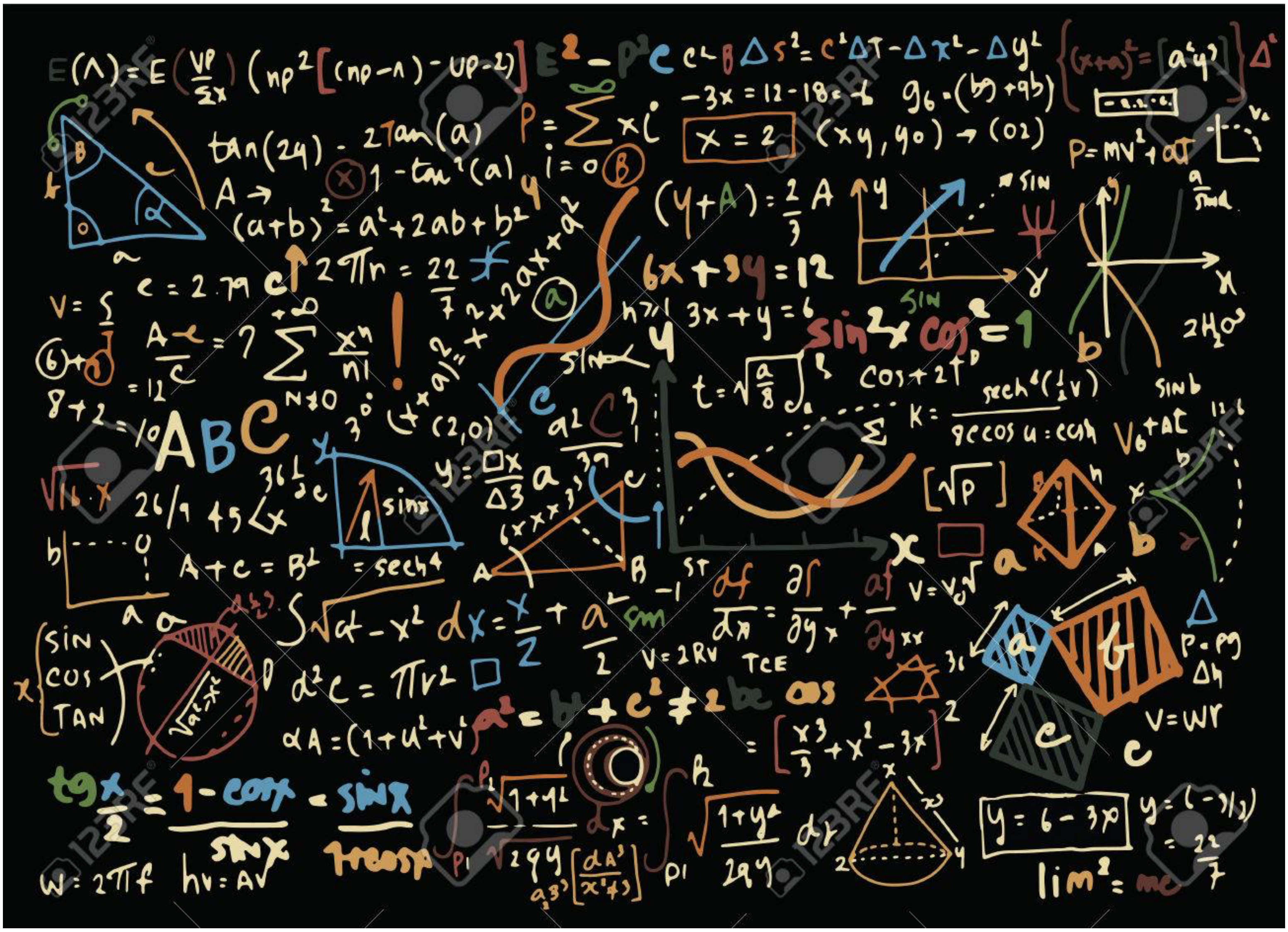
MATHS
FORMULA SHEET



CLASS-12

INVERSE TRIGONOMETRIC FUNCTIONS

FUNCTIONS	DOMAIN	RANGE (PRINCIPAL VALUE BRANCH)
$y = \sin^{-1}x$	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$y = \cos^{-1}x$	$[-1, 1]$	$[0, \pi]$
$y = \operatorname{cosec}^{-1}x$	$R - (-1, 1)$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$
$y = \sec^{-1}x$	$R - (-1, 1)$	$[0, \pi] - \{\frac{\pi}{2}\}$
$y = \tan^{-1}x$	R	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
$y = \cot^{-1}x$	R	$(0, \pi)$

① $\sin^{-1}(-x) = -\sin^{-1}x$

$\tan^{-1}(-x) = -\tan^{-1}x$

$\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1}x$

② $\cos^{-1}(-x) = \pi - \cos^{-1}x$

$\cot^{-1}(-x) = \pi - \cot^{-1}x$

$\sec^{-1}(-x) = \pi - \sec^{-1}x$

③ $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$

$\tan^{-1}x + \cot^{-1}x = \pi/2$

$\operatorname{cosec}^{-1}x + \sec^{-1}x = \pi/2$

④ $\tan^{-1}x \pm \tan^{-1}y = \tan^{-1}\frac{x \pm y}{1 \mp xy}$

⑤ $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

$\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$

⑥ $2\tan^{-1}\theta = \sin^{-1}\frac{2\theta}{1+\theta^2}$

$2\tan^{-1}\theta = \cos^{-1}\frac{1-\theta^2}{1+\theta^2}$

$2\tan^{-1}\theta = \tan^{-1}\left(\frac{2\theta}{1-\theta^2}\right)$

⑦ $1 - \cos\theta = 2\sin^2\theta/2$

$1 + \cos\theta = 2\cos^2\theta/2$

MATRICES

1. $kA = k[a_{ij}]_{m\times n} = [k(a_{ij})]_{m\times n}$
2. $-A = (-1)A$
3. If $A = [a_{ij}]_{m\times n}$, then A' or $A^T = [a_{ji}]_{n\times m}$
4. (i) $(A')' = A$ (iii) $(A+B)' = A' + B'$
   (ii) $(kA)' = kA'$ (iv) $(AB)' = B'A'$
5. A is a symmetric matrix if $A' = A$.
6. A is a skew symmetric matrix if $A' = -A$.

DETERMINANTS

1. Determinant of a matrix $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ is given by $|A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$
2. Determinant of a matrix $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$ is given by
$$|A| = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1\begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1\begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1\begin{vmatrix} a_2 & b_2 \\ a_3 & a_3 \end{vmatrix}$$
3. If $A = [a_{ij}]_{3\times 3}$ then $|k\cdot A| = k^3|A|$
4. $A^{-1} = \frac{1}{|A|}(\text{adj } A)$
5. Area of Δ with vertices $(x_1 y_1)(x_2 y_2)(x_3 y_3)$ is $\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

CONTINUITY AND DIFFERENTIABILITY

① $\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$

② $\frac{d}{dx}(\cos^{-1}x) = \frac{1}{\sqrt{1-x^2}}$

③ $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$

④ $\frac{d}{dx}(\cot^{-1}x) = \frac{-1}{1+x^2}$

⑤ $\frac{d}{dx}(\sec^{-1}x) = \frac{1}{x\sqrt{x^2-1}}$

⑥ $\frac{d}{dx}(\operatorname{cosec}^{-1}x) = \frac{-1}{x\sqrt{x^2-1}}$

⑦ $\frac{d}{dx}(e^x) = e^x$

⑧ $\frac{d}{dx}(\log x) = \frac{1}{x}$

⑨ $(f \pm g)(x) = f(x) \pm g(x)$ is continuous.

⑩ $(f \cdot g)(x) = f(x) \cdot g(x)$ is continuous.

⑪ $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ (wherever $g(x) \neq 0$) is continuous

APPLICATION OF DERIVATIVES

① A function is said to be:

(a.) increasing on an interval (a,b) if
$x_1 < x_2$ in $(a,b) \Rightarrow f(x_1) \leq f(x_2)$ for all $x_1, x_2 \in (a,b)$

(b.) decreasing on (a,b) if
$x_1 < x_2$ in $(a,b) \Rightarrow f(x_1) \geq f(x_2)$ for all $x_1, x_2 \in (a,b)$

② The equation of the tangent at (x_0, y_0) to the curve $y = f(x)$ is given by $y - y_0 = \left.\frac{dy}{dx}\right]_{(x_0 y_0)} (x - x_0)$

INTEGRALS

1. $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad n \neq -1$
2. $\int \cos x\,dx = \sin x + C$
3. $\int \sin x\,dx = -\cos x + C$
4. $\int \sec^2 x\,dx = \tan x + C$
5. $\int \operatorname{cosec}^2 x\,dx = -\cot x + C$
6. $\int \sec x \cdot \tan x\,dx = \sec x + C$
7. $\int \operatorname{cosec} x \cot x\,dx = -\operatorname{cosec} x + C$
8. $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C$
9. $\int \frac{dx}{\sqrt{1-x^2}} = -\cos^{-1} x + C$
10. $\int \frac{dx}{\sqrt{1+x^2}} = \tan^{-1} x + C$
11. $\int \frac{dx}{1+x^2} = -\cot^{-1} x + C$
12. $\int e^x\,dx = e^x + C$
13. $\int a^x\,dx = \frac{a^x}{\log a} + C$
14. $\int \frac{\cancel{x}\;dx}{x\sqrt{x^2-1}} = \sec^{-1} x + C$
15. $\int \frac{dx}{x\sqrt{x^2-1}} = -\operatorname{cosec}^{-1} x + C$
16. $\int \frac{1}{x}\,dx = \log|x| + C$
17. $\int \tan x\,dx = \log|\sec x| + C$
18. $\int \cot x\,dx = \log|\sin x| + C$
19. $\int \sec x\,dx = \log|\sec x + \tan x|$
20. $\int \operatorname{cosec} x\,dx = \log|\operatorname{cosec} x - \cot x|$
21. $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + C$
22. $\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + C$
23. $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$
24. $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + C$
25. $\int \frac{dx}{\sqrt{x^2-a^2}} = \log\left|x + \sqrt{x^2-a^2}\right| + C$
26. $\int \frac{dx}{\sqrt{x^2-a^2}} = \log\left|x + \sqrt{x^2+a^2}\right| + C$

$$\int f_1(x)\cdot f_2(x)\,dx = f_1(x)\int f_2(x)\,dx - \int\left[\frac{d}{dx} f_1(x)\cdot \int f_2(x)\,dx\right]dx$$

28. $\int e^x [f(x) + f'(x)]\, dx = \int e^x f(x)\, dx + C$

29. $\int \sqrt{x^2 - a^2}\, dx = \frac{x}{a}\sqrt{x^2 - a^2} - \frac{a^2}{2} \log\left|x + \sqrt{x^2 - a^2}\right| + C$

30. $\int \sqrt{x^2 + a^2}\, dx = \frac{x}{a}\sqrt{x^2 + a^2} + \frac{a^2}{2} \log\left|x + \sqrt{x^2 + a^2}\right| + C$

31. $\int \sqrt{a^2 - x^2}\, dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1}\frac{x}{a} + C$

APPLICATION OF INTEGRALS

1. Area bounded by the curve $y = f(x)$, x axis and the lines $x = a$, $x = b$ $(b > a)$ is $\int_a^b y\, dx = \int_a^b f(x)\, dx$

2. Area bounded by the curve $x = \phi(y)$, y axis and the lines $y = c$, $y = d$ is $\int_c^d x\, dy = \int_c^d \phi(y)\, dy$

3. If $f(x) \geq g(x)$ in $[a, c]$ and $f(x) \leq g(x)$ in $[c, b]$ $a < c < b$, then area $= \int_a^c [f(x) - g(x)]\, dx + \int_c^b [g(x) - f(x)]\, dx$

DIFFERENTIAL EQUATIONS

1. Order of a differential equation is the order of the highest order derivative occuring in the equation.

2. Degree of a differential equation is the highest power of the highest order derivative in it.

VECTOR ALGEBRA

1. Scalar product of two vectors $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$.
2. Cross product of two vectors $\vec{a} \times \vec{b} = |\vec{a}||\vec{b}| \sin\theta \hat{n}$
3. Position vector of a point $P(x, y, z)$ is $x\hat{i} + y\hat{j} + z\hat{k}$ and its magnitude by $\sqrt{x^2 + y^2 + z^2}$
4. Position vector of a point R dividing the line segment PQ in the ratio $m:n$
   - → internally, is given by $\dfrac{n\vec{a} + m\vec{b}}{m+n}$
   - → externally, is given by $\dfrac{m\vec{b} - n\vec{a}}{m-n}$

3D GEOMETRY

1. If θ is the actual angle between $\vec{r} = \vec{a_1} + \lambda \vec{b_1}$ and $\vec{r} = \vec{a_2} + \lambda \vec{b_2}$, then $\cos\theta = \left| \dfrac{\vec{b_1} \cdot \vec{b_2}}{|\vec{b_1}||\vec{b_2}|} \right|$
2. Shortest distance between $\vec{r} = \vec{a_1} + \lambda \vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu \vec{b_2}$ is $\left| \dfrac{(\vec{b_1} \times \vec{b_2})(\vec{a_2} - \vec{a_1})}{|\vec{b_1} \times \vec{b_2}|} \right|$
3. Shortest distance between the lines $\dfrac{x - x_1}{a_1} = \dfrac{y - y_1}{b_1} = \dfrac{z - z_1}{c_1}$ and $\dfrac{x - x_2}{a_2} = \dfrac{y - y_2}{b_2} = \dfrac{z - z_2}{c_2}$ is

$$\frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(b_1c_2 - b_2c_1)^2 + (c_1a_2 - c_2a_1)^2 + (a_1b_2 - b_1a_2)^2}}$$

④ Distance between parallel lines $\vec{r} = \vec{a_1} + \lambda \vec{b}$ and $\vec{r} = \vec{a_2} + \mu \vec{b}$ is $\left| \dfrac{\vec{b} \times (\vec{a_2} - \vec{a_1})}{|\vec{b}|} \right|$

⑤ Distance from a point (x_1, y_1, z_1) to the plane $Ax + By + Cz + D = 0$ is $\left| \dfrac{Ax_1 + By_1 + Cz_1 + D}{\sqrt{A^2 + B^2 + C^2}} \right|$

⑥ If $\dfrac{x - x_1}{l_1} = \dfrac{y - y_1}{m_1} = \dfrac{z - z_1}{n_1}$ and $\dfrac{x - x_2}{l_2} = \dfrac{y - y_2}{m_2} = \dfrac{z - z_2}{n_2}$ are the equations of two lines then the acute angle between the two lines is given by $\cos\theta = |l_1 l_2 + m_1 m_2 + n_1 n_2|$

PROBABILITY

① Conditional probability of an event E, given the occurance of the event F is given by $P(E|F) = \dfrac{P(E \cap F)}{P(F)}$

② $0 \leq P(E|F) \leq 1$ $\quad P(E'|F) = 1 - P(E|F)$

$P((E \cup F)|G) = P(E|G) + P(F|G) - P((E \cap F)|G)$

③ $P(E \cap F) = P(E)\, P(F|E),\ P(E) \neq 0$

$P(E \cap F) = P(F)\, P(E|F),\ P(F) \neq 0$

④ If E and F are independent, then

$P(E \cap F) = P(E)\, P(F)$

$P(E|F) = P(E),\ P(F) \neq 0$

$P(F|E) = P(F),\ P(E) \neq 0$

⑤ Baye's Theorem

$$P(E_i|A) = \frac{P(E_i)\, P(A|E_i)}{\sum_{j=1}^{n} P(E_j)\, P(A|E_j)}$$