1$$

$$\tan\theta_3 < \tan\theta_2 < \tan\theta_1$$

$$m_3 < m_2 < m_1$$

$$\left(\frac{dx}{dt}\right) = \text{Slope} = \text{velocity}$$

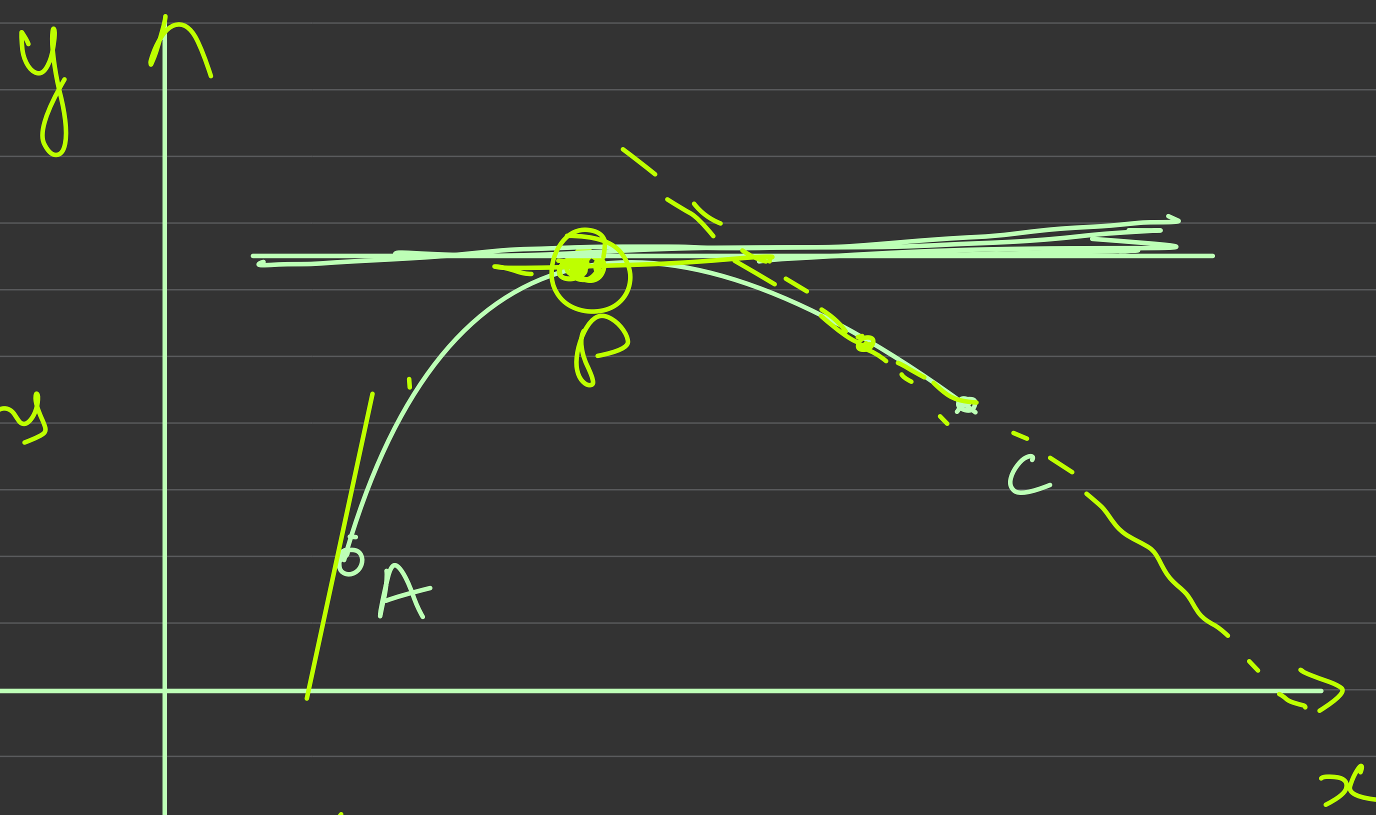
$$\Downarrow$$



"decreasing slope"

- Particle velocity is decreasing with time.
- Retardation of the particle

(3)

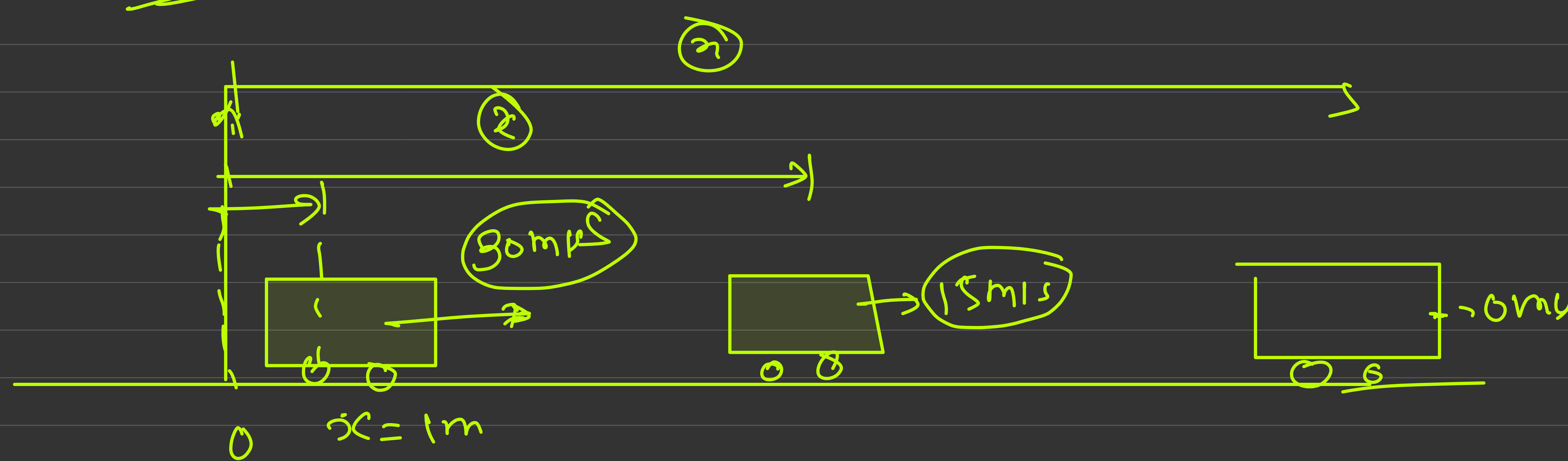


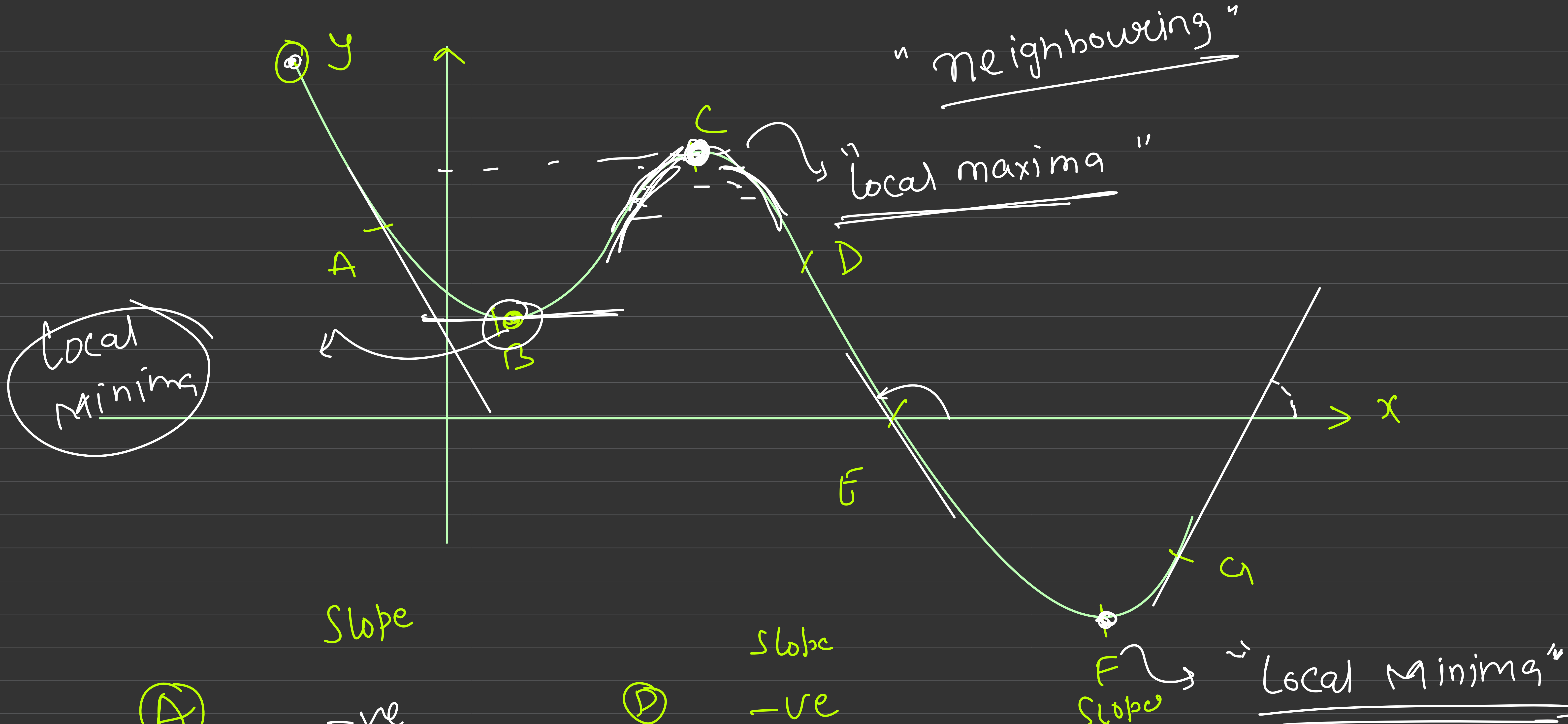
A - P
↳ decreasing slope

P - C
↳ magnitude of slope
⇓
Increasing

$$\text{Slope} = \tan\theta = \tan 0^\circ = \underline{\underline{0}}$$

$\boxed{\text{Slope at Point } P = 0}$

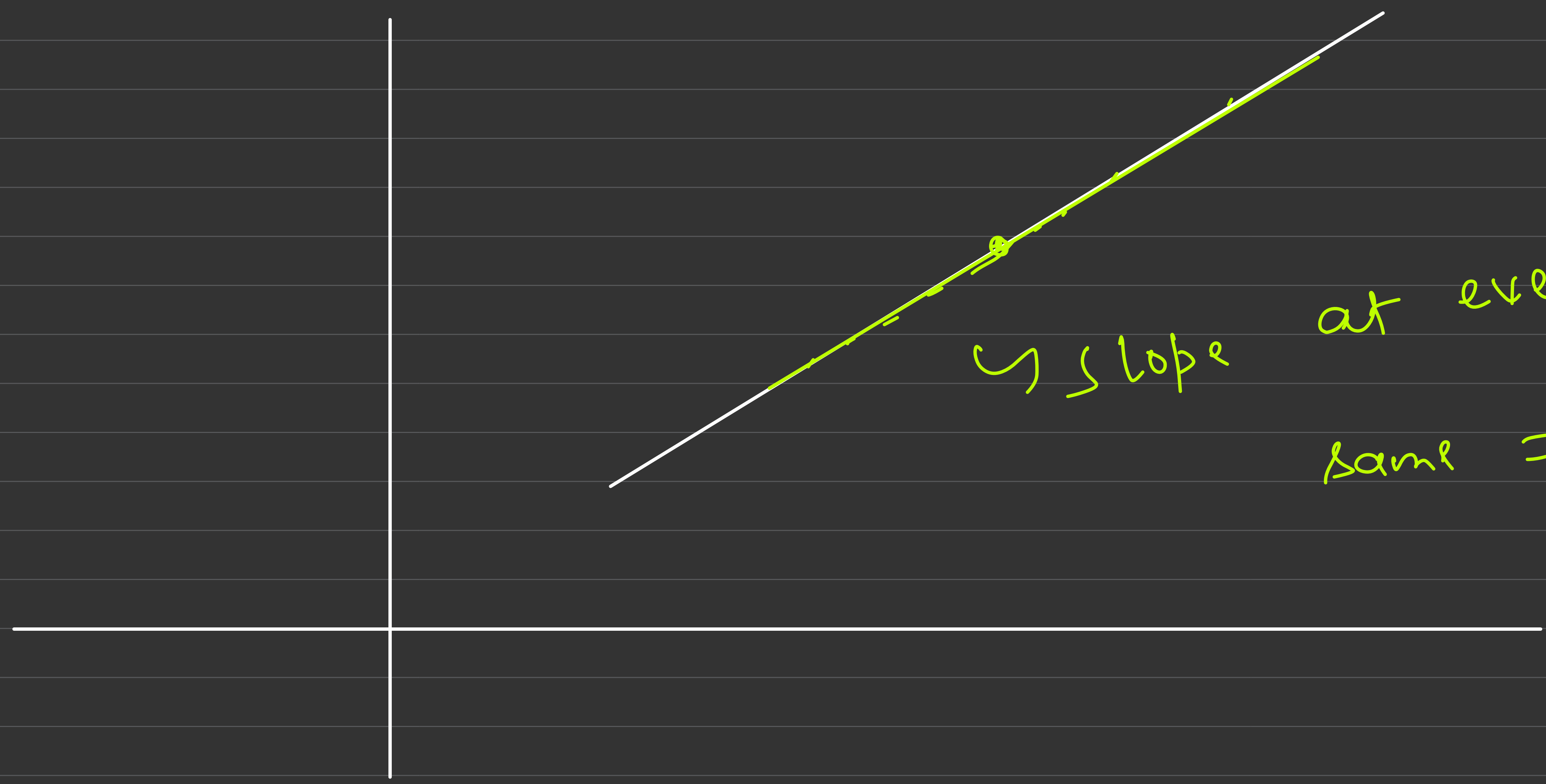




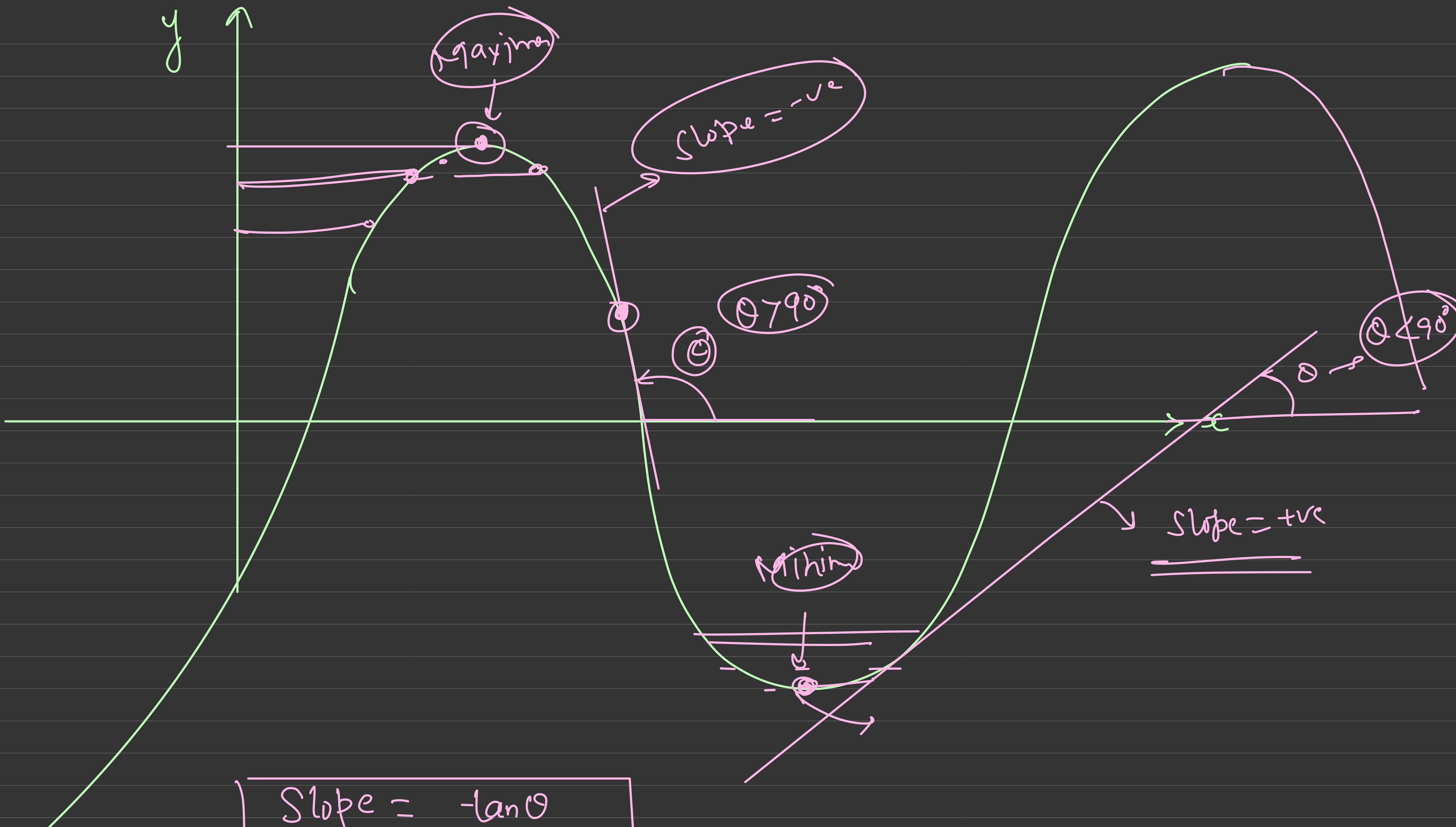
Point	Slope	Point	Slope	Point	Slope
A	−ve	D	−ve	G	+ve
B	0	E	−ve		
C	0	F	0		

** for maxima/ minima $\Rightarrow \dfrac{dy}{dx} = 0$ (Slope = 0)

=x=



↳ slope at every point is same ⇒ Slope = const



$$\boxed{\text{Slope} = \tan\theta}$$

Angle made by tangent from +ve x-axis

Maxima / Minima

$\Downarrow$ $\Downarrow$

Local Local

→ The value of y is greater than its neighbouring value

For $\underbrace{\text{maxima / minima}}$ [***]

$\Downarrow$

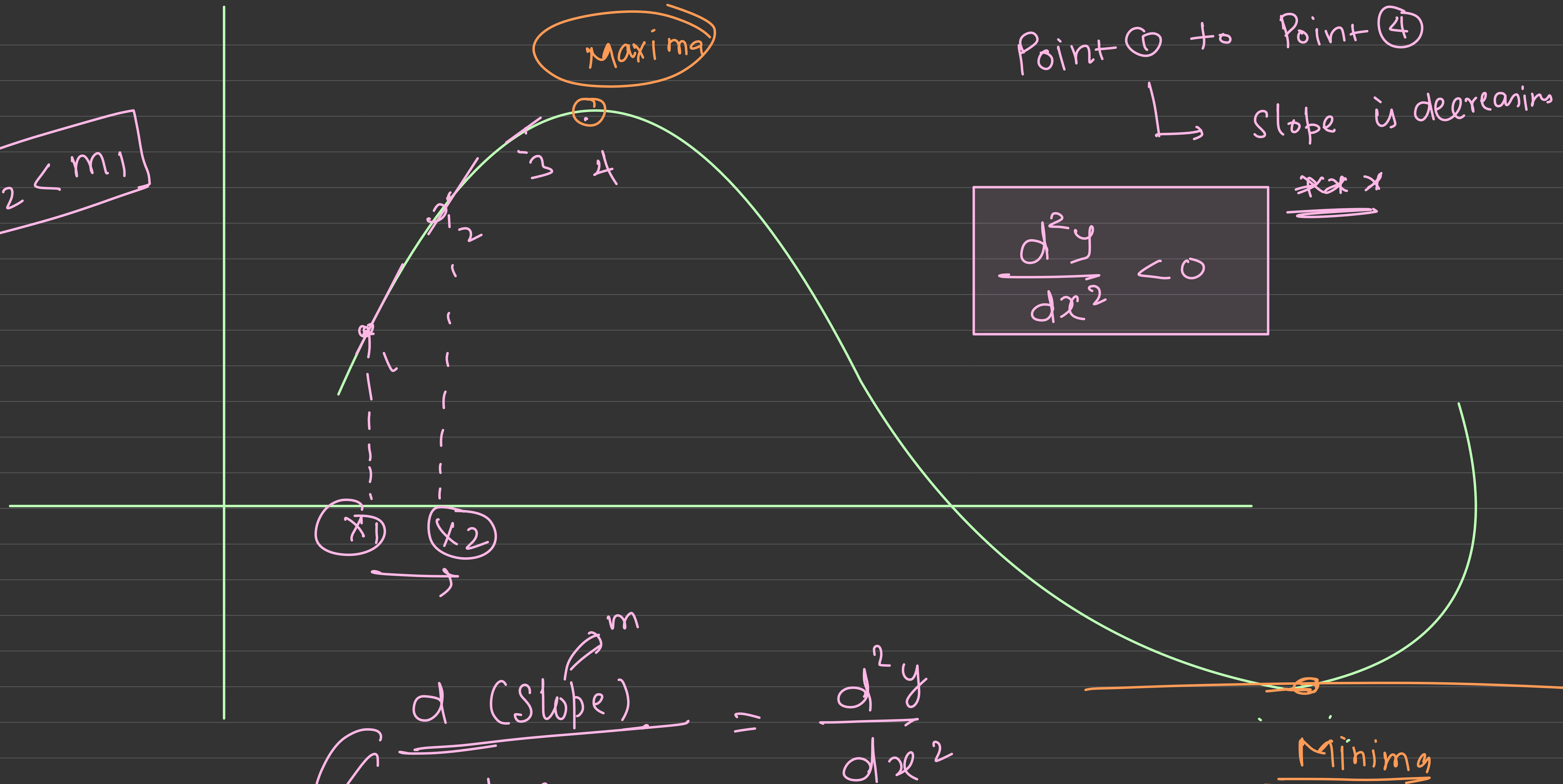
$$\frac{dy}{dx} = 0$$

$$\boxed{m_2 < m_1}$$

Point ① to Point ④

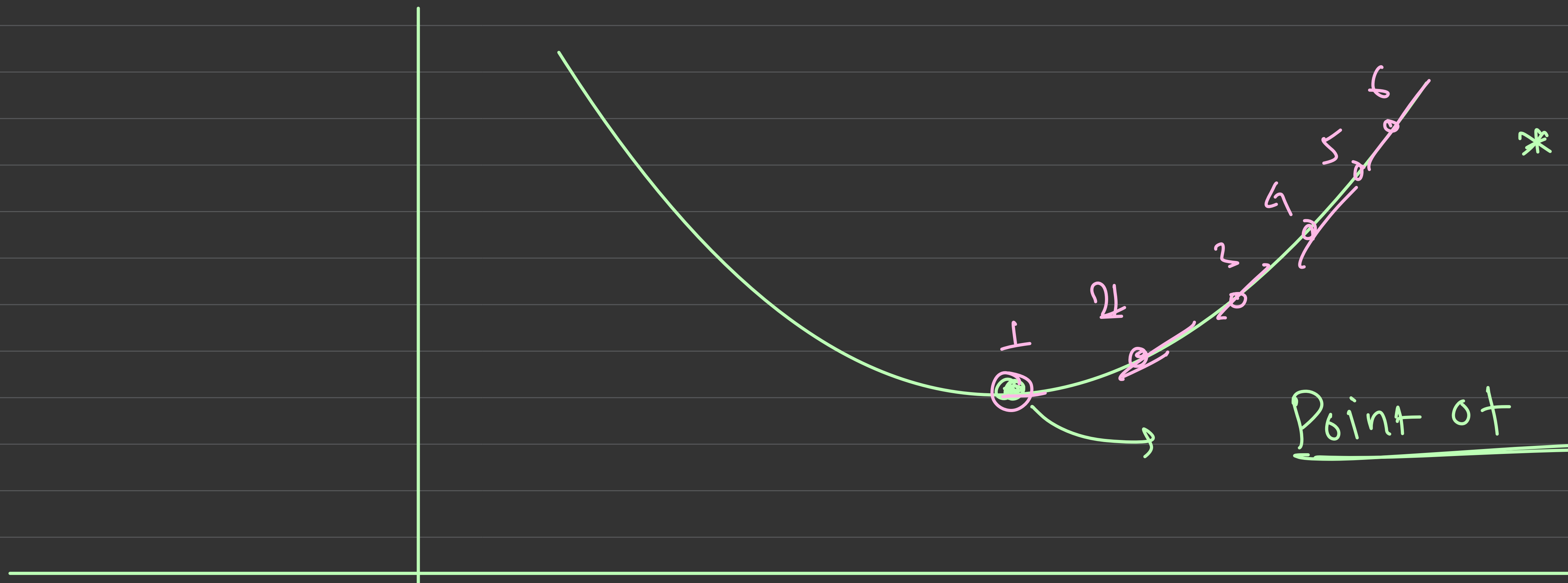
↳ slope is decreasing

$$\boxed{\frac{d^2y}{dx^2} < 0}$$



$$\frac{d\,(\overset{m}{\text{slope}})}{dx} = \frac{d^2y}{dx^2}$$

$$\frac{\Delta m}{\Delta x} = \frac{m_2 - m_1}{x_2 - x_1} < 0$$



* Slope is increasing

Point of minima

$$\frac{d^2y}{dx^2} = \frac{\Delta m}{\Delta x} = \frac{m_2 - m_1}{x_2 - x_1} = +ve$$

****

- For maxima; $\left.\frac{dy}{dx}\right|_P = 0$ and $\left.\frac{d^2y}{dx^2}\right|_P < 0$

  at Point 'P'

- For minima $\left.\frac{dy}{dx}\right|_P = 0$ and $\left.\frac{d^2y}{dx^2}\right|_P > 0$

  at point 'P' :-

Qu:- $x = t^3 - 3t^2 + 4$ find the position of maxima and minima.

Soln:- for maxima or minima, $\frac{dx}{dt} = 0$

$$\frac{dx}{dt} = 3t^2 - 6t$$

$$3t^2 - 6t = 0$$

$$3t(t-2) = 0$$

$$\frac{d^2x}{dt^2} = 6t - 6$$

$t = 0$ → Maxima

$t = 2$ → Minima

At $t=0$,

$$d^2x/dt^2 = -6 < 0$$

At $t=2$

$$\frac{d^2x}{dt^2} = 6 > 0$$

Qu:- $y = A\,x(2-x)$ $A = \text{Const}$

find the maximum value of y.

$$y = A(2x - x^2)$$

$$\frac{dy}{dx} = A(2 - 2x)$$

for maxima, $\frac{dy}{dx} = 0$

$$A\left[x(-1) + (2-x)(1)\right] = 0$$

$$\Rightarrow -x + 2 - x = 0$$

$$\boxed{x = 1}$$

$$\therefore y_{max} = A(1)(2-1) = \underline{A}$$

Qu:- $Z = A\sin\theta + B.\cos\theta$, Calculate Z_{max}.

$$\boxed{Z = f(\theta)}$$

for maxima, $\frac{dz}{d\theta} = 0$

$$A\cos\theta - B\sin\theta = 0 \Rightarrow \tan\theta = A/B$$

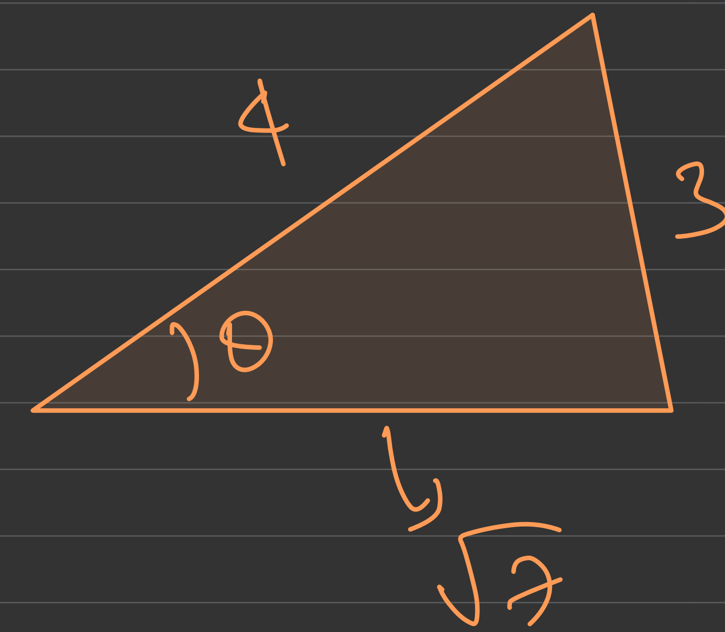
$$\sin\theta = \frac{A}{\sqrt{A^2+B^2}}$$

$$\cos\theta = \frac{B}{\sqrt{A^2+B^2}}$$

$$\therefore Z_{max} = A\frac{A}{\sqrt{A^2+B^2}} + B \cdot \frac{B}{\sqrt{A^2+B^2}}$$

$$= \sqrt{A^2+B^2}$$

$\sin\theta = 3/4$



$\tan\theta = 3/\sqrt{7}$

$\tan\theta = A/B$ $\quad \sqrt{A^2+B^2}$

$\sin\theta = \dfrac{A}{\sqrt{A^2+B^2}}$

Qu: $f(x) = 3x^2 - 4x$ find $f'(1)$ and $f''(0)$

$$f'(x) = \frac{d\,f(x)}{dx}$$

$$f''(x) = \frac{d^2 f(x)}{dx^2}$$

$$f'(x) = 6x - 4$$

$$f''(x) = 6$$

$$f'(1) = 2$$

$$f''(0) = 6$$

Qu:: $y^2 + x^2 = a^2$, $a =$ const.

find $\left.\dfrac{dy}{dx}\right|_{x=2}$

Sol^n:

$$y^2 + x^2 = a^2$$

$$\frac{d}{dx}(y^2) = \frac{d}{dy}(y^2)\cdot\frac{dy}{dx} = 2y\cdot\frac{dy}{dx}$$

$$2y\cdot\frac{dy}{dx} + 2x = 0$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

At $x=2$

$$y^2 = a^2 - 4$$

$$y = \pm\sqrt{a^2-4}$$

$$\left.\frac{dy}{dx}\right|_{x=2} = \frac{-2}{\sqrt{a^2-4}} \;\Big|\; \frac{2}{\sqrt{a^2-4}}$$

Qu:. $PV = C$ find the $\frac{dP}{dV}$ (Slope of P-V graph)

$\boxed{T = \text{const}}$

$PV = C$

$P \cdot \cancel{\frac{dV}{dV}}^{1} + V\frac{dP}{dV} = 0$

$\Rightarrow \boxed{\frac{dP}{dV} = -P/V}$

$P = \frac{C}{V} = CV^{-1}$

$\frac{dP}{dV} = C(-1)\,V^{-2}$

$= \frac{-C}{V^2}$

$= -\frac{P}{V}$

Qu:

$PV^\gamma = \text{const}$ $\qquad \gamma = \text{const}$, Calculate slope of

P–V– curve.

Soln:

$$PV^\gamma = C$$

$$P.\frac{d}{dV}(V^\gamma) + V^\gamma . \frac{dP}{dV} = 0$$

$$\Rightarrow \gamma P V^{\gamma-1} + V^\gamma . \frac{dP}{dV} = 0$$

$$\frac{dP}{dV} = -\frac{\gamma . P V^{\gamma-1}}{V^\gamma} = -\gamma \frac{P}{V}$$

$$P = \frac{C}{V^\gamma}$$

$$\frac{dP}{dV} = \frac{-\gamma C}{V^{\gamma+1}}$$

$$= \frac{-\gamma P V^\gamma}{V^{\gamma+1}}$$

$$= \frac{-\gamma P}{V}$$

#

$$y = \sqrt{x + \underbrace{\sqrt{x + \sqrt{x + \cdots \infty}}}_{y}} \qquad \frac{dy}{dx} = ?$$

$$y = \sqrt{x+y}$$

$$\Rightarrow \quad y^2 = x + y$$

Diffrentiating on both sides w.r.t. 'x'

$$\Rightarrow \quad 2y \cdot \frac{dy}{dx} = 1 + \frac{dy}{dx}$$

$$\Rightarrow \quad \boxed{\frac{dy}{dx} = \frac{1}{2y - 1}}$$

Qu: $x = a\cos\theta$, $y = a\sin\theta$, find $dy/dx = ?$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{a\cos\theta}{-a\sin\theta} = -\cot\theta$$

$$\boxed{\left(x + \frac{1}{x}\right) \geq 2 \quad \text{or} \quad \left(x + \frac{1}{x}\right) \leq -2}$$

$x = 1$ $\qquad$ $x = -1$

"Remember"

$$\left[\sin\theta + \frac{1}{\sin\theta}\right]_{\min} = 2 \qquad \boxed{0^\circ \leq \theta \leq 90^\circ}$$

$\sin\theta = 1 \Rightarrow \theta = 90^\circ$

Integration $\int$ (Indefinite Integration)

- Reverse Process of Differentiation
- Antiderivative

Differention

1) $\frac{d}{dx}(x^n) = n x^{n-1}$

2) $\frac{d}{dx}(c\,y) = c\frac{dy}{dx}$

3) $\frac{d}{dx}(\sin x) = \cos x$

4) $\frac{d}{dx}(\cos x) = -\sin x$

Integration

$\int n x^{n-1}\,dx = x^n + c$

or $\int x^{n-1}\,dx = \frac{x^n}{n} + c$

$$\frac{d}{dx}(\sin x) = \cos x$$

"Constant of Integration"

$$\int \cos x \, dx = \sin x + C$$

$$\frac{d}{dx}(\underbrace{\sin x + C}_{\Downarrow}) = \cos x$$

const

$$\frac{d}{dx}(\sin x) + \frac{dc}{dx} \to 0$$

$$\int x^{n-1}\,dx = \frac{x^n}{n} + C$$

*

$$\boxed{\int x^n\,dx = \frac{x^{n+1}}{n+1} + C}$$

$$\frac{d\left(x^{n+1}\right)}{dx} = (n+1)\,x^n$$

$$\Rightarrow \int \underline{(n+1)}\,x^n\,dx = x^{n+1} + C$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$$

$$\frac{d(x^n)}{dx} = n\,x^{n-1}$$

$$\boxed{\int x^n \, dx = \frac{x^{n+1}}{n+1} + C}$$

1) $\int x^{n}\,dx = \frac{x^{n+1}}{n+1} + C$; $\boxed{n \neq -1}$

2) $\int \cos x\,dx = \sin x + c.$

$\frac{d}{dx}(\sin x) = \cos x$

$\int -\sin x\,dx = \cos x + c'$

\# $\frac{d}{dx}(\cos x) = -\sin x$

$\int \sin x\,dx = -\cos x + c$

3. $\boxed{\int \sin x\,dx = -\cos x + C}$

4. $\boxed{\int \sec^2 x\,dx = \tan x + c}$

$\frac{d}{dx}(\tan x) = \sec^2 x$

5. $\boxed{\int \sec x \cdot \tan x = \sec x + c}$

$\frac{d}{dx}(\sec x) = \sec x \cdot \tan x$

6. $\int \operatorname{cosec}x \cdot \cot x \, dx = -\operatorname{cosec}x + c$

$\frac{d}{dx}(\operatorname{cosec}x) = -\operatorname{cosec}x \cdot \cot x$

7. $\int \operatorname{cosec}^2 x \, dx = -\cot x + c$

$\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$

8. $\int e^x \, dx = e^x + c$

$\frac{d}{dx}(e^x) = e^x$

**

$\boxed{9. \int \frac{1}{x} dx = \ln x + C}$

$\frac{d}{dx}(\ln x) = \frac{1}{x}$

** 10. $\int \frac{1}{1+x^2} dx = \tan^{-1}x + c$

$\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$

11. $\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1}x + c$

$\frac{d}{dx}(\sin^{-1}x) = \frac{+1}{\sqrt{1-x^2}}$

12 $\int \frac{-1}{\sqrt{1-x^2}} dx = \cos^{-1}x + c$

$\frac{d}{dx}(\cos^{-1}x) = \frac{-1}{\sqrt{1-x^2}}$

13. If $y = u \pm v \pm w$

$$\int y\,dx = \int u\,dx \pm \int v\,dx \pm \int w\,dx$$

14. $$\int f'(x)\,dx = f(x) + C$$

$$f'(x) = \frac{df}{dx}$$

15. $$\boxed{\int f'(ax+b)\,dx = \frac{f(ax+b)}{a} + C}$$

(coefficient of x)

Linear function $\Rightarrow$ Power of $x = 1$

16. $$\boxed{\int a\,dx = ax + C}$$

a = Constant

Ex: 1

$$\int \cos(4x+3)\,dx = \frac{\sin(4x+3)}{4} + C \quad ; \quad \int \cos x = \sin x + C$$

Ex: 2

$$\int e^{3x+5}\,dx = \frac{e^{3x+5}}{3} + C$$

Ex:

$$\int \frac{1}{3x+4}\,dx = \frac{\ln(3x+4)}{3} + C$$

Qu:

$$\int \frac{1}{x^2}\,dx = \int x^{-2}\,dx = \frac{x^{-2+1}}{-2+1} + C$$

$$= -\frac{1}{x} + C$$

Qu: $\int \sqrt{x}\, dx = \int x^{\frac{1}{2}}\, dx = \dfrac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + c$

$= \dfrac{2}{3} \cdot x^{3/2} + c$

Qu: $\int (\underline{3x+5})^2\, dx = \dfrac{(3x+5)^3}{3 \times 3} + c$

Qu: $\int \left(x + \dfrac{1}{x}\right)^2 dx = \int \left(x^2 + \dfrac{1}{x^2} + 2\right) dx$

$= \dfrac{x^3}{3} - \dfrac{1}{x} + 2x + c$

Qu: $\int \left(\sqrt{x} + \dfrac{1}{\sqrt{x}}\right)^2 dx = \int \left(x + \dfrac{1}{x} + 2\right) dx$

$= \underline{x^2/2 + \ln x + 2x + c}$

$$\int (3x+5)^2 \, dx = \int \left(9x^2 + \underline{25} + 30x\right) dx$$

$$= \frac{9x^3}{3} + 25x + \frac{30x^2}{2} + C$$

$$\int 25 \, dx =$$

$$25 \int x^0 \, dx = \frac{25x^{0+1}}{0+1} + C$$

$$= 25x + C$$

$\circ \quad \int \underline{\cos^2 x}\, dx =$

$\int \cos x\, dx = \sin x + c$

Linear function of $\cos x$?

$\int \cos 2x\, dx = \frac{\sin 2x}{2} + C$

$\cos 2x = 2\cos^2 x - 1$

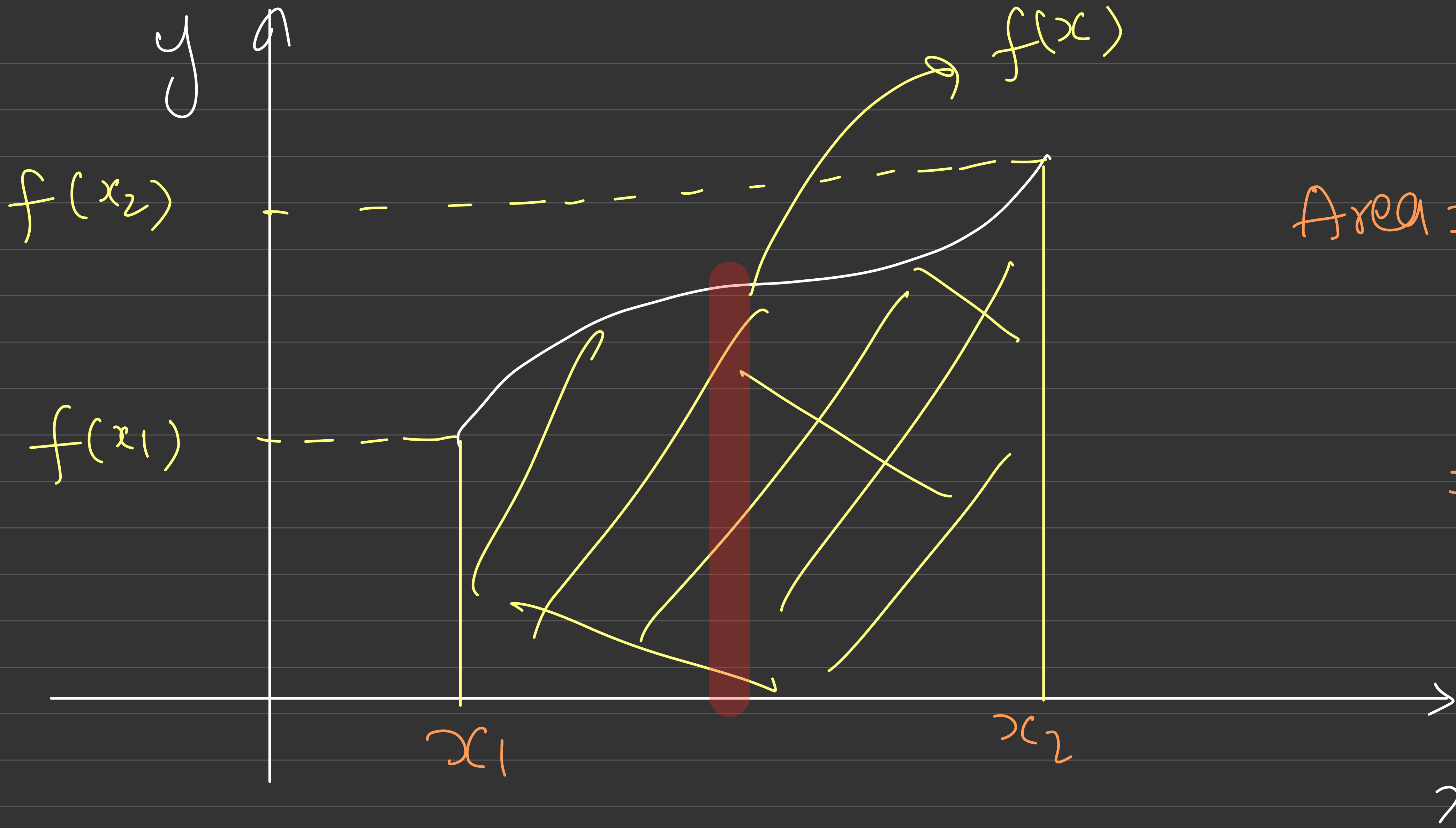
$\Rightarrow \boxed{\cos^2 x = \frac{1 + \cos 2x}{2}}$

$$\int \left(\frac{1 + \cos 2x}{2} \right) dx = \int \frac{1}{2}\, dx + \int \frac{\cos 2x}{2}\, dx$$

$$= \frac{x}{2} + \frac{\sin 2x}{2 \times 2} + c$$

$$\int \sin^2 x \, dx = \int \left(\frac{1 - \cos 2x}{2} \right) dx$$

$$= \frac{x}{2} - \frac{\sin 2x}{4} + C$$



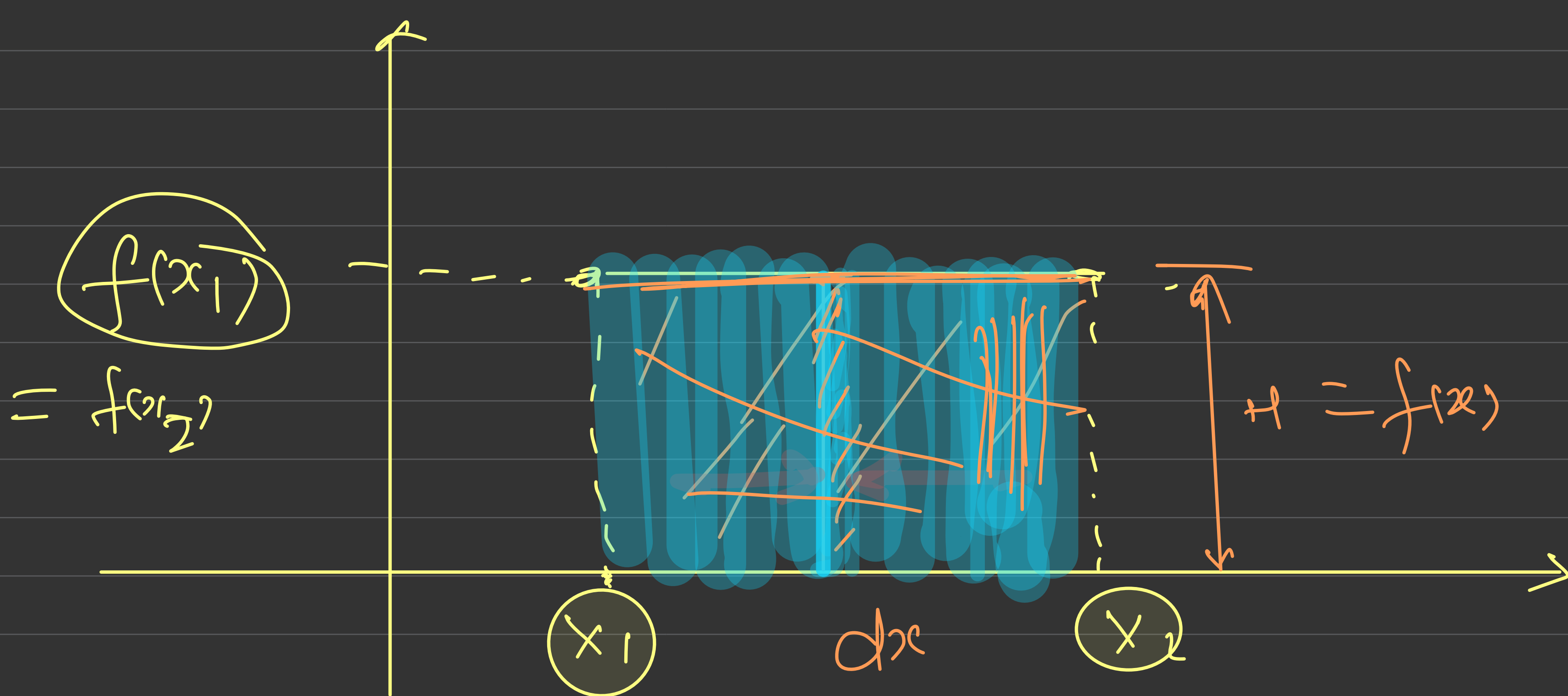
Area = $\displaystyle\int_{x_1}^{x_2} f(x)\,dx$

$= \Big[F(x)\Big]_{x_1}^{x_2}$

$= F(x_2) - F(x_1)$

elemental area

$= f(x)\,dx$



Total area under the curve

$= \displaystyle\int_{x_1}^{x_2} f(x)\,dx.$

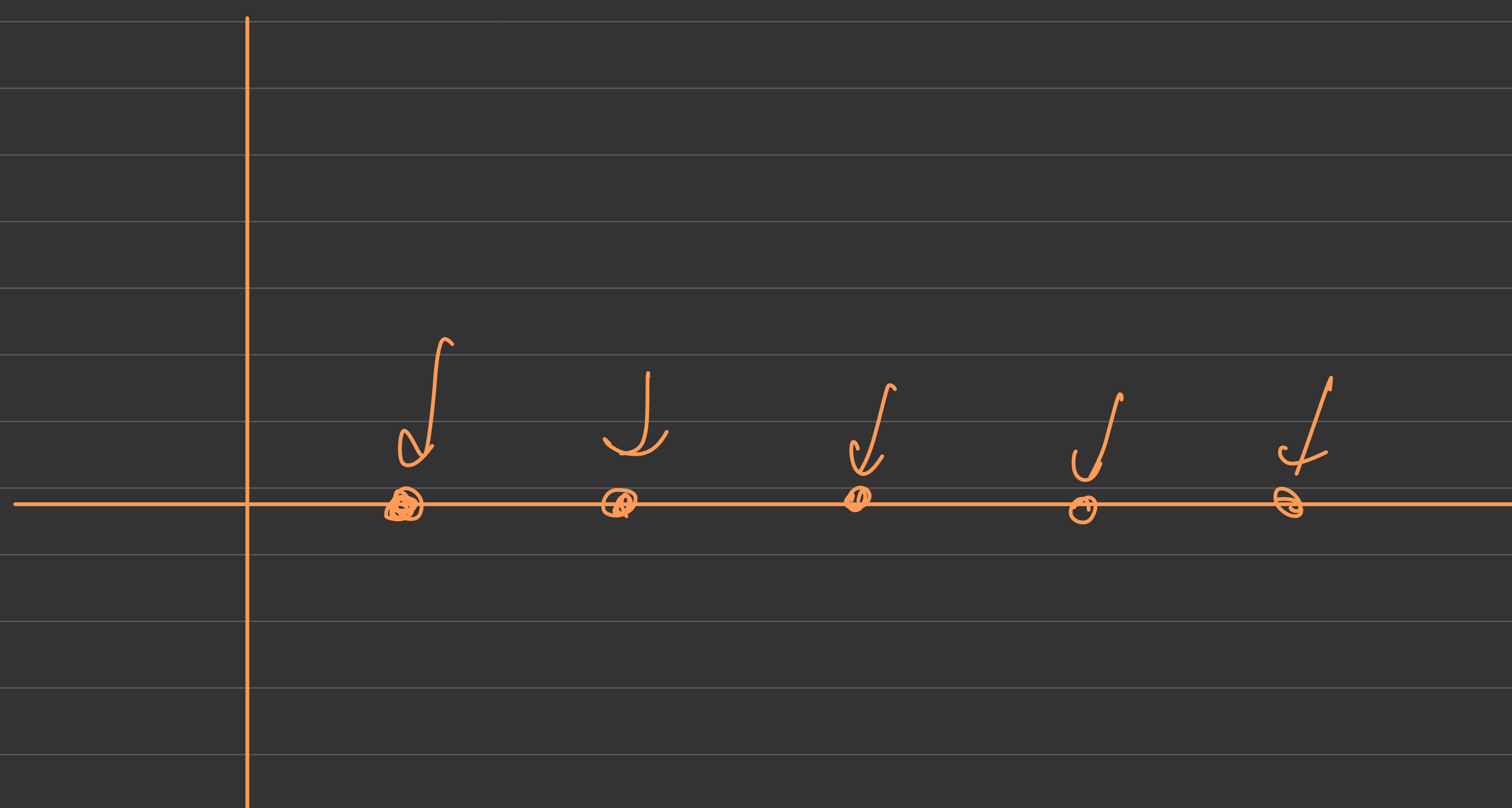
- Adding all these elemental area = Total area under the curve

upper limit

lower limit

$$1+2+3+4+5+ \cdots +10 = \sum_{n=1}^{n=10} n$$

discreate summation



Summation

For conituous summation of any-element

↳ <u>we use integration</u>

upper limit of integration

$$\int_{x_1}^{x_2} f(x)\, dx$$

lower limit of integration

Constant

$$= \int_{x_1}^{x_2} f(x_1)\, dx = f(x_1) \int_{x_1}^{x_2} dx$$

$$= f(x_1)\,[x]_{x_1}^{x_2}$$

Area $= f(x) \left[x \right]_{x_1}^{x_2}$

$= f(x_1) \left[x_2 - x_1 \right]$

$= (x_2 - x_1) \, f(x_1)$

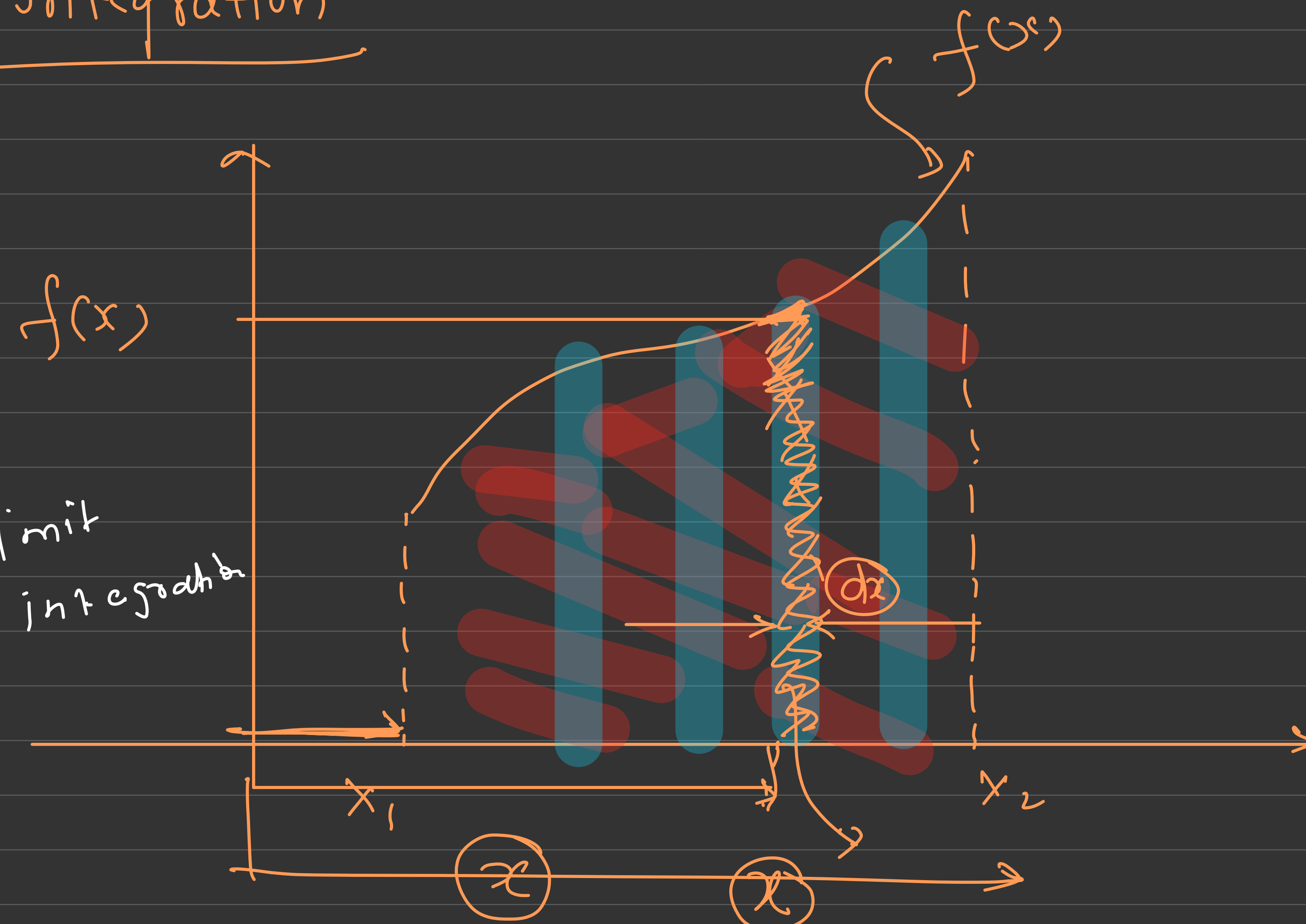
Definite Integration

- Area of elemental strip $= f(x)\,dx$

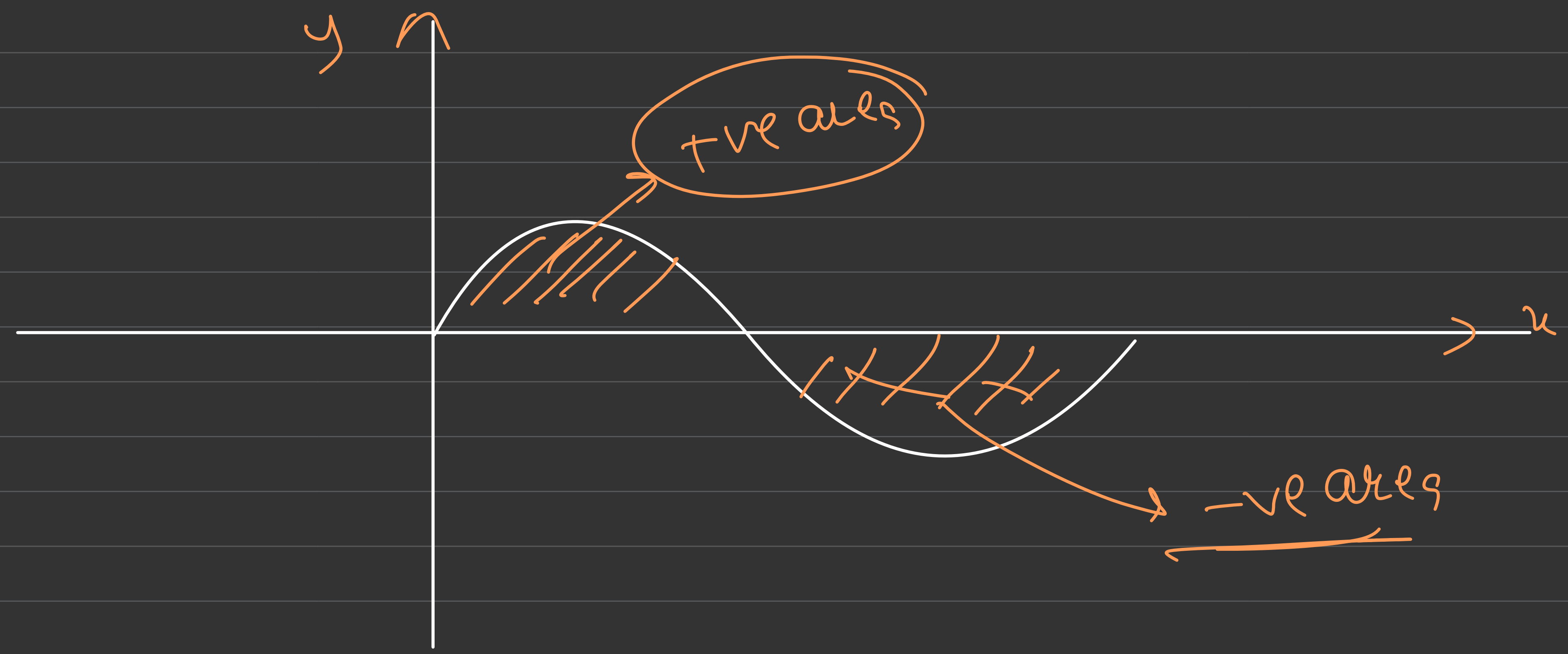
- Area under the curve $= \int_{x_1}^{x_2} f(x)\,dx$

x_2 — upper limit of integration

x_1 — lower limit of integration



- Definite integration gives area under the curve.



area above x- axis → +ve

" below " → -ve

$$I = \int_{-2}^{+4} x^2\,dx = \left[\frac{x^3}{3} + c\right]_{x=-2}^{x=+4}$$

$$= \left(\frac{64}{3} + \cancel{c}\right) - \left(-\frac{8}{3} + \cancel{c}\right)$$

$$= \frac{72}{3} = \underline{24}$$

$$= \left[\frac{x^3}{3}\right]_{-2}^{+4} = \frac{64}{3} - \left(-\frac{8}{3}\right) = \frac{72}{3}$$

Note: In definite integration we can avoid writing constant of integration.

Qu!. $\displaystyle\int_{-1}^{+1} x^3\,dx = \left[\frac{x^4}{4}\right]_{x=-1}^{x=1} = \left(\frac{1}{4}\right) - \left(\frac{1}{4}\right) = 0$

Qu: $\displaystyle\int_{0}^{4} \sqrt{x}\,dx = \left[\frac{2}{3}x^{3/2}\right]_0^4$

$= \frac{2}{3}\left[(4)^{3/2} - (0)^{3/2}\right]$

$= \frac{2}{3}(8) = \frac{16}{3}$

$\displaystyle\int \sqrt{x}\,dx =$

$\displaystyle\int x^{1/2}\,dx = \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}$

$= \frac{2}{3}\,x^{3/2}$

$(2^{\cancel{2}})^{3/\cancel{2}}$

Qu! $\displaystyle\int_{0}^{2\pi} \sin x\,dx$

Qu1 $\displaystyle\int_{0}^{\pi} \sin x\,dx$

Qu: $$\int_0^{2\pi} \sin x \, dx = \Big[-\cos x\Big]_0^{2\pi}$$

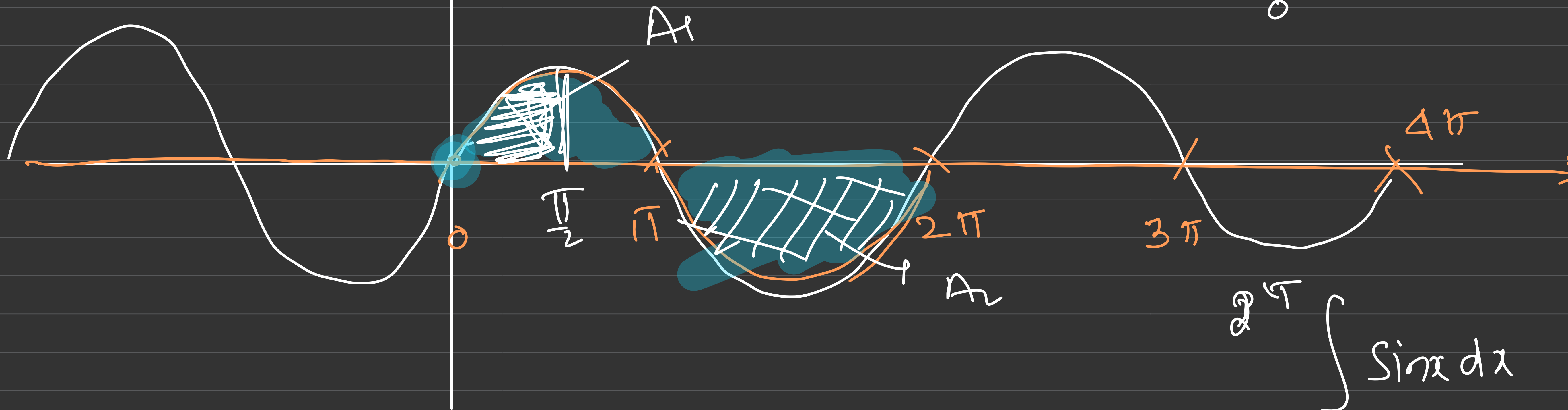
$$= -\Big[\cos 2\pi - \cos 0\Big]$$

$$= -\Big[1 - 1\Big] = 0$$

Qu: $$\int_0^{\pi} \sin x \quad dx = \Big[-\cos x\Big]_0^{\pi}$$

$$= -\Big[\cos \pi - \cos 0^\circ\Big]$$

$$= -\Big[-1 - 1\Big] = \underline{\underline{2}}$$



$$\int_0^{\pi} \sin x \, dx = 2$$

$$\int_{\pi}^{2\pi} \sin x \, dx = -2$$

$$\int_0^{2\pi} \sin x \, dx = A_1 + A_2$$

$$= A_1 + (-A_1) = 0$$

area of 1 loop of $\underline{\sin x \,/\, \cos x} = 2$ unit

$$\int_0^{\frac{\pi}{2}} \sin x \, dx = 1$$

$$\int_0^{\frac{\pi}{4}} \sin x \, dx = \Big[-\cos x\Big]_0^{\pi/4}$$

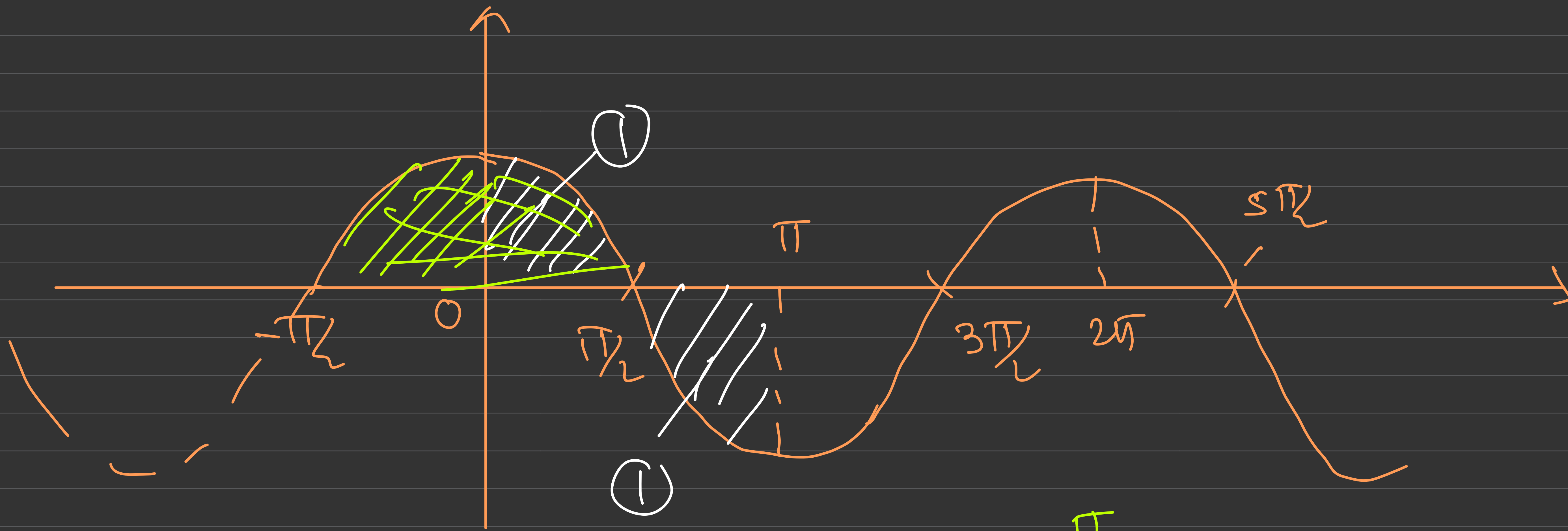
$$= -\left[\cos\frac{\pi}{4} - \cos 0^\circ\right]$$

$$= -\left[\frac{1}{\sqrt{2}} - 1\right]$$

$$= 1 - \frac{1}{\sqrt{2}} = \frac{\sqrt{2}-1}{\sqrt{2}}$$

$$\int_0^{\pi} \cos x \, dx = 0$$

$$\int_0^{2\pi} \cos x \, dx = 0$$



$$\int_{-\pi/2}^{+\pi/2} \cos x \, dx = 2$$

$$\int_{\pi/2}^{\pi} \cos x \, dx = -1$$

\# $\displaystyle\int_0^1 \frac{1}{1+x^2}\,dx = \left[\tan^{-1} x\right]_0^1$

$= \tan^{-1}(1) - \tan^{-1}(0)$

$= \frac{\pi}{4} - 0$

$= \pi/4$ Ans

$$\left.\begin{aligned} &\tan^{-1}(1) = \theta \\ \Rightarrow\ &\tan\theta = 1 \\ \Rightarrow\ &\boxed{\theta = \frac{\pi}{4}} \end{aligned}\right\}$$

Qu: $\int_0^{1/2} \frac{dx}{\sqrt{1-x^2}} = \left[\sin^{-1}x\right]_0^{1/2}$

$= \sin^{-1}(1/2) - \sin^{-1}(0)$

$= \frac{\pi}{6} - 0 = \frac{\pi}{6}$

$\sin^{-1}(1/2) = \theta$

$\Rightarrow \sin\theta = 1/2 \Rightarrow \left(\theta = \frac{\pi}{6}\right)$

Qu:

$$\int_0^1 \frac{dx}{(3x+4)^2} = -\frac{1}{3}\left[\frac{1}{3x+4}\right]_0^1$$

$$= -\frac{1}{3}\left[\frac{1}{7} - \frac{1}{4}\right]$$

$$= \frac{1}{28}$$

$$\boxed{\int (3x+4)^{-2}\,dx = \frac{(3x+4)^{-2+1}}{(-2+1)\times 3} = -\frac{1}{3}\left[\frac{1}{3x+4}\right]}$$

$$\int \frac{1}{(3x+4)^2}\,dx = \int (3x+4)^{-2}\,dx$$

$$= \frac{(3x+4)^{-2+1}}{(-2+1)\times 3}$$

$$\int x^2\,dx$$

$$x^3/3$$

Qu:

$$\int_0^1 \frac{dx}{\sqrt{3x+4}} = \frac{2}{3}\left[(3x+4)^{1/2}\right]_0^1$$

$$= \frac{2}{3}\left[\sqrt{7} - 2\right]$$

$$= \frac{2\sqrt{7} - 4}{3}$$

$$\int_0 \frac{1}{\sqrt{3x+4}}\, dx$$

$$= \int (3x+4)^{-1/2}\, dx$$

$$= \frac{(3x+4)^{-1/2+1}}{(-1/2+1) \times 3}$$

$$= \frac{2}{3}(3x+4)^{1/2}$$

**

Qu: $\int_0^{\frac{\pi}{2}} \cos^2 x \, dx = \left[\frac{x}{2} + \frac{\sin 2x}{4}\right]_0^{\pi/2}$

$= \frac{\pi}{4} + 0$

$\int \cos^2 x \, dx$

$= \int \left(\frac{1+\cos 2x}{2}\right) dx$

$= \frac{x}{2} + \frac{\sin 2x}{2 \times 2} + C$

**

Qu: $\int_0^{\pi} \sin^2 x \, dx$

$= \int_0^{\pi} \left(\frac{1 - \cos 2x}{2}\right) dx$

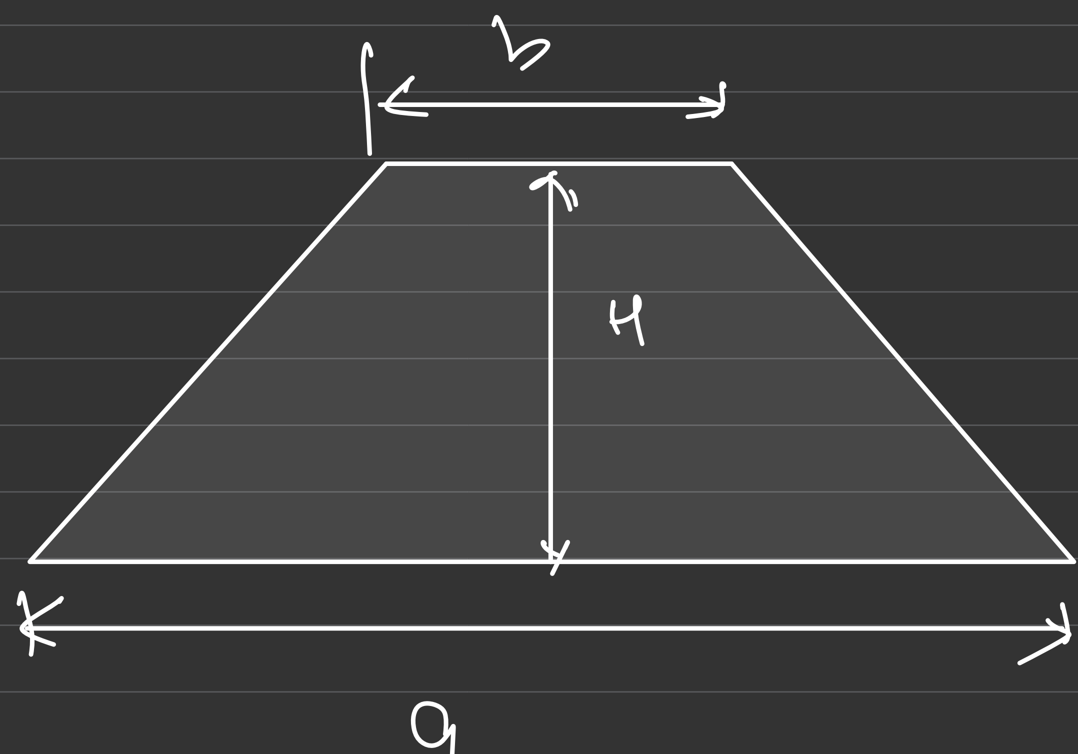
$= \left[\frac{x}{2} - \frac{\sin 2x}{4}\right]_0^{\pi}$

$1 - 2\sin^2 x =$

$\cos 2x = 2\cos^2 x - 1$

$\Rightarrow \cos^2 x = \frac{1+\cos 2x}{2}$

$= \pi/2$



area of trapezium

$$= \frac{1}{2}\left(\text{Sum of parallel sides}\right) \times h$$

$$= \frac{1}{2}(a+b)\,H$$

Qu: $\int \frac{dx}{x(x+2)}$ $\qquad$ $\int \frac{dx}{x} = \ln x$

$= \frac{1}{2} \int \frac{[(x+2) - x]}{(x+2)\, x}\, dx$ $\qquad$ $\frac{(x+2)}{(x+2)\, x} - \frac{x}{(x+2)x}$

$= \frac{1}{2} \int \left(\frac{1}{x} - \frac{1}{x+2} \right) dx$

$= \frac{1}{2} \left[\ln x - \ln (x+2) \right] + C = \frac{1}{2} \ln \left(\frac{x}{x+2} \right) + C$

Qu:- $$\int \frac{dx}{(2x+3)\,x}$$

Soln:-

$$= \frac{1}{3}\int \frac{(2x+3)-(2x)}{(2x+3)\,x}\,dx \qquad = \frac{1}{3}\ln\left(\frac{x}{2x+3}\right)+c$$

$$= \frac{1}{3}\int\left(\frac{1}{x}-\frac{2}{2x+3}\right)dx$$

$$= \frac{1}{3}\left[\ln(x) - \frac{\cancel{2}\,\ln(2x+3)}{\cancel{2}}\right]+c$$

#

$$\left\{\begin{array}{l} v = \text{Velocity} = \text{Rate of change of displacement} = \dfrac{dx}{dt} \\ a = \text{acceleration} = \text{Rate of change of velocity} = \dfrac{dv}{dt} \end{array}\right.$$

$$x \xrightarrow{\text{diff.}} v \xrightarrow{\text{diff.}} a$$

$$\Rightarrow \quad a = \frac{dv}{dt}$$

$$\text{at} \quad \left.\begin{array}{ll} t = 0, & v = u \\ t = t, & v = v \end{array}\right\}$$

$$\Rightarrow \int_u^v dv = \int_0^t a\,dt$$

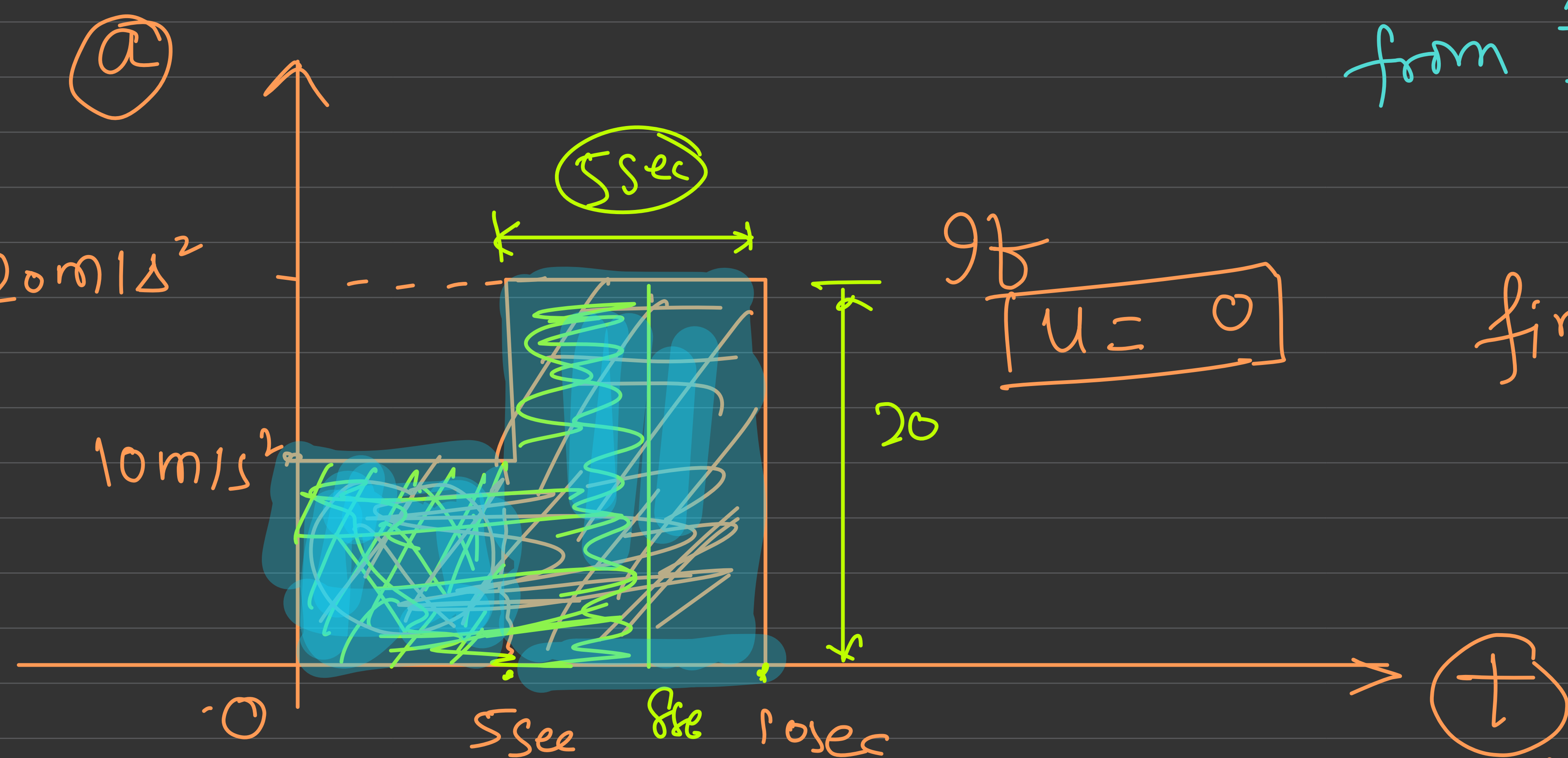
$$\Rightarrow \left[v\right]_u^v = \int_0^t a\,dt$$

$$\Rightarrow (v - u) = \int_0^t a\,dt$$

$(v-u)$ = change in velocity = $\int_0^t a\,dt$

area-under a–t graph from $t=0$ to t sec

Qu:



If $u = 0$

find v at
1. $t = 5$ sec
2. $t = 10$ sec
3. $t = 8$ sec

(1) $(v - \cancel{u}^{\,0}) = 50$

$\Rightarrow \boxed{v(t = 5\text{ sec}) = 50 \text{ m/s}}$

(2) $(v - \cancel{u}_{\,0}) = 150$

$v = 150$ m/s

(3) $v = 110$ m/s

#

$$v = \frac{dx}{dt}$$

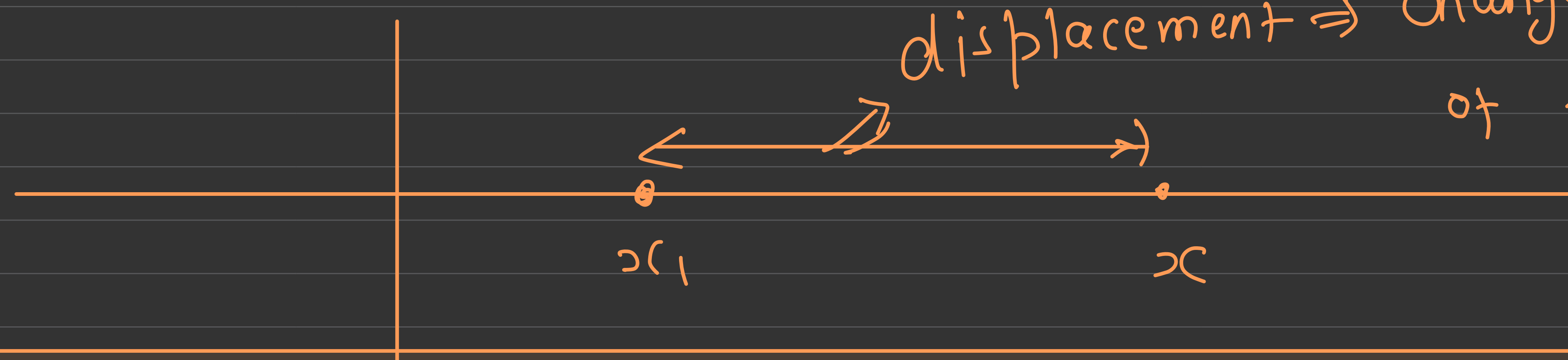
$t = 0, \quad x = x_1$

$t = t, \quad x = x$

$$\int_{x_1}^{x} dx = \int_{0}^{t} v\,dt$$

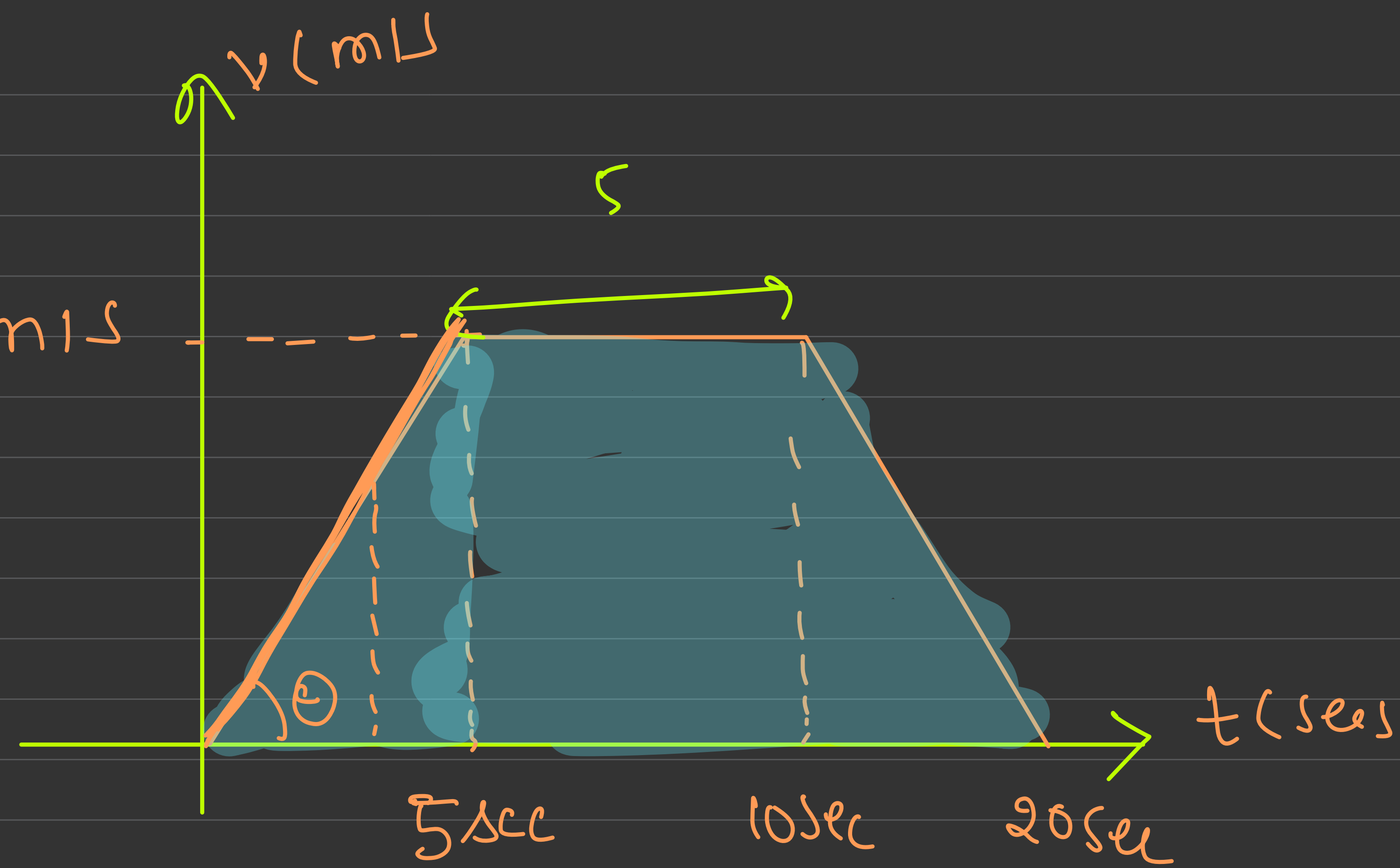
$(x - x_1)$ = area under v–t graph from "$t=0$" to "t"

displacement ⇒ "change in position of particle"



⇒ displacement = area - under v-t graph from $t=0$ $t=t$

Qu!.



$$a = \frac{dv}{dt}$$

If position of particle at $t = 0$ is $x = 0$

Then calculate

① position of particle at $t = 5$ sec and 20 sec

② acceleration of particle at $t = 4$ sec, $t = 7$ sec and $t = 15$ sec

① (a) $x - x_1 = \frac{1}{2} \times 5 \times 10 = 25$

$$\boxed{x = 25}$$

(b) $x - x_i = \frac{1}{2} \times (20 + 5) \times 10$

$$\boxed{x = 125}$$

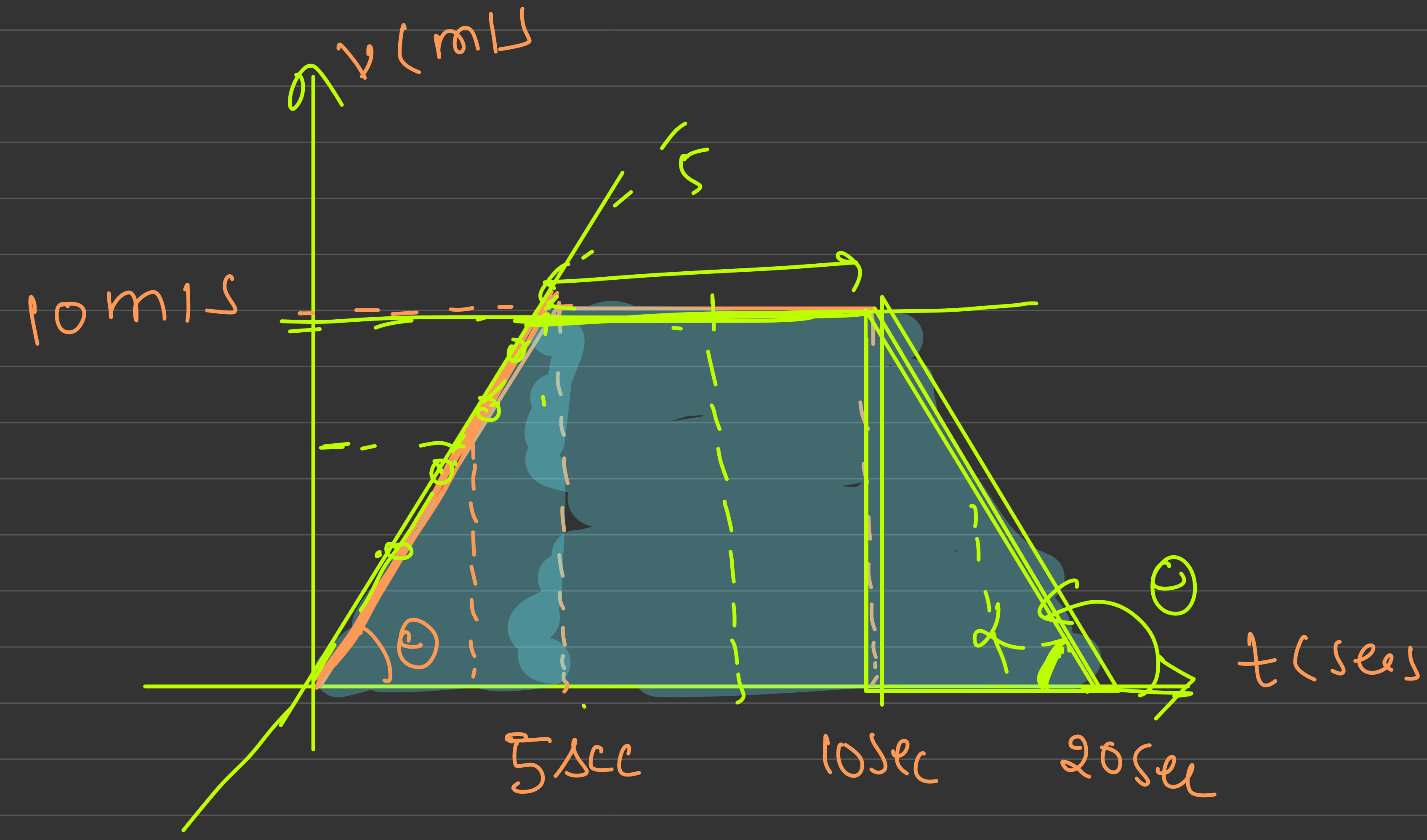
$x_f - x_{t_0}$

2(a)

At $t = 4\,sec$

$$\tan\theta = \frac{\cancel{10}^{2}}{\cancel{5}} = 2$$

$\therefore\; a = 2\,m/s^2$ ✓



At $t = 7\,sec$

Slope = 0

$$\boxed{a = 0}$$

At = 15 sec

$$\tan\alpha = \frac{10}{10} = 1$$

Slope = −1

$$\boxed{a = -1\,m/s^2}$$

Qu:- $a = 4t^3 - 3t^2$ and at $t=0$ $V = 2$ m/s

then calculate velocity at $t = 1$ sec

$$a = \frac{dv}{dt} = 4t^3 - 3t^2$$

$$\int_2^v dv = \int_0^1 (4t^3 - 3t^2)\, dt$$

$$\Rightarrow (v-2) = \left[\frac{\cancel{4}t^4}{\cancel{4}} - \frac{\cancel{3}t^3}{\cancel{3}}\right]_0^1$$

$$v - 2 = 0$$

$$\Rightarrow \boxed{v = 2 \text{ m/s}}$$

Qu:: $v = 3t^2 - 5t + 2$ and at $t=0$ $x = 1m$

then calculate

① x at $t = 2sec$

② a at $t = 0\,sec$ and $t = 1\,sec$

Solⁿ:: (1) $v = \dfrac{dx}{dt}$

$$\Rightarrow \int_1^x dx = \int_{t=0}^{t=2sec} v\,dt$$

$$\Rightarrow (x-1) = \int_0^2 (3t^2 - 5t + 2)\,dt = \left[t^3 - \frac{5t^2}{2} + 2t\right]_0^2$$

$$= \left(8 - \frac{5}{2} \times 4 + 4\right) - 0$$

$$= 2$$

$$\boxed{x = 3m}$$

② $a = \dfrac{dv}{dt} = 6t - 5$

$a(t = 0s) = -5\,m/s^2$

$a(t = 1s) = 6 \times 1 - 5 = 1\,m/s^2$

Integration by Substitution:

$$I = \int x^2 \sin(x^3)\, dx \longrightarrow \int \frac{3x^2}{3} \sin(x^3)\, dx$$

$$\int \sin(x^3)\, dx \quad \times$$

$$\int \sin x\, dx = -\cos x + C$$

Let $x^3 = t$

$$3x^2 = \frac{dt}{dx}$$

$$\Rightarrow \boxed{3x^2\, dx = dt}$$

$$\boxed{x^2\, dx = \frac{dt}{3}}$$

$$I = \int \sin t \left(\frac{dt}{3}\right)$$

$$= \frac{1}{3} \int \sin t\, dt$$

$$= \frac{1}{3}\left[-\cos t\right] + C$$

$$= -\frac{1}{3} \cos(x^3) + C$$

Qu!

$$\int \cos x \, e^{\boxed{\sin x}} \, dx$$

$\cancel{\int e^{\sin x} dx}$

Let $\boxed{\sin x = t}$ $\Rightarrow$ $\boxed{\cos x \, dx = dt}$

$dt/dx = \cos x$

$$I = \int e^{t} \, dt$$

$$= e^{t} + c = e^{\sin x} + c$$

Qu: $I = \int \frac{4y}{\sqrt{3y^2+4}}\,dy$

$$\int \frac{dy}{\sqrt{3y^2+4}}$$

$$3y^2+4 = t$$

$$\underline{\underline{6y\ dy}} = dt$$

$$y\,dy = \frac{dt}{6}$$

$$I = \frac{4}{6}\int \frac{dt}{\sqrt{t}}$$

$$= \frac{4}{\cancel{6}_3} \times \cancel{2}\sqrt{t} + C$$

$$= \frac{4}{3}\sqrt{3y^2+4} + c$$

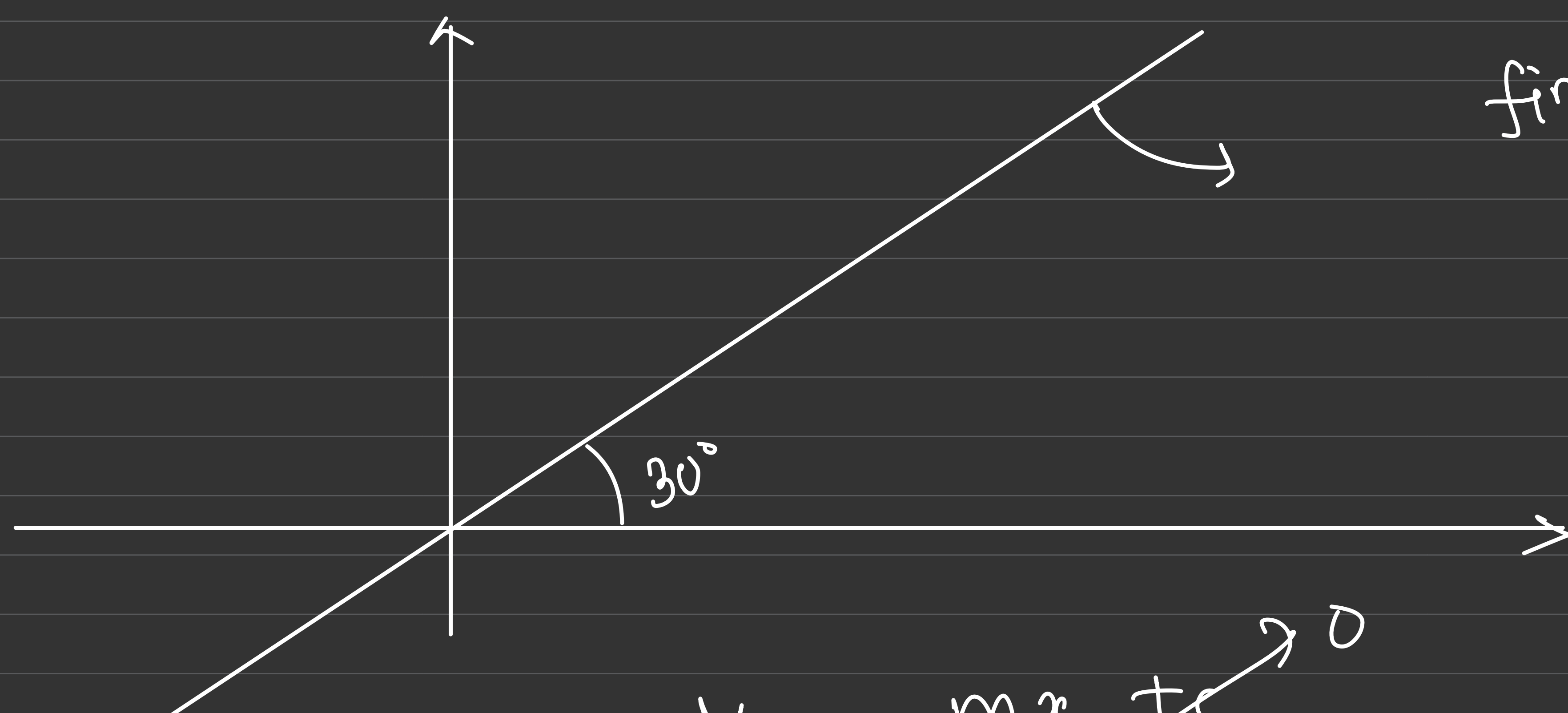
Graph:-

1) Stright- line

$$y = \overset{\text{slope}}{m}x + \overset{\text{Constant of Integration}}{c}$$

$$\frac{dy}{dx} = m = \text{Slope}$$

Que. 1.



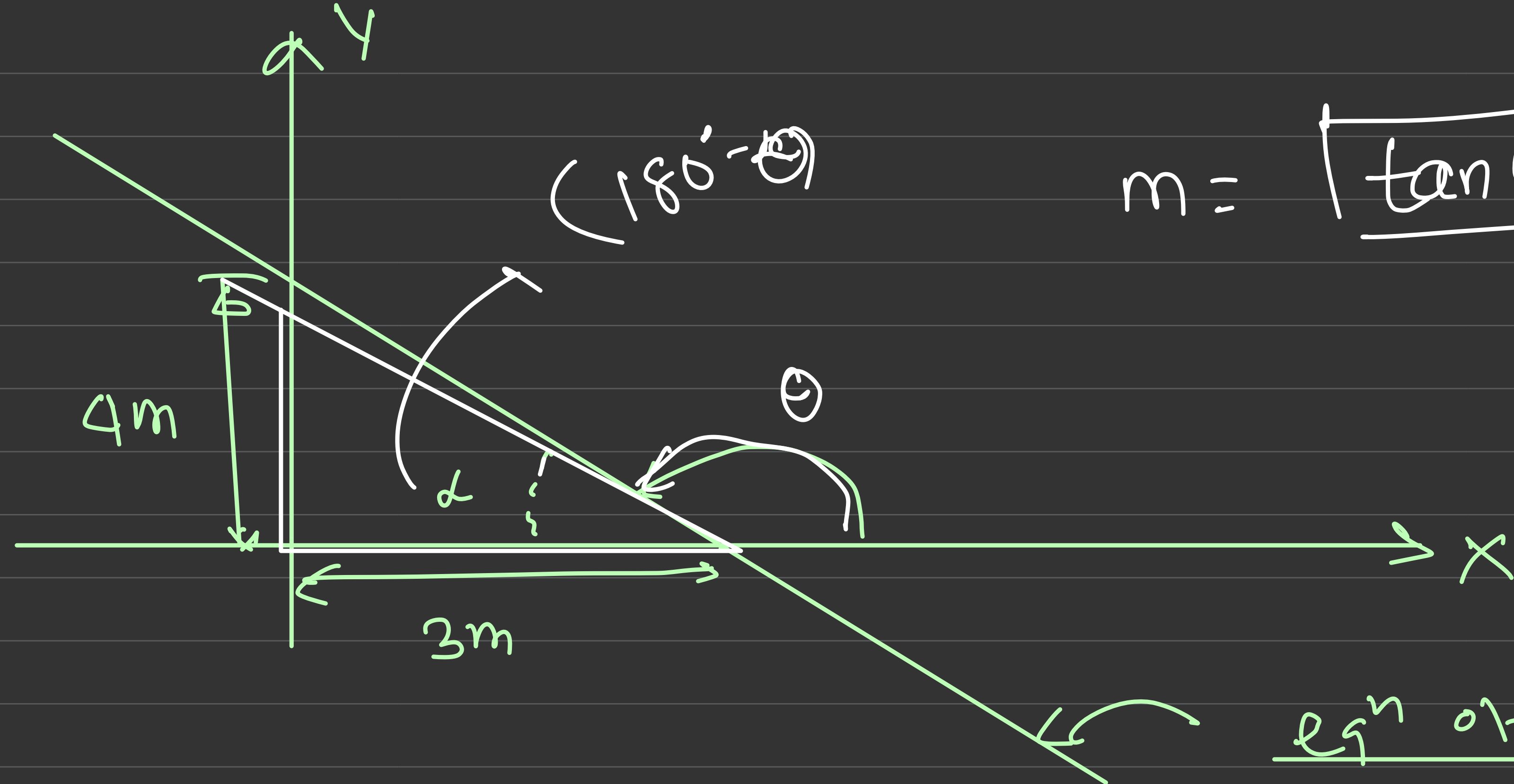
find eqn of stright line

$$m = \text{Slope} = \tan\theta$$
$$= \tan 30^\circ$$
$$= \frac{1}{\sqrt{2}}$$

$$y = mx + \cancel{c} \rightarrow 0$$

$$\boxed{y = \frac{x}{\sqrt{3}}}$$

Qu:



$m = \boxed{\tan\theta}$

$\tan(180^\circ - \theta) = 4/3$

$-\tan\theta = 4/3 \Rightarrow \tan\theta = -4/3$

eqn of straight line

$\tan\alpha = 4/3$

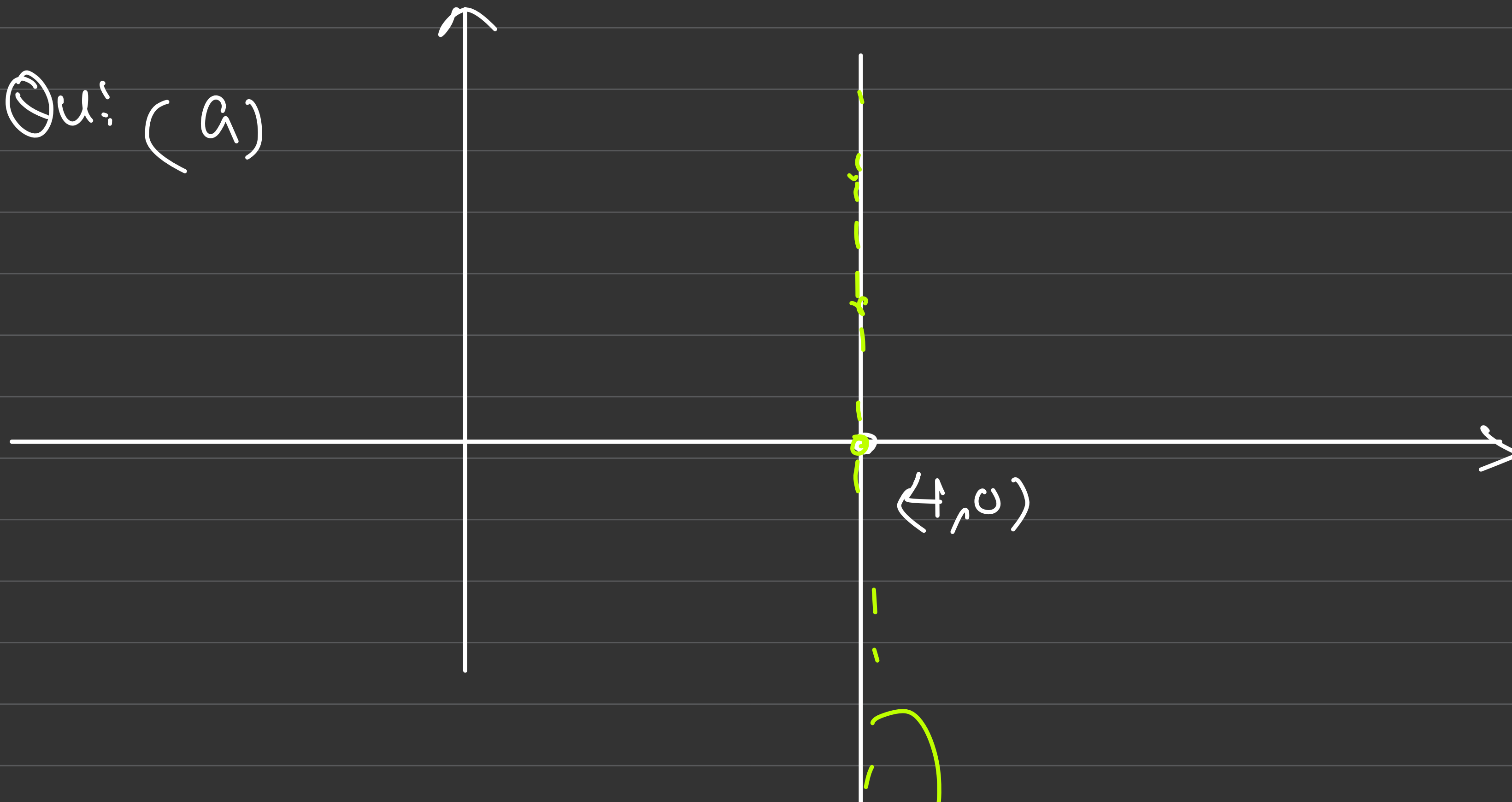
$m = \text{slope} = -ve$

$= -4/3$

$c = y\text{-intercept} = 4$

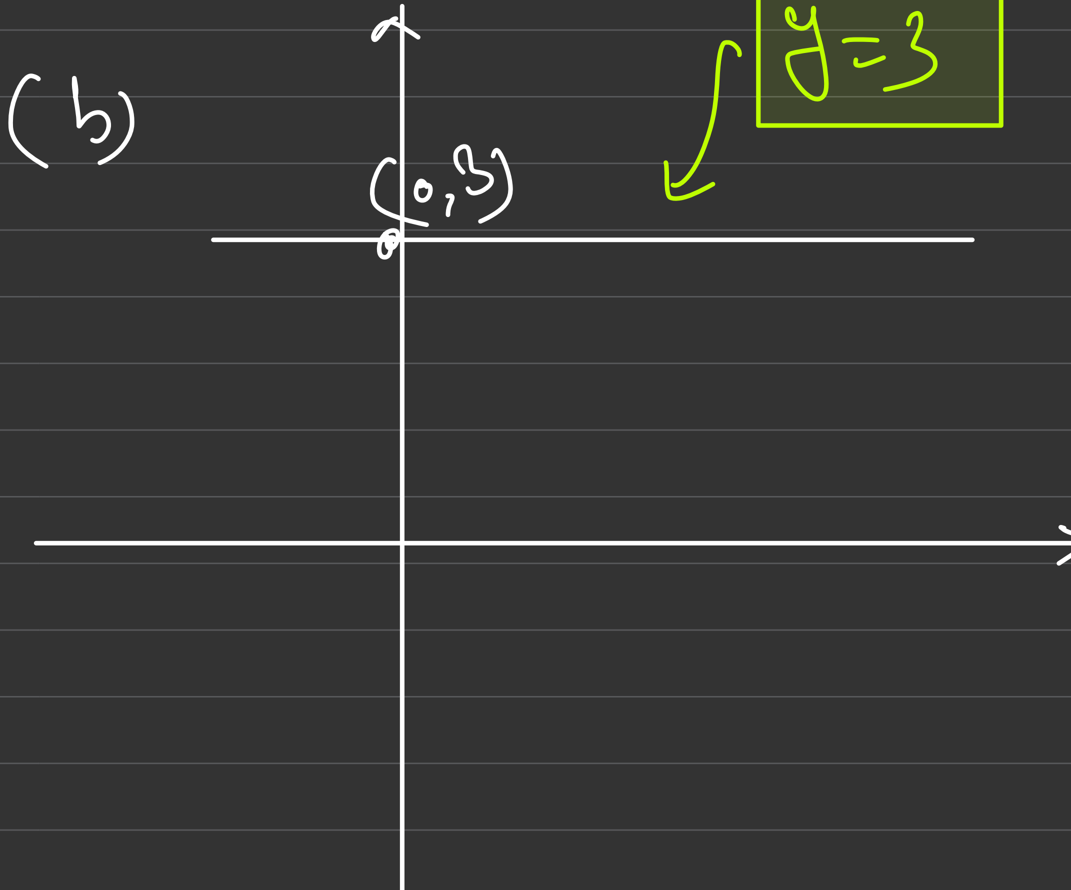
$\therefore \boxed{y = -\frac{4}{3}x + 4}$

Qu: (a)



$\boxed{x=4}$

(b)

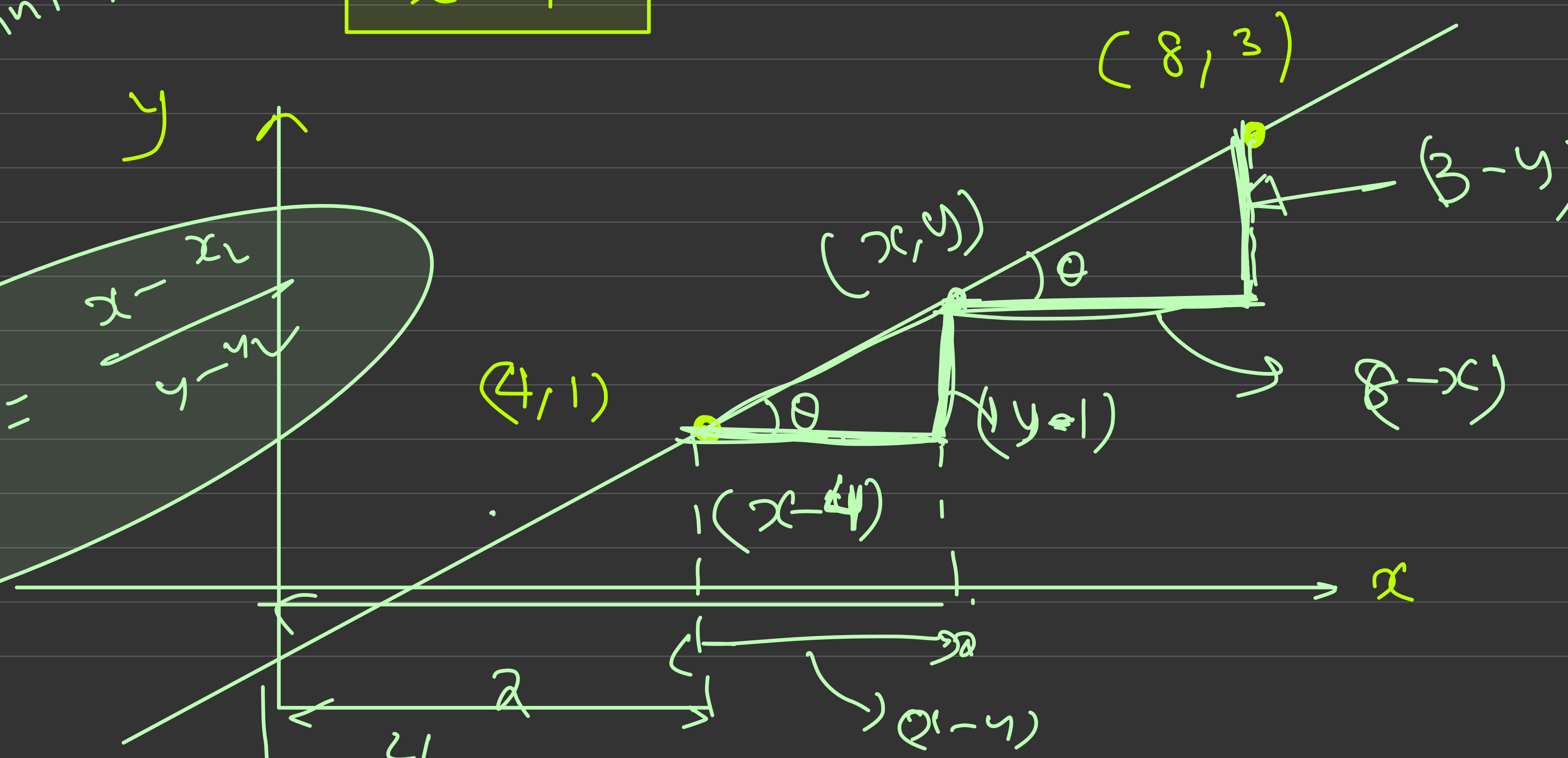


$\boxed{y=3}$

Two-point formula

Qu:

$$\frac{x-x_1}{y-y_1} = \frac{x-x_2}{y-y_2}$$



$m_1 = m_2$

$$\Rightarrow \frac{y-1}{x-4} = \frac{3-y}{8-x}$$

$\Downarrow$

2> Modulus function

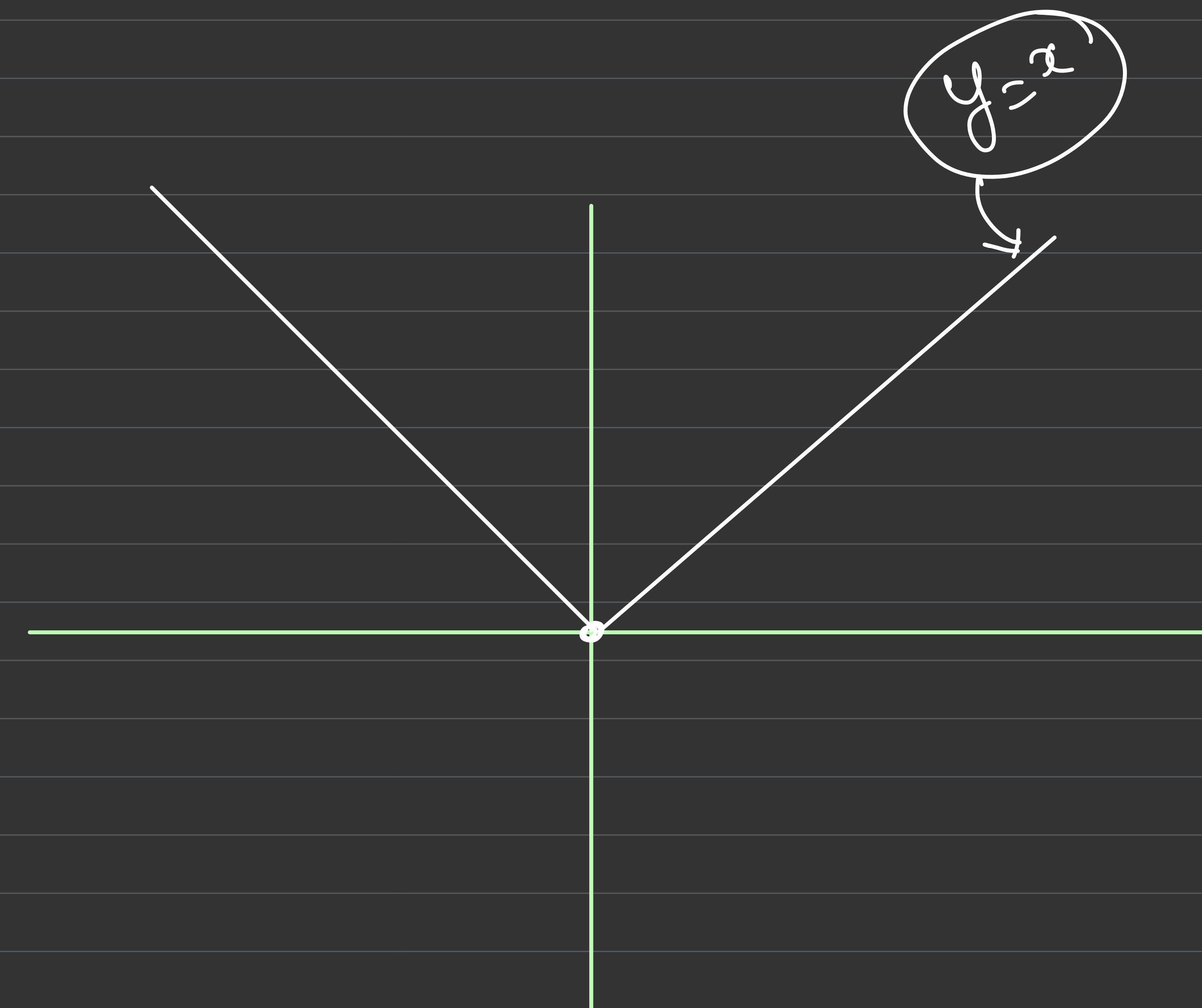
Absolute value

$|-3| = 3$, $|3| = 3$

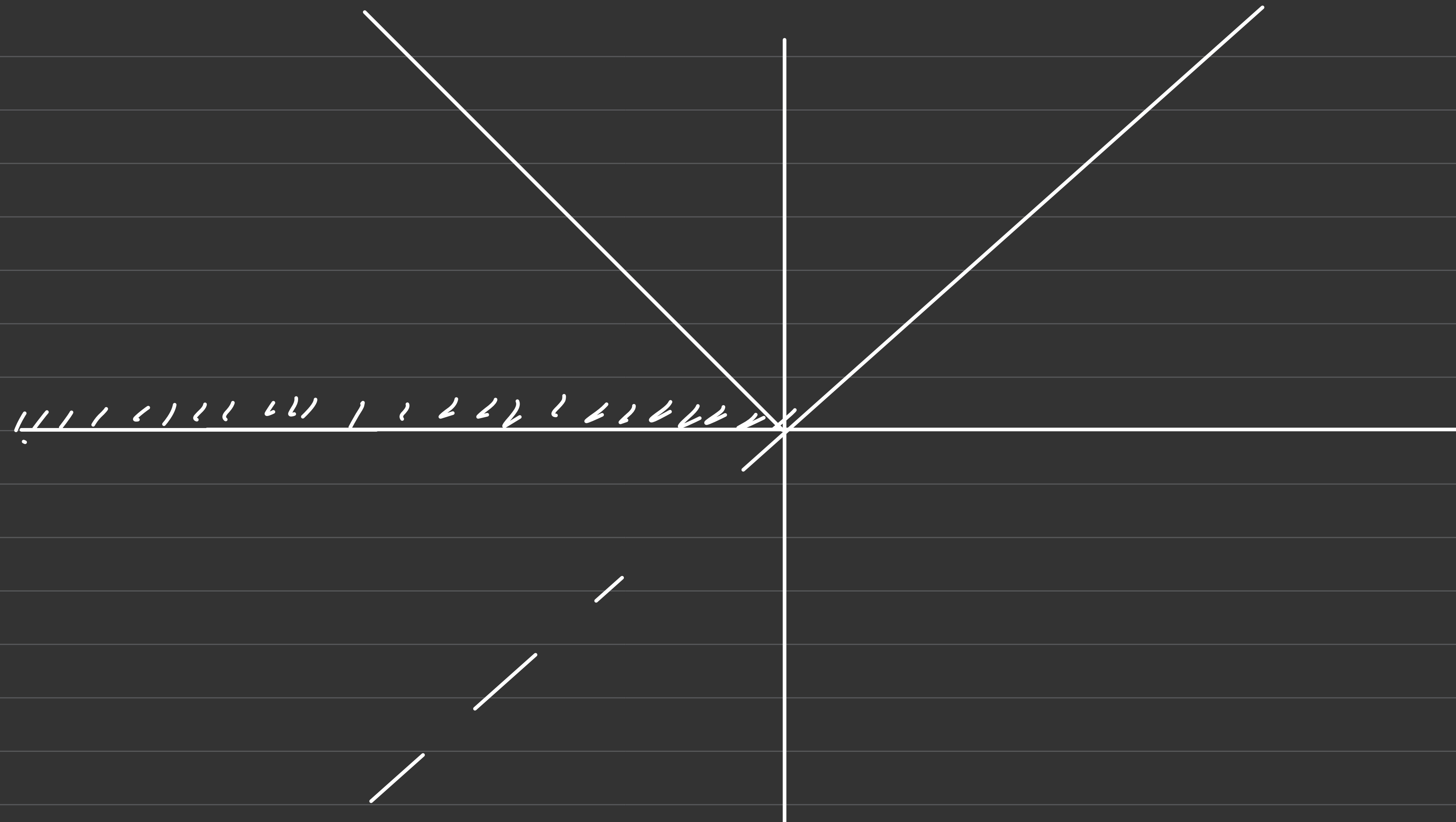
$-(-3)$

$$y = |x|$$

$$y = \begin{cases} x & , \; x > 0 \\ -x & , \; x < 0 \\ 0 & , \; x = 0 \end{cases}$$

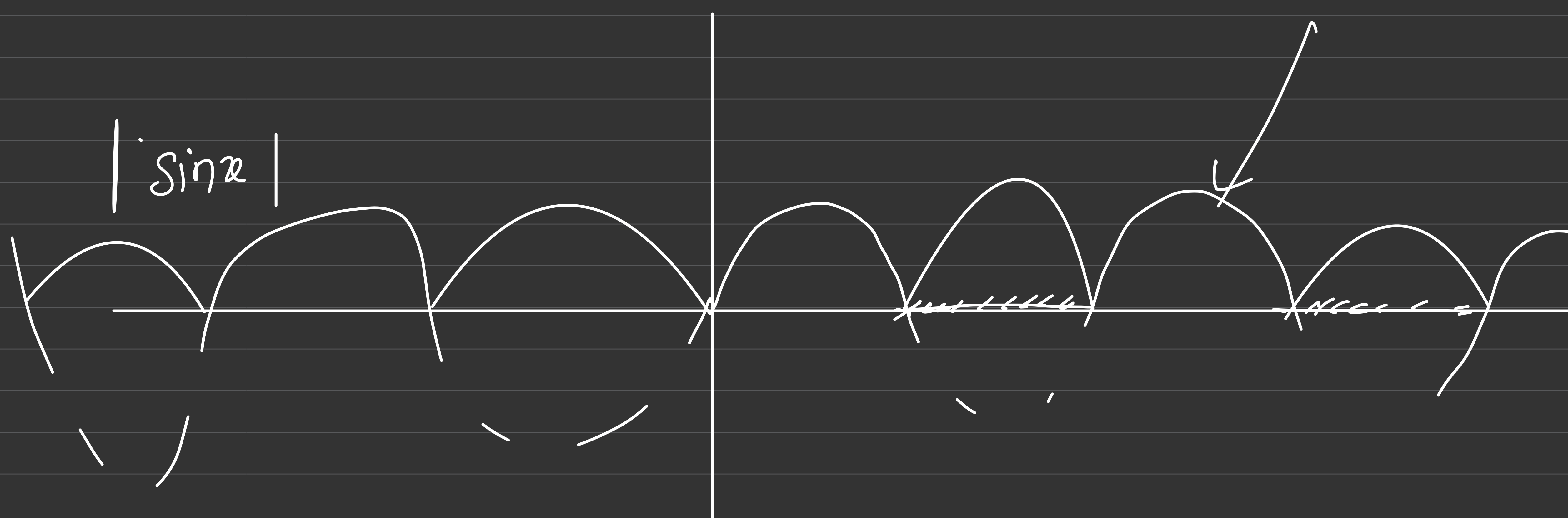


$y = |x|$

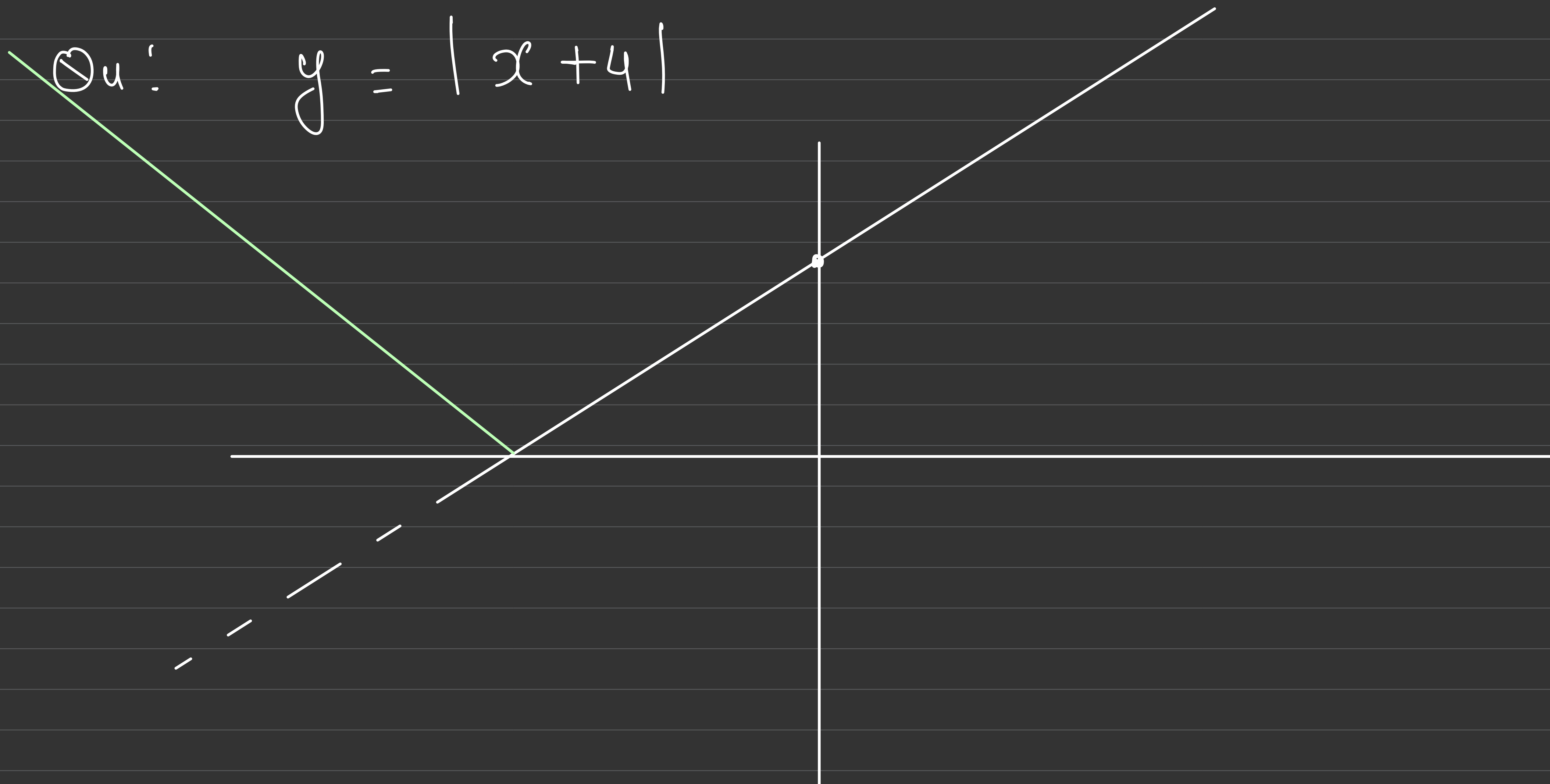


$y = |\sin x|$

$y = |\sin x|$



Ou : $y = |x+4|$



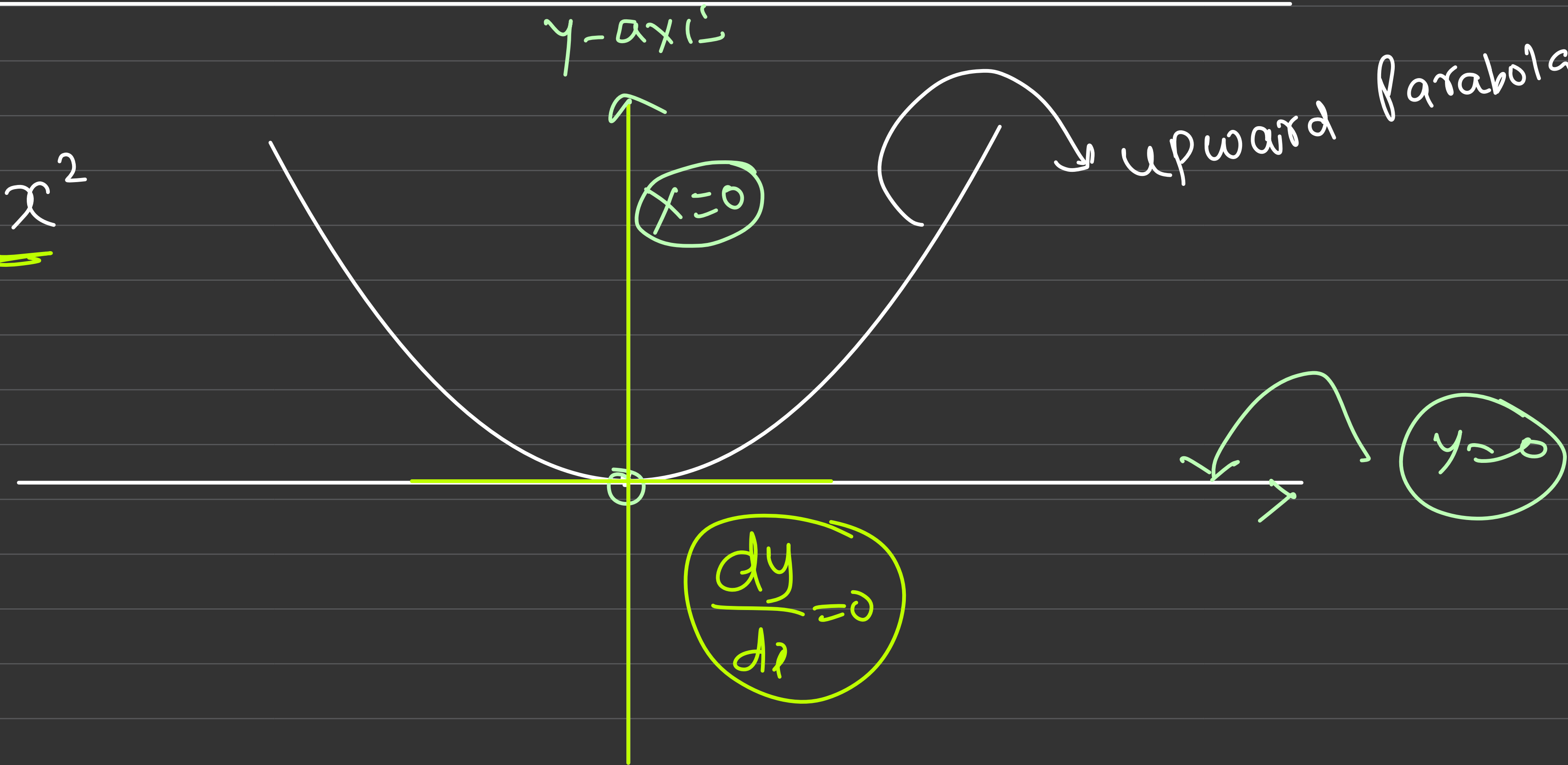
I) Graph of Quadratic Eqn / Parabola

$x^2 = 0$

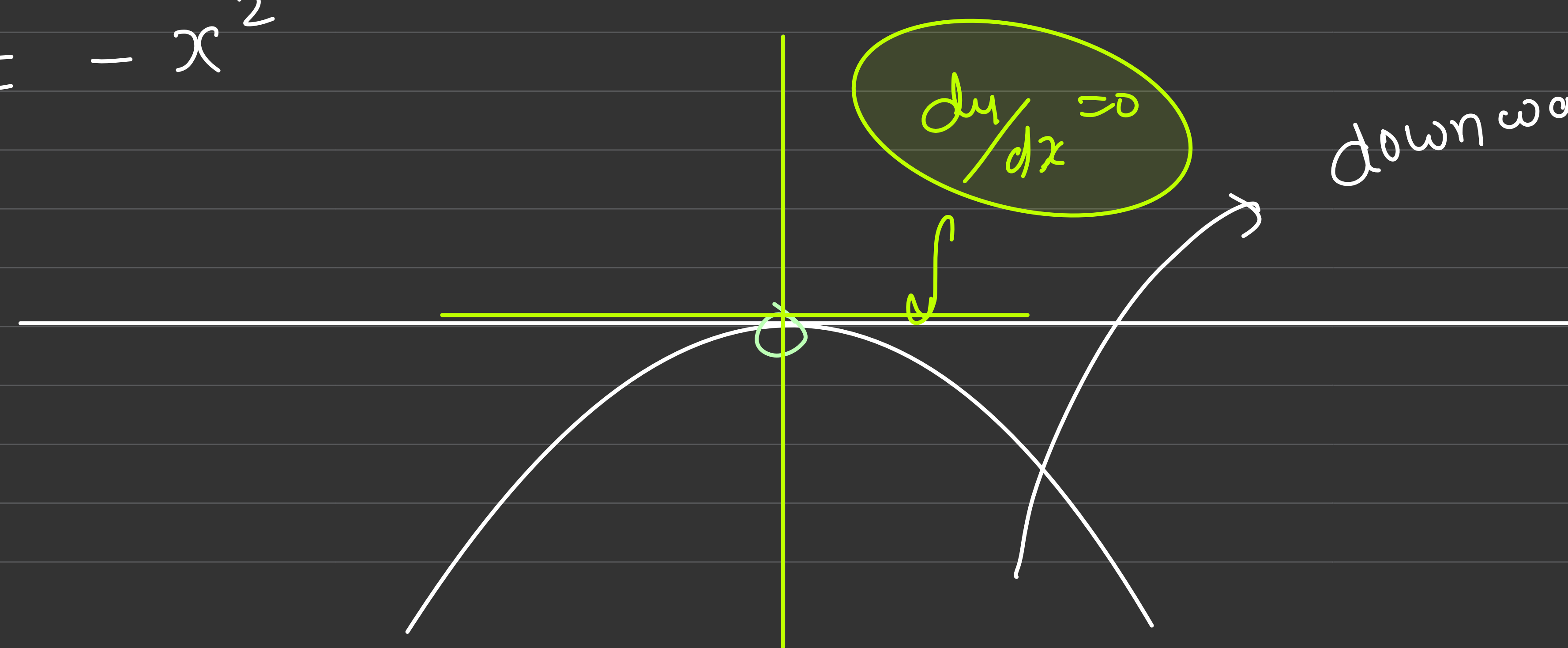
$\Downarrow$

$x = 0$

① $y = x^2$



② $y = -x^2$

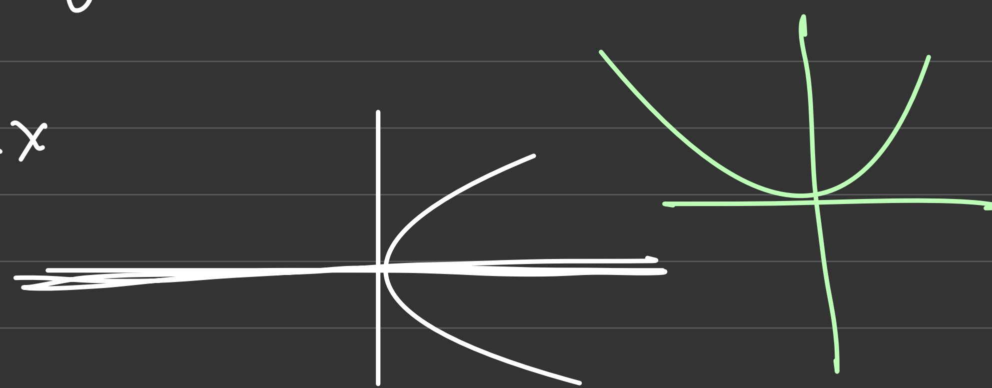


Note:-

1) If there is even power on 'x', graph will be symmetric about y- axis

2) If there is even power on 'y', graph will be symmetric about x-axis

$y^2 = 4ax$



Steps to draph Parabolic graph

1) If coefficient of $x^2 > 0 \Rightarrow$ upward parabola

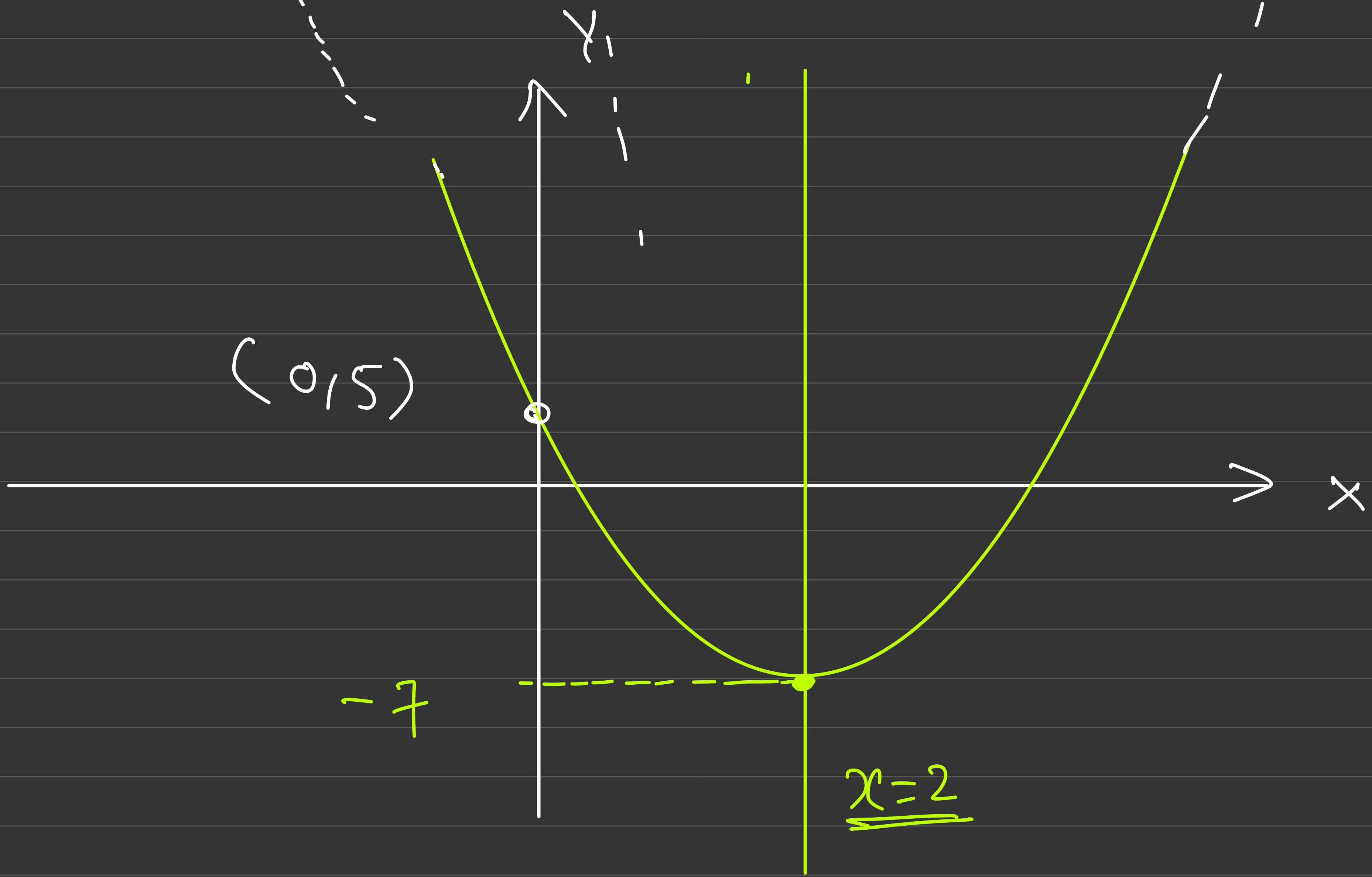
" " $< 0 \Rightarrow$ Downward parabola

2) find the Position of maxima / minima or graph will symmetric about this line.

3) find the value of y_{min} / y_{max}

Qu:

$$\boxed{y = 3x^2 - 12x + 5}$$



① upward Parabola

② Symmetric about x=2 line

③ $y_{min} = -7$

$$\frac{dy}{dx} = 6x - 12 = 0$$

$$\boxed{x = 2}$$

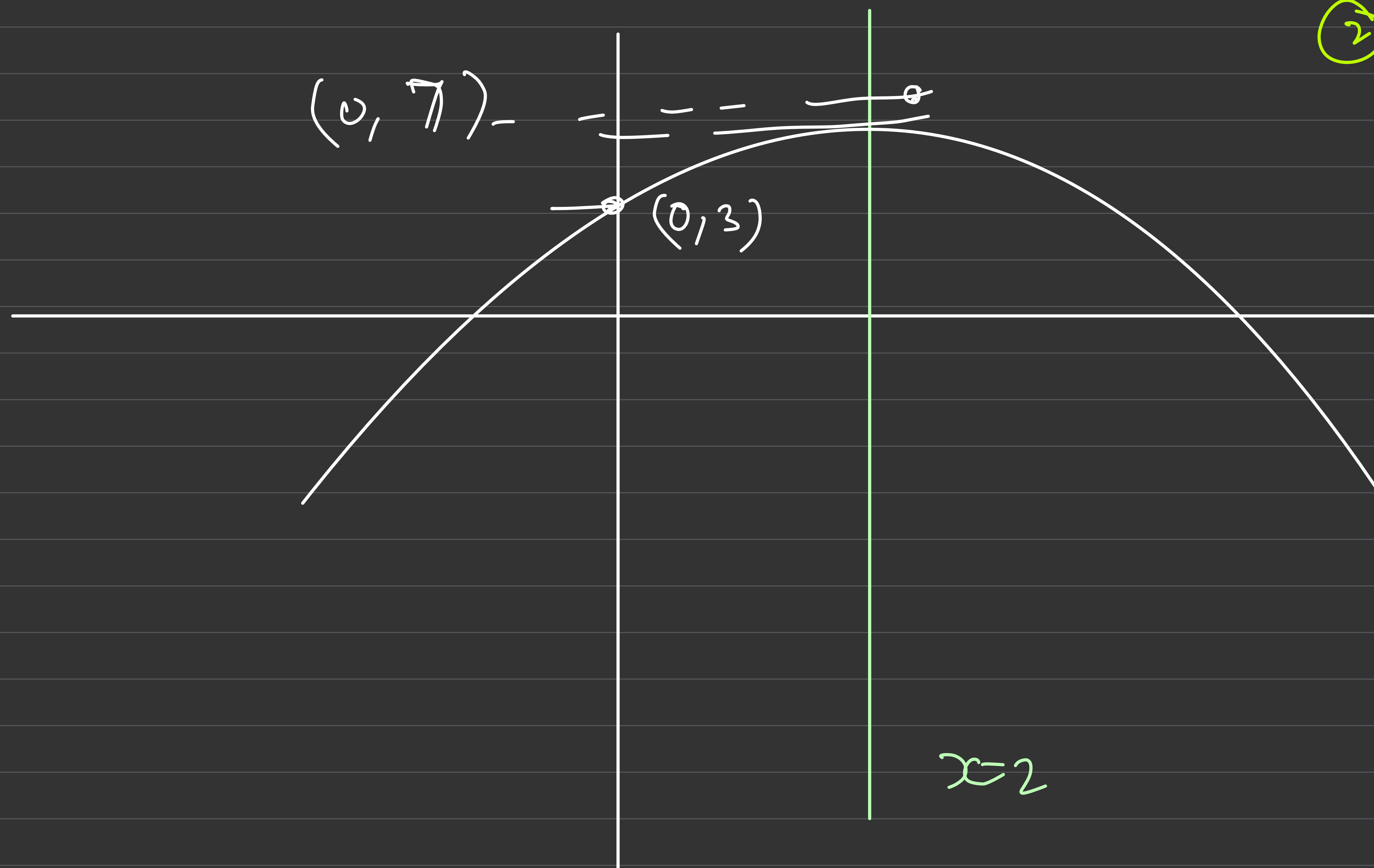
$$y_{min} = 3(2)^2 - 12(2) + 5$$

$$= 12 - 24 + 5$$

$$= -7$$

Qu:

$y = -x^2 + 4x + 3$



① Downward

② $\frac{dy}{dx} = 0$

$-2x + 4 = 0$

$\boxed{x = 2}$

③ $y_{max} =$

$-(2)^2 + 4(2) + 3$

$= 7$

$$y = -x^2 + 4x + 3$$

① downward Parabola

② $\dfrac{dy}{dx} = 0$

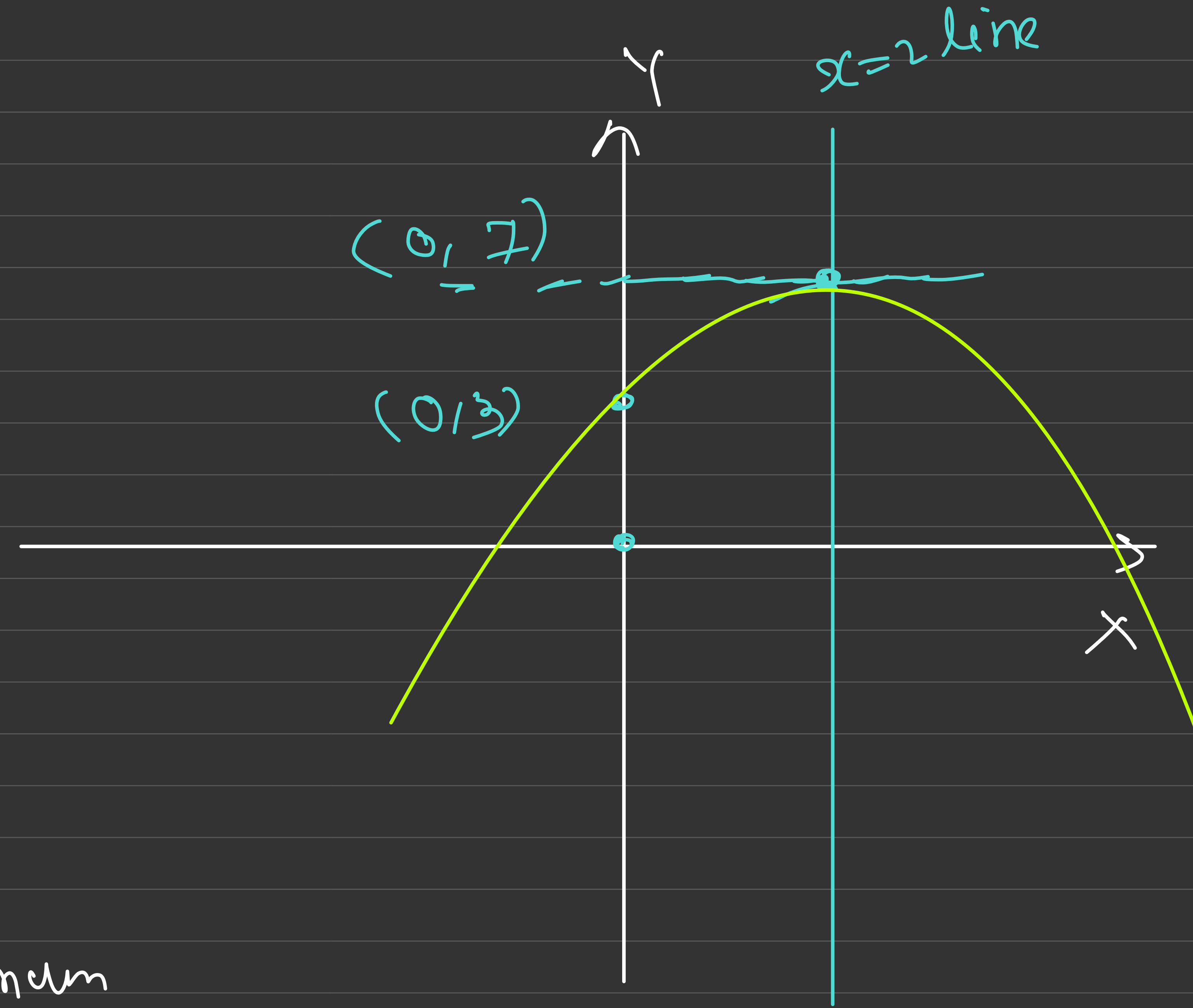
$\Rightarrow \quad -2x + 4 = 0 \quad \Rightarrow \boxed{x = 2}$

At $x = 2 \Rightarrow$ y has maximum value

③ $y_{max} = -(2)^2 + 4(2) + 3$

$= 7$

④ At $x = 0$, $y = 3$

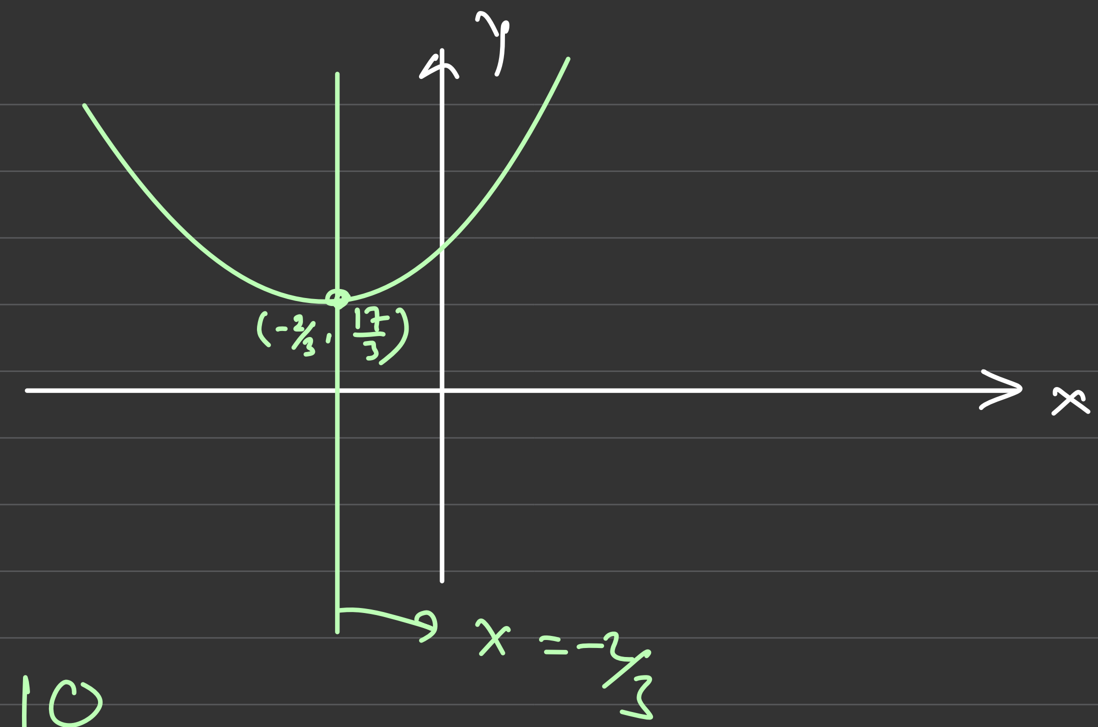


Qu:

$$y = 3x^2 + 4x + 7$$

$$\frac{dy}{dx} = 6x + 4 = 0$$

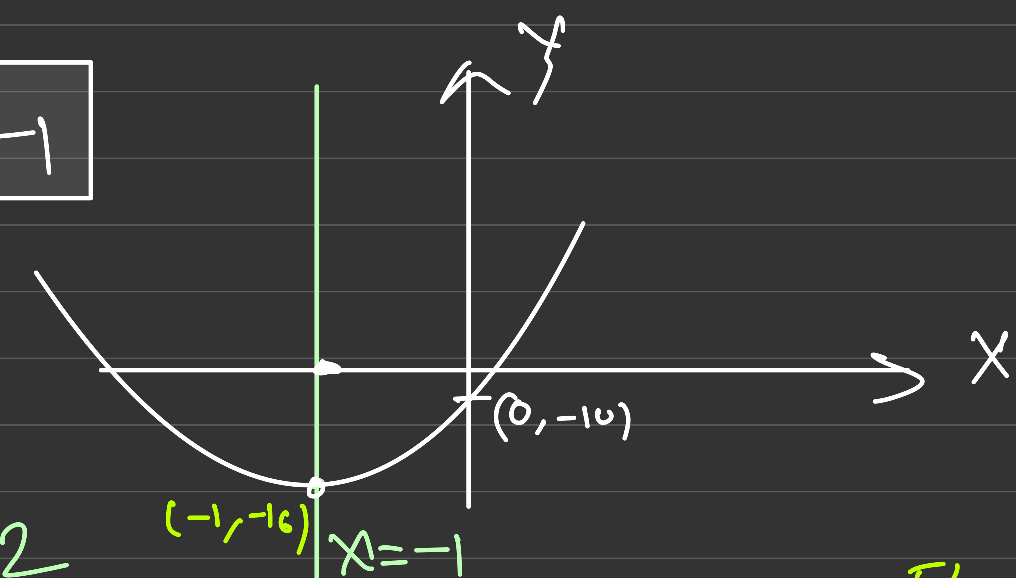
$$x = -4/6 = -2/3$$



Qu:

$$y = 6x^2 + 12x - 10$$

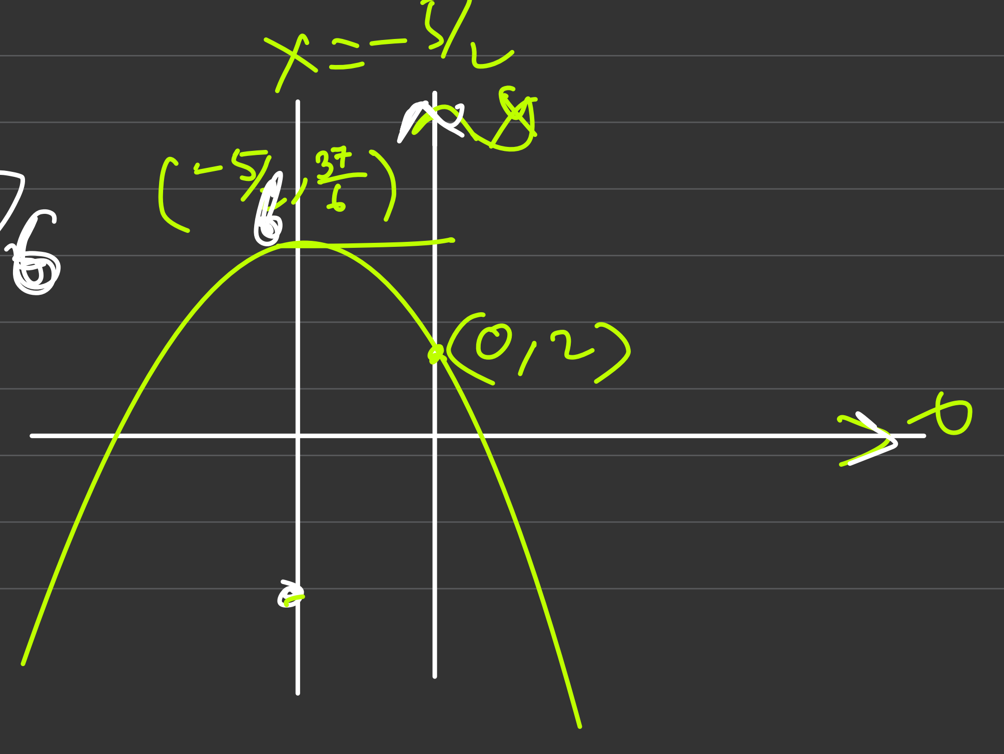
$$\frac{dy}{dx} = 12x + 12 = 0 \Rightarrow \boxed{x = -1}$$

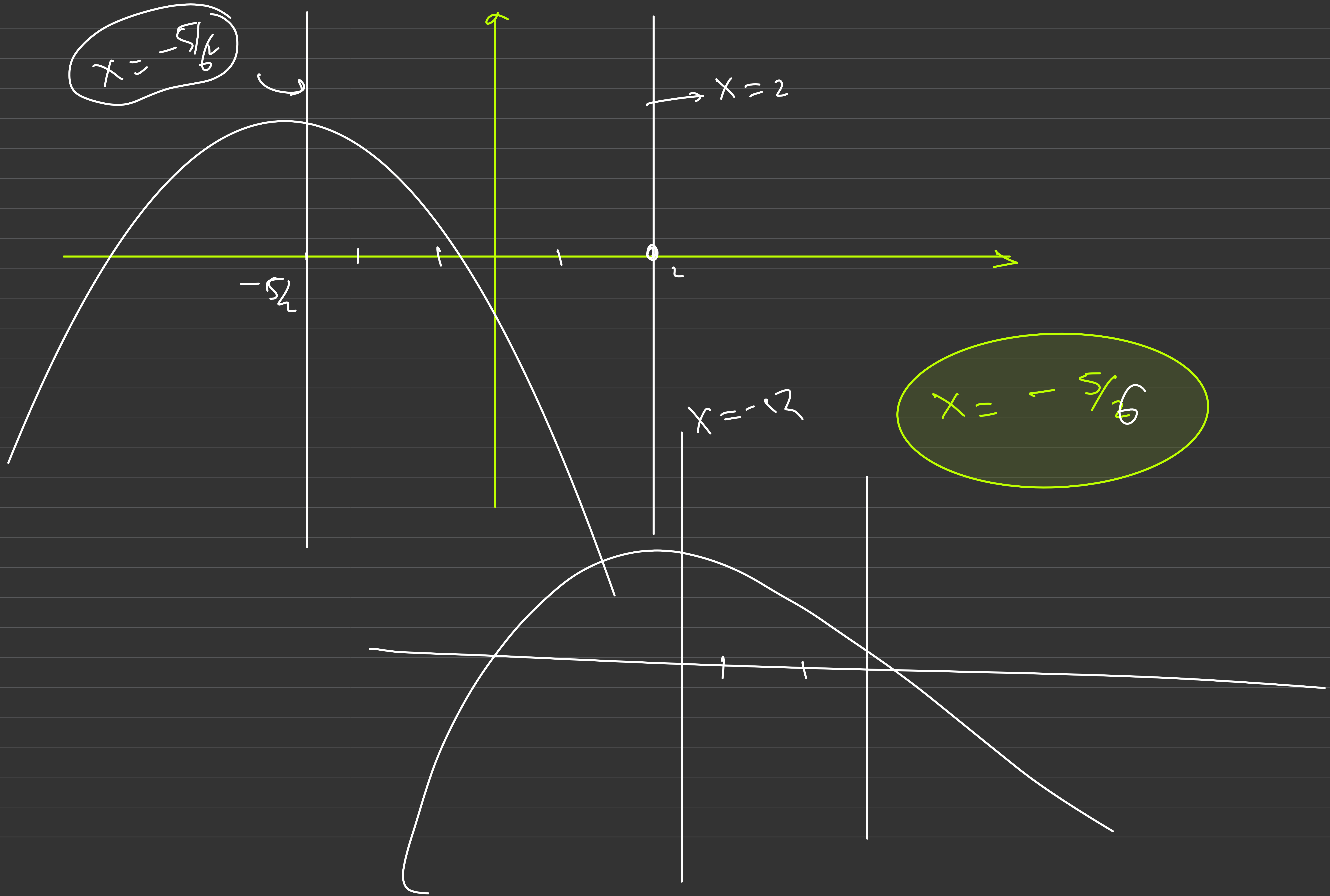


Qu:

$$y = -6x^2 - 10x + 2$$

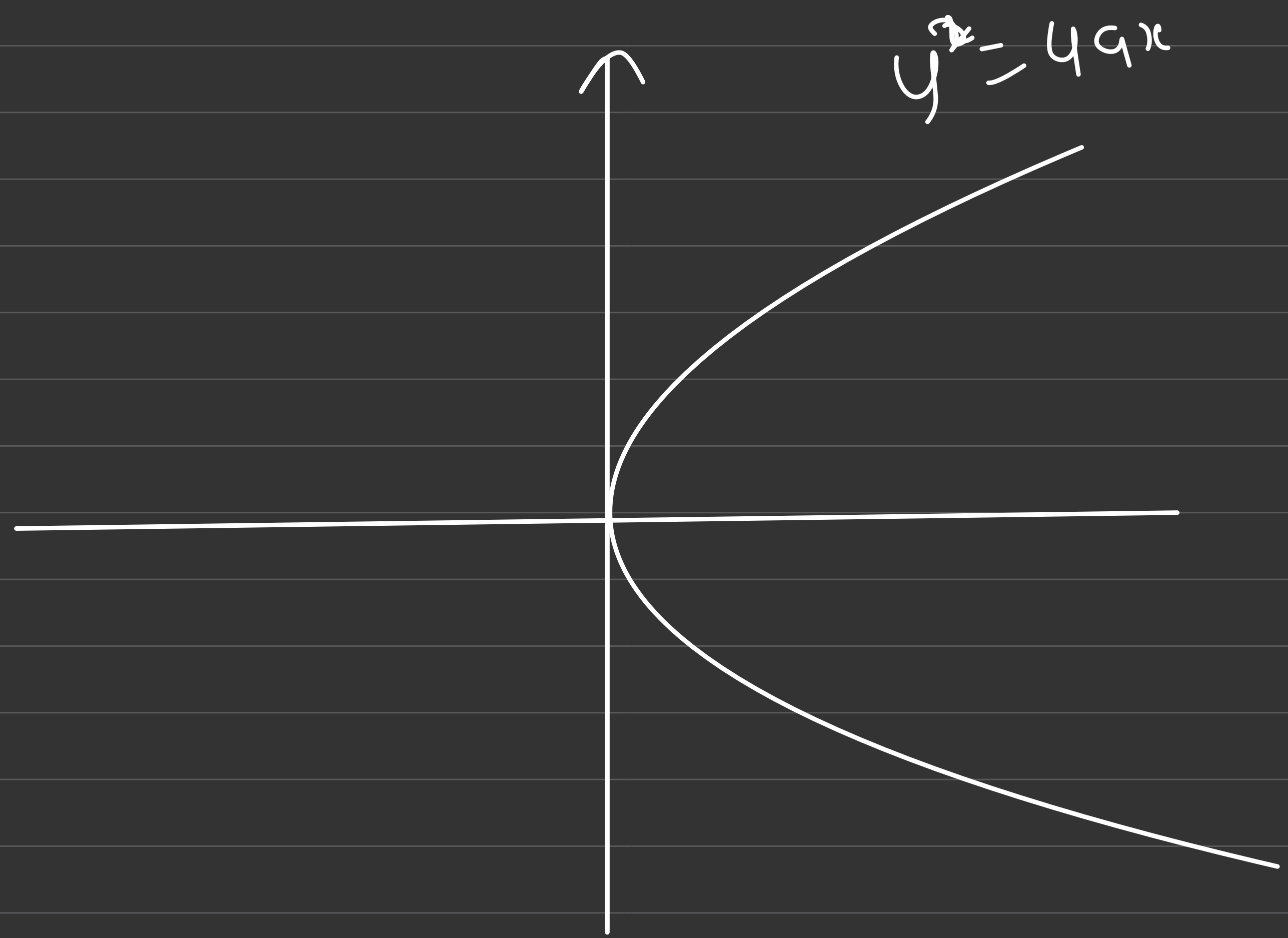
$$\frac{dy}{dx} = \underline{-12x - 10} = 0 ; x = \frac{-10}{12} = -5/6$$



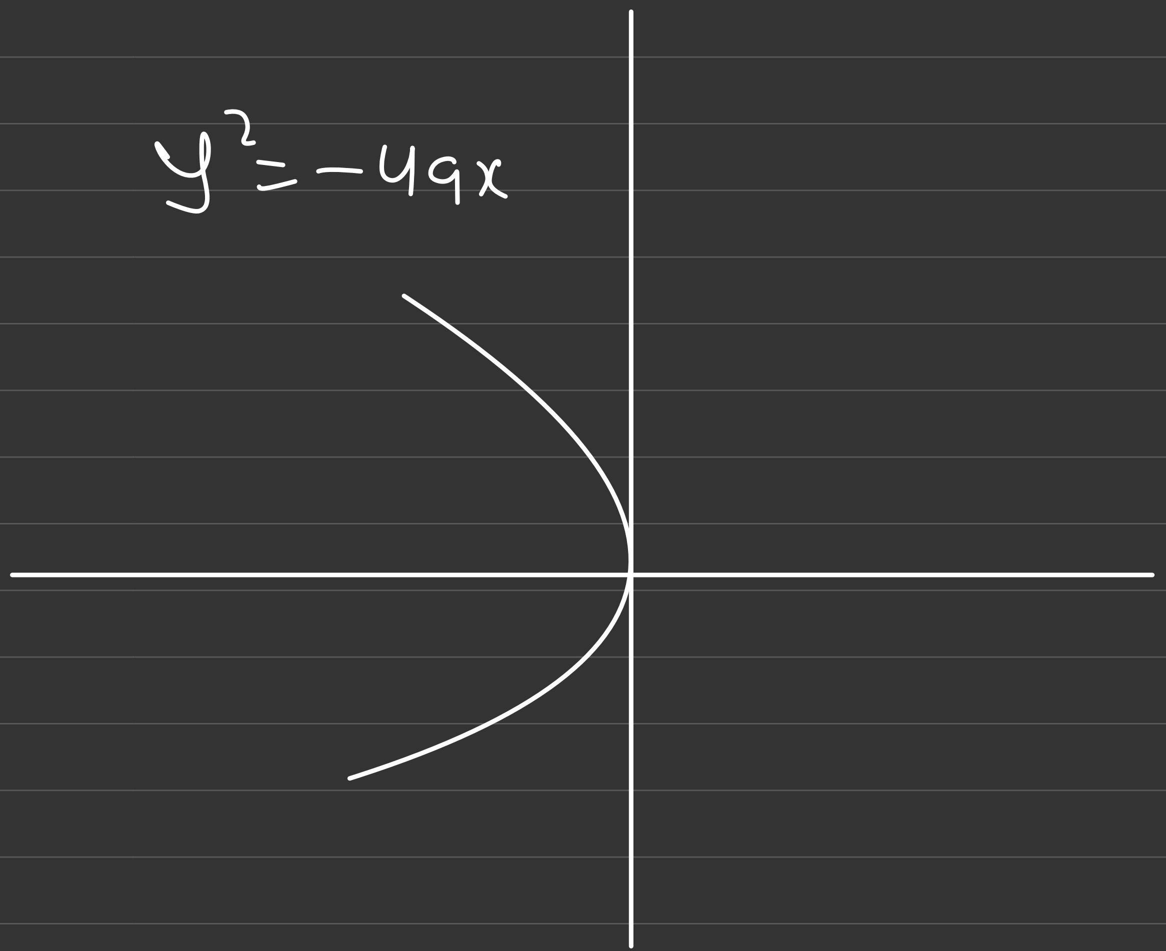


$x = -5/6$

$y^2 = 4ax$



Qu! $y^2 = -4ax$



If $\boxed{\underline{\text{acceleration} = \text{constant}} = a}$

$t = 0, \quad v = u$
$t = t, \quad v = v$

$$\frac{dv}{dt} = a$$

$$\Rightarrow \int dv = \int a \, dt \qquad (a = \text{const})$$

$$\Rightarrow \int_u^v dv = a \int_{t=0}^{t} dt$$

$$\Rightarrow (v - u) = a(t - 0)$$

$$\Rightarrow \boxed{v = u + at}$$

$$v = \frac{dx}{dt} = u + at$$

At $t = 0,\ x = x_0$

$t = t,\ x = x$

$$\int_{x_0}^{x} dx = \int_{t=0}^{t} (u + at)\, dt = \int_0^t u\, dt + \int_0^t at\, dt$$

$$(x - x_0) = u(t - 0) + \frac{at^2}{2}$$

$$\Rightarrow$$

$$\Rightarrow \quad (x - x_0) = ut + \frac{1}{2}at^2$$

$$\Rightarrow \quad \boxed{s = ut + \frac{1}{2}at^2}$$

Equation of Motion :

If $a = \text{Const}$

1) $v = u + at$

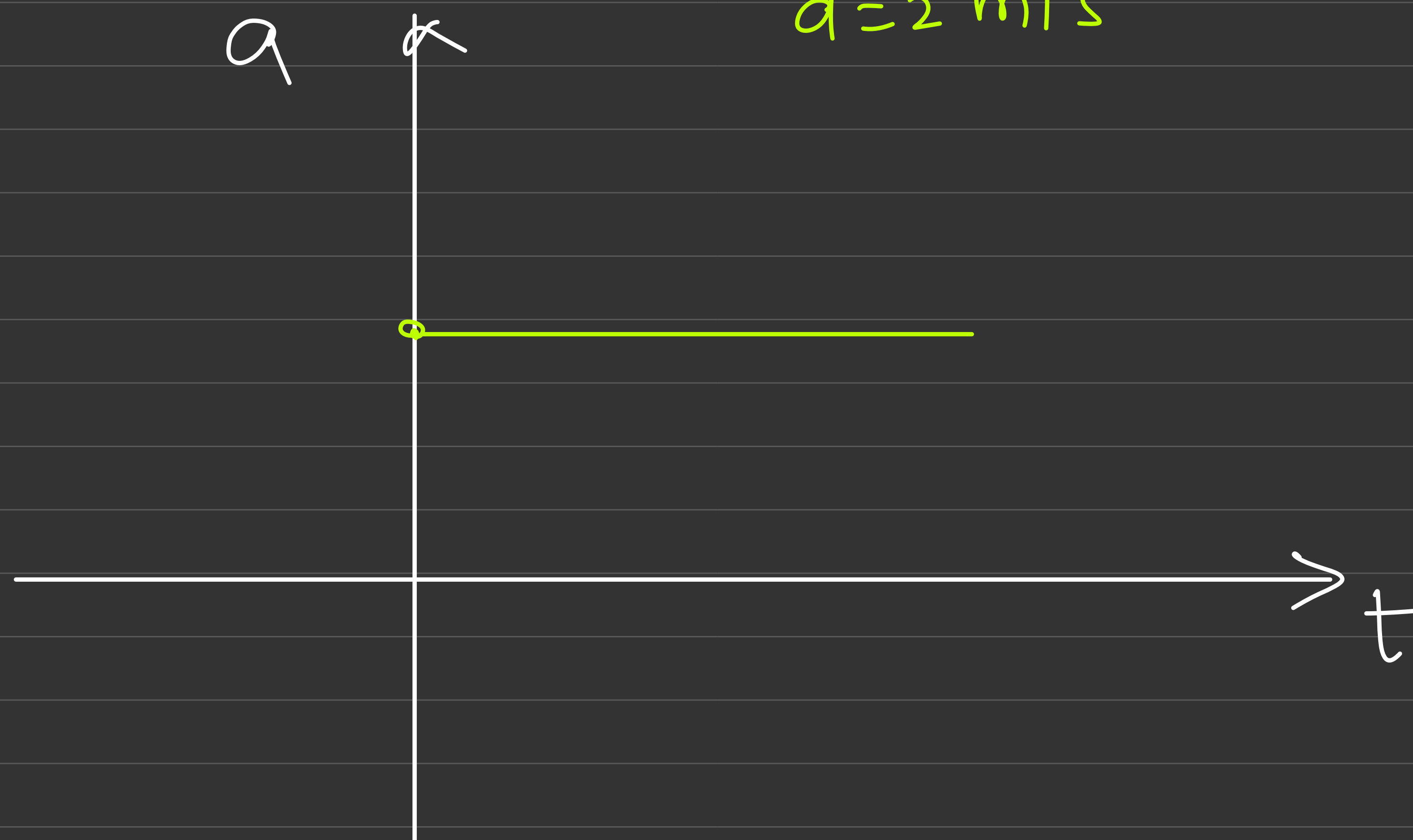
2) $S = ut + \frac{1}{2}at^2$

** 3) $v^2 = u^2 + 2aS$

1) $a-t$ graph

$a = \text{Const}$

$a = 2\ m/s^2$



time can't be negative

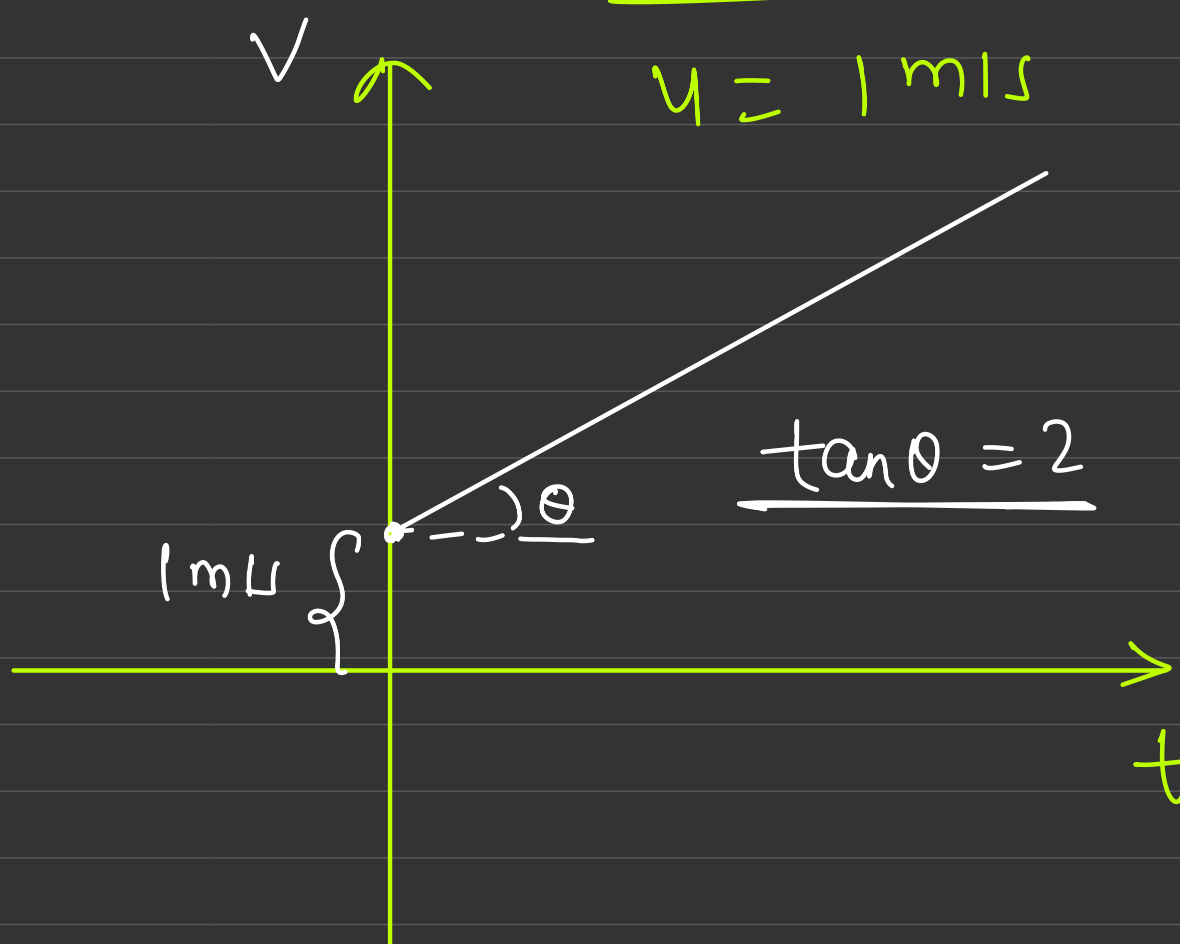
So graph can't be extended

on left - side.

② $v-t$ graph

$a = 2\ m/s^2$

$u = 1\ m/s$



$\tan\theta = 2$

$$\boxed{v = 1 + 2t}$$

③ $x - t$ graph

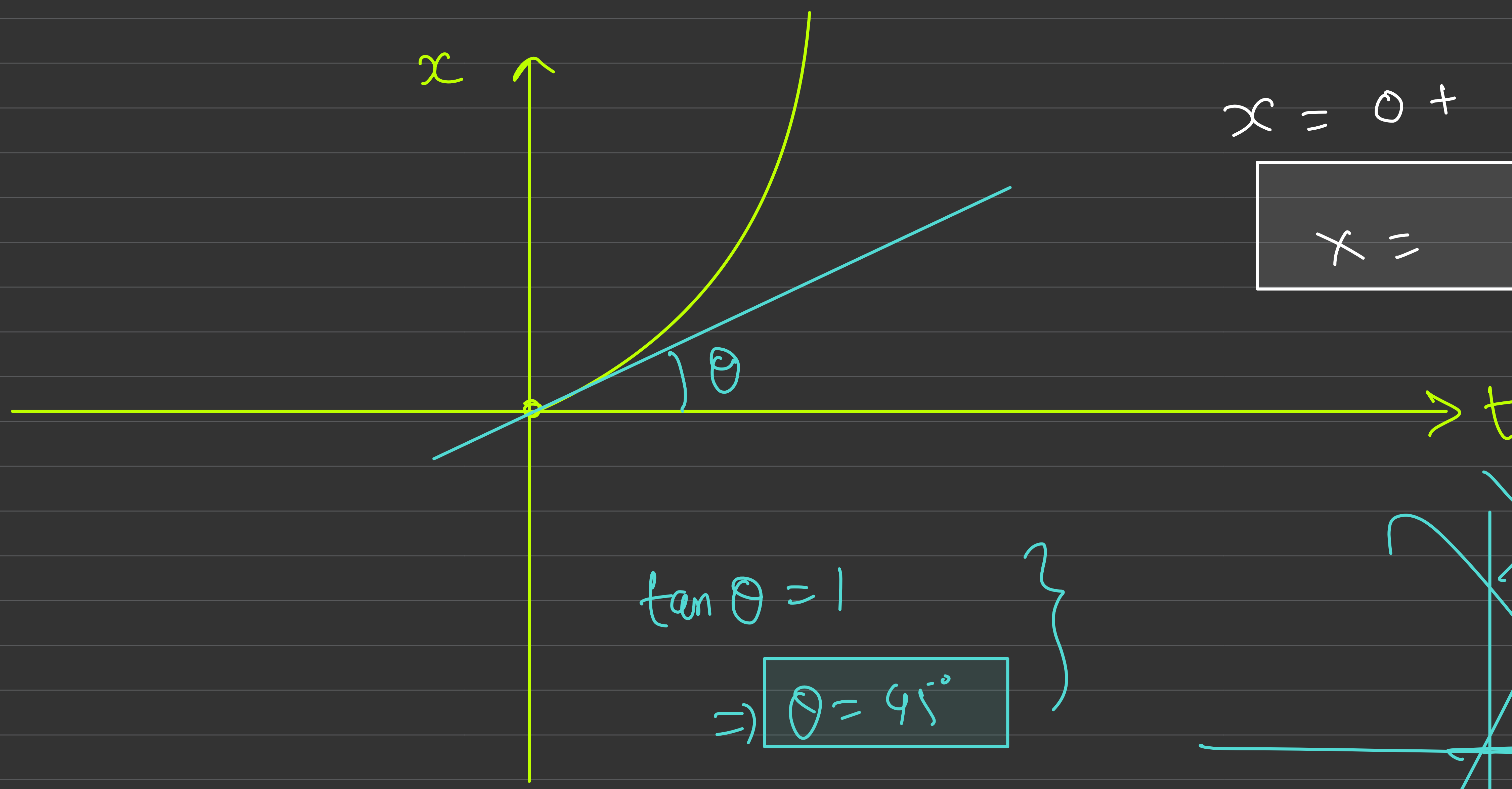
$$\boxed{x = x_0 + ut + \frac{1}{2}at^2}$$

H.W.

$x_0 = 0$, $u = 1\ m/s$ $a = 2\ m/s^2$

$x = 0 + t + \frac{1}{2} \times 2t^2$

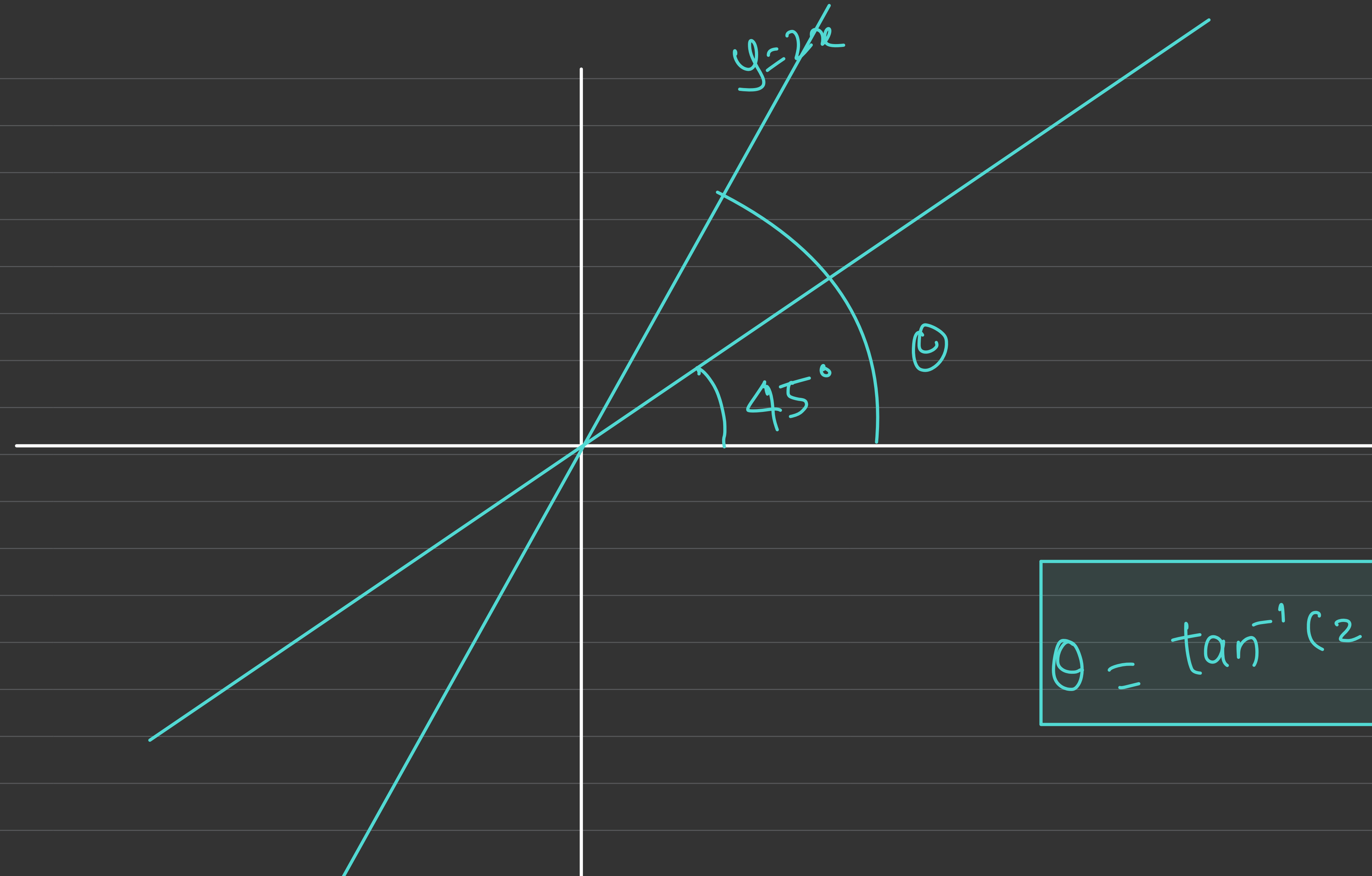
$$\boxed{x = t^2 + t}$$



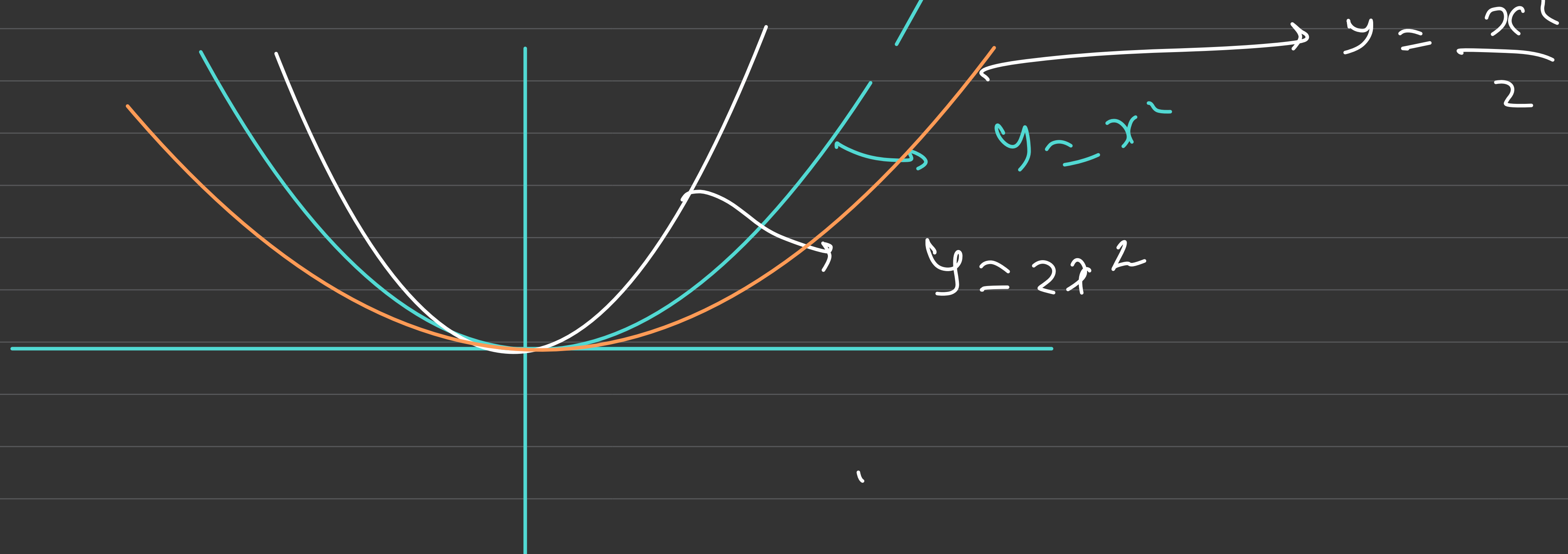
$\tan\theta = 1$

$\Rightarrow \boxed{\theta = 45^\circ}$

$y = 2x$



$$\boxed{\theta = \tan^{-1}(2)}$$



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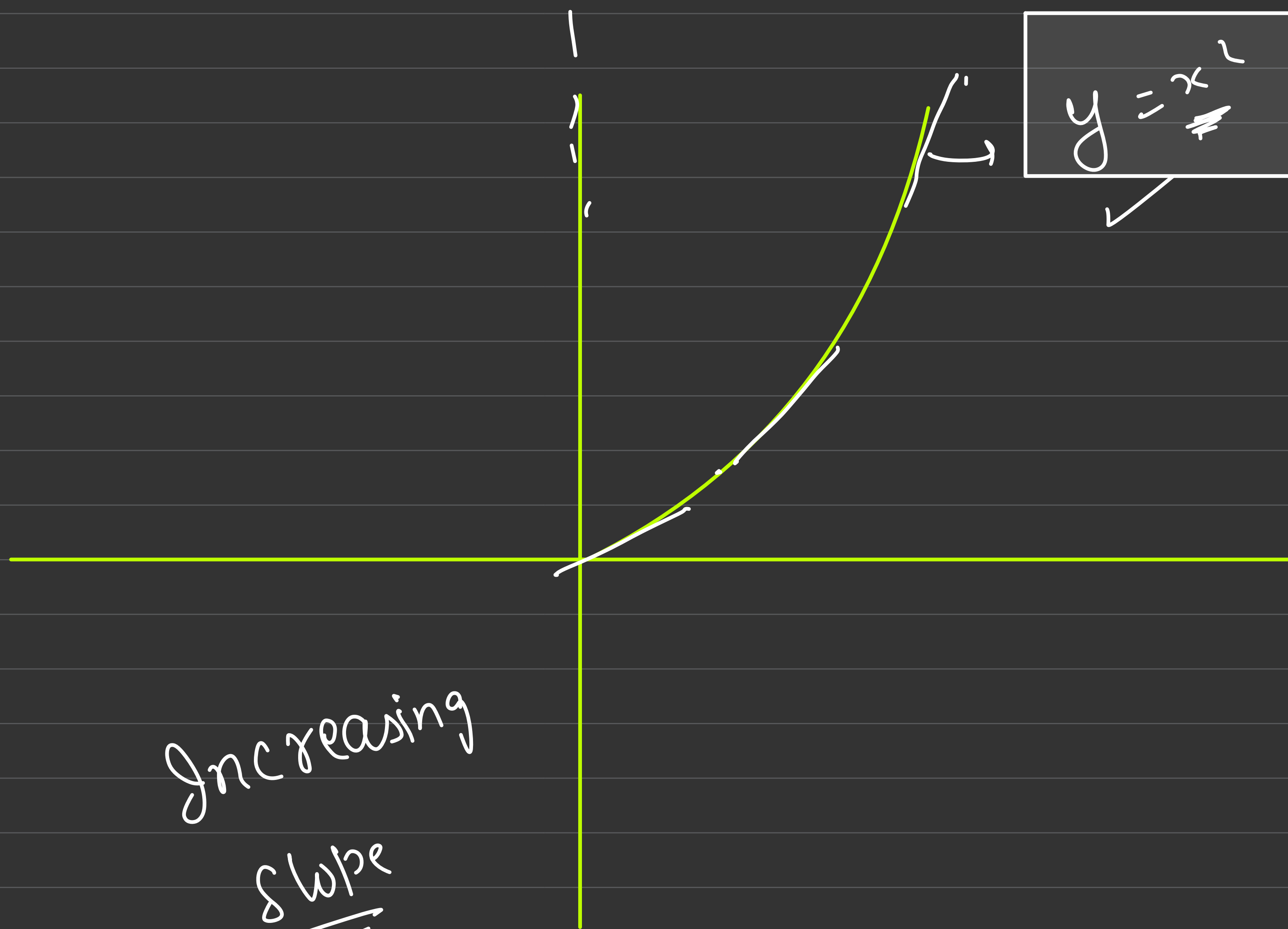
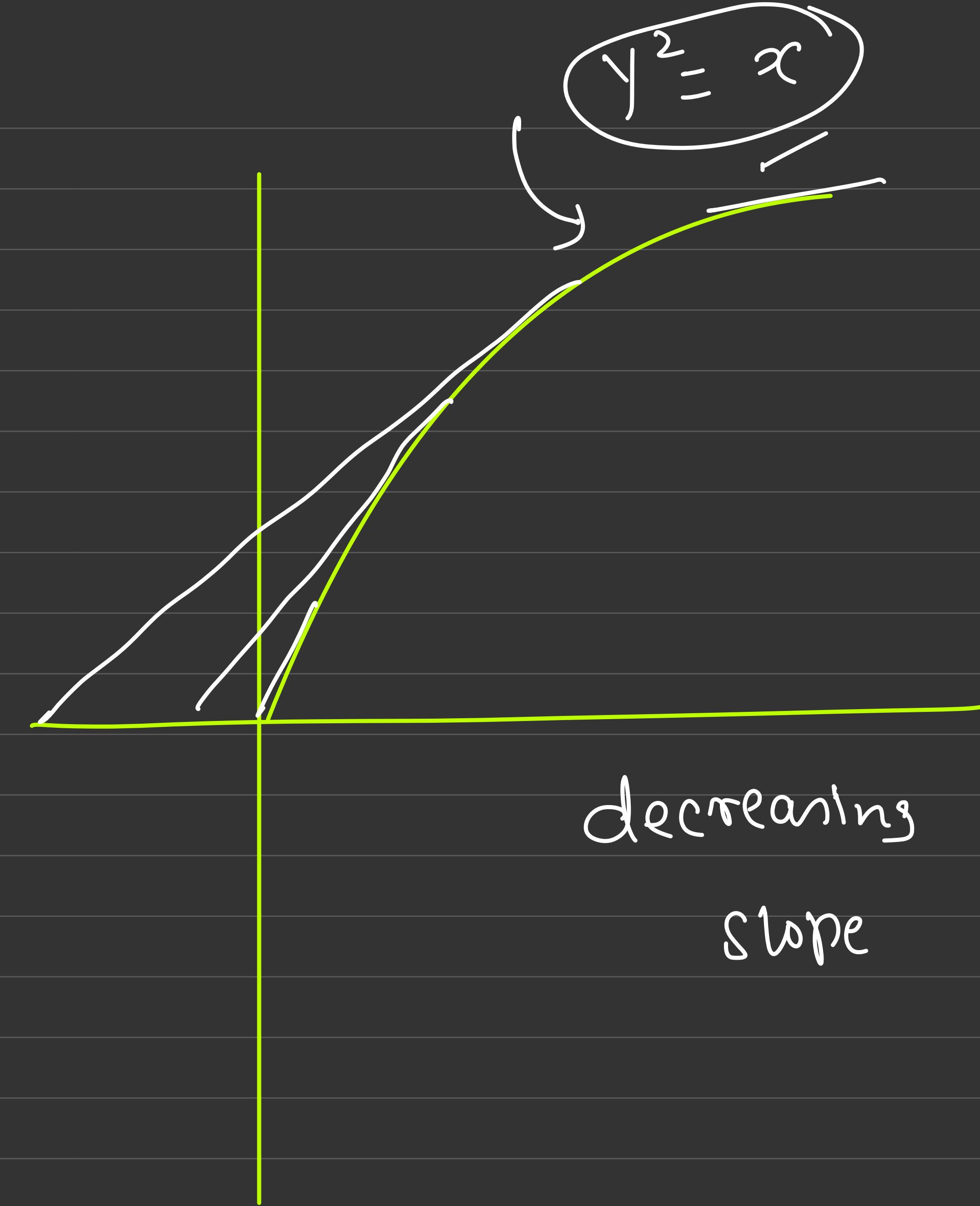


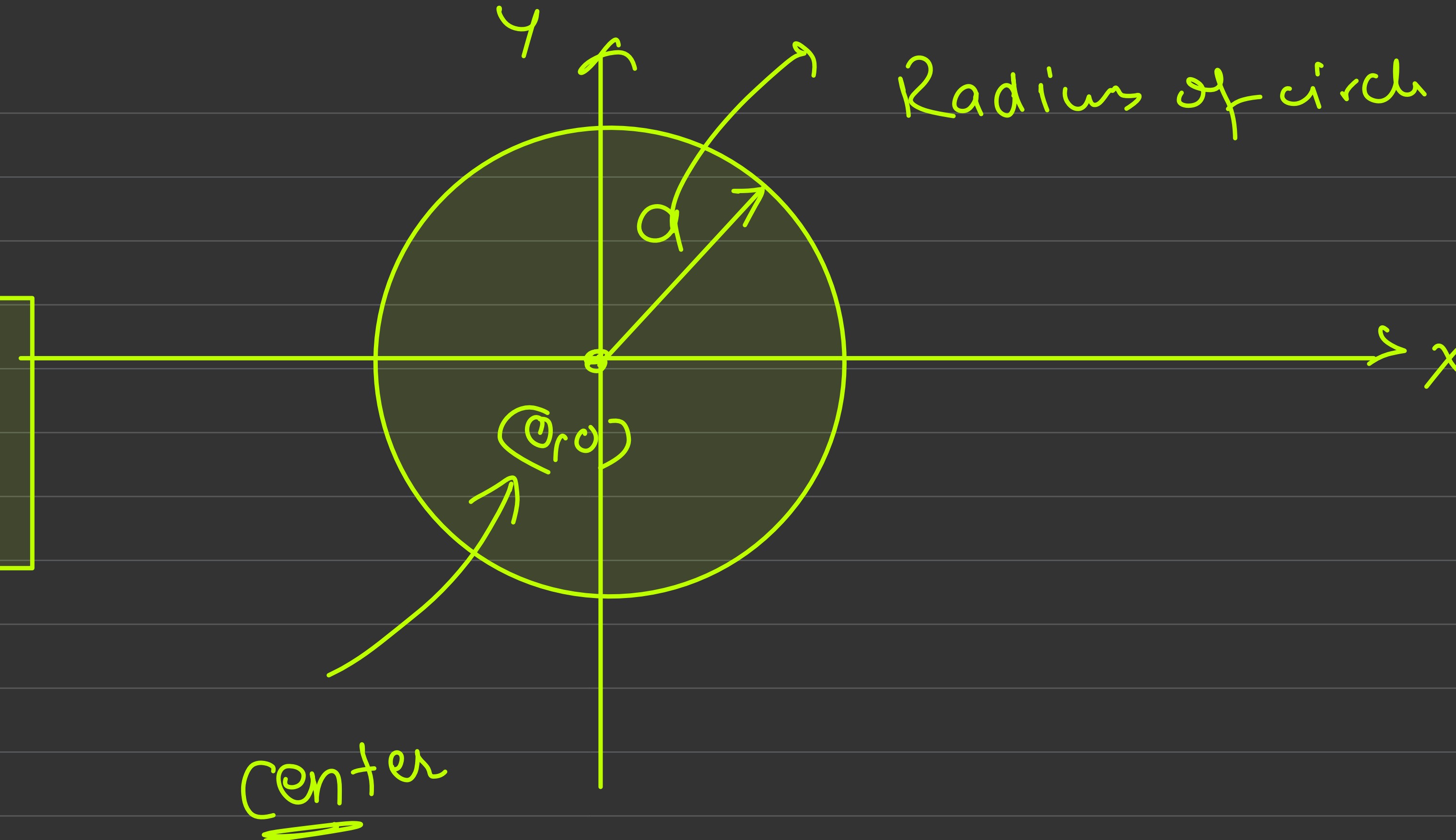
Increasing slope



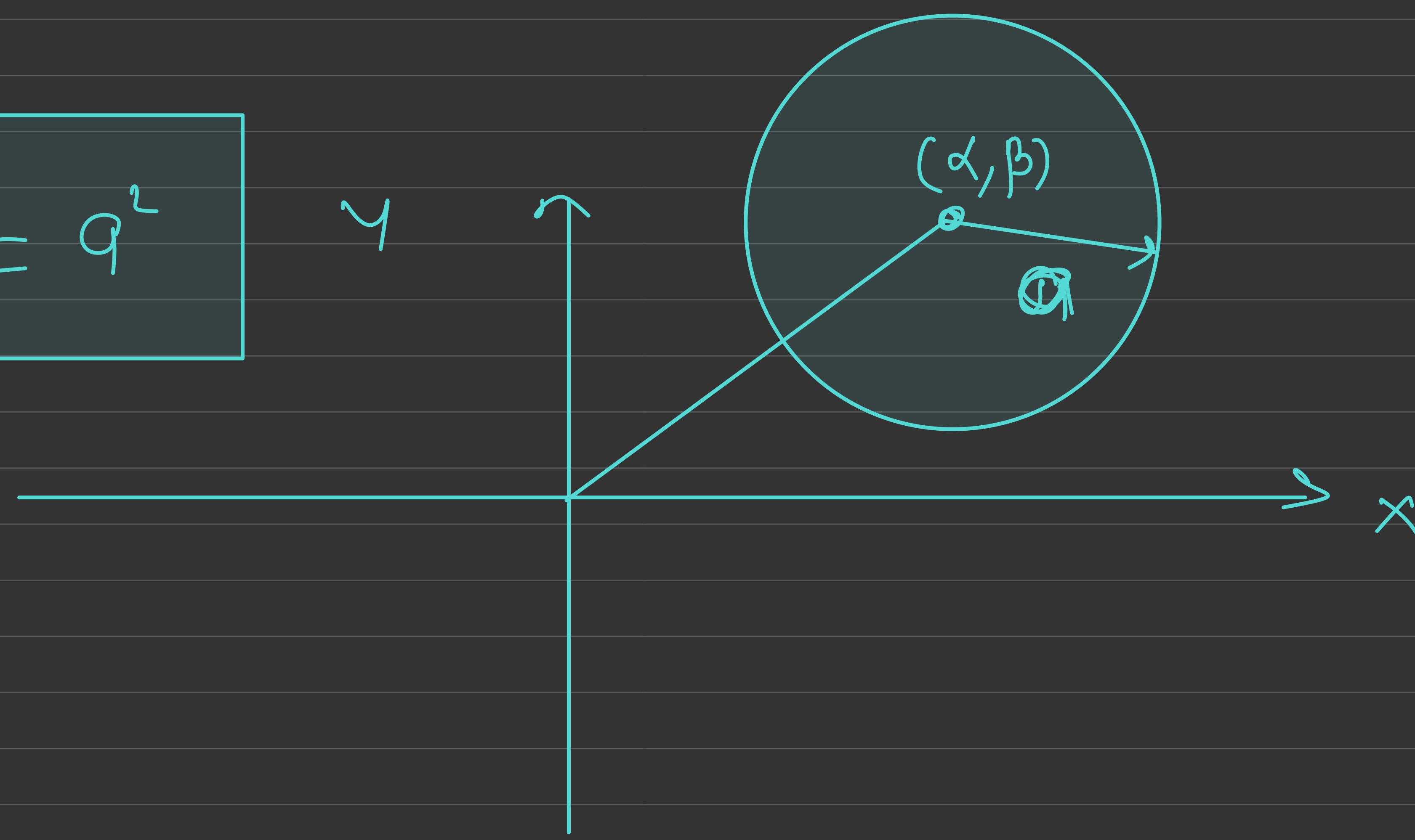
decreasing slope

37) Circle

$$\boxed{(x-0)^2 + (y-0)^2 = a^2}$$ ✓



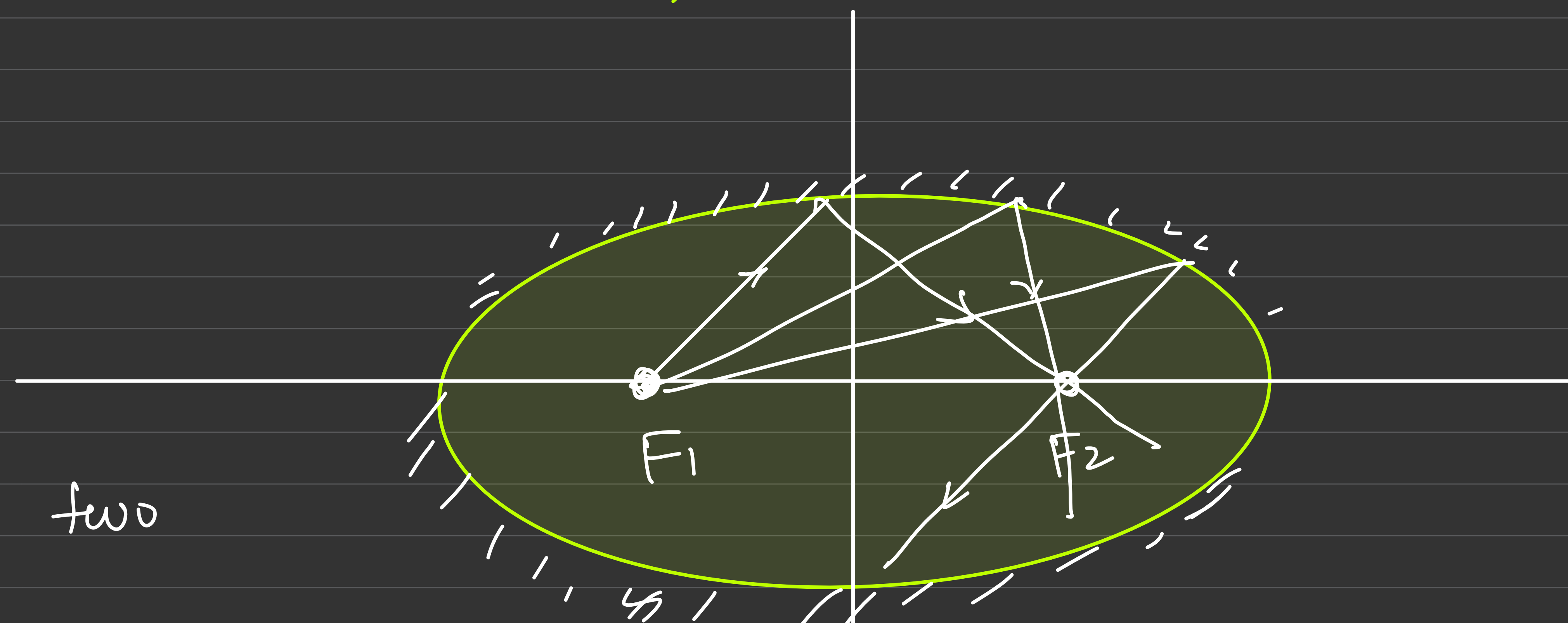
$$\boxed{(x-\alpha)^2 + (y-\beta)^2 = a^2}$$



- Area of Circle = πa^2

 Perimeter of Circle = $2\pi a$

4) Ellipse

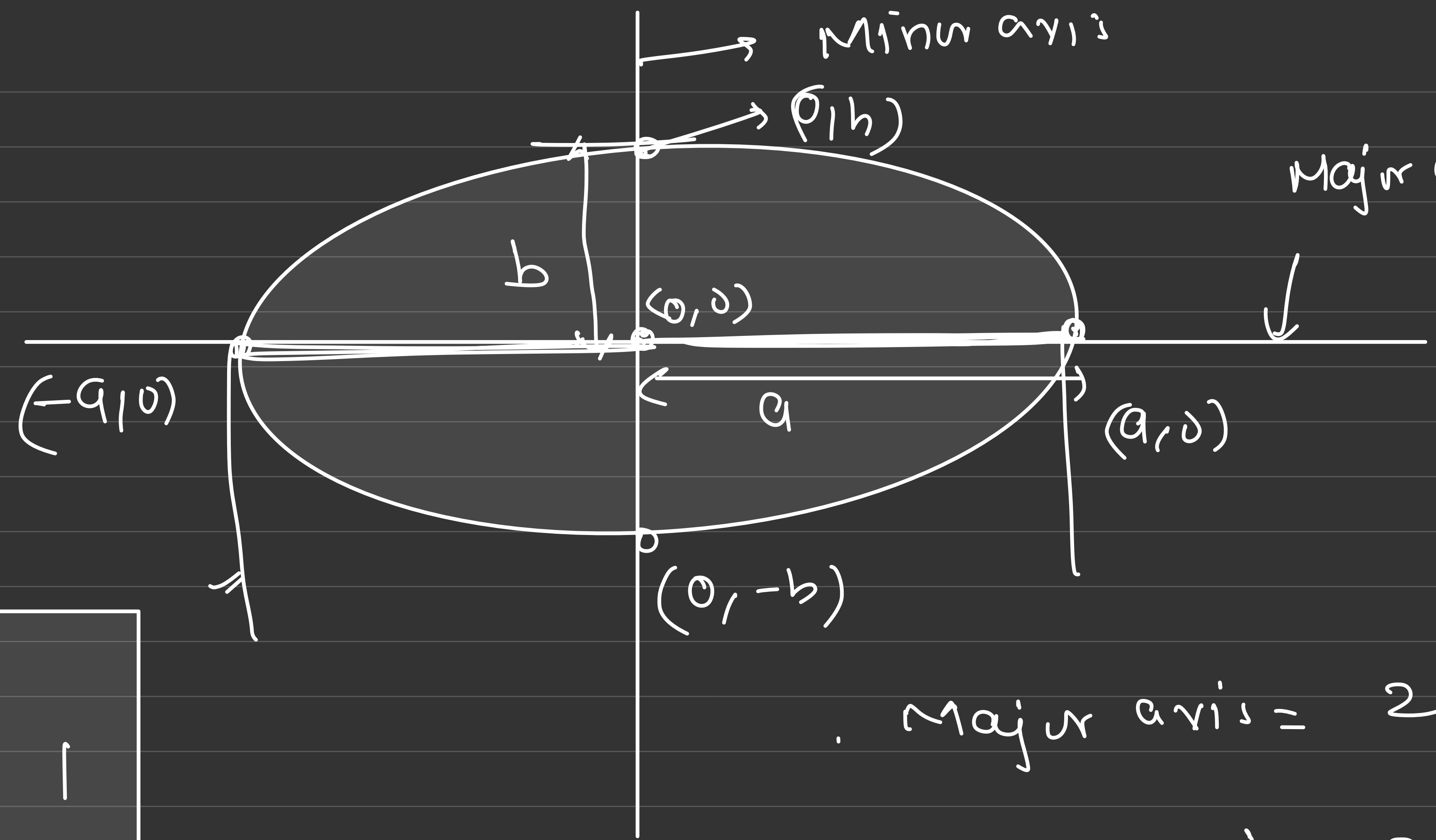


→ ellipse has two focus.

→ If we pass light from one focus and elliptical surface behaves as mirror then reflected light will pass through other focus.

a: Semi-major axis

b: Semi-minor axis



$$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1}$$

- Major axis = $2a$
- Minor axis = $2b$

- Circle is a subset of ellipse whose semi-major axis and semi-minor axis are same.

$\boxed{\text{Area of ellipse} = \pi ab}$

Perimeter of ellipse = $2\pi\sqrt{\left(\frac{a^2+b^2}{2}\right)}$

ex:-

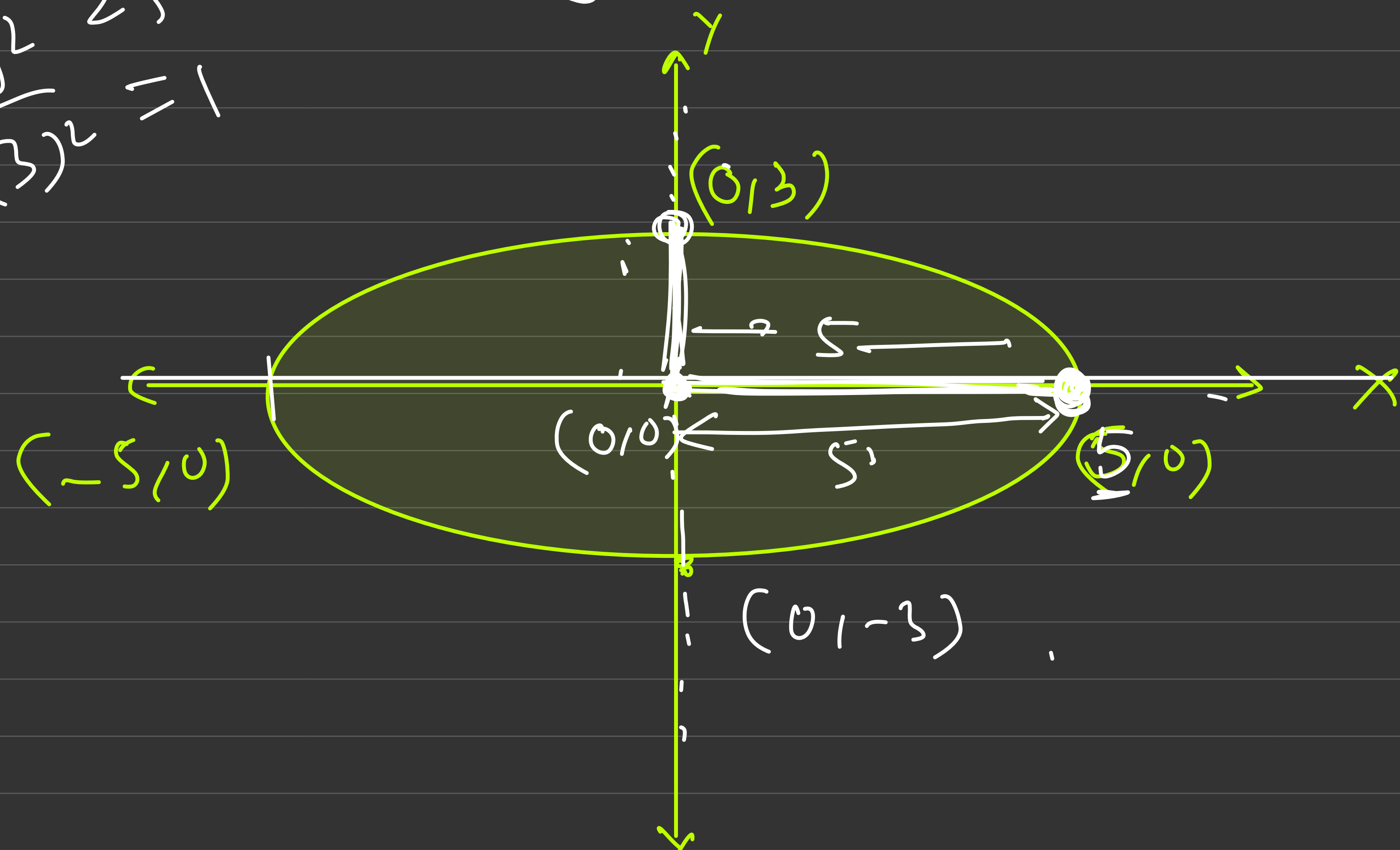
$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

① Draw the graph

$$\frac{x^2}{(5)^2} + \frac{y^2}{(3)^2} = 1$$

Semi-major axis = 5

Semi-minor " = 3

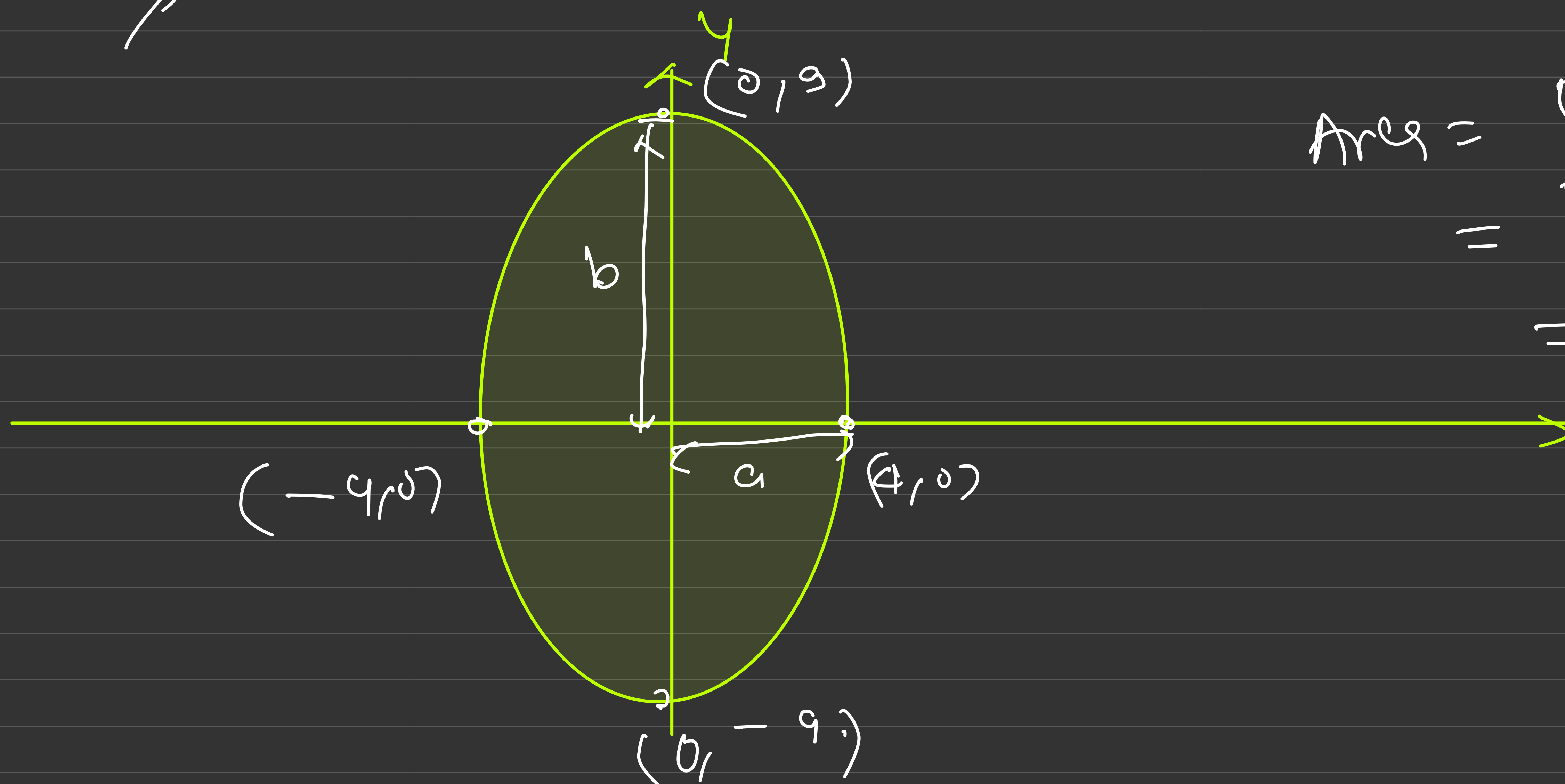


ex :-

$$\frac{x^2}{16} + \frac{y^2}{81} = 1$$

Area of ellipse = ?

$$\frac{x^2}{(4)^2} + \frac{y^2}{(9)^2} = 1$$



Area = $\pi a b$

$= \pi (4)(9)$

$= 36\pi$

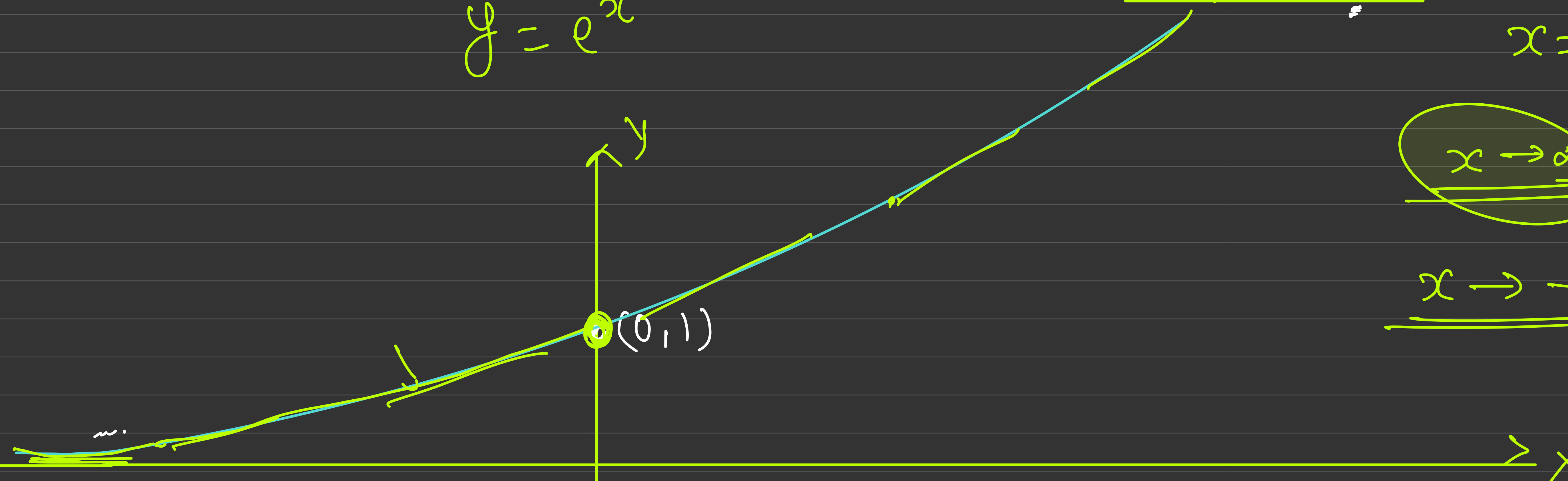
Exponential function

$$y = e^x$$

$$\boxed{e \approx 2.73}$$

$$\boxed{e^0 = 1}$$

$x = 0, \; y = 1$



$x \to \infty \quad y = e^{\infty} \to \infty$

$x \to -\infty \quad y = e^{-\infty}$

$$= \frac{1}{e^{\infty}} = \frac{1}{\infty} \to 0$$

→ Increasing slope

→ Increasing function

→ Asymptote → $y = 0$ (x-axis)

Curve is actually not intersecting the curve but appears to touch the curve

$x \to \infty, \quad e^x \to \infty$

$x \to -\infty, \quad e^x \to$

$$e^{-\infty} = \frac{1}{e^{\infty} \to \infty} \to 0$$

$$\boxed{y = e^{-x}}$$

$x = 0, \quad y = e^{-0} = 1$

$x \to \infty, \quad y = e^{-\infty} \to 0$

$x \to -\infty, \quad y = e^{-(-\infty)} = e^{\infty} \to \infty$

⇒ Decreasing function

⇒ Magnitude of slope → decreasing (+ve to zero)

⇒ Slope → increasing (−ve to zero)

⇒ Asymptote is $\boxed{x\text{-axis} \Rightarrow y = 0}$

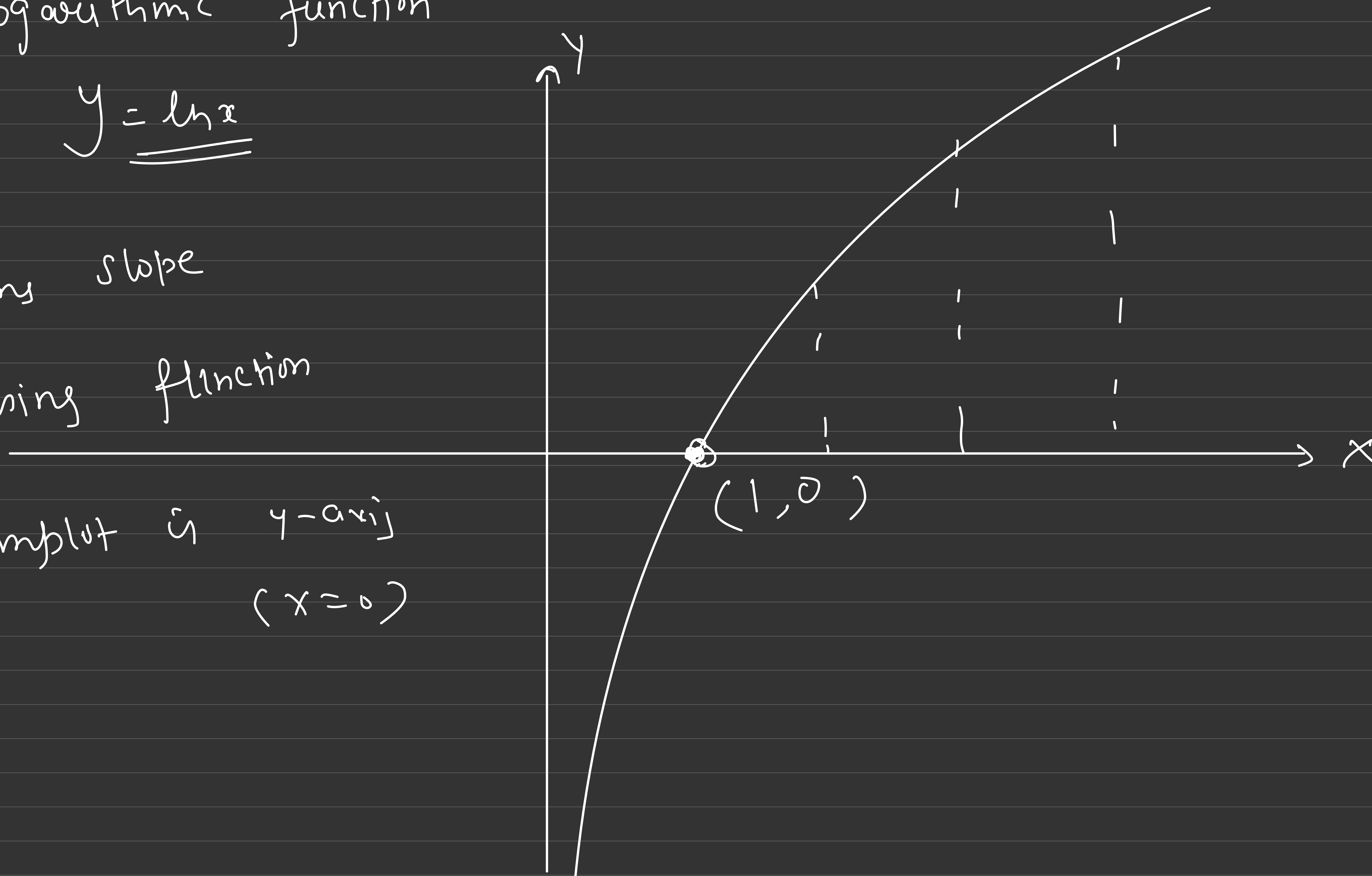
logarithmic function

$y = \ln x$

→ decreasing slope

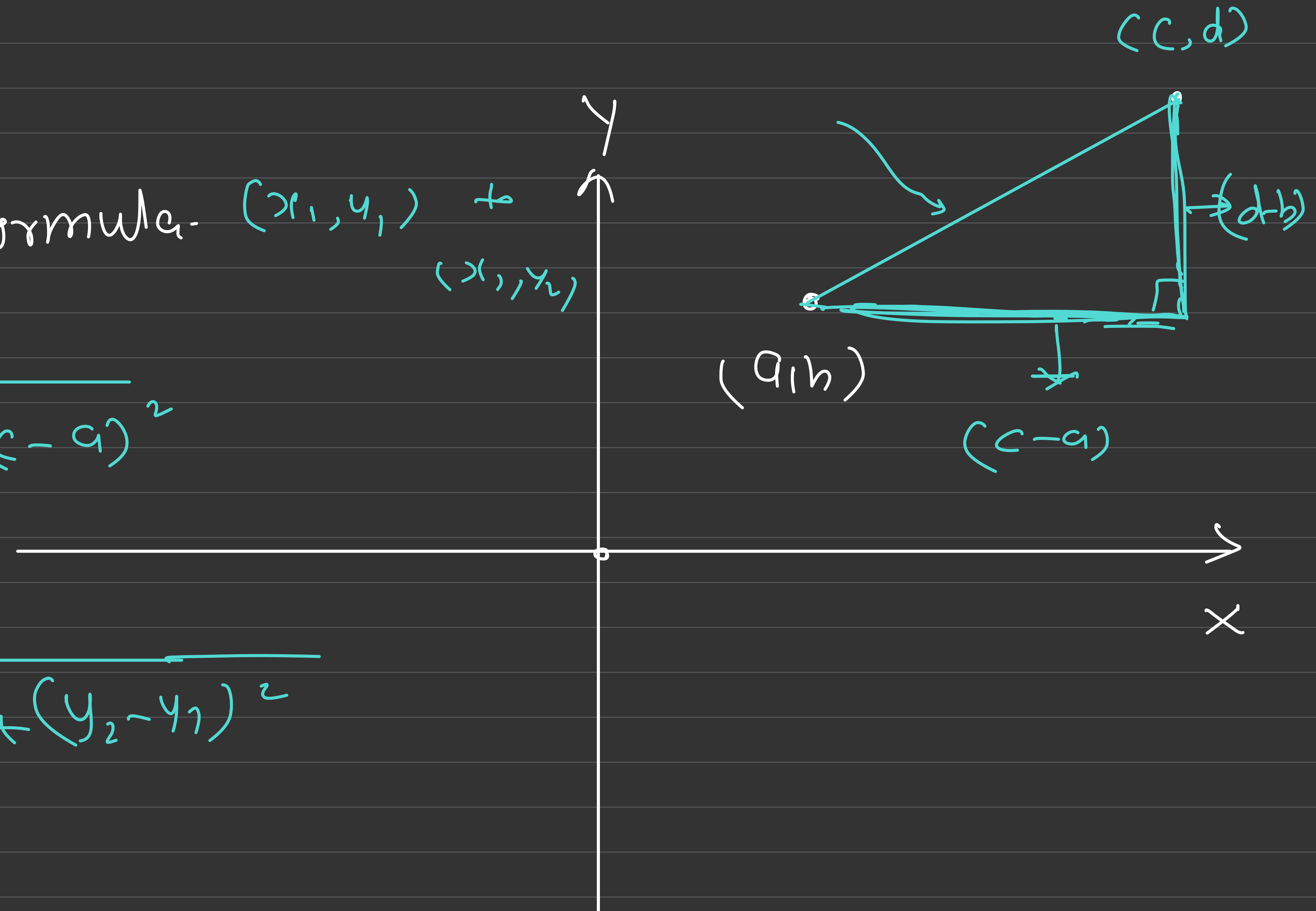
→ increasing function

→ Asymptot is y-axis ($x = 0$)



Co-ordinate

Distance formula (x_1, y_1) to (x_2, y_2)



$$= \sqrt{(d-b)^2 + (c-a)^2}$$

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

* $(x_1, y_1, z_1) \longrightarrow (x_2, y_2, z_2)$

$$D = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

y - cordinate $\Rightarrow$ Ordinate

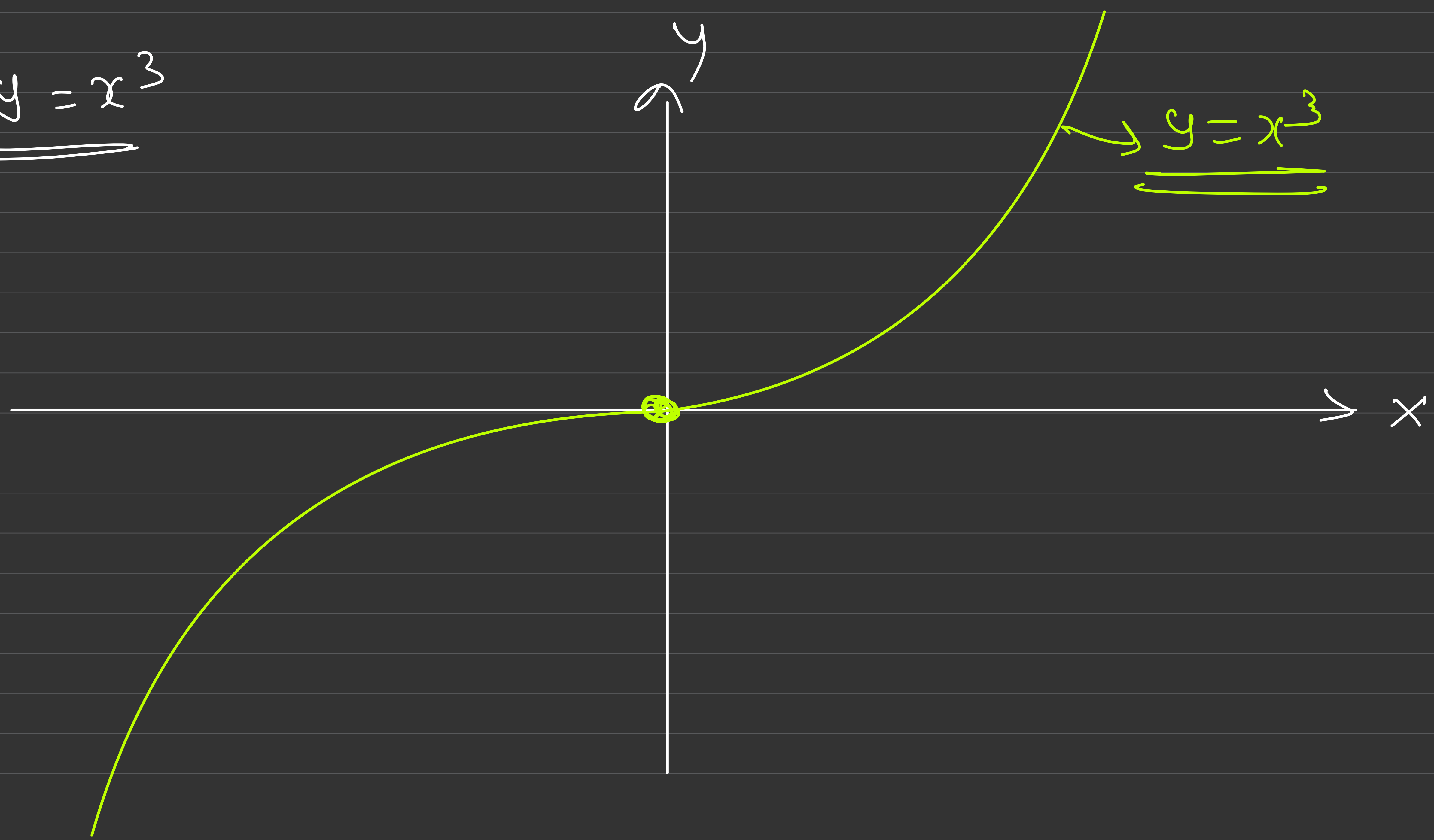
x - Cordinate $\Rightarrow$ Abscissa

$x = 0,\ y = 0$

$x \to \infty,\ y \to \infty$

$x \to -\infty,\ y \to -\infty$

$y = x^3$



Test - Portion

- Mathematical Tool
- Trigonometry
- Algebra
  - AP
  - GP
  - Log.
- Coordinate
  - Distance for
  - Slope
- Calculus
  - Differentiation
  - Integration

Qu: $y = 2\cos\theta - 3\sin\theta = 2\cos\theta + (-3)\sin\theta$

$a = 2, \quad b = -3$

∘ $y_{max} = \sqrt{a^2 + b^2}$

$= \sqrt{(2)^2 + (-3)^2}$

$= \sqrt{4 + 9} = \sqrt{13}$

∘∘ $y_{min} = -\sqrt{13}$

$y = \sqrt{13}\sin(\theta + \phi)$

#

$$y = 1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \cdots \infty$$

$$r = -\frac{1}{2} \qquad \frac{\frac{1}{4}}{-\frac{1}{2}} = -\frac{1}{2}$$

G.P. series

$$S_n = \frac{a}{1-r} = \frac{1}{1-\left(-\frac{1}{2}\right)} = 2$$