

Tricky Question

Question from "Concepts of Physics" by

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$$\int \frac{dx}{\sqrt{ax - x^2}} = a^n \sin^{-1}\left(\frac{x}{a}\right) + C$$

What is the value of "n".

This question *prima facie* looks like that of "maths". Fine, you can do it mathematically.

But now use your "Lateral Thinking".

L.N.S.

$$\int \frac{dx}{\sqrt{ax - x^2}}$$

$$\begin{aligned} & \rightarrow [dx] = [x] \\ & \rightarrow [ax] = [x^2] \end{aligned}$$

$ax - x^2$ \Rightarrow ax & x^2 must have

Same dimensions, then only
they can be added/subtracted.

$$\text{So } \left[\sqrt{ax - x^2} \right] = \left[\sqrt{x^2} \right] = [x]$$

Hence $\int \frac{dx}{\sqrt{ax - x^2}}$ is Dimensionless

So RHS should also be dimensionless

$\sin^{-1}\left(\frac{x}{a}\right)$ is angle hence it is dimensionless:

$a^n \sin^{-1}\left(\frac{x}{a}\right)$ will be dimensionless

only when

$$n = 0$$

Ans.

