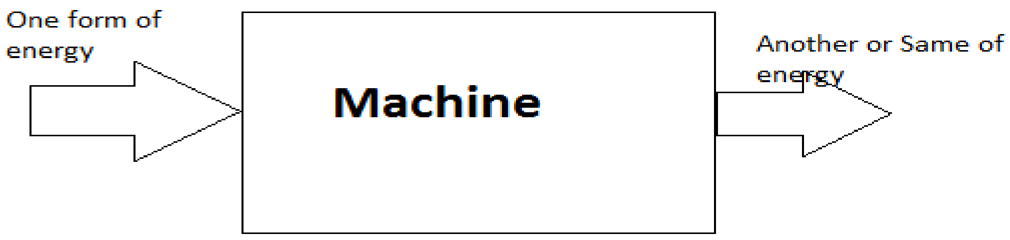
UNIT-III

TRANSFORMERS

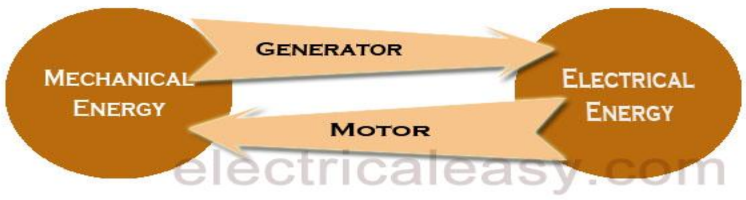
Machine:

A machine is a device which converts one form energy into another form or same form of energy.

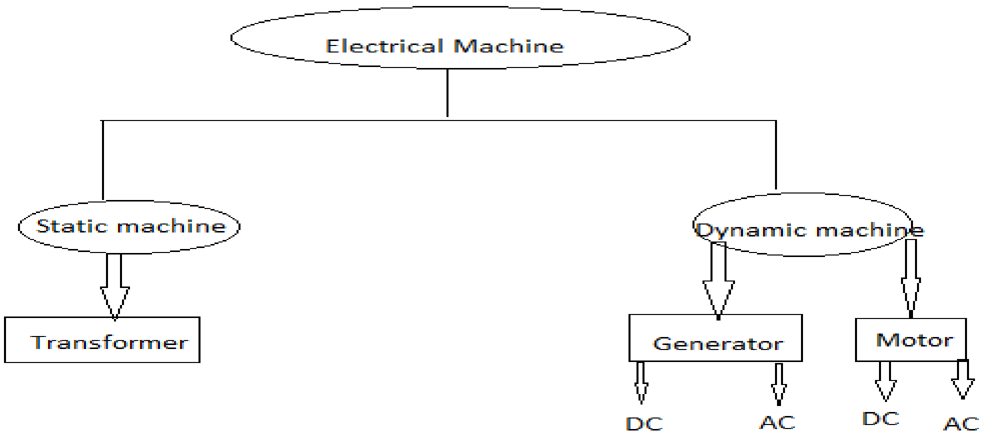


Electrical machine:

An electrical machine is a device which converts mechanical energy into electrical energy or vice versa. Electrical machines also include transformers, which do not actually make conversion between mechanical and electrical form but they convert AC current from one voltage level to another voltage level.



Classification of Electrical Machines:



Faraday's Laws of Electromagnetic Induction

Faraday's First Law

Faraday's first law of electromagnetic induction states that "EMF is induced in a coil when there is a change in the flux linking the coil".

Faraday's Second Law

Faraday's second law of electromagnetic induction states that "the magnitude of induced EMF in a coil is directly proportional to the rate of change of flux linking the coil".

$$e = N \, d\phi/dt$$

Where

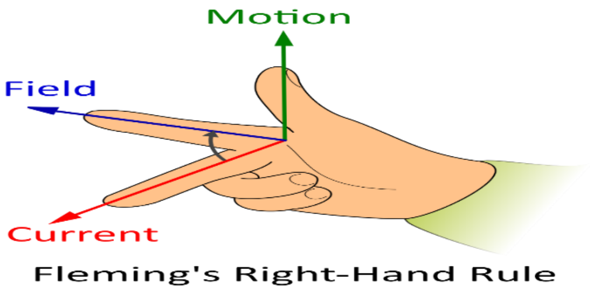
e = Induced EMF, N = the number of turns, $d\phi$ = Change in flux, dt = Change in time

Lenz's Law

Lenz's law determines the direction of an induced EMF in a coil can be . "It thus states that the direction of induced EMF is such that it opposes the change causing it.

Fleming's Right Hand Rule

It states that "if the thumb, the forefinger and the middle finger are held in such a way that they are mutually perpendicular to each other (makes 90° of Angles), then the forefinger points the direction of the field, the thumb points the direction of motion of the conductor and the middle finger points the direction of the induced Current (from EMF)



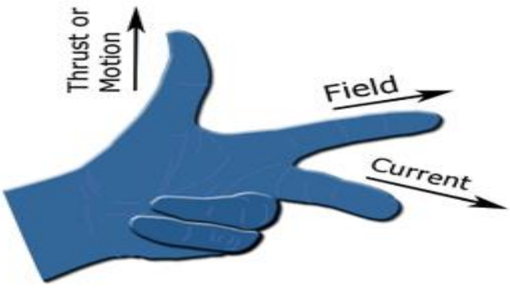
Fleming's Left Hand Rule

Fleming's left hand rule helps in determining the motion of the conductor.

Fore finger - direction of magnetic **f**ield (N-S)

Centre finger - direction of **c**urrent (positive to negative)

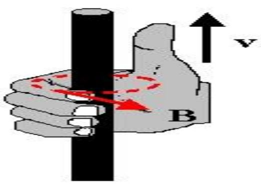
Thumb - **m**ovements of the wire



Fleming's Thumb Rule

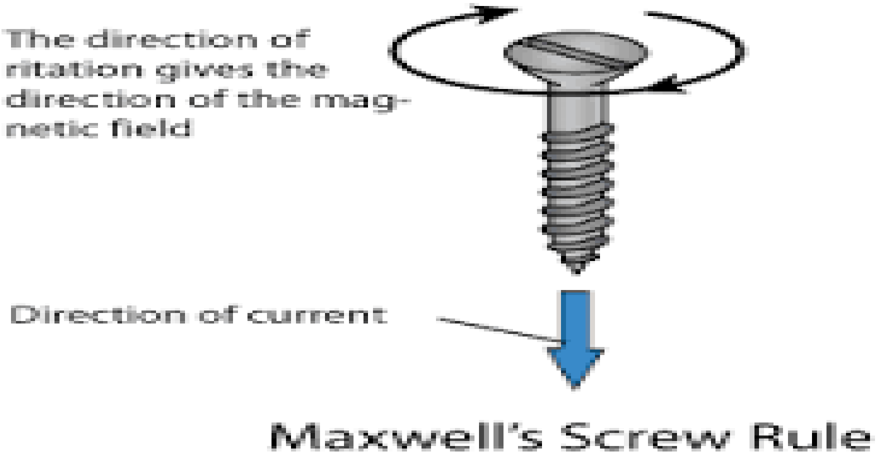
It determines the direction of the magnetic field around a current-carrying wire and vice-versa

Using your right-hand:
Curl your fingers into a half-circle around the wire, they point in the direction of the magnetic field, B Point your thumb in the direction of the conventional current.



<u>Maxwell's Cork screw rule:</u> It determines the direction of the magnetic field around a current-carrying wire and vice-versa

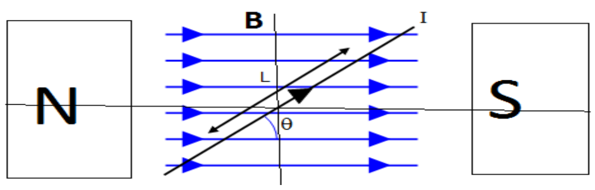
X--- Represents downward direction .---- Represents Upward direction



<u>Lorentz Force Equation</u>

The force F on the wire in Figure 4 can be shown to be proportional to
(a) the current on the wire I,
(b) the length of the conductor in the field L,
(c) the sine of the angle θ that the conductor makes with the field , and
(d) the strength of the field - this is measured by a quantity known as the magnetic flux density B of the field The units for B are tesla (T).The force is given by the equation:

$$F=BIL \sin \theta$$



Requirements for inducing an EMF in a conductor:

1. A Conductor
2. Magnetic field
3. Relative motion between conductor and magnetic field.

Induced Emfs are classified as follows

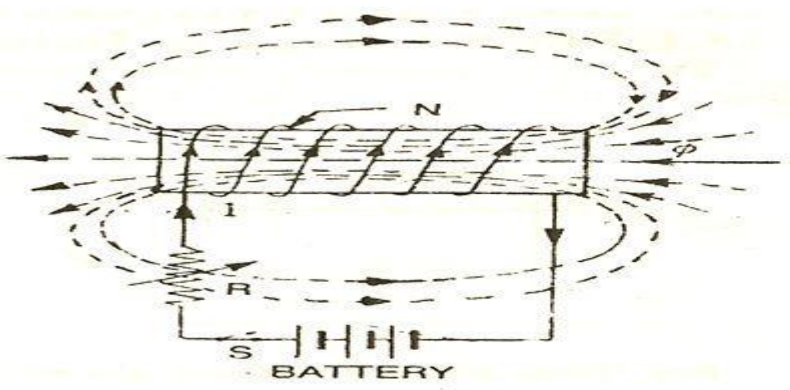
1. Statically induced EMF i) Self-induced EMF ii) Mutually induced EMF
2. Dynamically induced EMF

Statically induced EMF:

It is the emf induced in the conductor when the conductor stationary. These are classified into two types

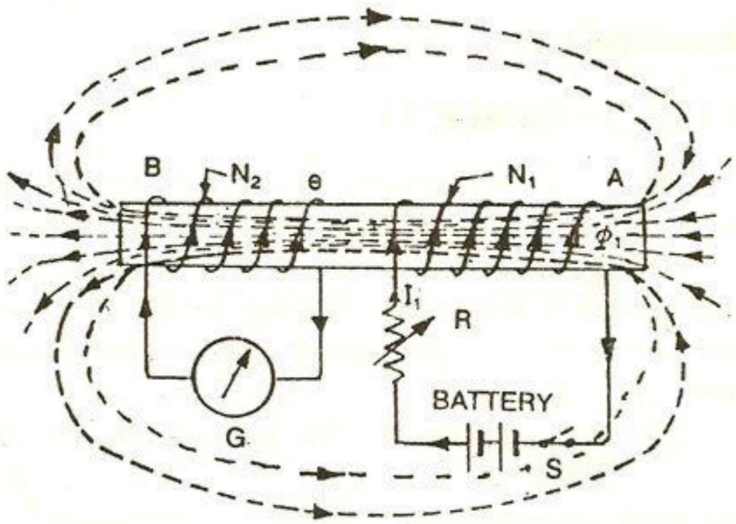
Self-induced EMF :

Self-induced emf is the e.m.f induced in the coil due to the change of flux produced by linking it with its own turns.



Mutually Induced EMF:

The emf induced in a coil due to the change of flux produced by another neighbouring coil linking to it, is called **Mutually Induced emf.**



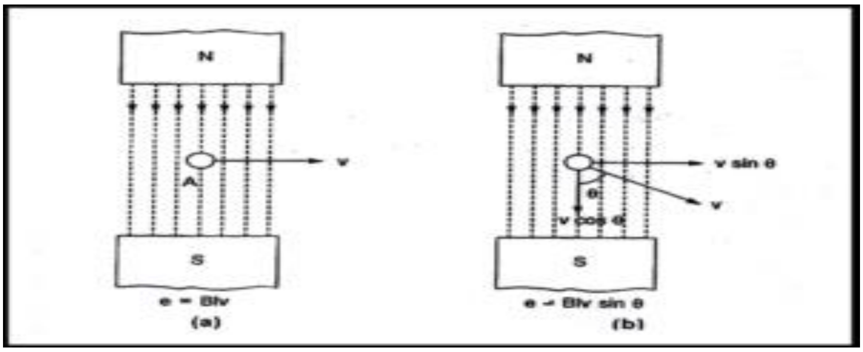
Dynamically induced EMF:

An emf induced due to a physical movement of either conductor or flux. Here field is stationary and conductors cut across it. Either the coil or magnet moves.

Magnitude of dynamically induced emf is as shown the conductor of length l is placed in the magnetic field produced by a permanent magnet.

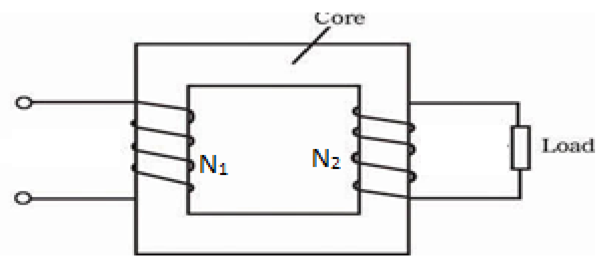
$$E_d = BLv \sin\theta$$

The conductor moves in a plane which is parallel to the plane of the magnetic flux. Therefore, induced emf is zero. Now if the plane of direction of motion of the conductor is perpendicular to the plane of magnetic flux then the induced emf is maximum. So the expression for magnitude of emf is given by:



Introduction:

1. Static (no moving parts) AC machine.
2. Transfers electrical energy from one electrical circuit to the another electrical circuit without changing frequency, electrical power and with or without changing the voltage with the corresponding change of current
3. There is no electrical connection between two circuits but they are coupled by magnetic medium.
4. Transformer has two windings
    - High Voltage (H V) winding --- the winding which has more turns
    - Low Voltage (L V) winding --- the winding which has less turns



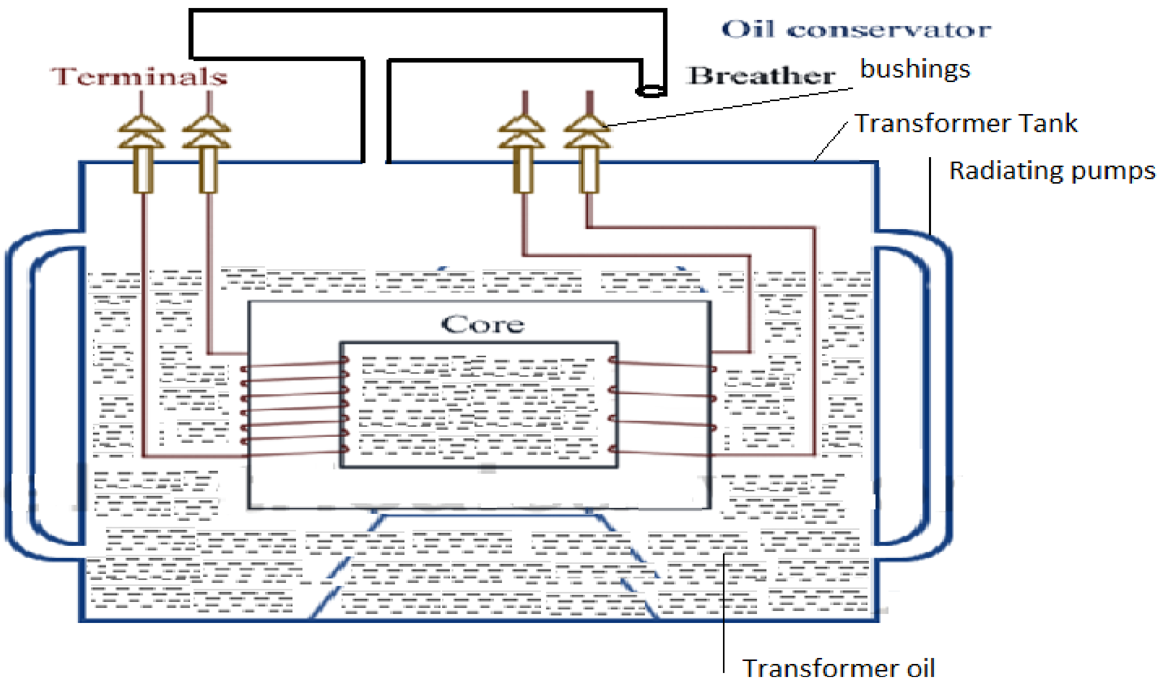
5. The winding which is connected to the source is called Primary winding.
6. The winding which is connected to the load is called Secondary winding.

If N_1--- number of turns in the primary

N_2--- number of turns in the secondary

7. If $N_1>N_2$---- Step down transformer
8. If $N_1<N_2$---- Step up transformer
9. If N1=N2---- Isolation transformer
10. It works based on Faraday's law of electromagnetic mutual induction principle.

Construction details of a Transformer:



The main parts of a transformer are

1. Transformer core
2. Transformer windings
3. Transformer tank
4. Transformer oil (cooling oil)
5. Conservator Tank
6. Breather
7. Bushings
8. Primary and Secondary leads (Terminals)
9. Radiating Pumps (Pipes)

Transformer core:

1) The main purpose of core in a tranformer is to provide a low reluctance path between primary and secondary.
2) Transformer core is made up of silicon steel or sheet steel. Silicon is added to the steel to reduce the conductivity of the core. Only 3 to 5% of silicon added should to the steel otherwise the brittleness of the core reduces.
3) Silicon sheets will be laminated to reduce iron losses (eddy current losses) in the transformer core.
4) CRGO (Cold Rolled Grain Orientation) cores used for transformer
5) Based on the construction of core transformers are two types
   a. Core type transformer
   b. Shell type transformer

Transformer windings:

1. Transformer windings are used to produce the flux and to induce the emf the windings are necessary
2. Windings are made up of stranded copper conductors
3. The insulation will be provided by using the impregnated paper between the windings and between windings to the core.
4. Every Transformer has two windings 1. Primary 2. Secondary

Transformer Tank:

1. It acts as an outer frame of a transformer
2. It avoids the exposure of core and windings to the atmosphere
3. It is manufactured with iron sheets and outer part of the transformer tank is painted with enamel paint to avoid the shocks. **Transformer oil:**

1. The purpose of the transformer oil is to provide the cooling as well as the insulation between the windings and the transformer tank.
2. For the transformer cooling provided in three ways
   i. Natural cooling
   ii. Oil filled cooling
   iii. Air blast cooling
3. The oil using in the transformer must have high dielectric strength.

Conservator Tank:

Main function of conservator tank of transformer is to provide adequate space for expansion of oil inside the transformer when the temperature inside the transformer increases due to the loading on the transformer.

Breather:

The conservator tank is fitted with a 'breather' through which air is expelled (breathed out) when transformer is loaded with the losses causing the oil temperature to increase and expand. During light load conditions oil cools down and contracts in volume so air is breathed in from the atmosphere. This air contains moisture and the moisture is absorbed by 'silica gel' crystals kept in the breather and then allowed to enter the conservator tank.

Bushings:

A bushing is an insulated device that allows an electrical conductor to pass safely through a grounded conducting barrier such as a transformer.

For low voltages ---- Ceramic Type bushings

For high voltages--- Condenser Type bushings

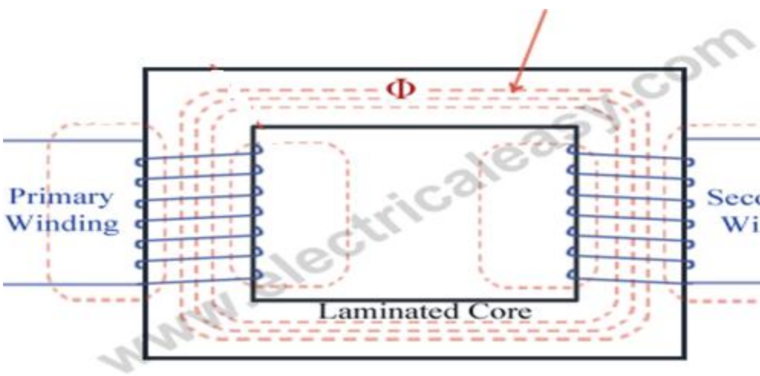
Contact Leads:

1. These are used to bring out the primary and secondary windings from the transformer tank
2. These are made up of copper material.

Radiating Pipes:

These are used for providing the extra space for the expansion of transformer oil.

Working principle of transformer



The basic principle behind working of a transformer is the phenomenon of mutual induction between two windings linked by common magnetic flux.

When, primary winding is connected to a source of alternating voltage, an alternating current flows through the primary winding which produces alternating around the winding. The core provides magnetic path for the flux, to get linked with the secondary winding. As the flux produced is alternating (the direction of it is continuously changing), EMF gets induced in the secondary winding according to Faraday's law of electromagnetic mutual induction. This emf is called 'mutually induced emf', and the frequency of mutually induced emf is same as that of supplied emf. If the secondary winding is closed circuit, then mutually induced current flows through it, and hence the electrical energy is transferred from one circuit (primary) to another circuit (secondary).

The alternating flux produced in the core also links the primary winding which results am emf is induced in the primary winding according to Faraday's law of electromagnetic Self-induction. And this emf is called self-induced emf and the direction this emf is given by the lenz's law

According to lenz's law the direction of the induced emf opposes the cause producing it. The induced emf in the primary opposing the supply voltage so it is called as back emf or counter emf or opposing emf.

Ideal Transformer:

Assumptions of ideal transformer: -

1. Core has infinite permeability
2. Windings has no ohmic resistance
3. Core does not have leakage reactance
4. No copper losses
5. No core losses

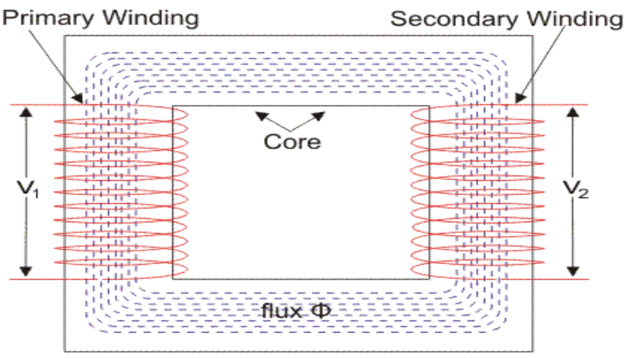
An ideal transformer is an imaginary transformer which has
- no copper losses (no winding resistance)
- no iron loss in core
- no leakage flux

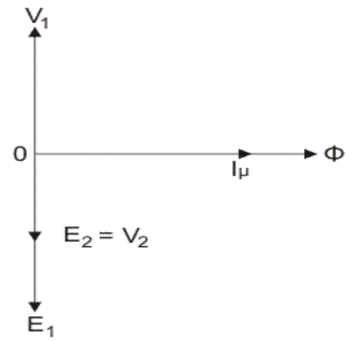
In other words, an ideal transformer gives output power exactly equal to the input power. The efficiency of an idea transformer is 100%. Actually, it is impossible to have such a transformer in practice, but ideal transformer model makes problems easier.

Characteristics of ideal transformer

- Zero winding resistance: It is assumed that, resistance of primary as well as secondary winding of an ideal transformer is zero. That is, both the coils are purely inductive in nature.
- Infinite permeability of the core: Higher the permeability, lesser the mmf required for flux establishment. That means, if permeability is high, less magnetizing current is required to magnetize the transformer core.
- No leakage flux: Leakage flux is a part of magnetic flux which does not get linked with secondary winding. In an ideal transformer, it is assumed that entire amount of flux get linked with secondary winding (that is, no leakage flux).

- <u>100% efficiency</u>: An ideal transformer does not have any losses like hysteresis loss, eddy current loss etc. So, the output power of an ideal transformer is exactly equal to the input power. Hence, 100% efficiency.

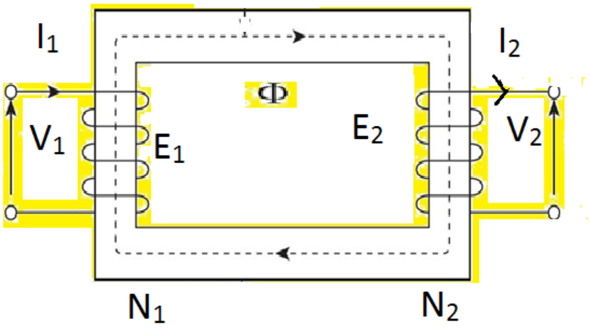


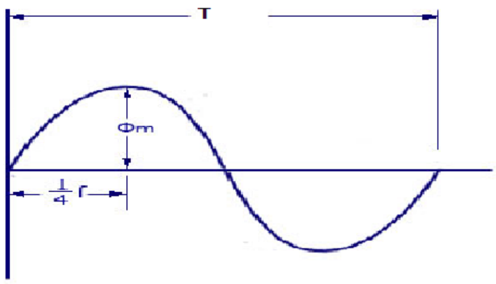


Now, if an alternating voltage V_1 is applied to the primary winding of an ideal transformer, transformer draws magnetising current (I_μ) which is Exactly 90 degrees lagging to the supply voltage V_1. This magnetizing current $I\mu$ produces alternating magnetic flux Φ . This flux Φ gets linked with the secondary winding and emf E_2 gets induced by mutual induction, Counter emf E_1 will be induced in the primary winding. As windings are purely inductive, this induced emf E_1 will be exactly equal to the applied voltage but in 180-degree phase opposition Faraday's law of electromagnetic induction. This mutually induced emf E_2 is in phase with E_2

For an ideal transformer, $E_1I_1 = E_2I_2$.

E.M.F Equation of a Transformer





Let,

N_1 = Number of turns in primary

N_2 = Number of turns in secondary

$Ø_{max}$ = Maximum flux in the core in webers = B_{max} X A

f = Frequency of alternating current input in hertz (Hz)

E_1 = Emf induced in the Primary winding in volts

E_2 = Emf induced in the Secondary winding in volts

According to the Faradays laws of Electromagnetic induction principle the average induced Emf is proportional to the rate of change of flux linkages.

As shown in figure above, the core flux increases from its zero value to maximum value $Ø_{max}$ $_{in}$ one quarter of the cycle , that is in ¼ frequency second.

Therefore, average rate of change of flux = $Ø_{max}$/ ¼ f

$$= 4f\ Ø_{max} \text{Wb/s}$$

Now, rate of change of flux per turn means induced electro motive force in volts.

Therefore,

average electro-motive force induced/turn = 4f $Ø_{max}$volt

If flux Ø varies sinusoidal, then r.m.s value of induced e.m.f is obtained by multiplying the average value with form factor.

Form Factor = r.m.s. value/average value = 1.11

Therefore, r.m.s value of e.m.f/turn = 1.11 X 4f $Ø_{max}$

$$= 4.44f\ Ø_{max}$$

Now, r.m.s value of induced e.m.f in the whole of primary winding

= (induced e.m.f./turn) X Number of primary turns

Therefore,

$$E_1 = 4.44f\ N_1 Ø_{max}$$

$$= 4.44 f N_A B_m A$$

Similarly, r.m.s value of induced e.m.f in secondary is

$$E_2 = 4.44f\ N_B\ Ø_{max} = 4.44 f N_2 B_m A$$

In an ideal transformer on no load,

$$V_1 = E_1 \text{ and } V_2 = E_{2,}$$

where V_B is the terminal voltage

Voltage Transformation Ratio (K)

From the above equations we get

$E_2/E_1 = V_2/V_1 = N_2/N_1 = K$

This constant K is known as voltage transformation ratio.

(1) If $N_2 > N_1$, that is $K > 1$, then transformer is called step-up transformer.

(2) If $N_2 < N_1$, that is $K < 1$, then transformer is known as step-down transformer.

Again for an ideal transformer,

Input VA = output VA

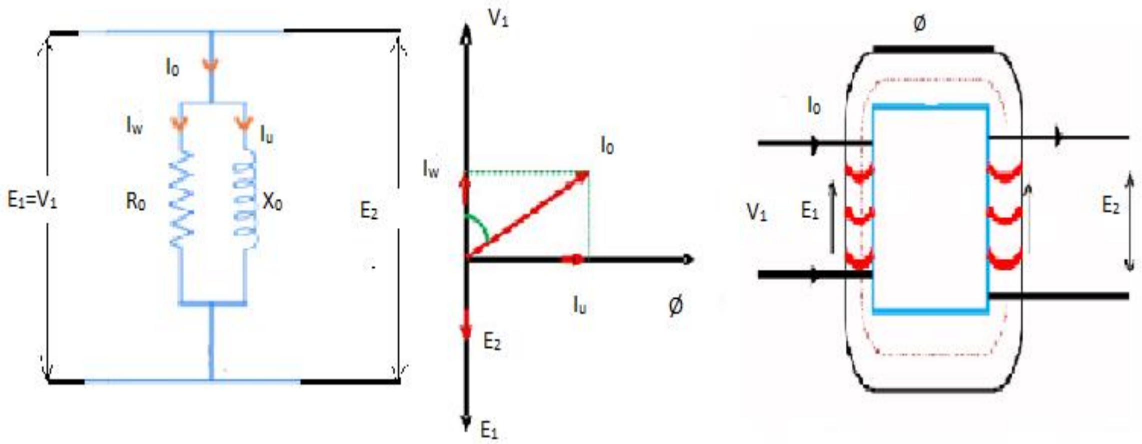
$V_1 I_1 = V_2 I_2$

Or, $I_2/I_1 = V_1/V_2 = 1/K$

Hence, currents are in the inverse ratio of the (voltage) transformation ratio.

Practical Transformer:Transformer on no load:

When the transformer is operating at no load, the secondary winding is open circuited, which means there is no load on the secondary side of the transformer and, therefore, current in the secondary will be zero, while primary winding carries a small current I_0 called no load current which is 2 to 10% of the rated current. This current is responsible along with the magnetisation of the core for supplying the iron losses (hysteresis and eddy current losses) in the core and a very small amount of copper losses in the primary winding. The phase angle is less than 90 degrees and it depends upon the losses in the transformer and it is less than $\phi_0 < 90$ degrees. The power factor is very low and varies from 0.1 to 0.15.



The no load current consists of two components

- Reactive or magnetizing component I_m
  (It is in quadrature with the applied voltage V_1. It produces flux in the core and does not consume any power)

- Active or power component I_w, also known as working component
  (It is in phase with the applied voltage V_1. It supplies the iron losses and a small amount of primary copper loss)

The following steps are given below to draw the phasor diagram

1. The function of the magnetizing component is to produce the magnetizing flux, and thus, it will be in phase with the flux.
2. Induced emf in the primary and the secondary winding lags the flux ϕ by 90 degrees.
3. The primary copper loss is neglected, and secondary current losses are zero as $I_2 = 0$. Therefore, the current I_0 lags behind the voltage vector V_1 by an angle ϕ_0 called no-load power factor angle shown in the phasor diagram above.
4. The applied voltage V_1 is drawn equal and opposite to the induced emf E_1 because the difference between the two, at no load, is negligible.
5. Active component I_w is drawn in phase with the applied voltage V_1.
6. The phasor sum of magnetizing current I_m and the working current I_w gives the no load current I_0.
7. From the phasor diagram and equivalent circuit drawn above, the following conclusions are made

The working component of no load current $I_w = I_o \cos \phi_0$

The working component of no load current $I_\mu = I_o \sin \phi_0$

No load current $I_o = (I_w^2 + I_\mu^2)^{1/2}$

Input power under no load $W_o = V_1 I_o \cos \phi_0$

The no load phase angle $\phi_0 = \tan^{-1}(I\mu / I_w)$

The no load parameters $R_0 = V_1 / I_w$

$X_0 = V_1 / I\mu$

Transformer on load but having no Winding Resistance and Leakage Reactance:

Now we will examine the behaviour of above said transformer on load, that means load is connected to the secondary terminals.

Consider, transformer having core loss but no copper loss and leakage reactance. Whenever load is connected to the secondary winding, load current will start to flow through the load as well as secondary winding. This load current solely depends upon the characteristics of the load and also upon secondary voltage of the transformer. This current is called secondary current or load current, here it is denoted as I_2. As I_2 is flowing through the secondary, a self mmf in secondary winding will be produced. Here it is N_2I_2, where, N_2 is the number of turns of the secondary winding of transformer.

This mmf or magneto motive force in the secondary winding produces flux φ_2. This φ_2 will oppose the main magnetizing flux and momentarily weakens the main flux and tries to reduce primary self-induced emf E_1. If E_1 falls down below the primary source voltage V_1, there will be an extra current flowing from source to primary winding. This extra primary current I_2' produces extra flux φ' in the core which will neutralize the secondary counter flux φ_2. Hence the main magnetizing flux of core, Φ remains unchanged irrespective of load.

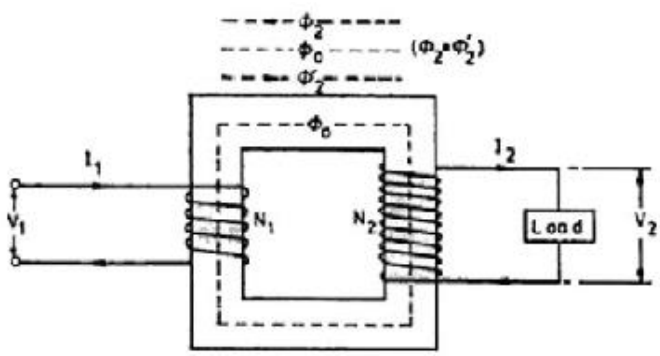
So total current, this transformer draws from source can be divided into two components, first one is utilized for magnetizing the core and compensating the core loss i.e. I_o. It is no-load component of the primary current.

Second one is utilized for compensating the counter flux of the secondary winding. It is known as load component of the primary current.

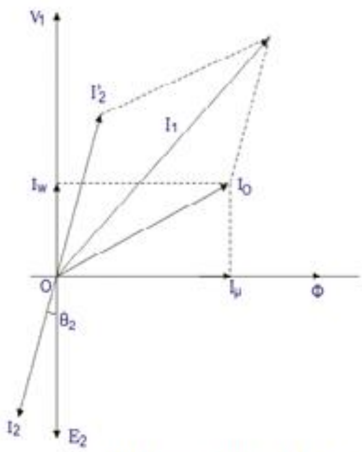
Hence total load primary current I_1 of an electrical power transformer having no winding resistance and leakage reactance can be represented as follows

$$I_1 = I_0 + I_2'$$

Where, θ_2 is the angle between Secondary Voltage and Secondary Current of transformer.
Now we will proceed one further step toward more practical aspect of a transformer.



transformer on load



Phasor diagram for on load of a transformer

Transformer on load with Resistance and Leakage Reactance in Transformer Windings:

All the flux in transformer will not be able to link with both the primary and secondary windings. A small portion of flux will link either winding but not both. This portion of flux is called leakage flux.

Due to this leakage flux in transformer, there will be a self-reactance in the concerned winding. This self-reactance of transformer is alternatively known as leakage reactance of transformer. This self-reactance associated with resistance of transformer is impedance. Due to this impedance of transformer, there will be voltage drops in both primary and secondary transformer windings.

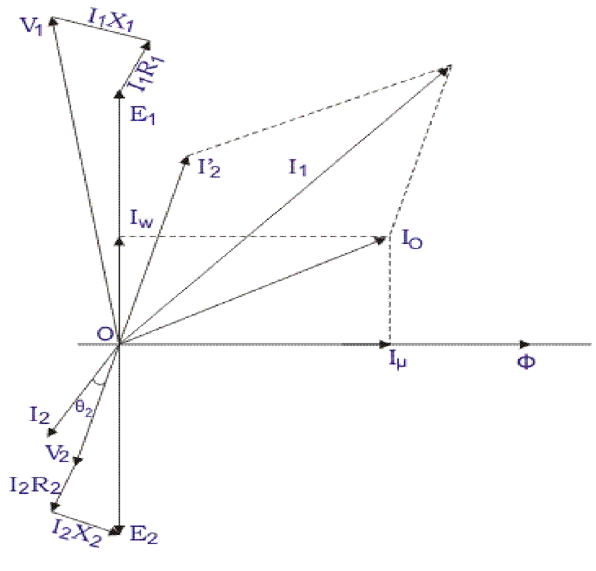
Let leakage reactance of primary and secondary windings of the transformer are X_1 and X_2 respectively. Hence total impedance of primary and secondary winding of transformer with resistance R_1 and R_2 respectively, can be represented as,

$$Z_1 = R_1 + jX_1 \ (impedance\ of\ primary\ winding)$$

$$Z_2 = R_2 + jX_2 \ (impedance\ of\ secondary\ winding)$$

The voltage equation of a transformer on load, with resistances and leakage reactance in the windings, voltage drop occurs in the winding not only because of resistance, it is because of impedance of transformer windings. Hence, actual voltage equation of a transformer can easily be determined by the knowing the drops due to the Z_1 and Z_2.

Therefore, the voltage equations are, Resistance drops are in the direction of current vector but, reactive drop will be perpendicular to the current vector as shown in the above vector diagram of transformer.



$$V_1 = E_1 + I_1Z_1 \ \& \ V_2 = E_2 - I_2Z_2$$

$$V_1 = E_1 + I_1(R_1 + jX_1)$$

$$\Rightarrow V_1 = E_1 + I_1R_1 + jI_1X_1$$

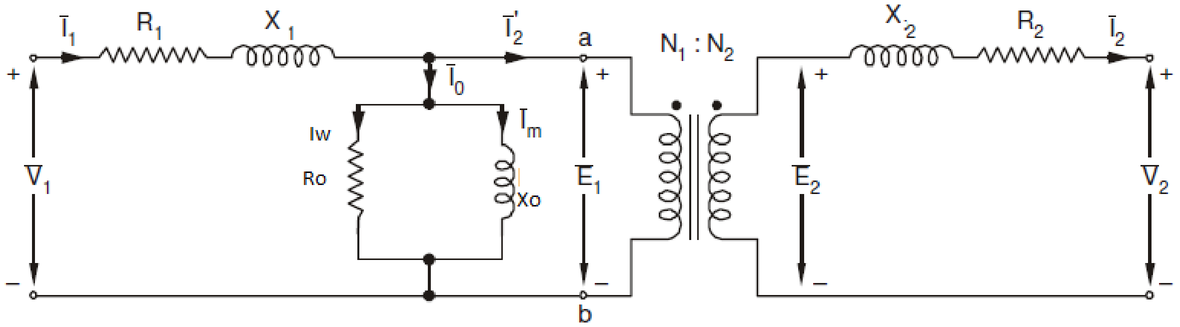
$$V_2 = E_2 - I_2(R_2 + jX_2)$$

$$\Rightarrow V_2 = E_2 - I_2R_2 - jI_2X_2$$

Equivalent circuit of a Transformer:

Equivalent impedance of transformer is essential to be calculated because the electrical power transformer is an electrical power system equipment for estimating different parameters of electrical power system which may be required to calculate total internal impedance of an electrical power transformer, viewing from primary side or secondary side as per requirement. This calculation requires equivalent circuit of transformer referred to primary or equivalent circuit of transformer referred to secondary sides respectively. \

Percentage impedance is also very essential parameter of transformer. Special attention is to be given to this parameter during installing a transformer in an existing electrical power system. Percentage impedance of different power transformers should be properly matched during parallel operation of power transformers. The percentage impedance can be derived from equivalent impedance of transformer so, it can be said that equivalent circuit of transformer is also required during calculation of % impedance.



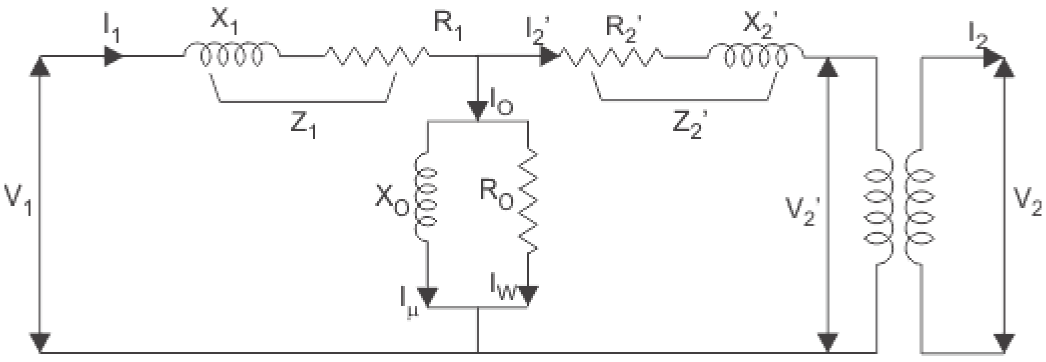
The equivalent circuit of a transformer

Referred to Primary side:

Let us consider the transformation ratio be,

$$K = \frac{N_1}{N_2} = \frac{E_1}{E_2}$$

The complete equivalent circuit of transformer is shown below.



Equivalent Circuit of Transformer referred to Primary

Now if we see the voltage drop in secondary from primary side, then it would be ′K′ times greater and would be written as K Z_2. I_2.

Again I_2'. $N_1 = I_2.N_2$

$$\Rightarrow I_2 = I_2' \frac{N_1}{N_2}$$

$$\Rightarrow I_2 = KI_2'$$

Therefore,

$$KZ_2I_2 = KZ_2KI_2' = K^2Z_2I_2'$$

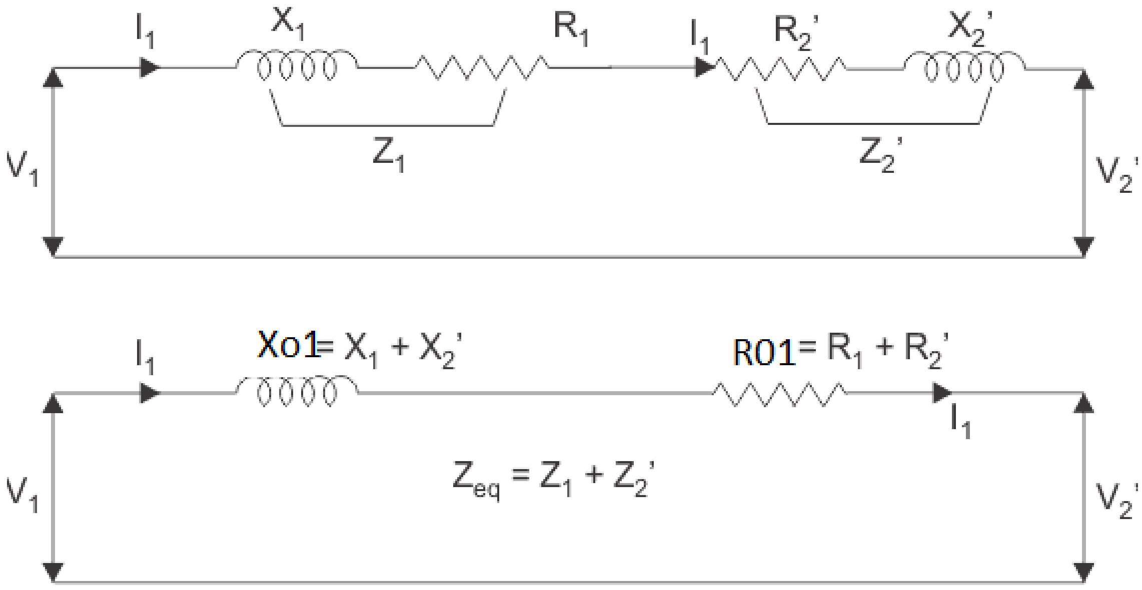
From above equation, secondary impedance of transformer referred to primary is,

$$Z_2' = K^2Z_2$$

$$Hence,\ R_2' = K^2R_2\ and\ X_2 = K^2X_2$$

Since I_o is very small compared to I_1, it is less than 5% of full load primary current, I_o changes the voltage drop insignificantly. Hence, it is good approximation to ignore the excitation circuit in approximate equivalent circuit of transformer. The winding resistance and reactance being in series can now be combined into equivalent resistance and reactance of transformer, referred on the primary side.

$$Here,\ V_2' = KV_2$$



Approximate Equivalent Circuit of Transformer referred to Primary

The Equivalent Resistance Referred to primary side is $R_{01} = R_1 + R_2^1$

The Equivalent Reactance Referred to primary side is $X_{01} = X_1 + X_2^1$

The Equivalent Impedance Referred to primary side is $Z_{01} = Z_1 + Z_2^1$

Referred to Secondary side:

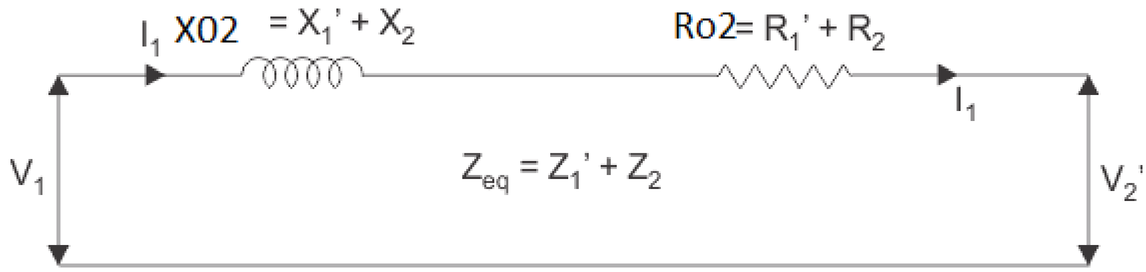
In similar way, approximate equivalent circuit of transformer referred to secondary can be drawn. Where equivalent impedance of transformer referred to secondary, can be derived as

$$Z_1^{1} = \frac{Z_1}{K^2}$$

$$Therefore,\ R_1' = \frac{R_1}{K^2}$$

$$X_1' = \frac{X_1}{K^2}$$

$$Here,\ V_1' = \frac{V_1}{K}$$



Approximate Equivalent Circuit of Transformer referred to Secondary

The Equivalent Resistance Referred to primary side is $R_{02}= R_2+R_1^1$

The Equivalent Reactance Referred to primary side is $X_{02}= X_2+X_1^1$

The Equivalent Impedance Referred to primary side is $Z_{02}= Z_2+Z_1^1$

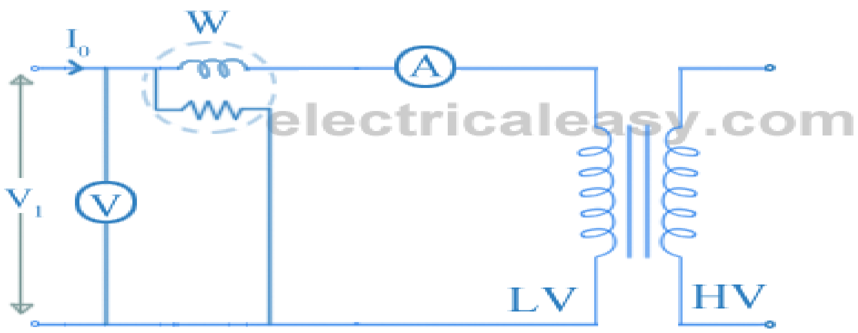
Why transformer is rated in KVA:

Copper losses (I^2R) depends on Current which passing through transformer winding while Iron Losses or Core Losses or Insulation Losses depends on Voltage. It does not depend on the power factor. Therefore, the Transformer Rating may be expressed in kVA, Not in kW.

OC and SC tests of a Transformer:

Open circuit or No load test on Transformer:

Open circuit test or no load test on a transformer is performed to determine 'no load loss (core loss)' and 'no load current I_0'. The circuit diagram for open circuit test is shown in the figure below.



Usually high voltage (HV) winding is kept open and the low voltage (LV) winding is connected to its normal supply. A wattmeter (W), ammeter (A) and voltmeter (V) are connected to the LV winding as shown in the figure. Now, applied voltage is slowly increased from zero to normal rated value of the LV side with the help of a variac. When the applied voltage reaches to the rated value of the LV winding, readings from all the three instruments are taken.

The ammeter reading gives the no load current I_0

The input power is indicated by the wattmeter (W). the wattmeter reading gives the core losses of the transformer.

Sometimes, a high resistance voltmeter is connected across the HV winding. Though, a voltmeter is connected, HV winding can be treated as open circuit as the current through the voltmeter is negligibly small. This helps in to find voltage transformation ration (K).

The two components of no load current can be given as,

$$I_\mu = I_0 \sin\Phi_0 \quad \text{and}$$

$$I_w = I_0 \cos\Phi_0.$$

$\cos\Phi_0$ (no load power factor) = $W / (V_1 I_0)$. ... (W = wattmeter reading)

From this, shunt parameters of equivalent circuit parameters of equivalent circuit of transformer (X_0 and R_0) can be calculated as

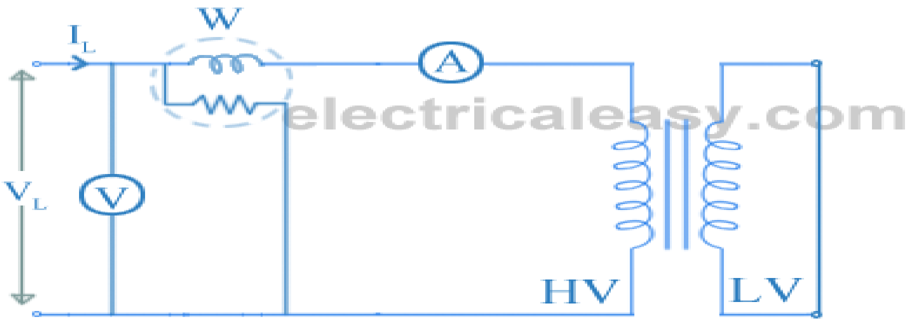
$$X_0 = V_1/I_\mu \text{ and } R_0 = V_1/I_w.$$

(These values are referring to LV side of the transformer.)
Hence, it is seen that open circuit test gives core losses of transformer and shunt parameters of the equivalent circuit.

Short circuit or Impedance test on Transformer

The connection diagram for short circuit test or impedance test on transformer is as shown in the figure below. The LV side of transformer is short circuited and wattmeter (W), voltmeter (V) and ammeter (A) are connected on the HV side of the transformer.

Voltage is applied to the HV side and increased from the zero until the ammeter reading equals the rated current. All the readings are taken at this rated current.



The ammeter reading gives primary equivalent of full load current (I_{sc}).
The voltage applied for full load current is very small as compared to rated voltage. Hence, core loss due to small applied voltage can be neglected. Thus, the wattmeter reading can be taken as copper loss in the transformer.

Therefore, $W = I_{sc}^2 R_{eq}$.......

where R_{eq} is the equivalent resistance of transformer)

$$Z_{eq} = V_{sc}/I_{sc}.$$

Therefore, equivalent reactance of transformer can be calculated from the formula

$$Z_{eq}^2 = R_{eq}^2 + X_{eq}^2.$$

These, values are referred to the HV side of the transformer.
Hence, it is seen that the short circuit test gives copper losses of transformer and approximate equivalent resistance and reactance of the transformer.

Losses in transformer

In any electrical machine, 'loss' can be defined as the difference between input power and output power. An electrical transformer is a static device, hence mechanical losses (like windage or friction losses) are absent in it. A transformer only consists of electrical losses (iron losses and copper losses). Transformer losses are similar to losses in a DC machine, except that transformers do not have mechanical losses.

Losses in transformer are explained below -

(i) Core losses or Iron losses

Eddy current loss and hysteresis loss depend upon the magnetic properties of the material used for the construction of core. Hence these losses are also known as core losses or iron losses.

- **Hysteresis loss in transformer**: Hysteresis loss is due to reversal of magnetization in the transformer core. This loss depends upon the volume and grade of the iron, frequency of magnetic reversals and value of flux density. It can be given by,

  Steinmetz formula:

$$W_h = \eta B_{max}^{1.6} f V \text{ (watts)}$$

where, η = Steinmetz hysteresis constant

V = volume of the core in m^3

- **Eddy current loss in transformer:**

  In transformer, AC current is supplied to the primary winding which sets up alternating magnetizing flux. When this flux links with secondary winding, it produces induced emf in it. But some part of this flux also gets linked with other conducting parts like steel core or iron body or the transformer, which will result in induced emf in those parts, causing small circulating current in them. This current is called as eddy current. Due to these eddy currents, some energy will be dissipated in the form of heat.

(ii) Copper loss in transformer

Copper loss is due to ohmic resistance of the transformer windings. Copper loss for the primary winding is $I_1^2R_1$ and for secondary winding is $I_2^2R_2$. Where, I_1 and I_2 are current in primary and secondary winding respectively, R_1 and R_2 are the resistances of primary and secondary winding respectively. It is clear that Cu loss is proportional to square of the current, and current depends on the load. Hence copper loss in transformer varies with the load.

Efficiency of Transformer

Just like any other electrical machine, efficiency of a transformer can be defined as the output power divided by the input power. That is

$$\text{efficiency} = \text{output} / \text{input}.$$

Transformers are the most highly efficient electrical devices. Most of the transformers have full load efficiency between 95% to 98.5%. As a transformer being highly efficient, output and input are having nearly same value, and hence it is impractical to measure the efficiency of transformer by using output / input. A better method to find efficiency of a transformer is using,

efficiency = (input - losses) / input

= 1 - (losses / input).

Condition for maximum efficiency

Let,

Copper loss = $I_1^2 R_1$

Iron loss = W_i

$$\text{efficiency} = 1 - \frac{\text{losses}}{\text{input}} = 1 - \frac{I_1^2 R_1 + W_i}{V_1 I_1 \cos\Phi_1}$$

$$\eta = 1 - \frac{I_1 R_1}{V_1 \cos\Phi_1} - \frac{W_i}{V_1 I_1 \cos\Phi_1}$$

differentiating above equation with respect to I_1

$$\frac{d\eta}{dI_1} = 0 - \frac{R_1}{V_1 \cos\Phi_1} + \frac{W_i}{V_1 I_1^2 \cos\Phi_1}$$

η will be maximum at $\frac{d\eta}{dI_1} = 0$

Hence efficiency η will be maximum at

$$\frac{R_1}{V_1 \cos\Phi_1} = \frac{W_i}{V_1 I_1^2 \cos\Phi_1}$$

$$\frac{I_1^2 R_1}{V_1 I_1^2 \cos\Phi_1} = \frac{W_i}{V_1 I_1^2 \cos\Phi_1}$$

$$I_1^2 R_1 = W_i$$

Hence, efficiency of a transformer will be maximum when copper loss and iron losses are equal.

That is Copper loss = Iron loss.

Voltage Regulation

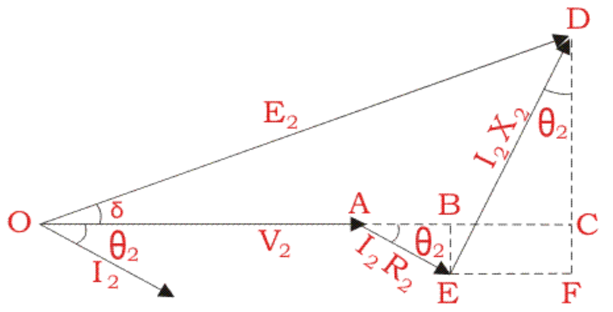
The voltage regulation is the percentage of voltage difference between no load and full load voltages of a transformer with respect to its full load voltage.

Voltage Regulation of Transformer, represented in percentage, is

$$Voltage\ regulation(\%) = \frac{E_2 - V_2}{V_2} \times 100\%$$

Voltage Regulation of Transformer for Lagging Power Factor:

Now we will derive the expression of voltage regulation in detail. Say lagging power factor of the load is $\cos\theta_2$, that means angle between secondary current and voltage is θ_2



Voltage Regulation at Lagging Power Factor

$$OC = OA + AB + BC$$

$$Here,\ OA = V_2$$

$$Here,\ AB = AE\cos\theta_2 = I_2R_2\cos\theta_2$$

$$and,\ BC = DE\sin\theta_2 = I_2X_2\sin\theta_2$$

Here, from the above diagram, Angle between OC and OD may be very small, so it can be neglected and OD is considered nearly equal to OC i.e.

$$E_2 = OC = OA + AB + BC$$

$$E_2 = OC = V_2 + I_2R_2\cos\theta_2 + I_2X_2\sin\theta_2$$

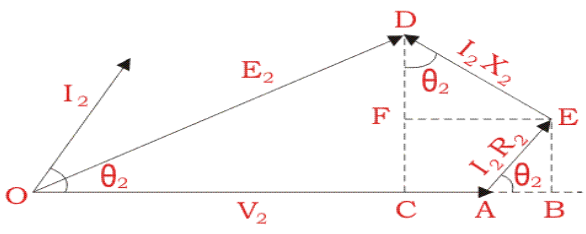
Voltage regulation of transformer at lagging power factor,

$$Voltage\ regulation\ (\%) = \frac{E_2 - V_2}{V_2} \times 100(\%)$$

$$= \frac{I_2R_2\cos\theta_2 + I_2X_2\sin\theta_2}{V_2} \times 100(\%)$$

Voltage Regulation of Transformer for Leading Power Factor

Let's derive the expression of voltage regulation with leading current, say leading power factor of the load is $\cos\theta_2$, that means angle between secondary current and voltage is θ_2.



Voltage Regulation at Leading Power Factor

Here, from the above diagram, Angle between OC and OD may be very small, so it can be neglected and OD is considered nearly equal to OC i.e.

$$E_2 = OC = OA + AB - BC$$

$$E_2 = OC = V_2 + I_2R_2\cos\theta_2 - I_2X_2\sin\theta_2$$

Voltage regulation of transformer at leading power factor,

$$Voltage\ regulation\ (\%) = \frac{E_2 - V_2}{V_2} \times 100(\%)$$
$$= \frac{I_2R_2\cos\theta_2 - I_2X_2\sin\theta_2}{V_2} \times 100(\%)$$

UNIT-IV

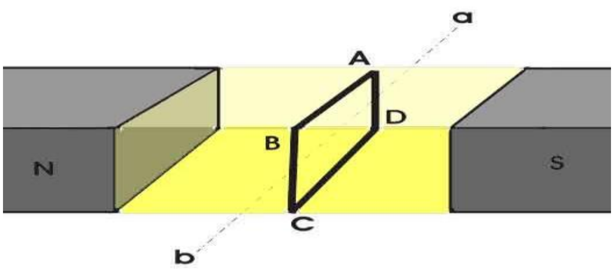
D.C AND A.C MACHINES

Generator converts mechanical power to electrical power. There are two types of generators, one is ac generator and other is dc generator.

Principle of operation of a DC Generator:

The working principle of a generator can be explained by using a simple loop generator. it is also called as single loop generator.

Single Loop DC Generator



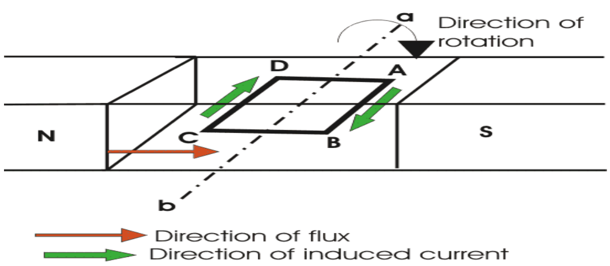
In the figure above, a single loop of conductor of rectangular shape is placed between two opposite poles of magnet. Let's us consider, the rectangular loop of conductor is ABCD which rotates inside the magnetic field about its own axis ab.

Let us take this position of the conductor is taken as a reference and rotation of the conductor can be calculated from this position.

At vertical position shown in fig 1 the rate of flux linkages by the conductor is minimum so the induced emf is zero.

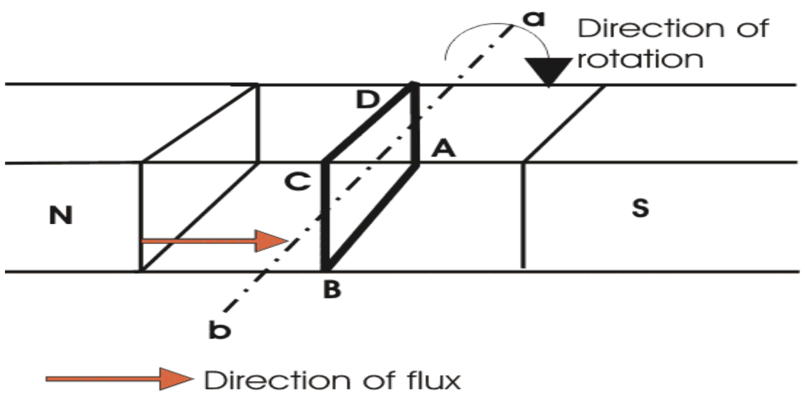
When the loop rotates from its vertical position to its horizontal position, rate of flux linkages by the conductor increases so the emf induced increases according to the faradays laws of electromagnetic induction principle. As the loop is closed there will be a current circulating through the loop. The direction of the current can be determined by Fleming's right hand Rule.

When the loop is at horizontal position, rate of flux linkages by the conductor maximum so the emf induced maximum.



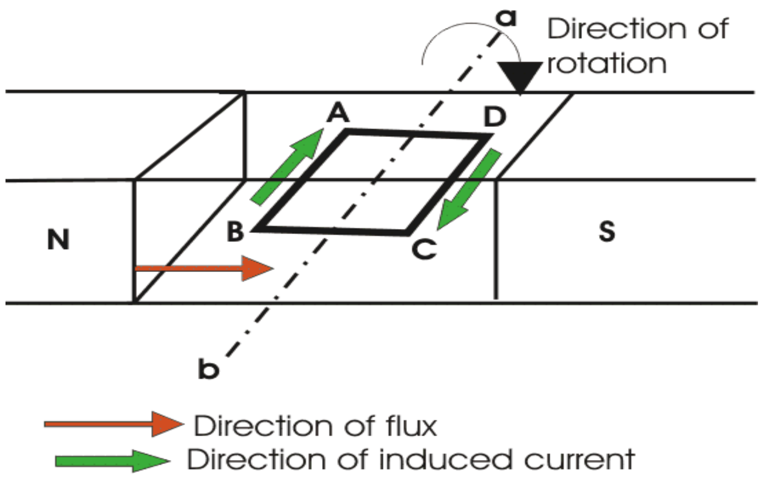
Now if we allow the loop to move further, rate of flux linkages by the conductor decreases so the emf induced decreases.

when it reaches to its vertical position, upper side of the loop will be CD and lower side will be AB (just opposite of the previous vertical position). At this position1 the rate of flux linkages by the conductor is minimum so the induced emf is zero. Therefore, there will be no current in the loop.



If the loop rotates further, rate of flux linkages by the conductor increases so the emf induced increases. But, the direction of induced emf is reversed.

When it reaches to horizontal position, rate of flux linkages by the conductor maximum so the emf induced maximum. But it is in the opposite direction.

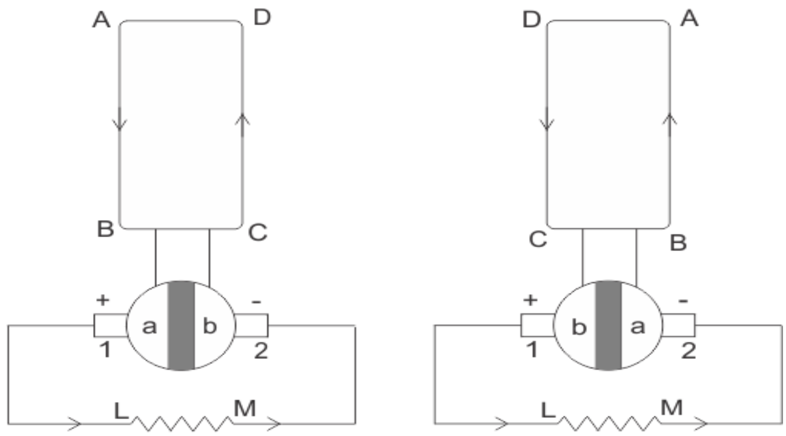


Further rotation of loop, rate of flux linkages by the conductor decreases so the emf induced decreases.

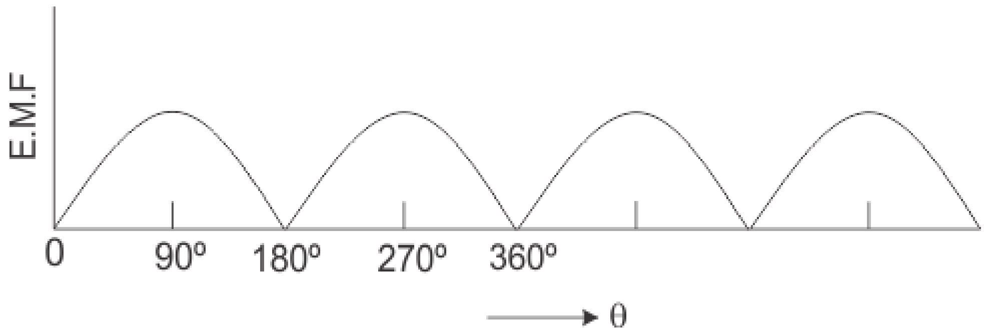
when it reaches to its vertical position, it comes back to the initial original position.

If we observe this phenomenon in different way, it can be concluded, that each side of the loop comes in front of N pole, the current will flow through that side in same direction i.e. downward to the reference plane and similarly each side of the loop comes in front of S pole, current through it flows in same direction i.e. upwards from reference plane.

Now the loop is opened and connect it with a split ring as shown in the figure below. Split ring are made out of a conducting cylinder which cuts into two halves or segments insulated from each other. The external load terminals are connected with two carbon brushes which are rest on these split slip ring segments.



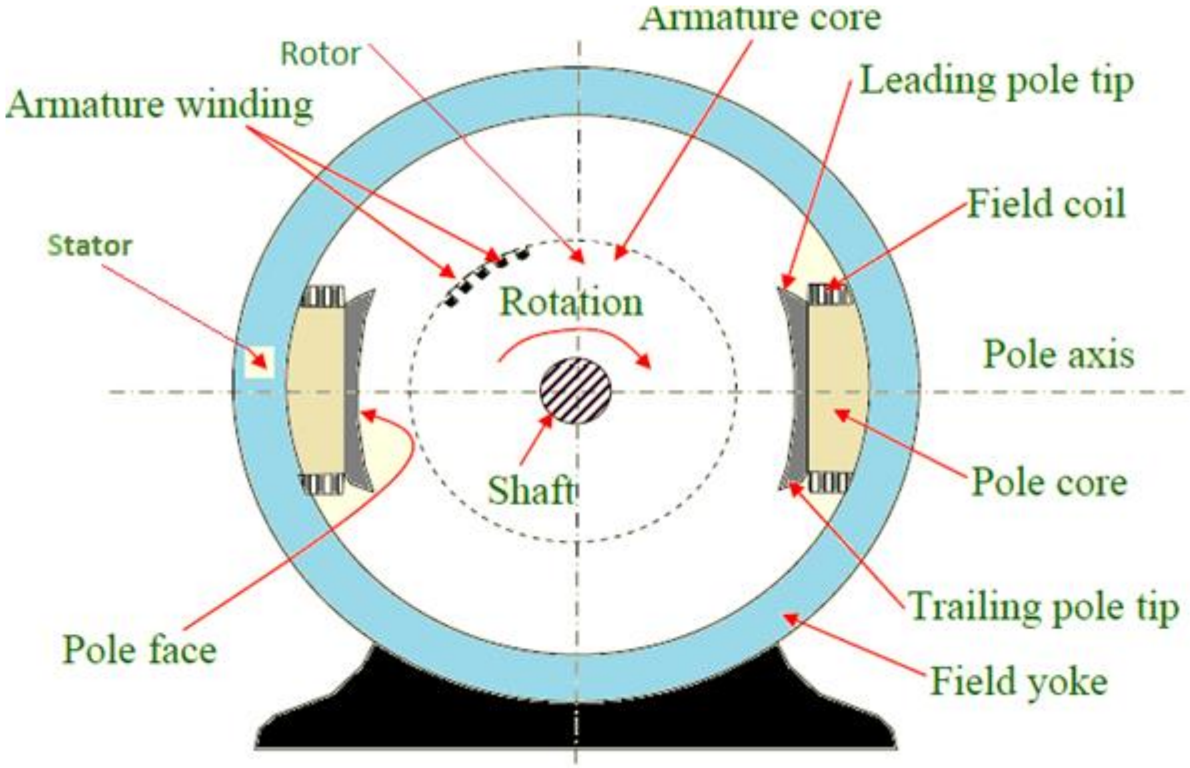
It is seen that in the first half of the revolution current flows always along ABLMCD i.e. brush no 1 in contact with segment a. In the next half revolution, in the figure the direction of the induced current in the coil is reversed. But at the same time the position of the segments a and b are also reversed which results that brush no 1 comes in touch with that segment b. Hence, the current in the load resistance again flows from L to M. The wave from of the current through the load circuit is as shown in the figure. This current is unidirectional.



This is basic working principle of DC generator, explained by single loop generator model. The position of the brushes of DC generator is so arranged that the changeover of the segments a and b from one brush to other takes place when the plane of rotating coil is at right angle to the plane of the lines of force. It is so become in that position, the induced emf in the coil is zero.

Construction of DC Generator:
A DC generator has the following parts

1. Yoke
2. Pole of generator
3. Field winding
4. Armature of DC generator
5. Brushes of generator
6. Bearing



Construction of a DC Generator

Yoke of DC Generator
Yoke of DC generator serves two purposes,

1. It holds the magnetic pole cores of the generator and acts as cover of the generator.
2. It carries the magnetic field flux.

In small generator, yoke is made up of cast iron. Cast iron is cheaper in cost but heavier than steel. But for large construction of DC generator, where weight of the machine is concerned, lighter cast steel or rolled steel is preferable for constructing yoke of DC generator. Normally larger yokes are formed by rounding a rectangular steel slab and the edges are welded together at the bottom. Then feet, terminal box and hangers are welded to the outer periphery of the yoke frame.

Pole Cores and Pole Shoes of DC Generator
There are mainly two types of construction available.

1) Solid pole care, where it made of a solid single piece of cast iron or cast steel.
2) Laminated pole core, where it made of numbers of thin, limitations of annealed steel which are riveted together. The thickness of the lamination is in the range of 0.04" to 0.01". The pole core is fixed to the inner periphery of the yoke by means of bolts through the yoke and into the pole body.

The pole shoes are so typically shaped, that, they spread out the magnetic flux in the air gap and reduce the reluctance of the magnetic path. Due to their larger cross - section they hold the pole coil at its position.

Pole Coils: The field coils or pole coils are wound around the pole core. These are a simple coil of insulated copper wire or strip, which placed on the pole which placed between yoke and pole shoes as shown.

Armature Core of DC Generator

The purpose of armature core is to hold the armature winding and provide low reluctance path for the flux through the armature from N pole to S pole. Although a DC generator provides direct current but induced current in the armature is alternating in nature.

cylindrical or drum shaped armature core is build-up of circular laminated sheet. In every circular lamination, slots are either die - cut or punched on the outer periphery and the key way is located on the inner periphery as shown.

Air ducts are also punched of cut on each lamination for circulation of air through the core for providing better cooling. Up to diameter of 40", the circular stampings are cut out in one piece of lamination sheet. But above 40", diameter, number of suitable sections of a circle is cut. A complete circle of lamination is formed by four or six or even eight such segment.

Armature Winding of DC Generator

Armature winding are generally formed wound. These are first wound in the form of flat rectangular coils and are then pulled into their proper shape in a coil puller. Various conductors of the coils are insulated from each other. The conductors are placed in the armature slots, which are lined with tough insulating material. This slot insulation is folded over above the armature conductors placed in it and secured in place by special hard wooden or fiber wedges.

Commutator of DC Generator

The commutator plays a vital role in DC generator.

It collects current from armature and sends it to the load as direct current.

It actually takes alternating current from armature and converts it to direct current and then send it to external load.

It is cylindrical structured and is build-up of wedge - shaped segments of high conductivity, hard drawn or drop forged copper. Each segment is insulated from the shaft by means of insulated commutator segment shown below. Each commutator segment is connected with corresponding armature conductor through segment riser or lug.

Brushes of DC Generator

The brushes are made of carbon. These are rectangular block shaped.

The only function of these carbon brushes of DC generator is to collect current from commutator segments.

The brushes are housed in the rectangular box shaped brush holder. As shown in figure, the brush face is placed on the commutator segment with attached to the brush holder.

Bearing of DC Generator

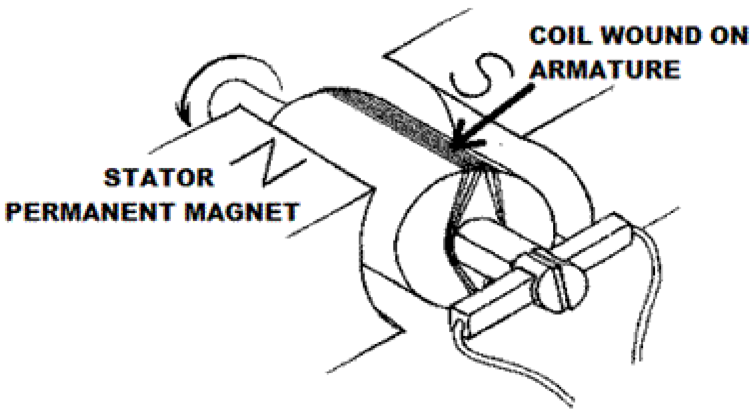
For small machine, ball bearing is used and for heavy duty DC generator, roller bearing is used. The bearing must always be lubricated properly for smooth operation and long life of generator.

Types of DC Generators:

Generally, DC generators are classified according to the ways of excitation of their fields. There are three methods of excitation.

1. Field coils excited by permanent magnets – Permanent magnet DC generators.
2. Field coils excited by some external source – Separately excited DC generators.
3. Field coils excited by the generator itself – Self excited DC generators.

Permanent Magnet DC Generator



PERMANENT MAGNET DC GENERATOR

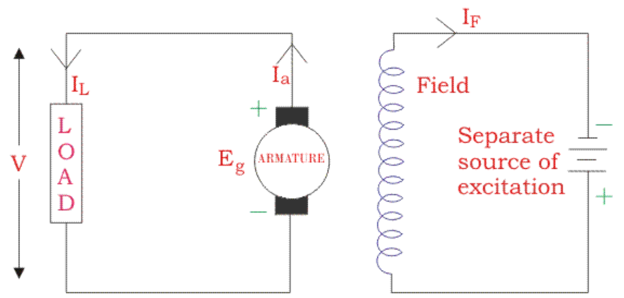
When the flux in the magnetic circuit is established by the help of permanent magnets then it is known as Permanent magnet DC generator.

It consists of an armature and one or several permanent magnets situated around the armature. This type of dc generators generates very low power. So, they are rarely found in industrial applications. They are normally used in small applications like dynamos in motor cycles.

Separately Excited DC Generator

These are the generators whose field magnets are energized by some external dc source such as battery .

A circuit diagram of separately excited DC generator is shown in figure.



Separately Excited DC Generator

I_a = Armature current

I_L = Load current

V = Terminal voltage

E_g = Generated emf

Voltage drop in the armature = $I_a \times R_a$

Where, Ra= is the armature resistance

Let, $I_a = I_L = I$ (say)

Then, voltage across the load, $V = IR_a$

Power generated, $P_g = E_g \times I$

Power delivered to the external load, $P_L = V \times I$.

Self-excited DC Generators

These are the generators whose field magnets are energized by the current supplied by themselves. In these type of machines field coils is internally connected with the armature.

Due to residual magnetism some flux is always present in the poles. When the armature is rotated some emf is induced. Hence some induced current is produced. This small current flows through the field coil as well as the load and thereby strengthening the pole flux. As the pole flux strengthened, it will produce more armature emf, which cause further increase of current through the field. This increased field current further raises armature emf and this cumulative phenomenon continues until the excitation reaches to the rated value. According to the position of the field coils the Self-excited DC generators may be classified as…

1. Series wound generators
2. Shunt wound generators
3. Compound wound generators

Series Wound Generator

In these type of generators, the field windings are connected in series with armature conductors as shown in figure below. So, whole current flows through the field coils as well as the load.

As series field winding carries full load current it is designed with relatively few turns of thick wire. The electrical resistance of series field winding is therefore very low (nearly 0.5Ω).

Let, R_{sc} = Series winding resistance

I_{sc} = Current flowing through the series field

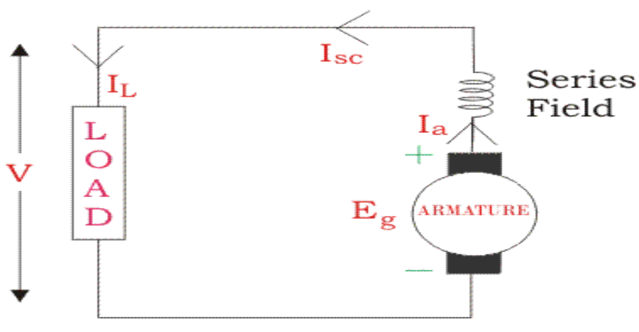
R_a = Armature resistance

I_a = Armature current

I_L = Load current

V = Terminal voltage

E_g = Generated emf



Series Wound Generator

Then, $I_a = I_{sc} = I_L = I$ (say)

Voltage across the load, $V = E_g - I(I_a \times R_a)$

Power generated, $P_g = E_g \times I$

Power delivered to the load, $P_L = V \times I$

Shunt Wound DC Generators

In these type of DC generators the field windings are connected in parallel with armature conductors as shown in figure below.

In shunt wound generators the voltage in the field winding is same as the voltage across the terminal.

Let, R_{sh} = Shunt winding resistance

I_{sh} = Current flowing through the shunt field

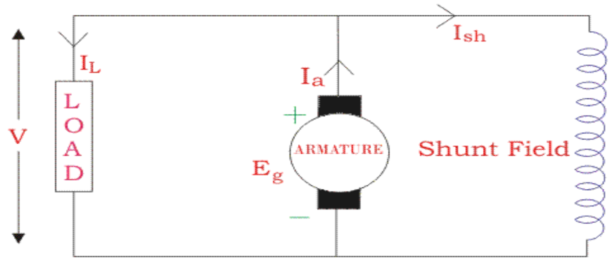
R_a = Armature resistance

I_a = Armature current

I_L = Load current

V = Terminal voltage

E_g = Generated emf



Shunt Wound Generator

Here armature current I_a is dividing in two parts, one is shunt field current I_{sh} and another is load current I_L.

So, $I_a = I_{sh} + I_L$

The effective power across the load will be maximum when I_L will be maximum. So, it is required to keep shunt field current as small as possible. For this purpose, the resistance of the shunt field winding generally kept high (100 Ω) and large no of turns are used for the desired emf.

Shunt field current, $I_{sh} = V/R_{sh}$

Voltage across the load, $V = E_g - I_a R_a$

Power generated, $P_g = E_g \times I_a$

Power delivered to the load, $P_L = V \times I_L$

Compound Wound DC Generator

In series wound generators, the output voltage is directly proportional with load current. In shunt wound generators, output voltage is inversely proportional with load current.

A combination of these two types of generators can overcome the disadvantages of both. This combination of windings is called compound wound DC generator. Compound wound generators have both series field winding and shunt field winding. One winding is placed in series with the armature and the other is placed in parallel with the armature.

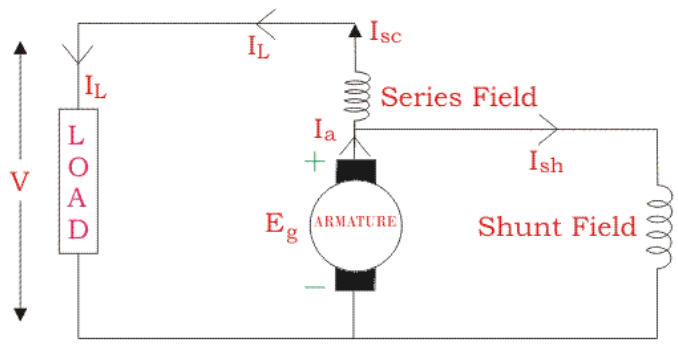
This type of DC generators may be of two types-

short shunt compound wound generator and

long shunt compound wound generator.

Short Shunt Compound Wound DC Generator

The generators in which only shunt field winding is in parallel with the armature winding as shown in figure.



Short Shunt Compound Wound Generator

Series field current, $I_{sc} = I_L$

Shunt field current, $I_{sh} = (V + I_{sc} R_{sc})/R_{sh}$

Armature current, $I_a = I_{sh} + I_L$

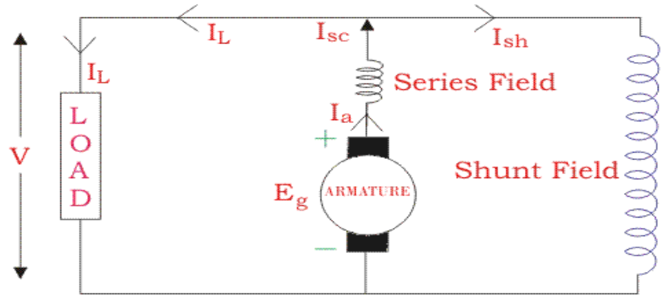
Voltage across the load, $V = E_g - I_a R_a - I_{sc} R_{sc}$

Power generated, $P_g = E_g \times I_a$

Power delivered to the load, $P_L = V \times I_L$

Long Shunt Compound Wound DC Generator

The generators in which shunt field winding is in parallel with both series field and armature winding as shown in figure.



Long Shunt Compound Wound Generator

Shunt field current, $I_{sh}=V/R_{sh}$

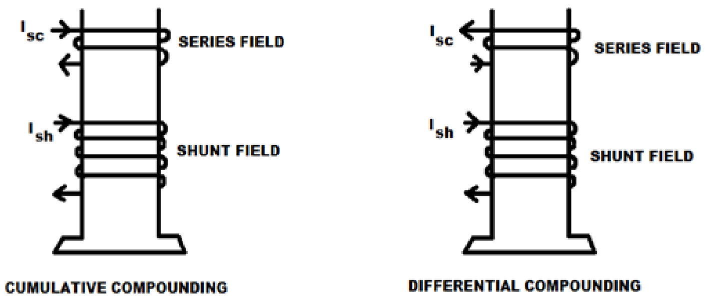
Armature current, I_a= series field current, $I_{sc}= I_L+ I_{sh}$

Voltage across the load, $V=E_g-I_a R_a-I_{sc} R_{sc}=E_g-I_a (R_a+ R_{sc})$ [$\therefore I_a= I_{cs}$]

Power generated, $P_g= E_g\times I_a$

Power delivered to the load, $P_L=V\times I_L$

In a compound wound generator, the shunt field is stronger than the series field. When the series field assists the shunt field, generator is said to be commutatively compound wound. On the other hand, if series field opposes the shunt field, the generator is said to be differentially compound wound.



EMF equation for DC generator

For one revolution of the conductor,

Let, Φ = Flux produced by each pole in weber (Wb) and

P = number of poles in the DC generator.

N = speed of the armature conductor in rpm.

Z = total numbers of conductor

A = number of parallel paths

therefore,

Total flux produced by all the poles=ØXP

And, Time taken to complete one revolution=60/N

Now,

According to Faraday's law of induction, the induced emf of the armature one conductor is denoted by "e" which is equal to rate of cutting the flux. Therefore,

$$e = \frac{d\phi}{dt} \ \ and \ e = \frac{total \ flux}{time \ take}$$

Induced emf of one conductor is

$$e = \frac{\phi P}{\frac{60}{N}} = \phi P \frac{N}{60}$$

Let us suppose there are Z total numbers of conductor in a generator, and arranged in such a manner that all parallel paths are always in series Then, Z/A = number of conductors connected in series

Therefore, Induced emf of DC generator

E = emf of one conductor × number of conductor connected in series.

Induced emf of DC generator is

$$e = \phi P \frac{N}{60} X \frac{Z}{A} \ volts$$

Simple wave wound generator,

Numbers of parallel paths are only A = 2

Therefore,

Induced emf for wave type of winding generator is

$$\frac{\phi PN}{60} X \frac{Z}{2} = \frac{\phi ZPN}{120} \ volts$$

Simple lap-wound generator,

number of parallel paths is equal to number of conductors in one path i.e. P = A

Therefore,

Induced emf for lap-wound generator is

$$E_g = \frac{\phi ZN}{60} X \frac{P}{A} volt$$

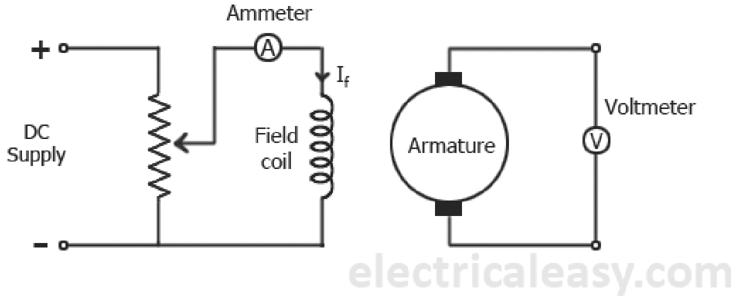
Characteristics of DC Generators:

Generally, following three characteristics of DC generators are taken into considerations:

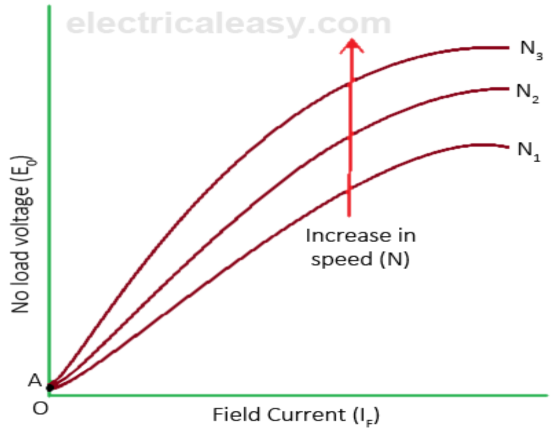
(i) Open Circuit Characteristic (O.C.C.),
(ii) Internal or Total Characteristic and
(iii)External Characteristic.

1. Open Circuit Characteristic (O.C.C.) (E_0/I_f)

Open circuit characteristic is also known as magnetic characteristic or no-load saturation characteristic. This characteristic shows the relation between generated emf at no load (E_0) and the field current (I_f) at a given fixed speed. The O.C.C. curve is just the magnetization curve and it is practically similar for all type of generators. The data for O.C.C. curve is obtained by operating the generator at no load and keeping a constant speed. Field current is gradually increased and the corresponding terminal voltage is recorded. The connection arrangement to obtain O.C.C. curve is as shown in the figure below. For shunt or series excited generators, the field winding is disconnected from the machine and connected across an external supply.



Now, from the emf equation of dc generator, we know that Eg = kϕ. Hence, the generated emf should be directly proportional to field flux (and hence, also directly proportional to the field current). However, even when the field current is zero, some amount of emf is generated (represented by OA in the figure below). This initially induced emf is due to the fact that there exists some residual magnetism in the field poles. Due to the residual magnetism, a small initial emf is induced in the armature. This initially induced emf aids the existing residual flux, and hence, increasing the overall field flux. This consequently increases the induced emf. Thus, O.C.C. follows a straight line. However, as the flux density increases, the poles get saturated and the ϕ becomes practically constant. Thus, even we increase the I_f further, ϕ remains constant and hence, Eg also remains constant. Hence, the O.C.C. curve looks like the B-H characteristic.



Open Circuit Characteristic (O.C.C.)

The above figure shows a typical no-load saturation curve or open circuit characteristics for all types of DC generators.

2. Internal or Total Characteristic (E/I_a)

An internal characteristic curve shows the relation between the on-load generated emf (Eg) and the armature current (I_a). The on-load generated emf Eg is always less than E_0 due to the armature reaction. Eg can be determined by subtracting the drop due to demagnetizing effect of armature reaction from no-load voltage E_0. Therefore, internal characteristic curve lies below the O.C.C. curve.

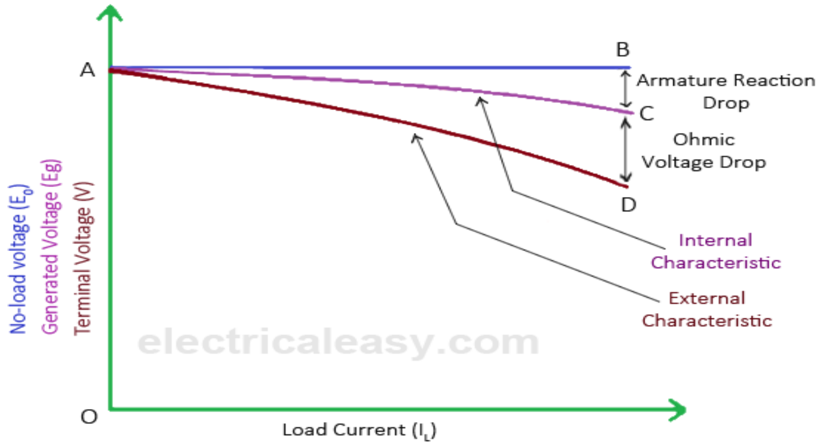
3. External Characteristic (V/I_L)

An external characteristic curve shows the relation between terminal voltage (V) and the load current (I_L). Terminal voltage V is less than the generated emf Eg due to voltage drop in the armature circuit. Therefore, external characteristic curve lies below the internal characteristic curve. External characteristics are very important to determine the suitability of a generator for a given purpose.

Therefore, this type of characteristic is sometimes also called as performance characteristic or load characteristic.

Internal and external characteristic curves:

Characteristics of separately excited DC Generator

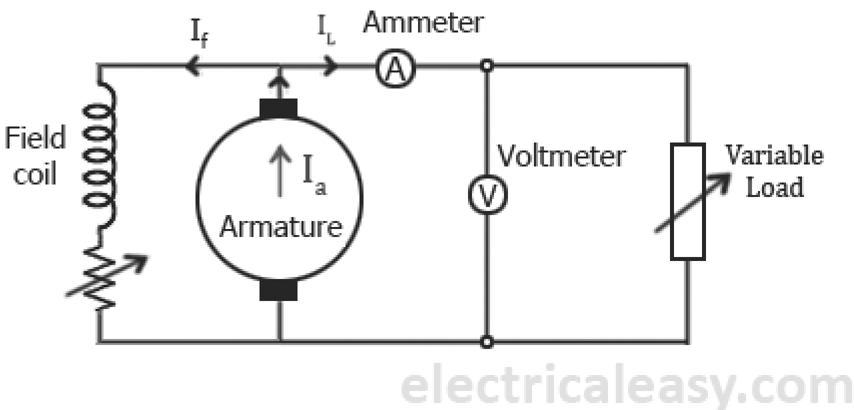


Characteristics of separately excited DC generator

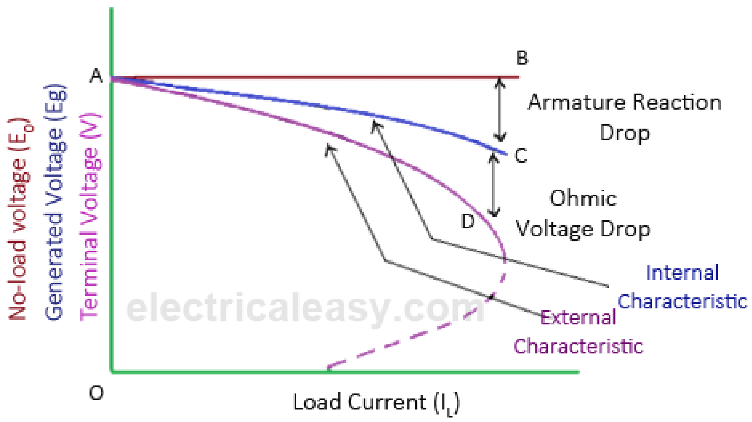
If there is no armature reaction and armature voltage drop, the voltage will remain constant for any load current. Thus, the straight line AB in above figure represents the no-load voltage vs. load current I_L. Due to the demagnetizing effect of armature reaction, the on-load generated emf is less than the no-load voltage. The curve AC represents the on-load generated emf Eg vs. load current I_L i.e. internal characteristic (as $I_a = I_L$ for a separately excited dc generator). Also, the terminal voltage is lesser due to ohmic drop occurring in the armature and brushes. The curve AD represents the terminal voltage vs. load current i.e. external characteristic.

Characteristics of DC Shunt Generator

To determine the internal and external load characteristics of a DC shunt generator the machine is allowed to build up its voltage before applying any external load. To build up voltage of a shunt generator, the generator is driven at the rated speed by a prime mover. Initial voltage is induced due to residual magnetism in the field poles. The generator builds up its voltage as explained by the O.C.C. curve. When the generator has built up the voltage, it is gradually loaded with resistive load and readings are taken at suitable intervals. Connection arrangement is as shown in the figure below.



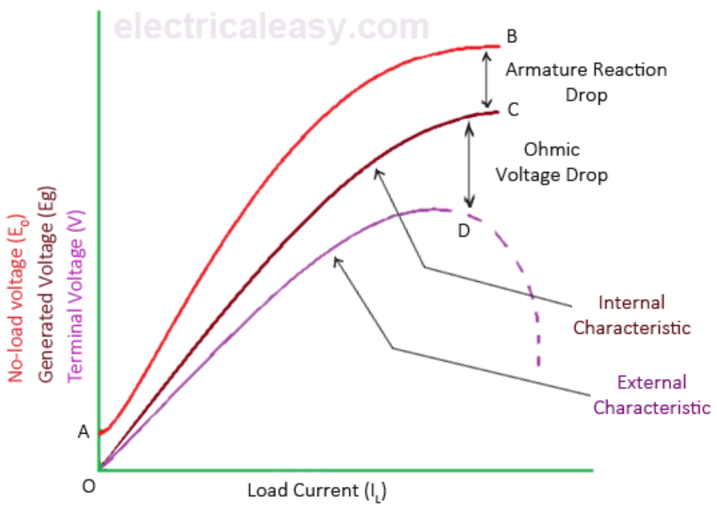
Unlike, separately excited DC generator, here, $I_L \neq I_a$. For a shunt generator, $I_a = I_L + I_f$. Hence, the internal characteristic can be easily transmitted to Eg vs. I_L by subtracting the correct value of I_f from I_a.



Characteristics of DC shunt generator

During a normal running condition, when load resistance is decreased, the load current increases. But, as we go on decreasing the load resistance, terminal voltage also falls. So, load resistance can be decreased up to a certain limit, after which the terminal voltage drastically decreases due to excessive armature reaction at very high armature current and increased I^2R losses. Hence, beyond this limit any further decrease in load resistance results in decreasing load current. Consequently, the external characteristic curve turns back as shown by dotted line in the above figure.

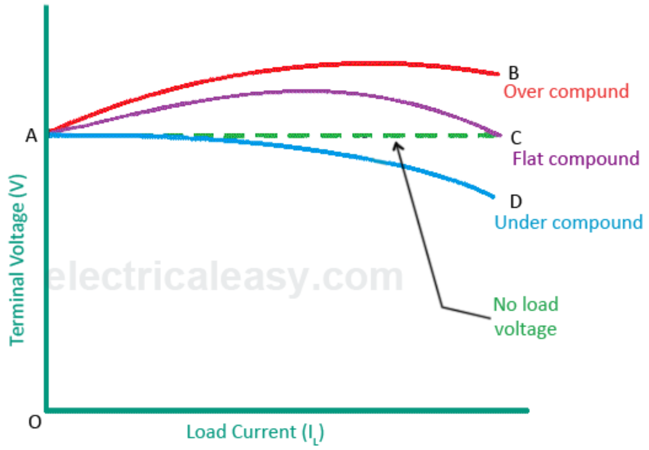
Characteristics of DC Series Generator



Characteristics of DC series generator

The curve AB in above figure identical to open circuit characteristic (O.C.C.) curve. This is because in DC series generators field winding is connected in series with armature and load. Hence, here load current is similar to field current (i.e. $I_L = I_f$). The curve OC and OD represent internal and external characteristic respectively. In a DC series generator, terminal voltage increases with the load current. This is because, as the load current increases, field current also increases. However, beyond a certain limit, terminal voltage starts decreasing with increase in load. This is due to excessive demagnetizing effects of the armature reaction.

Characteristics of DC Compound Generator



External characteristic of DC compound generator

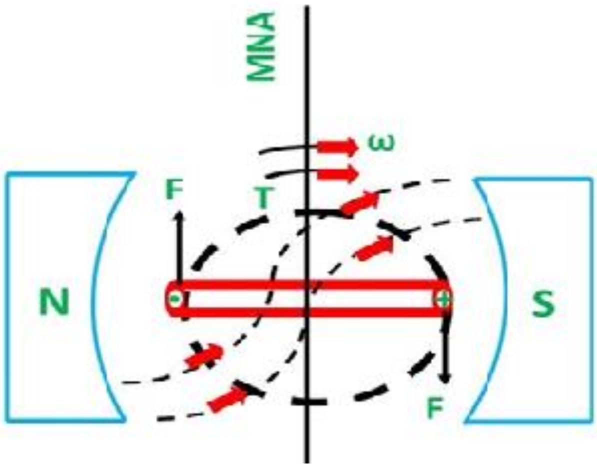
The above figure shows the external characteristics of DC compound generators. If series winding amp-turns are adjusted so that, increase in load current causes increase in terminal voltage then the generator is called to be over compounded. The external characteristic for over compounded generator is shown by the curve AB in above figure. If series winding amp-turns are adjusted so that, the terminal voltage remains constant even the load current is increased, then the generator is called to be flat compounded. The external characteristic for a flat compounded generator is shown by the curve AC. If the series winding has lesser number of turns than that would be required to be flat compounded, then the generator is called to be under compounded. The external characteristics for an under compounded generator are shown by the curve AD.

DC Motor:

A device that converts DC electrical energy to a mechanical energy.

Principle of operation DC Motor:

In a basic DC motor, an armature is placed in between magnetic poles. If the armature winding is supplied by an external DC source, current starts flowing through the armature conductors. As the conductors are carrying current inside a magnetic field, they will experience a force according to Lorentz force equation, which tends to rotate the armature.



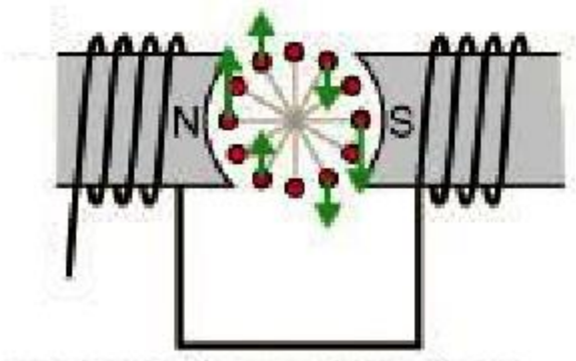
The force Experienced in a conductor F=BIL SinΘ

Suppose armature conductors under N poles of the field magnet, are carrying current downwards (crosses) and those under S poles are carrying current upwards (dots). By applying Fleming's Left Hand Rule, the direction of force F, experienced by the conductor under N poles and the force experienced by the conductors under S poles can be determined.

It is found that at any instant the forces experienced by the conductors are in such a direction that they tend to rotate the armature.

Again, due this rotation the conductors under N – poles come under S – pole and the conductors under S – poles come under N – pole. While the conductors go form N – poles to S – pole and S – poles to N – pole, the direction of current through them, is reversed by means of commutator.

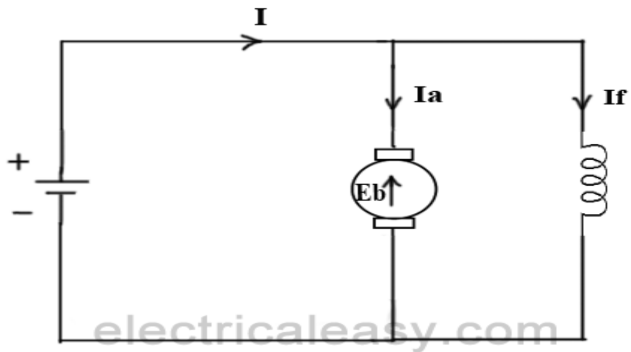
Due to this reversal of current, all the conductors come under N - poles carry current in downward direction and all the conductors come under S – poles carry current in upward direction as shown in the figure. Hence, every conductor comes under N – pole experiences force in same direction and same is true for the conductors come under S – poles. This phenomenon helps to develop continuous and unidirectional torque.



Back EMF

According to fundamental laws of nature, no energy conversion is possible until there is something to oppose the conversion. In case of generators this opposition is provided by magnetic drag, but in case of dc motors there is back emf.

When the armature of the motor is rotating, the conductors are also cutting the magnetic flux lines and hence according to the Faraday's law of electromagnetic induction, an emf induces in the armature conductors. The direction of this induced emf is such that it opposes the armature current (I_a). The circuit diagram below illustrates the direction of the back emf and armature current. Magnitude of Back emf can be given by the emf equation of DC generator.



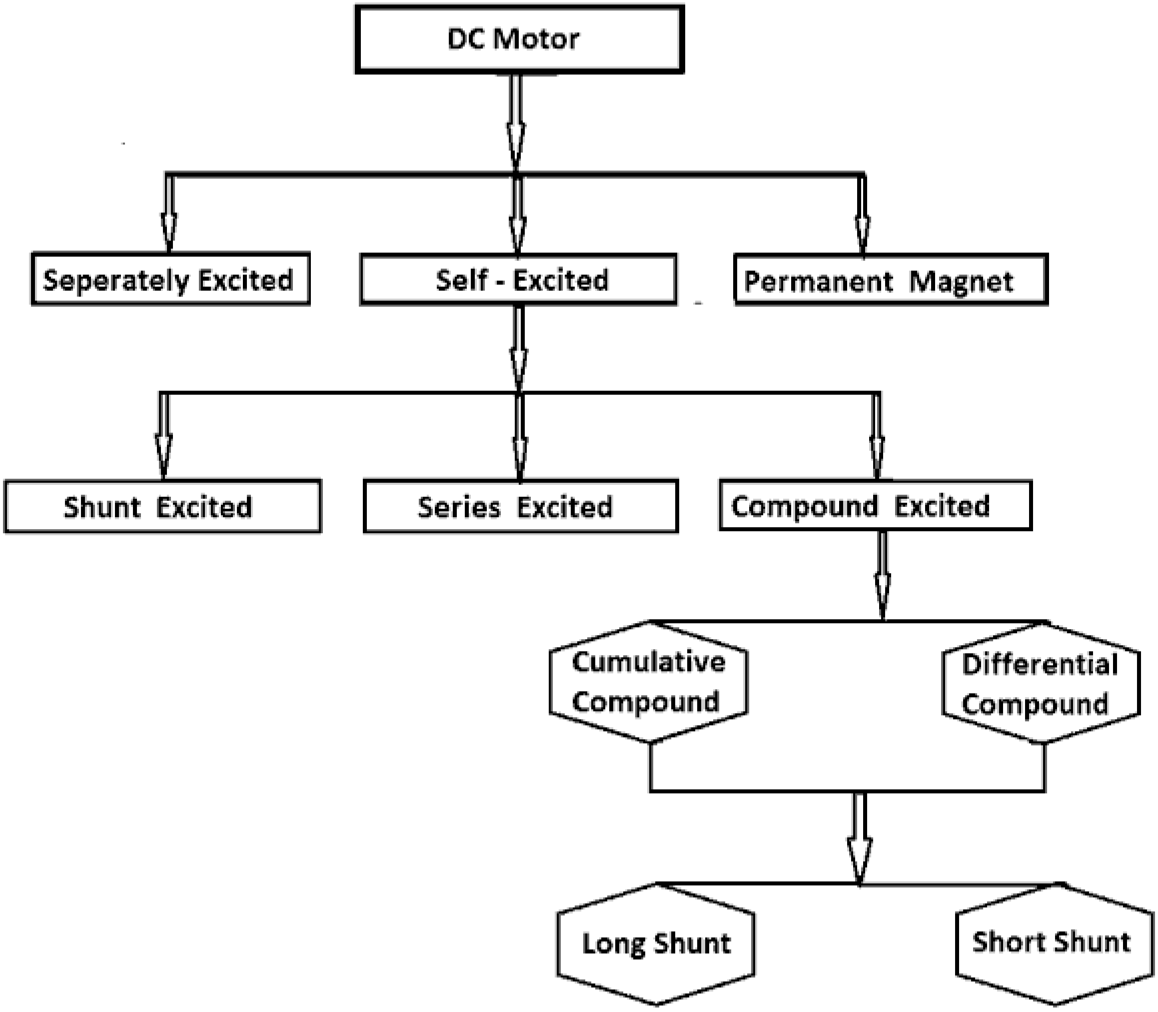
Significance of back emf:

Magnitude of back emf is directly proportional to speed of the motor. Consider the load on a dc motor is suddenly reduced. In this case, required torque will be small as compared to the current torque. Speed of the motor will start increasing due to the excess torque. Hence, being proportional to the speed, magnitude of the back emf will also increase. With increasing back emf armature current will start decreasing. Torque being proportional to the armature current, it will also decrease until it becomes sufficient for the load. Thus, speed of the motor will regulate.

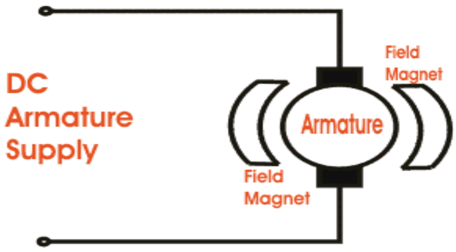
On the other hand, if a dc motor is suddenly loaded, the load will cause decrease in the speed. Due to decrease in speed, back emf will also decrease allowing more armature current. Increased armature current will increase the torque to satisfy the load requirement. Hence, presence of the back emf makes a dc motor ‘self-regulating’.

Types of DC Motors:

DC motors classification is shown in fig



Permanent Magnet DC Motor:

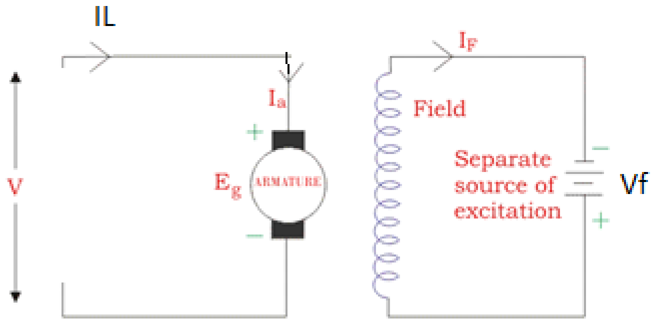


Permanent Magnet DC Motor

The permanent magnet DC motor consists of an armature winding as in case of an usual motor, but does not necessarily contain the field windings. The construction of these types of DC motor are such that, radially magnetized permanent magnets are mounted on the inner periphery of the stator core to produce the field flux. The rotor on the other hand has a conventional DC armature with commutator segments and brushes. The diagrammatic representation of a permanent magnet DC motor is given below.

Separately excited DC motor:

•The armature and field winding are electrically separate from each other.
•The field winding is excited by a separate DC source.



Separately Excited DC Generator

I_a = Armature current

I_f = Field current

V = Supply voltage

E_b = Back emf

Supply Voltage $V = E_b + (I_a \times R_a)$

Where, Ra= is the armature resistance

Let, $I_a = I_L = I$ (say)

Mechanical Power developed, $P_m = E_b \times I_a$

Electrical power input, $P_L = V \times I$.

Series wound DC motor:

In these type of motor, the field windings are connected in series with armature conductors as shown in figure below. So, whole current flows through the field coils as well as the load.

As series field winding carries full load current it is designed with relatively few turns of thick wire. The electrical resistance of series field winding is therefore very low (nearly 0.5Ω).

Let, R_{sc} = Series winding resistance

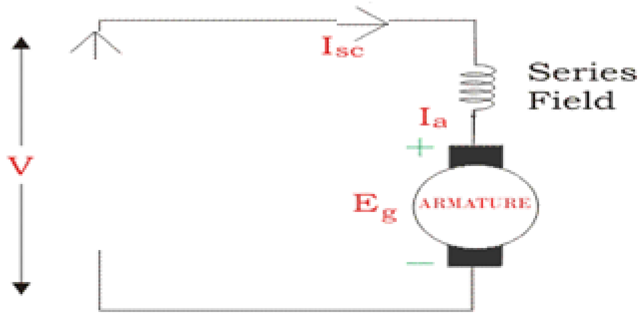
I_{sc} = Current flowing through the series field

R_a = Armature resistance

I_a = Armature current

I_L = line current

V = supply voltage



Then, $I_a = I_{sc} = I_L = I$ (say)

Voltage across the load, $V = E_b + I(R_a + R_{se})$

Mechanical Power developed, $P_m = E_b \times I$

Electrical Power input, $P = V \times I$

Shunt Wound DC Motor:

In these type of DC motors the field windings are connected in parallel with armature conductors as shown in figure below.

In shunt wound generators the voltage in the field winding is same as the voltage across the terminal.

Let, R_{sh} = Shunt winding resistance

I_{sh} = Current flowing through the shunt field

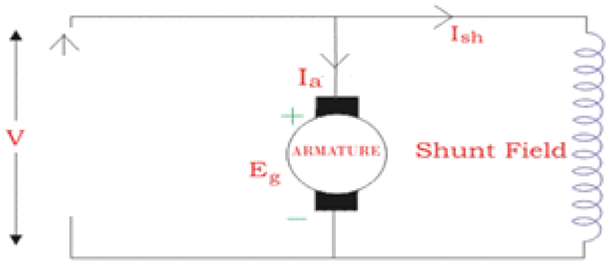
R_a = Armature resistance

I_a = Armature current

I_L = line current

V = supply voltage

E_g = Generated emf



Here armature current I_a is dividing in two parts, one is shunt field current I_{sh} and another is load current I_L.

So, $I_a = I_l - I_{sh}$

The effective power across the load will be maximum when I_L will be maximum. So, it is required to keep shunt field current as small as possible. For this purpose, the resistance of the shunt field winding generally kept high (100 Ω) and large no of turns are used for the desired emf.

Shunt field current, $I_{sh} = V/R_{sh}$

Voltage across the load, $V = E_b + I_a R_a$

Mechanical Power developed, $P_m = E_b \times I_a$

Electrical Power input, $P = V \times I_L$

Compound Wound DC motor

In series wound motor, the output voltage is directly proportional with load current. In shunt wound motors, output voltage is inversely proportional with load current.

A combination of these two types of motors can overcome the disadvantages of both. This combination of windings is called compound wound DC motor. Compound wound motors have both series field winding and shunt field winding. One winding is placed in series with the armature and the other is placed in parallel with the armature.

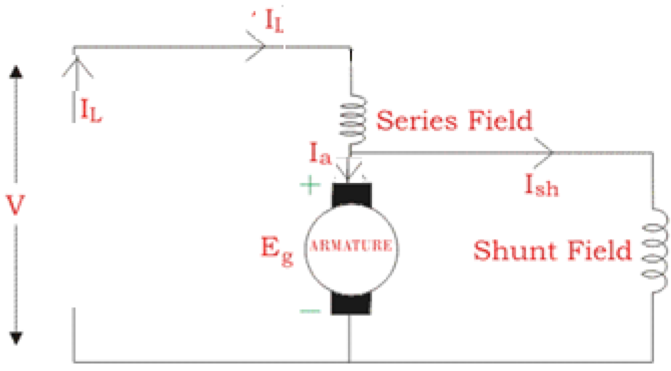
This type of DC motors may be of two types-

short shunt compound wound motor and

long shunt compound wound motor.

Short Shunt Compound Wound DC motor:

The motors in which only shunt field winding is in parallel with the armature winding as shown in figure.



Series field current, $I_{sc} = I_L$

Shunt field current, $I_{sh} = (V\text{-}I_{sc}\ R_{sc})/R_{sh}$

Armature current, $I_a = I_L\text{-}\ I_{sh}$

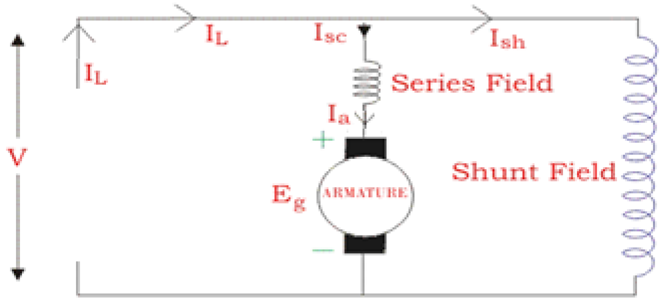
Voltage across the load, $V = E_b\text{+}\ I_a\ R_a + I_{sc}\ R_{sc}$

Power generated, $P_g = E_g \times I_a$

Power delivered to the load, $P_L\text{=}V \times I_L$

Long Shunt Compound Wound DC motor

The motors in which shunt field winding is in parallel with both series field and armature winding as shown in figure.



Shunt field current, $I_{sh}\text{=}V/R_{sh}$

Armature current, I_a= series field current, $I_{sc}\text{=}\ I_L\text{-}I_{sh}$

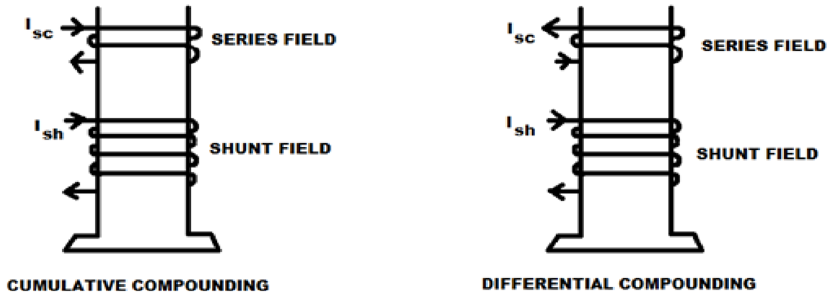
Voltage across the load, $V\text{=}\ E_g + I_a\ R_a + I_{sc}\ R_{sc}\text{=}E_g + I_a\ (R_a\text{+}R_{sc})$ [$\therefore I_a\text{=}I_{se}$]

Mechanical Power developed, $P_m = E_b \times I_a$

Electrical Power input, $P = V \times I_L$

Commutatively compound wound and Differentially compound wound.

In a compound wound motor, the shunt field is stronger than the series field. When the series field assists the shunt field, motor is said to be commutatively compound wound. On the other hand, if series field opposes the shunt field, the motor is said to be differentially compound wound.



Torque equation of a DC motor:

When armature conductors of a DC motor carry current in the presence of stator field flux, a mechanical torque is developed between the armature and the stator. Torque is given by the product of the force and the radius at which this force acts.

Torque $T = F \times r$ (N-m) …where, F = force and r = radius of the armature

Work done by this force in once revolution = Force × distance = $F \times 2\pi r$

where, $2\pi r$ = circumference of the armature

Net power developed in the armature = word done / time

= (force × circumference × no. of revolutions) / time

= $(F \times 2\pi r \times N) / 60$ (Joules per second) ....(1)

But, $F \times r = T$ and $2\pi N/60$ = angular velocity ω in radians per second.

Putting these in the above equation (1)

Net power developed in the armature = $P = T \times \omega$ (Joules per second)

Armature torque (Ta)

The power developed in the armature can be given as, $Pa = Ta \times \omega = Ta \times 2\pi N/60$

The mechanical power developed in the armature is converted from the electrical power,

Therefore, mechanical power = electrical power

That means,

$$T_a \times 2\pi N/60 = E_b . I_a$$

We know,

$$E_b = P\Phi NZ / 60A$$

Therefore,

$$T_a \times 2\pi N/60 = (P\Phi NZ / 60A) \times I_a$$

Rearranging the above equation,

$$T_a = (PZ / 2\pi A) \times \Phi . I_a \text{ (N-m)}$$

The term $(PZ / 2\pi A)$ is practically constant for a DC machine. Thus, armature torque is directly proportional to the product of the flux and the armature current i.e. $T_a \propto \Phi . I_a$

Shaft Torque (Tsh)

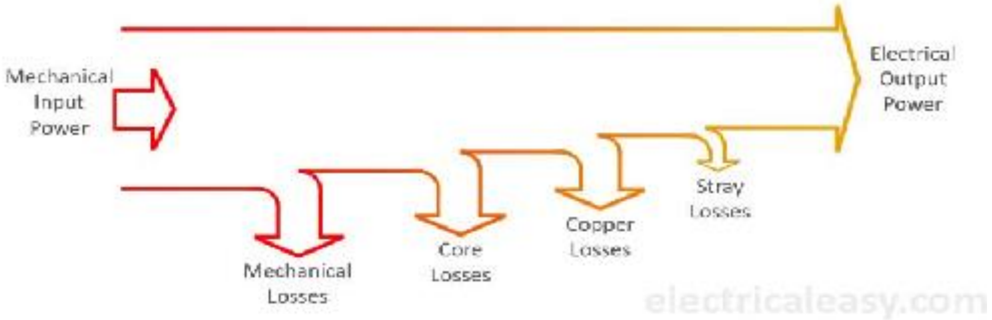
Due to iron and friction losses in a dc machine, the total developed armature torque is not available at the shaft of the machine. Some torque is lost, and therefore, shaft torque is always less than the armature torque.

Shaft torque of a DC motor is given as,

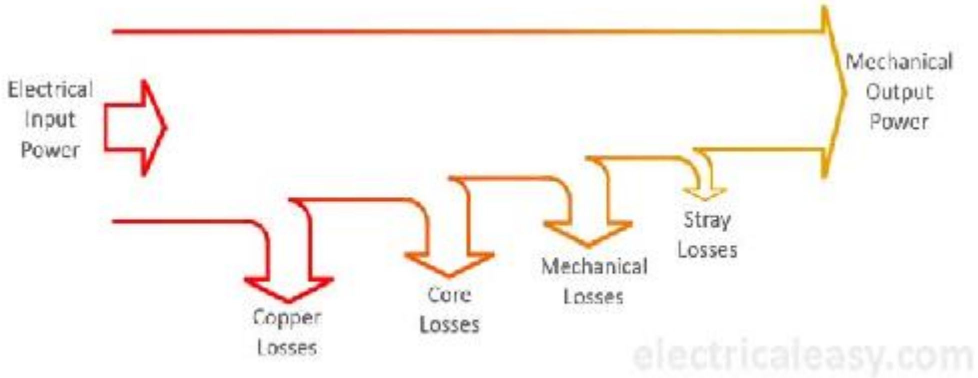
$$T_{sh} = \text{output in watts} / (2\pi N/60) \text{ .... (where, N is speed in RPM)}$$

Power Flow Diagram

The most convenient method to understand these losses in a dc generator or a dc motor is using the power flow diagram. The diagram visualizes the amount of power that has been lost in various types of losses and the amount of power which has been actually converted into the output. Following are the typical power flow diagrams for a dc generator and a dc motor.



Power flow diagram of a DC generator



Power flow diagram of a DC motor

Losses in a rotating DC machine

- Copper losses
  - Armature Cu loss
  - Field Cu loss
  - Loss due to brush contact resistance
- Iron Losses
  - Hysteresis loss
  - Eddy current loss
- Mechanical losses
  - Friction loss
  - Windage loss

The above tree categorizes various types of losses that occur in a dc generator or a dc motor. Each of these is explained in details below.

Copper losses

These losses occur in armature and field copper windings. Copper losses consist of Armature copper loss, Field copper loss and loss due to brush contact resistance.

Armature copper loss = $I_a^2R_a$

where, I_a = Armature current and

R_a = Armature resistance

This loss contributes about 30 to 40% to full load losses. The armature copper loss is variable and depends upon the amount of loading of the machine.

Field copper loss = $I_f^2 R_f$

where, I_f = field current and

R_f = field resistance

In the case of a shunt wounded field, field copper loss is practically constant. It contributes about 20 to 30% to full load losses.

Brush contact resistance also contributes to the copper losses. Generally, this loss is included into armature copper loss.

Iron losses (Core losses)

As the armature core is made of iron and it rotates in a magnetic field, a small current gets induced in the core itself too. Due to this current, eddy current loss and hysteresis loss occur in the armature iron core. Iron losses are also called as Core losses or magnetic losses.

Hysteresis loss

Hysteresis loss is due to the reversal of magnetization of the armature core. When the core passes under one pair of poles, it undergoes one complete cycle of magnetic reversal. The frequency of magnetic reversal if given by,

$f = P.N/120$

where, P = no. of poles and

N = Speed in rpm)

The loss depends upon the volume and grade of the iron, frequency of magnetic reversals and value of flux density. Hysteresis loss is given by,

Steinmetz formula: $W_h = \eta B_{max}^{1.6} f V$ (watts)

where, η = Steinmetz hysteresis constant
V = volume of the core in m^3

Eddy current loss:

When the armature core rotates in the magnetic field, an emf is also induced in the core (just like it induces in armature conductors), according to the Faraday's law of electromagnetic induction. Though this induced emf is small, it causes a large current to flow in the body due to the low resistance of the core. This current is known as eddy current. The power loss due to this current is known as eddy current loss.

Mechanical Losses:

Mechanical losses consist of the losses due to friction in bearings and commutator. Air friction loss of rotating armature also contributes to these. These losses are about 10 to 20% of full load losses.

Stray Losses:

In addition to the losses stated above, there may be small losses present which are called as stray losses or miscellaneous losses. These losses are difficult to account. They are usually due to inaccuracies in the designing and modelling of the machine. Most of the times, stray losses are assumed to be 1% of the full load.

Efficiency of DC Machines:

Generator:

1. Mechanical efficiency (η_m)

$$\eta_m = \frac{\text{Electrical power developed by armature}}{\text{Total mechanical power input}}$$

$$= \frac{E_g I_a}{\text{B. H. P. of prime mover} \times 735.5}$$

2. Electrical efficiency (η_e)

$$\eta_e = \frac{\text{Useful electrical power output}}{\text{Electrical power developed}}$$

$$= \frac{VI}{E_g I_a}$$

3.Overall or commercial efficiency ($\eta_g = \eta_m \times \eta_e$)

$$\eta_{og} = \frac{\text{Useful electrical power output}}{\text{Total mechanical power input}}$$

$$= \frac{VI}{\text{B. H. P. of prime mover} \times 735.5}$$

The overall efficiency of generator can also be expressed as follows:

$$\eta_{og} = \frac{\text{Useful power output}}{\text{Useful power output} + \text{total losses}}$$

$$= \frac{VI}{VI + \text{total losses}}$$

where E_g = generated e.m.f. V = terminal voltage

I = load current, I_a = armature current.

For good generators the value of overall or commercial efficiency may be as high as 95%.

Motor:

1. Electrical efficiency (η_e)

$$\eta_e = \frac{\text{Mechanical power developed}}{\text{Total electrical power input}}$$

$$= \frac{E_b I_a}{VI} \qquad \dots (21)$$

2. Mechanical efficiency (η_m)

$$\eta_m = \frac{\text{Useful mechanical power output}}{\text{Mechanical power developed}}$$

$$= \frac{\text{B. H. P. of motor} \times 735.5}{E_b I_a}$$

3. Overall or commercial efficiency ($\eta_{om} = \eta_m \times \eta_e$)

$$\eta_{om} = \frac{\text{Useful mechanical power output}}{\text{Total electrical power input}}$$

$$= \frac{\text{B. H. P. of motor} \times 735.5}{VI}$$

η_{om} can also be expressed as follows:

$$\eta_{om} = \frac{\text{Useful power output}}{\text{Total power input}}$$

$$= \frac{\text{Total power input} - \text{total losses}}{\text{Total power input}}$$

$$= \frac{\text{Total power input} - \text{total losses}}{\text{Total power input}}$$

$$= \frac{VI - \text{total losses}}{VI}$$

where E_b = back e.mf.

V = supply voltage

I = load current

I_a = armature current.

Condition for maximum efficiency:

Condition of maximum efficiency for both generator and motor is same. The condition for maximum efficiency is derived as follows:

Generator power output = VI,

If flux and speed are constant all losses except armature copper loss are constant, Losses = armature copper loss + constant loss

$= (I + I_{sh})^2 R_a + P_c$

$= I^2 R_a + P_c$

[Neglecting I_{sh} in comparison with load current I]

where I_{sh} is the shunt field current and P, denotes constant losses which include iron loss, field winding loss and mechanical loss,

$$\text{Efficiency,} \quad \eta = \frac{\text{Output}}{\text{Input}} = \frac{\text{Output}}{\text{Output + losses}}$$

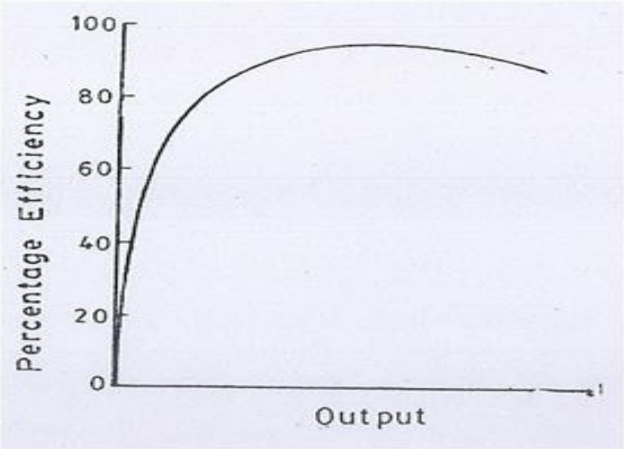
$$= \frac{VI}{VI + I^2 R_a + P_c} \qquad \ldots (25)$$

$$\eta \text{ is maximum when } \frac{d\eta}{dI} = 0 = \frac{(VI + I^2 R_a + P_c)V - VI(V + 2IR_a)}{(VI + I^2 R_a + P_c)^2}$$

or $I^2 R_a = P_c$

Hence, efficiency will be maximum when variable losses are equal to constant losses

Further (from eqn. 26) $I = \sqrt{\dfrac{P_c}{R_a}}$



Three phase induction motors

Introduction:

The three phase induction motor is the most widely used electrical motor. Almost 80% of the mechanical power used by industries is provided by three phase induction motors because of its simple and rugged construction, low cost, good operating characteristics, absence of commutator and good speed regulation.

In three phase induction motor the power is transferred from stator to rotor winding through induction. The Induction motor is also called Asynchronous motor as it runs at a speed other than the synchronous speed.

Construction of Three Phase Induction Motor:

Like any other electrical motor induction motor also have two main parts namely

- Stator and
- rotor.

Stator of Three Phase Induction Motor

The stator of the three phase induction motor consists of three main parts:

1. Stator frame,
2. Stator core,
3. Stator winding or field winding.

Stator Frame

It is the outer most part of the three phase induction motor. Its main function is to support the stator core and the field winding. It acts as a covering and it provide protection and mechanical strength to all the inner parts of the induction motor. The frame is either made up of die cast or fabricated steel. The frame of three phase induction motor should be very strong and rigid as the air gap length of three phase induction motor is very small, otherwise rotor will not remain concentric with stator, which will give rise to unbalanced magnetic pull.

Stator Core

The main function of the stator core is to carry the alternating flux. In order to reduce the eddy current loss, the stator core is laminated. These laminated types of structure are made up of stamping which is about 0.4 to 0.5 mm thick. All the stamping is stamped together to form stator core, which is then housed in stator frame. The stamping is generally made up of silicon steel, which helps to reduce the hysteresis loss occurring in motor.

Stator Winding or Field Winding

The slots on the periphery of stator core of the three phase induction motor carries three phase windings. This three phase winding is supplied by three phase ac supply. The three phases of the winding are connected either in star or delta depending upon which type of starting method is used. The squirrel cage motor is mostly started by star – delta starter and hence the stator of squirrel cage motor is delta connected. The slip ring three phase induction motor are started by inserting resistances so, the stator winding of slip ring induction motor can be connected either in star or delta. The winding wound on the stator of three phase induction motor is also called field winding and when this winding is excited by three phase ac supply it produces a rotating magnetic field.

The rotor of the three phase induction motor is further classified as

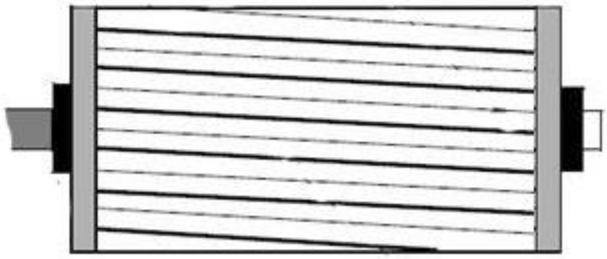
1. Squirrel cage rotor,
2. Slip ring rotor or wound rotor or phase wound rotor.

Depending upon the type of rotor construction used the

1. Squirrel cage induction motor,
2. Slip ring induction motor or wound induction motor or phase wound induction motor.

Squirrel cage three phase induction motor:

The rotor of the squirrel cage three phase induction motor is cylindrical in shape and have slots on its periphery. The slots are not made parallel to each other but are bit skewed (skewing is not shown in the figure of squirrel cadge rotor beside) as the skewing prevents magnetic locking of stator and rotor teeth and makes the working of motor more smooth and quieter. The squirrel cage rotor consists of aluminium, brass or copper bars (copper bras rotor is shown in the figure beside). These aluminium, brass or copper bars are called rotor conductors and are placed in the slots on the periphery of the rotor. The rotor conductors are permanently shorted by the copper or aluminium rings called the end rings. In order to provide mechanical strength these rotor conductors are braced to the end ring and hence form a complete closed circuit resembling like a cage and hence got its name as "squirrel cage induction motor". The squirrel cage rotor winding is made symmetrical. As the bars are permanently shorted by end rings, the rotor resistance is very small and it is not possible to add external resistance as the bars are permanently shorted. The absence of slip ring and brushes make the construction of Squirrel cage three phase induction motor very simple and robust and hence widely used three phase induction motor. These motors have the advantage of adapting any number of pole pairs. The below diagram shows squirrel cage induction rotor having aluminium bars short circuit by aluminium end rings.



Squirrel Cage Rotor
electricaleasy.com

Advantages of squirrel cage induction rotor-

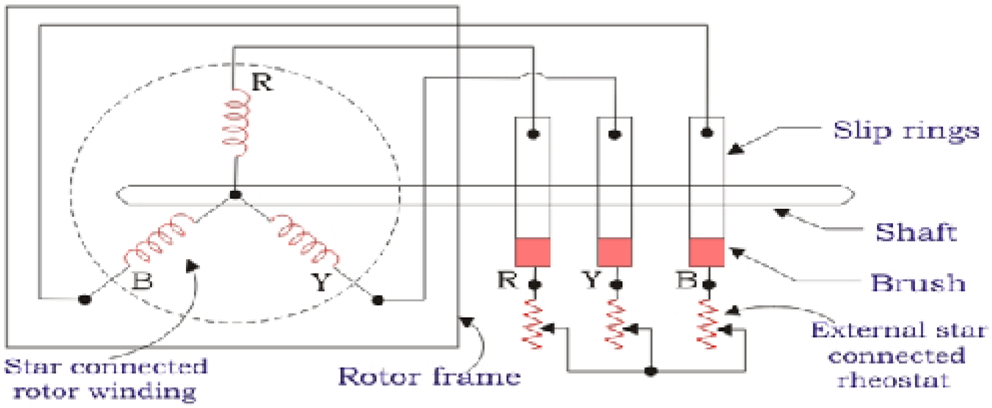
1. Its construction is very simple and rugged.
2. As there are no brushes and slip ring, these motors require less maintenance.

Applications:

Squirrel cage induction motor is used in lathes, drilling machine, fan, blower printing machines etc.

Slip ring or wound three phase induction motor:

In this type of three phase induction motor the rotor is wound for the same number of poles as that of stator but it has less number of slots and has less turns per phase of a heavier conductor. The rotor also carries star or delta winding similar to that of stator winding. The rotor consists of numbers of slots and rotor winding are placed inside these slots. The three end terminals are connected together to form star connection. As its name indicates three phase slip ring induction motor consists of slip rings connected on same shaft as that of rotor. The three ends of three phase windings are permanently connected to these slip rings. The external resistance can be easily connected through the brushes and slip rings and hence used for speed control and improving the starting torque of three phase induction motor. The brushes are used to carry current to and from the rotor winding. These brushes are further connected to three phase star connected resistances. At starting, the resistance is connected in rotor circuit and is gradually cut out as the rotor pick up its speed. When the motor is running the slip ring are shorted by connecting a metal collar, which connect all slip ring together and the brushes are also removed. This reduces wear and tear of the brushes. Due to presence of slip rings and brushes the rotor construction becomes somewhat complicated therefore it is less used as compare to squirrel cage induction motor.



Slip Ring Three Phase Induction Motor

Advantages of slip ring induction motor -

1. It has high starting torque and low starting current.
2. Possibility of adding additional resistance to control speed.

Application:

Slip ring induction motor are used where high starting torque is required i.e. in hoists, cranes, elevator etc.

Working principle of Three Phase Induction Motor

The stator of the motor consists of overlapping winding offset by an electrical angle of 120°. When the primary winding or the stator is connected to a 3 phase AC source, it establishes a rotating magnetic field $Ør = Ø_m/2$ which rotates at the synchronous speed (N_s).

The rotational speed of the rotating magnetic field is called as synchronous speed.

$$N_s = \frac{120 \times f}{P} \quad (RPM)$$

where, f = frequency of the supply

P = number of poles

According to Faraday's law an emf induced in any circuit is due to the rate of change of magnetic flux linkage through the circuit. As the rotor winding in an induction motor are either closed through an external resistance or directly shorted by end ring, and cut the stator rotating magnetic field, an emf is induced in the rotor copper bar and due to this emf a current flows through the rotor conductor. Here the relative speed between the rotating flux and static rotor conductor is the cause of current generation; hence as per Lenz's law the rotor will rotate in the same direction to reduce the cause i.e. the relative velocity.

Thus from the working principle of three phase induction motor it may have observed that the rotor speed should not reach the synchronous speed produced by the stator. If the speeds equals, there would be no such relative speed, so no emf induced in the rotor, & no current would be flowing, and therefore no torque would be generated. Consequently, the rotor cannot reach the synchronous speed.

The difference between the stator (synchronous speed) and rotor speeds is called the slip. The rotation of the magnetic field in an induction motor has the advantage that no electrical connections need to be made to the rotor. Thus the three phase induction motor is:

- Self-starting.
- Less armature reaction and brush sparking because of the absence of commutators and brushes that may cause sparks.
- Robust in construction.
- Economical.
- Easier to maintain.

Difference between Slip Ring and Squirrel Cage Induction Motor

Slip ring or phase wound Induction motor	**Squirrel cage induction motor**
Construction is complicated due to presence of slip ring and brushes	Construction is very simple
The rotor winding is similar to the stator winding	The rotor consists of rotor bars which are permanently shorted with the help of end rings
We can easily add rotor resistance by using slip ring and brushes	Since the rotor bars are permanently shorted, it's not possible to add external resistance
Due to presence of external resistance high starting torque can be obtained	Staring torque is low and cannot be improved
Slip ring and brushes are present	Slip ring and brushes are absent
Frequent maintenance is required due to presence of brushes	Less maintenance is required
The construction is complicated and the presence of brushes and slip ring makes the motor costlier	The construction is simple and robust and it is cheap as compared to slip ring induction motor
This motor is rarely used only 10 % industry uses slip ring induction motor	Due to its simple construction and low cost. The squirrel cage induction motor is widely used
Rotor copper losses are high and hence less efficiency	Less rotor copper losses and hence high efficiency
Speed control by rotor resistance method is possible	Speed control by rotor resistance method is not possible

Slip ring induction motor are used where high starting torque is required i.e. in hoists, cranes, elevator etc.	Squirrel cage induction motor is used in lathes, drilling machine, fan, blower printing machines etc.

Slip:

Rotor tries to catch up the synchronous speed of the stator field, and hence it rotates. But in practice, rotor never succeeds in catching up. If rotor catches up the stator speed, there won't be any relative speed between the stator flux and the rotor, hence no induced rotor current and no torque production to maintain the rotation. However, this won't stop the motor, the rotor will slow down due to loss of torque, the torque will again be exerted due to relative speed. That is why the rotor rotates at speed which is always less the synchronous speed.

The difference between the synchronous speed (N_s) and actual speed (N) of the rotor is called as slip.

$$\% \text{ slip } \quad s = \frac{Ns - N}{Ns} \times 100$$

Torque Equation of three phase induction motor

The torque produced in the induction motor depends on the following factors:

1. The part of rotating magnetic field which reacts with rotor and is responsible to produce induced e.m.f. in rotor.

2. The magnitude of rotor current in running condition.

3. The power factor of the rotor circuit in running condition.

$T \propto \phi I_2 \cos\phi_2$ OR $T = k \phi I_2 \cos\phi_2$.

where, ϕ = flux per stator pole,

I_2 = rotor current at standstill,

ϕ_2 = angle between rotor emf and rotor current,

k = a constant.

Now, let E_2 = rotor emf at standstill

we know, rotor emf is directly proportional to flux per stator pole, i.e. $E_2 \propto \phi$.

therefore, $T \propto E_2 I_2 \cos\phi_2$ OR $T = k_1 E_2 I_2 \cos\phi_2$.

Starting torque

The torque developed at the instant of starting of a motor is called as starting torque. Starting torque may be greater than running torque in some cases, or it may be lesser.

We know, $T = k_1 E_2 I_2 \cos\phi_2$.

let, R2 = rotor resistance per phase

X2 = stand still rotor reactance

$$Z_2 = \sqrt{(R_2^2 + X_2^2)} \quad = \text{rotor impedence per phase at standstill}$$

then,

$$I_2 = \frac{E_2}{Z_2} = \frac{E_2}{\sqrt{(R_2^2 + X_2^2)}} \quad \text{and} \quad \cos\phi_2 = \frac{R_2}{Z_2} = \frac{R_2}{\sqrt{(R_2^2 + X_2^2)}}$$

Therefore, starting torque can be given as,

$$Tst = k_1 E_2 \frac{E_2}{\sqrt{(R_2^2 + X_2^2)}} \times \frac{R_2}{\sqrt{(R_2^2 + X_2^2)}} = \frac{k_1 E_2^2 R_2}{R_2^2 + X_2^2}$$

The constant k1 = 3 / 2πNs

$$Tst = \frac{3}{2\pi Ns} \frac{E_2^2 R_2}{R_2^2 + X_2^2}$$

Torque under running condition

$$T \propto \phi \, I_r \cos\phi_2 .$$

where, E_r = rotor emf per phase under running condition = sE_2. (s=slip)

I_r = rotor current per phase under running condition

reactance per phase under running condition will be = sX_2

therefore,

$$I_r = \frac{E_r}{Z_r} = \frac{sE_2}{\sqrt{(R_2^2 + (sX_2)^2)}} \quad \text{and} \quad \cos\phi_2 = \frac{R_2}{Z_r} = \frac{R_2}{\sqrt{(R_2^2 + (sX_2)^2)}}$$

$$T = \frac{k \phi s E_2 R_2}{\sqrt{(R_2^2 + (s X_2)^2)}}$$

as, $\phi \propto E_2$.

$$T = \frac{k_1 s E_2^2 R_2}{\sqrt{(R_2^2 + (s X_2)^2)}} = \frac{3}{2\pi N_s} \frac{s E_2^2 R_2}{\sqrt{(R_2^2 + (s X_2)^2)}}$$

UNIT-V

BASIC INSTRUMENTS

Measurement is the process by which one can convert physical parameters in to meaningful numbers. The measurement of a given quantity is the result of comparison between the quantity to be measured, and a definite standard. The instruments which are used for such measurements are called measuring instruments.

Some important terms in ‘measurement’ are:

Instrument

A device for finding the value, or magnitude of a quantity or variable.

Accuracy

It is the nearness of the measured value towards the true value. ie, the measure of conformity to the true value.

Precision

It refers to the degree of agreement within a group of measurements or instruments. i.e. The measure of repeatability or reproducibility. Precision has two characteristics: conformity and the no. of significant figures to which measurements may be made.

Resolution

It is defined as the smallest change in input that can be detected by an instrument.

Sensitivity

It is the ratio of output signal to a change of input.

Or

It is the ratio of response of an instrument to a change in measured variable.

True value

It is the average of the infinite no. of measurements, when the average deviation tends to become zero.

Error

An error is the deviation from the true value of the measured variable.

Classification of Instruments

Instruments can be broadly classified in to

i. Absolute instruments
ii. Secondary instruments

Absolute instruments

Absolute instruments give the magnitude of the quantity under measurement in terms of physical constants of the instruments.

e.g: - Tangent galvanometer, Rayleigh's current balance.

Secondary instruments

In secondary instruments, the quantity under measurement can only be measured by observing the output of the instrument. The secondary instruments should be calibrated by comparing with an absolute instrument or another secondary instrument which has already been calibrated against an absolute instrument.

e.g.: - Voltmeter, pressure gauge.

The secondary instruments are the commonly used instruments compared to the absolute instruments.

Electrical measuring instruments may be classified according to their functions as;

(i) Indicating instruments
(ii) (ii)Integrating instruments
(iii) (iii) Recording instruments

Indicating instruments: -

These instruments directly indicate the value of the electrical quantity at the time when it is being measured. In these instruments, a pointer moving over a graduated scale directly gives the value of the electrical quantity being measured.

e.g.: - Ammeter, voltmeter, wattmeter.

Integrating instruments

The instruments which measure the total quantity of electricity (in Ampere hours) or electrical energy (in Watt hours) in a given time are called integrating instruments. In such instruments, there are a set of dials and pointers which register the total quantity of electricity or electrical energy supplied to the load.

e.g.: - Ampere- Hour meter, Watt-hour meter.

Recording instruments

Recording instruments give a continuous record of the variations of the electrical quantity to be measured. A recording instrument is merely an indicating instrument with a pen attached to its pointer. The pen rests lightly on a chart wrapped over a drum moving with a slow uniform speed. The motion of the drum is in a direction perpendicular to the direction of the pointer. The path traced out by the pen indicates the manner in which the quantity being measured, has varied during the time of the record.

e.g.: - Recording voltmeters, Recording ammeters in supply stations.

Essentials of Indicating instruments

An indicating instrument essentially consists of a moving system pivoted in jewel bearings. A pointer is attached to the moving system which indicates the electrical quantity to be measured, on a graduated scale. In order to ensure the proper operation of the indicating instruments, the following three torques are required.

i. Deflecting (or operating) torque.
ii. Controlling (or restoring) torque.
iii. Damping torque

The deflecting torque is produced by utilising the various effects (magnetic effect, induction effect, thermal effect, hall effect) of electric current or voltage, and causes the moving system and hence the pointer to move from zero position.

The controlling torque is produced by spring or gravity and opposes the deflecting torque. The pointer comes to rest at a position, where these two opposing torques are equal.

Damping torque is provided by air friction or eddy currents. It ensures that, the pointer comes to the final position, without oscillations, thus enabling accurate and quick readings **to be taken.**

Deflecting torque(T_d): -

The deflecting torque causes the moving system to move from zero position to indicate the value of the electrical quantity being measured on a graduated scale. The actual method of producing the deflecting torque depends upon the type of instrument.

Controlling torque(T_c): -

If the deflecting torque were acting alone, the pointer will continue to move indefinitely and would swing over to the maximum deflected position irrespective of the magnitude of the electrical quantity to be measured. This necessitates providing some form of controlling or opposing torque. This controlling torque should increase with the deflection of the moving system. The pointer will be brought to rest at a position where the two opposing torques are equal. i.e., $T_d = T_c$.

The controlling torque performs two functions.

1. It increases with the deflection of the moving system so that, the final position of the pointer on the scale will be according to the magnitude of the electrical quantity to be measured.
2. It brings the pointer back to zero position, when the deflecting torque is removed. If it were not provided, the pointer once deflected would not return to zero position on removing the deflecting torque.

Controlling torque is generally provided by two ways

1. Gravity control
2. Spring control

Gravity Control

This type of control consists of a small weight attached to the moving system whose position is adjustable. This weight produces a controlling torque due to gravity. This weight is called control weight.

The Fig. 1 shows the gravity control system. At the zero position of the pointer, the controlling torque is zero. This position is shown as position A of the weight in the Fig. 2. if the system deflects, the weight positions also changes, as shown in the Fig.2.

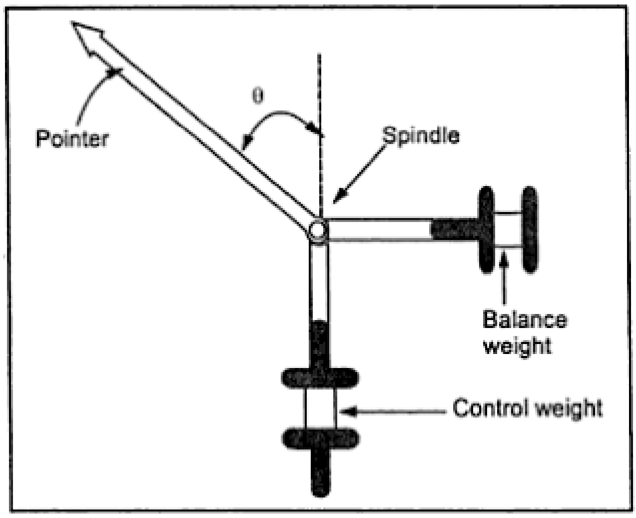


Fig. 1 Gravity Control

The system deflects through an angle θ. The control weight acts at a distance *l* from the centre. The component Wsinθ of this weight tries to restore the pointer back to the zero position. This is nothing but the controlling torque T_C.

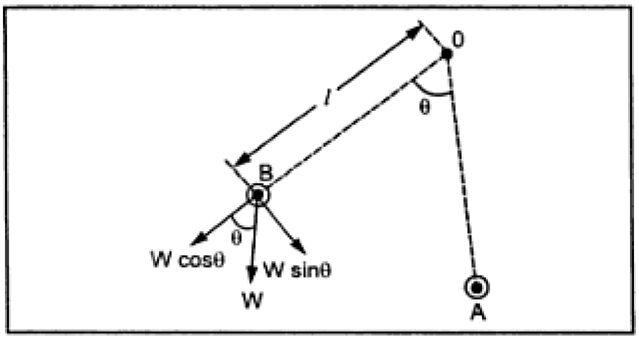


Fig. 2

Thus,

controlling torque $T_C = W\sin\theta \times l$

$= K\sin\theta$

Here $K = W\,l$

= gravity constant

Now, generally all meters are current sensing meters where,

Deflecting torque $T_d = K_t I$

where K_t= another constant.

In equilibrium position, $T_d = T_C$

$\therefore \quad K_t \; I = K\sin\theta$

$\therefore \quad I \; \alpha \; \sin\theta$

Thus the deflection is proportional to current i.e. quantity to be measured. But as it is a function of sin θ , the scale for the instrument using gravity control is not uniform.

Its advantages are:

1. Its performance is not time dependent.
2. It is simple and cheap.
3. The controlling torque can be varied by adjusting the position of the control weight.
4. Its performance is not temperature dependent.

Its dis advantages are:

1. The scale is non-uniform causing problems to record accurate readings.
2. The system must be used in vertical position only and must be properly levelled.
3. Otherwise it may cause serious errors in the measurement.
4. As delicate and proper levelling required, in general it is not used for indicating instruments and portable instruments.

Spring Control

Two hair springs are attached to the moving system which exerts controlling torque. To employ spring control to an instrument, following requirements are essential.

1) The spring should be non-magnetic.
2) The spring should be free from mechanical stress.
3) The spring should have a small resistance, sufficient cross-sectional area.
4) It should have low resistance temperature co-efficient.

The arrangement of the springs is shown in the Fig.3

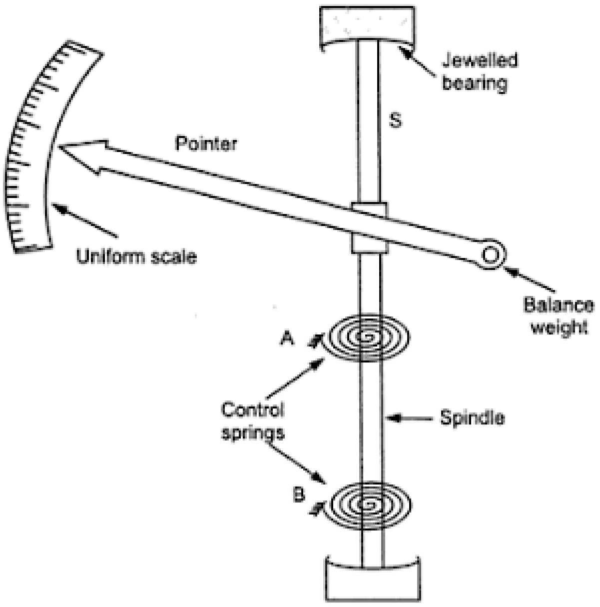


Fig. 3 Spring Control

The springs are made up of non-magnetic materials like silicon bronze, hard rolled silver copper, platinum silver and german silver. For most of the instruments, phosphor bronze spiral is provided. Flat spiral springs are used in almost all indicating instruments.

The inner of the spring is attached to the spindle while the outer end is attached to a lever or arm which is actuated by a set of screw mounted at the front of the instrument. So zero setting can be easily done. The controlling torque provided by the instrument is directly proportional to the angular deflection of the pointer.

The controlling torque produced by spiral springs is given by,

$$T_c = \frac{E\,b\,t^3}{12\,L}\theta = K_s\theta$$

where

E = Young's modulus of spring material in N/m^2

t = thickness in metres

b = depth in metres

L = length in metres

K_s = spring constant = $\frac{Ebt^3}{12L}$

$\therefore \quad T_c \propto \theta$

Now deflecting torque is proportional to current.

$\therefore \quad T_d \propto I$

At equilibrium, $T_d = T_c$

$\therefore \quad I \propto \theta$

Thus the deflection is proportional to the current. Hence the scale of the instrument using spring control is uniform. When the current is removed, due to spring force the pointer comes back to initial positions. The spring control is very popular and is used in almost all indicating instruments.

1.3 Comparison of Controlling Systems

	Gravity Control	Spring Control
1.	Adjustable small weight is used which produces the controlling torque.	Two hair springs are used which exert controlling torque.
2.	Controlling torque can be varied.	Controlling torque is fixed.
3.	The performance is not temperature dependent.	The performance is temperature dependent.
4.	The scale is nonuniform.	The scale is uniform.
5.	The controlling torque is proportional to sin θ.	The controlling torque is proportional to θ.
6.	The readings can not be taken accurately.	The readings can be taken very accurately.
7.	The system must be used in vertical position only.	The system need not be necessarily in vertical position.
8.	Proper levelling is required as gravity control.	The levelling is not required.
9.	Simple, cheap but delicate.	Simple, rigid but costlier compared to gravity control.
10.	Rarely used for indicating and portable instruments.	Very popularly used in most of the instruments.

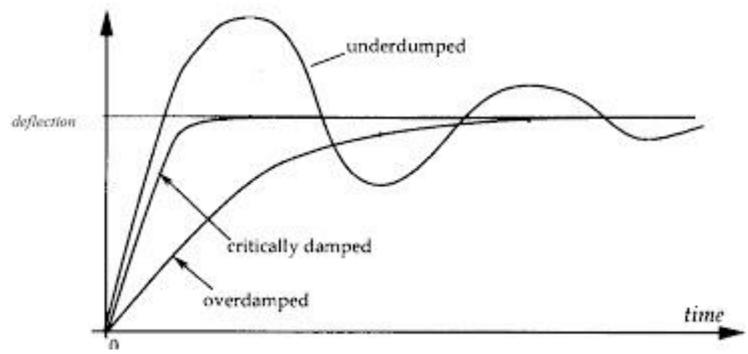
Damping torque (T_{damp}): -

If the moving system is acted upon by deflecting and controlling torques alone, then due to inertia, the pointer will oscillate about its final deflected position for some time before coming to rest. This oscillation makes it difficult to obtain quick and accurate reading. In order to avoid these oscillations of the pointer and to bring it quickly to its final deflected position, a damping torque is provided in the indicating instruments. The damping does not affect the stationary pointer, as the damping torque acts only when the pointer is in motion and always opposes the motion.

The damping torque in indicating instruments can be provided by,

- Air- friction damping
- Fluid friction damping
- Eddy currents damping

The behaviour of the moving system is decided by the degree of damping. The fig. given below shows the graph for under damping, over damping, and critical damping.



Under damped moving system: - The pointer will oscillate about the final position for some time, before coming to rest.

Over damped: - The pointer will become slow and lethargic.

Critically damped/ dead beat: - The degree of damping is so that, the pointer comes up to the correct reading quickly without passing beyond it or oscillating about it.

1. Air damping:

Fig. 4 shows an arrangement for obtaining air damping. It consists of a thin metal vane MV attached to the spindle S; the vane moves in a sector-shaped box B. Any tendency of the moving system to oscillate is damped by the action of the air on the vane.

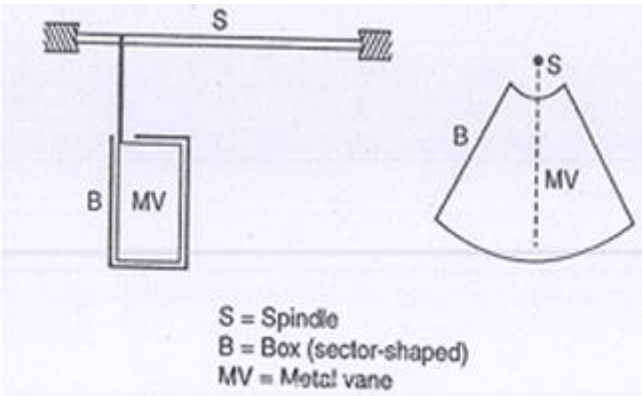


Fig. 4. Air damping.

Eddy current damping:

This method of damping is based on the principle that when a conducting non-magnetic material is moved in a magnetic field an e.m.f is induced in it which causes

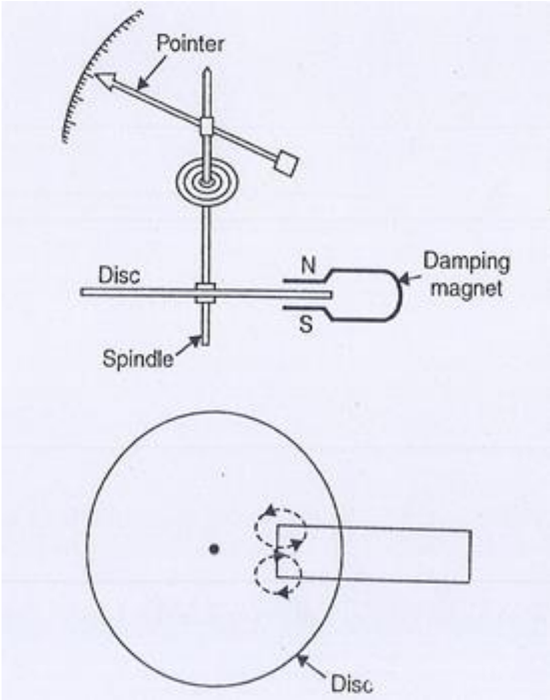


Fig. 5. Eddy current damping.

currents called the eddy currents. Due to these eddy currents a force exists between them and the field. Due to Lenz's law this force is always in opposition to the force causing rotation of the conducting, material, thus, it provides the necessary damping.

- One form of eddy-current damping is shown in Fig. 5. Here a copper or aluminium disc, curried by a spindle, can move between the poles of a permanent magnet. If the disc moves clockwise, the e.m.f.s induced in the disc circulate eddy currents as shown dotted. It follows from Lenz's law that these currents exert a force opposing the motion producing them, namely the clockwise movement of the disc.
- Another form of providing damping is used in the moving coil instruments using permanent magnet. The moving coil is mounted over a metallic former. When the coil is deflected eddy e.m.f.s are induced in the two sides of the former, causing eddy forces as shown in Fig. 6.

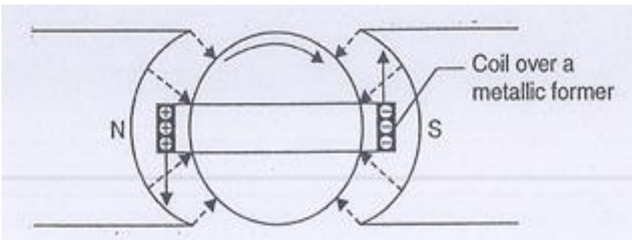


Fig. 6. Eddy current damping with permanent magnet.

3.Fluid friction damping:

Fig. 7 shows the method of fluid friction damping. Here light vanes are attached to the spindle of the moving system. The vanes are dipped into a pot of damping oil and are completely submerged by the oil. The motion of the moving system is always opposed by the friction of the damping oil on the vanes. The damping force thus created always increases with the increase in velocity of vanes. There is no damping force when the vanes are stationary:

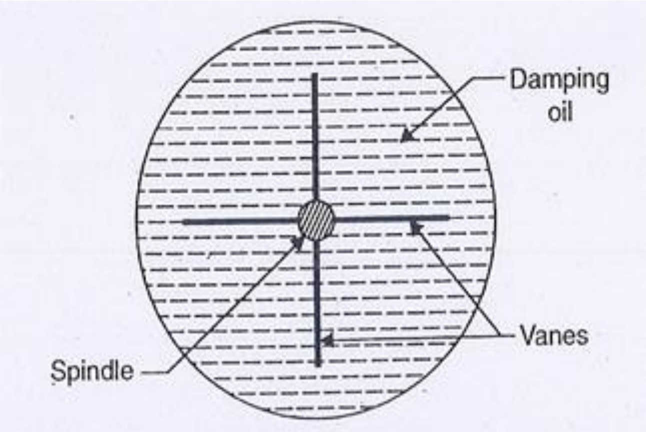


Fig. 7. Fluid friction damping.

The damping oil used must have the following properties:

1. Must be a good insulator.
2. Should be non-evaporating.
3. Should not have corrosive action upon the metal of the vane.
4. The viscosity of the oil should not change with the temperature.

Though in this method of damping, no case is required as in the air friction damping but it is not much used due to the following disadvantages:

1. Objectionable creeping of oil.
2. Using the instrument always in the vertical position and its obvious unsuitability for use in portable instruments.

Types of instruments

(1) Permanent Magnet Moving Coil (PMMC) instruments.
(2) Moving iron instruments.
(3) Electrodynamometer instruments.
(4) Thermal instruments
(5) Induction instruments.
(6) Electrostatic type instruments.
(7) Rectifier type instruments.

The PMMC instruments can be used for direct measurement only, and the induction type for alternating current measurements only.

The moving iron(MI) and moving coil(MC) types both depend for their action upon the magnetic effect of current. The MI instruments can be used for either direct or alternating current measurements, and is the cheapest.

Electrodynamometer type of instruments can be used both on a.c as well as on d.c. They are useful as “transfer instruments”, as their calibration for both d.c and a.c is the same.

The calibration for d.c and a.c is same for the thermal instruments also. They are particularly suited for a.c measurements. This is because, the deflection of the thermal instruments depends directly upon the heating effect of the alternating current. i.e., upon the rms value of the current.

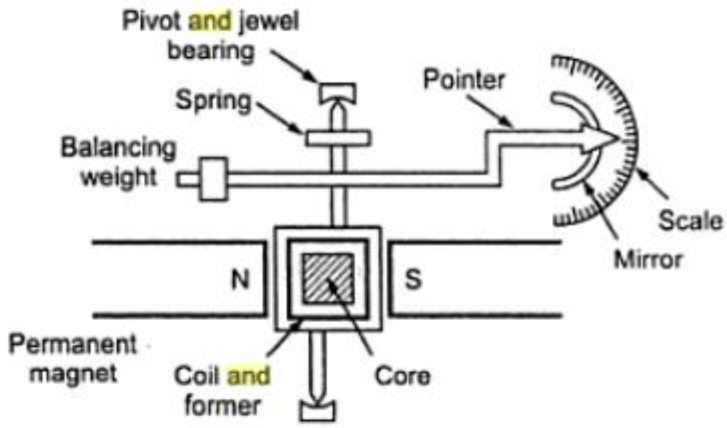
For the electrostatic instruments, the electrostatic principle is only directly applicable to voltage measurements. They have the advantage that, their power consumption is extremely small.

The induction principle is more generally used for Watt- hour meters than for ammeters and voltmeters.

(1) PMMC Instruments

These instruments are used either as ammeters or voltmeters and are suitable for d.c work only. PMMC instruments work on the principle that, when a current carrying conductor is placed in a magnetic field, a mechanical force acts on the conductor. The current carrying coil, placed in magnetic field is attached to the moving system. With the movement of the coil, the pointer moves over the scale to indicate the electrical quantity being measured. This type of movement is known as D’ Arsenoval movement.

Construction: -



It consists of a light rectangular coil of many turns of fine wire wound on an aluminium former inside which is an iron core as shown in fig. The coil is delicately pivoted upon jewel bearings and is mounted between the poles of a permanent horse shoe magnet. Two soft-iron pole pieces are attached to these poles to concentrate the magnetic field. The current is led in to and out of the coils by means of two control hair- springs, one above and other below the coil, as shown in fig (b). These springs also provide the controlling torque. The damping torque is provided by eddy currents induced in the aluminium former as the coil moves from one position to another.

Working: -

When the instrument is connected in the circuit to measure current or voltage, the operating current flows through the coil. Since the current carrying coil is placed in the magnetic field of the permanent magnet, a mechanical torque acts on it. As a result of this torque, the pointer attached to the moving system moves in clockwise direction over the graduated scale to indicate the value of current or voltage being measured.

This type of instruments can be used to measure direct current only. This is because, since the direction of the field of permanent magnet is same, the deflecting torque also gets reversed, when the current in the coil reverses. Consequently, the pointer will try to deflect below zero. Deflection in the reverse direction can be prevented by a “stop” spring.

Deflecting torque equation: -

The magnetic field in the air gap is radial due to the presence of soft iron core. Thus, the conductors of the coil will move at right angles to the field. When the current is passed through the coil, forces act on its both sides which produce the deflecting torque.

Let, B = flux density, Wb /m^2

l = length or depth of coil, m

b = breadth of the coil.

N = no. of turns of the coil.

If a current of ‘I’ Amperes flows in the coil, then the force acting on each coil side is given by,

Force on each coil side, F = BIlN Newtons.

Deflecting torque, T_d = Force × perpendicular distance

= (BIlN) × b

T_d = BINA Newton metre.

Where, A = l × b, the area of the coil in m^2.

Thus, $T_d \alpha I$

The instrument is spring controlled so that, $T_c \alpha \theta$

The pointer will come to rest at a position, where $T_d = T_c$

Therefore, $\theta \alpha I$

Thus, the deflection is directly proportional to the operating current. Hence, such instruments have uniform scale.

Advantages: -

1. Uniform scale.ie, evenly divided scale.
2. Very effective eddy current damping.
3. High efficiency.
4. Require little power for their operation.
5. No hysteresis loss (as the magnetic field is constant).
6. External stray fields have little effects on the readings (as the operating magnetic field is very strong).
7. Very accurate and reliable.

Disadvantages: -

1. Cannot be used for a.c measurements.
2. More expensive (about 50%) than the moving iron instruments because of their accurate design.
3. Some errors are caused due to variations (with time or temperature) either in the strength of permanent magnet or in the control spring.

Applications: -

1. In the measurement of direct currents and voltages.
2. In d.c galvanometers to detect small currents.
3. In Ballistic galvanometers used for measuring changes of magnetic flux linkages.

(2) Moving Iron Instruments

M.I instruments are mainly used for the measurement of alternating currents and voltages, though it can also be used for d.c measurements.

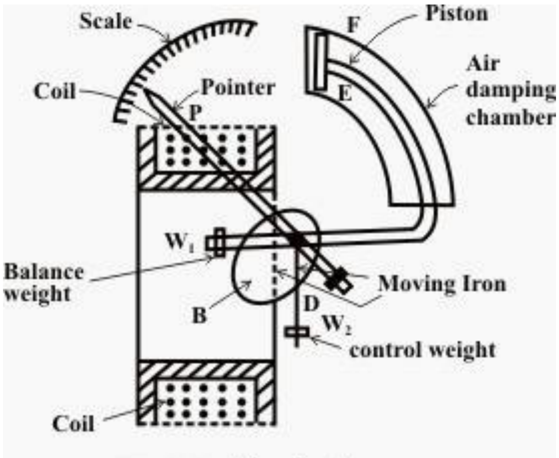
The general principle of a M.I instrument can be explained under:

Let a plate or vane of soft iron or of high permeability steel forms the moving element of the system. The iron vane is situated so as, it can move in a magnetic field produced by a stationary coil. The coil is excited by the current or voltage under measurement. When the coil is excited, it becomes an electromagnet and the iron vane moves in such a way so as to increase the flux of the electromagnet. Thus, the vane tries to occupy a position of minimum reluctance. Thus, the force produced is always in such a direction so as to increase the inductance of the coil.

There are two types of Moving- iron instruments.

i. Attraction type: -

In this type of instrument, a single soft iron vane (moving iron) is mounted on the spindle, and is attracted towards the coil when operating current flows through it.



Attraction type

Deflecting torque equation

The force F, pulling the soft -iron piece towards the coil is directly proportional to;

a) Field strength H, produced by the coil.

b) pole strength 'm' developed in the iron piece.

$$F \alpha mH$$

Since, $m \alpha H$,

$$F \alpha H^2$$

Instantaneous deflecting torque αH^2

Also, the field strength $H = \mu i$

If the permeability(μ) of the iron is assumed constant,

Then, $H \alpha i$

Where, i instantaneous coil current, Ampere

Instantaneous deflecting torque αi^2

Average deflecting torque, $T_d \alpha$ mean of i^2 over a cycle.

Since the instrument is spring controlled,

$$T_c \alpha \theta$$

In the steady position of deflection, $T_d = T_c$

$$\theta \propto \text{mean of } i^2 \text{ over a cycle}$$

$$\propto I^2$$

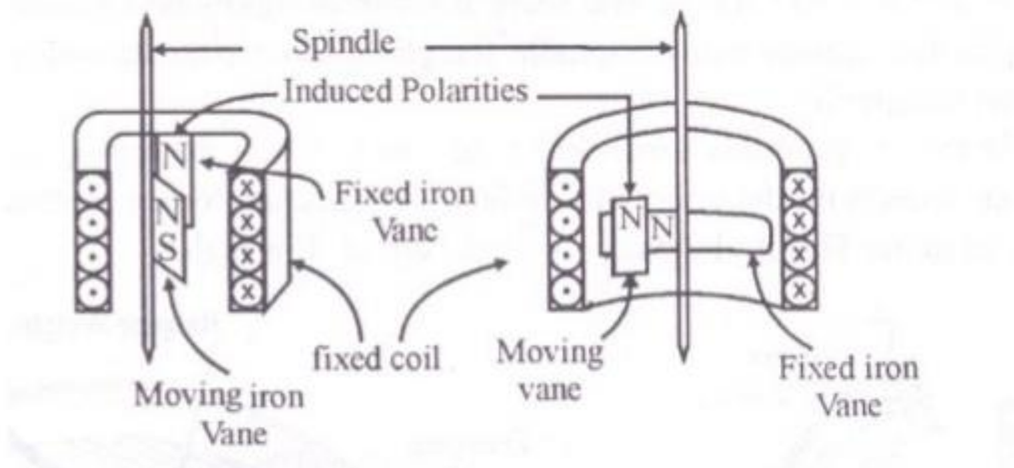
Since the deflection is proportional to the square of coil current, the scale of such instruments is non-uniform (being crowded in the beginning and spread out near the finishing end of the scale).

ii.Repulsion type: -

In this two soft iron vanes are used; one fixed and attached the stationary coil, while the other is movable (moving iron), and mounted on the spindle of the instrument. When operating current flows through the coil, the two vanes are magnetised, developing similar polarity at the same ends. Consequently, repulsion takes place between the vanes and the movable vane causes the pointer to move over the scale.

Two types

1. Radial vane type: - vanes are radial strips of iron.
2. Co-axial vane type: -vanes are sections of coaxial cylinders.



(b) Radial vane type **Fig.4.8** (c) Co–Axial vane type

Deflecting torque: -

The deflecting torque results due to repulsion between the similarly charged soft- iron pieces or vanes. If the two pieces develop pole strength of m_1 and m_2 respectively, then;

$$\text{Instantaneous deflecting torque } \propto m_1 m_2 \propto H^2$$

If the permeability of iron is assumed constant, then; $H \propto i$, where, i is the coil current

$$\text{Instantaneous deflecting torque } \propto i^2$$

$$\text{Average deflecting torque, } T_d \propto \text{mean of } i^2 \text{ over a cycle.}$$

Since the instrument is spring controlled, $T_c \alpha \theta$

In the steady position of deflection, $T_d = T_c$

θ α mean of i^2 over a cycle.

α I^2

Thus, the deflection is proportional to the square of the coil current. The scale of the instrument is non- uniform; being crowded in the beginning and spread out near the finish end of the scale. However, the non- linearity of the scale can be corrected to some extent by the accurate shaping and positioning of the iron vanes in relation to the operating coil.

Comparison between MC and MI Instruments

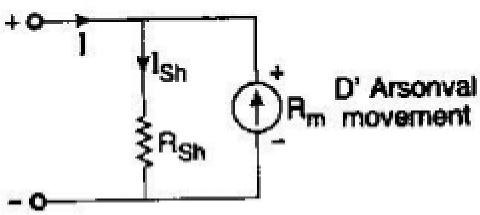
S.No	M.C Instruments	M.I Instruments
1	More accurate	Less accurate
2	Costly	cheap
3	Uniform scale	Non-uniform scale (scale cramped at beginning and finishing)
4	Very sensitive	Robust in construction
5	Low power consumption	Slightly high power consumption
6	Eddy current damping is used	Air friction damping is used
7	Can be used only for D.C	Can be used on A.C as well as on D.C
8	Controlling torque is provided by spring	Controlling torque is provided by gravity or spring
9	θ α I	θ α I^2
10	Errors are set due to aging of control springs, permanent magnet (i.e. No Hysteresis loss)	Errors are set due to hysteresis and stray fields (i.e. hysteresis loss takes place).

Extension of instrument ranges

Shunts are used for the extension of range of Ammeters. So a good shunt should have the following properties: -

1- The temperature coefficient of shunt should be low

2- Resistance of shunt should not vary with time
3- They should carry current without excessive temperature rise
4- They should have thermal electromotive force with copper
* ‘Manganin’ is used for DC shunt and ‘Constantan’ as AC shunt.



Ammeter: -

PMMC is used as indicating device. The current capacity of PMMC is small.
It is impractical to construct a PMMC coil, which can carry a current greater than 100 mA. Therefore, a shunt is required for measurement of large currents.
Rm = Internal resistance of movement (coil) in Ω
Rsh = Resistance of shunt in Ω
Im = Ifs = Full scale deflection current of movement in Amperes
Ish = Shunt current in Amperes
I = Current to be measured in Amperes
Since the shunt resistance is in parallel with the meter movement, the voltage drop across shunt and movement must be same.

$$I_{sh}R_{sh} = I_m R_m$$

$$R_{sh} = I_m R_m / I_{sh}$$

$$I_{sh} = I - I_m$$

$\therefore$ We can write $R_{sh} = I_m R_m / (I - I_m)$

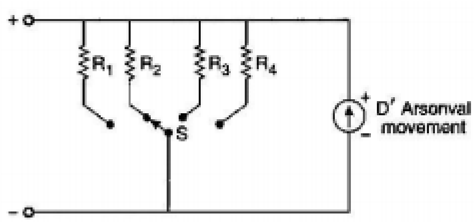
$$\frac{I}{I_m} - 1 = \frac{R_m}{R_{sh}}$$

$$\frac{I}{I_m} = 1 + \frac{R_m}{R_{sh}}$$

$\frac{I}{I_m} = m$ is known as 'multiplying power' of shunt

Resistance of shunt $R_{sh} = R_m / (m-1)$

Or $R_{sh} = R_m / \left(\frac{I}{I_m} - 1\right)$



Multi Range Ammeter:- Let m_1, m_2, m_3, m_4 be the shunt multiplying powers for current I_1, I_2, I_3, I_4.

$$R_{sh1} = R_m / (m_1 - 1)$$

$$R_{sh2} = R_m / (m_2 - 1)$$

$$R_{sh3} = R_m / (m_3 - 1)$$

$$R_{sh4} = R_m / (m_4 - 1)$$

Voltmeter:-

For measurement of voltage a series resistor or a multiplier is required for extension of range.

I_m = Deflection current of movement

R_m = Internal resistance of movement

R_s = Multiplier resistance

V = Full range voltage of instrument

$$V = I_m(R_s + R_m)$$

$$R_s = \frac{V - I_m R_m}{I_m} = \frac{V}{I_m} - R_m$$

* For more than 500 V multiplier is mounted outside the case.

Multi Range Voltmeter:

$$R_{s1} = \frac{V_1}{I_m} - R_m$$

$$R_{s2} = \frac{V_2}{I_m} - R_m$$

$$R_{s3} = \frac{V_3}{I_m} - R_m$$

$$R_{s4} = \frac{V_4}{I_m} - R_m$$