ELECTROMAGNETIC INDUCTION

6.1 Introduction: We know from the study of electrostatic charges, magnetic fields and magnetism till this point, that a magnetic field is associated with the flow of charge (current) and have its peculiar properties in the space in which it exists. First of all, the great scientist, *Michael Faraday* thought that a magnetic field must also be associated with an electric current, *i.e.* an electric current may be produced with the help of an external magnetic field. He experimented for a long duration, about 20 years of his life, but got failed again and again. One day he threw the whole setup of his experimentation, when became totally fed up of the tedious experimentation. The magnet passed from the vicinity of the coil at a very high speed and he observed the deflection in the galvanometer, and accidentally he came to know that it is not the magnetic field but the continuous change in magnetic field, which will generate the electric current. So, he named this branch of physics as ***Electromagnetic Induction***. We will go through the generation of electric current with the help of electromagnetic induction in detail in this chapter.

6.2 Magnetic Flux (ϕ): *The total number of magnetic lines of force passing normally through any surface* (*cross-sectional area*) *is known as magnetic flux passing through that surface*. It may be expressed and measured as the product of the component of the magnetic field normal to the surface and the surface area. The *magnetic flux* (ϕ) is a scalar quantity and its unit is *Weber* (*Wb*).

If a uniform magnetic field ($\vec{B}$) crosses normally through a plane surface of area *A*, as shown in the Fig. 6.1 (a), then the magnetic flux through this surface may be given as:

$$\phi = B \times A \qquad (6.1)$$

If a uniform magnetic field ($\vec{B}$) makes an angle θ with the normal to the surface and crosses this plane surface of area *A*, as shown in the Fig. 6.1 (b), then the magnetic flux through this surface may be given as:

$$\phi = B \cos\theta \times A = B\,A \cos\theta = \vec{B}\,.\,\vec{A} \qquad (6.2)$$

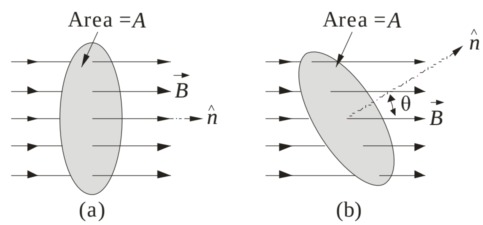

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Fig. 6.1

In general, the magnetic field ($\vec{B}$) over a surface area ($\vec{A}$) may be non uniform. However, if we consider a large number of infinitesimally small areas in that surface, the non-uniform magnetic field may be approximated as the uniform magnetic field over each incremental area ($d\,\vec{A}$), as shown in the Fig. 6.2. If angle between the magnetic field ($\vec{B}$) and the normal to the incremental area ($d\,\vec{A}$) is θ, then the component of $\vec{B}$ along $d\,\vec{A}$ will be *B* cos θ. So, the flux through the small incremental area ($d\,\vec{A}$) may be given as:

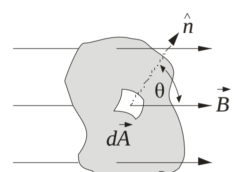


Fig. 6.2

$$d\phi = B \cos\theta \times dA = B\,dA \cos\theta = \vec{B}\,.\,d\,\vec{A} \qquad (6.3)$$

Now, flux through the complete surface area *A* may be given as:

$$\phi = \int_A \vec{B}.d\,\vec{A} \qquad (6.4)$$

Dimensions of Magnetic Flux: We know that the flux passing through a surface area may be given as:

$$\phi = B\,A = \frac{F}{q\,v \sin\theta} \times A$$

So, dimensions of flux, $\phi = \dfrac{M\,LT^{-2}}{C.LT^{-1}.1}\,.\,L^2 = [\mathrm{M\,L^2\,A^{-1}\,T^{-2}}]$ (6.5)

SI Unit of Magnetic Flux: The SI unit of magnetic flux is *Weber* (*Wb*). *The flux through a unit surface area* (*A*) *in a unit magnetic field* (*B*) *is said to be one Weber, if the flux is passing normally through the surface*.

i.e. 1 Weber = 1 Tesla × 1 meter^2 (6.6)

CGS Unit of Magnetic Flux: The CGS unit of magnetic flux is *Maxwell* (*Mx*). *The flux through a unit surface area* (*A*) *in a unit magnetic field* (*B*) *is said to be one Maxwell, if the flux is passing normally through the surface*.

i.e. 1 Maxwell = 1 Gauss × 1 cm^2

So, 1 Maxwell = $(1 \times 10^{-4}$ Tesla$) \times (1 \times 10^{-4}$ m$^2) = 10^{-8}$ Tm$^2 = 10^{-8}$ Wb

or, 1 Weber = 10^8 Maxwell

Positive and Negative Flux: If the angle between magnetic field (*B*) and the normal to the surface area (*A*) is given as $\theta = 0°$, as shown in the Fig. 6.3 (a), ***the flux is said to be positive***. On the other hand, if the angle between magnetic field (*B*) and the normal to the surface area (*A*) is given as $\theta = 180°$, as shown in the Fig. 6.3 (b), ***the flux is said to be negative***.

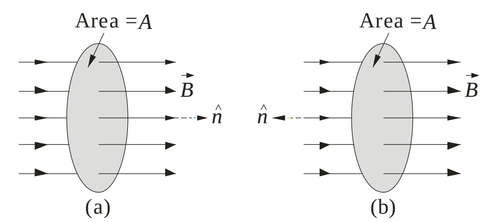


Fig. 6.3

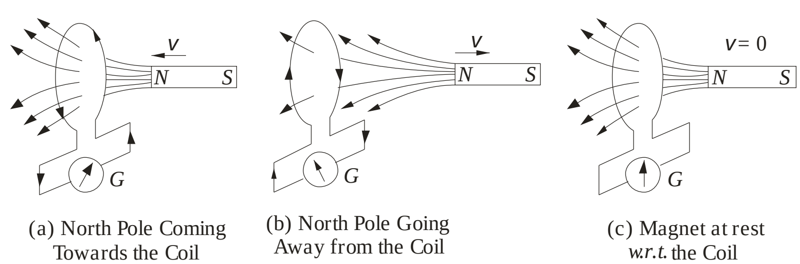
6.3 Electromagnetic Induction: The flow of electric charge is associated with a magnetic field around it. So, an electric current and the magnetism are intimately inter-related. In the early part of nineteenth century various experiments and postulates of many scientists established the fact that, moving charges produce a magnetic field around them. The converse effect is also true, *i.e.* moving magnets or more precisely changing magnetic fields can produce electric current. The great scientist *Michael Faraday* in *England* and *Joseph Henry* in U.S.A. discovered in 1831 that, currents can be produced in the conducting loops, if a magnet is suddenly moved towards the loop or away from the loop, such that the flux associated with the conducting loop changes. This induced current in the conducting loop exists as long as the flux associated with the loop is changing. This phenomenon of inducing the current with the help of changing magnetic fields is known as ***Electromagnetic Induction***.

So, "*the phenomenon of production of induced emf and induced currents due to a changing magnetic field linked with a closed circuit is known as* ***Electromagnetic Induction***".

The phenomenon of electromagnetic induction is of great practical significance, as all the electrical machines around us works on the principals of electromagnetic induction, for easy and luxurious life of human being, whether they are generators for bulk production of electricity or transformers for changing the voltage levels for distribution and transmission or electric motors fitted in the home appliances of great importance in our daily life.

6.4 Faraday's Experiments for Electromagnetic Induction: The phenomenon of electromagnetic induction may be understood with the help of following experiments performed by *Michael Faraday* and *Joseph Henry*.

EXPERIMENT NUMBER 1; Induced emf with a Stationary Coil and Moving Magnet: Take a circular coil of thick insulated copper wire of several turns connected to a sensitive galvanometer to form a closed circuit, as shown in the Fig. 6.4 (a).



(a) North Pole Coming Towards the Coil
(b) North Pole Going Away from the Coil
(c) Magnet at rest *w.r.t.* the Coil

Fig. 6.4

i) If the North-pole of a strong bar magnet is moved quickly towards the coil, as shown in the Fig. 6.4 (a), the galvanometer shows a deflection towards the right to the zero mark.

ii) If the North-pole of a strong bar magnet is moved quickly away from the coil, as shown in the Fig. 6.4 (b), the galvanometer shows a deflection towards the left to the zero mark.

iii) If the above experiment is repeated by bringing quickly the South-pole of a strong bar magnet towards the coil or away from the coil, the galvanometer shows a deflection opposite to that obtained in the case of North-pole.

iv) If the magnet is held stationary, as shown in the Fig. 6.4 (c), the galvanometer shows no deflection at all.

Explanation: When the bar magnet is placed near the coil, a number of magnetic lines of force cross through the coil. As the magnet moves quickly towards the coil, the magnetic flux (the total number of magnetic lines of force passing through the coil) linked with the coil increases at a rapid rate, as a result of which an induced emf and hence an induced current is setup in the coil in a direction, so as to oppose the cause due to which it is being induced. Now, when the magnet moves quickly away from the coil, the magnetic flux (the total number of magnetic lines of force passing through the coil) linked with the coil decreases at a rapid rate, as a result of which an induced emf and hence an induced current is setup in the coil in another direction, so as to oppose the cause due to which it is being induced. When the magnet is at rest, there is no change in the magnetic flux linked with the coil, so the emf induced and hence the current induced in the coil is zero and the galvanometer shows no deflection.

EXPERIMENT NUMBER 2; Induced emf with a Stationary Magnet and Moving Coil: Similar results, as in experiment number 1, may be obtained in an arrangement, if the magnet is held stationary while the coil is kept moving, as shown in the Fig. 6.5. If the relative motion between the magnet and the coil is at faster rate, a large deflection may be obtained in the galvanometer, on the other hand if the relative motion between the magnet and the coil is at a slower rate, a small deflection will be obtained in the galvanometer. If the relative motion between the coil and the magnet is ceased, the deflection in the galvanometer become zero, *i.e.* no induced emf and hence no induced current.

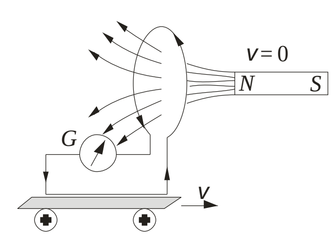


Fig. 6.5

We may conclude that, "*the faster is the relative motion between the coil and the magnet, greater is the rate of change of the magnetic flux linked with the coil and hence larger is the induced emf and induced current in the coil*".

EXPERIMENT NUMBER 3; Induced emf by Varying Current in the Neighboring Coil: Two coils, *P* and *S*, are wound on a cylindrical support in vicinity of each other. The coil *P*, known as *primary coil*, is connected to a battery (emf *E*) through a rheostat (R_h) and a tapping key (*k*). The coil *S*, known as *secondary coil*, is connected to a sensitive galvanometer, as shown in the Fig. 6.6.

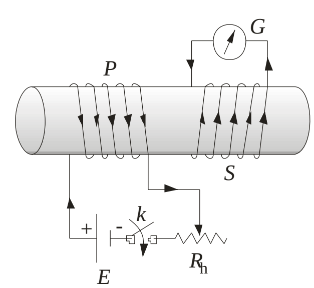


Fig. 6.6

i) If the tapping key *k* is closed from its open position, the galvanometer shows a sudden and momentary deflection in one direction, which disappears quickly. Now, if the tapping key is released from its close position, the galvanometer again shows a momentary deflection in opposite direction to earlier, which also disappears quickly.

ii) If the tapping key is kept pressed continuously and a steady current flows through the primary coil *P*, the galvanometer does not show any deflection.

iii) If the current through the primary coil *P* is increased with the help of the rheostat (R_h), the induced current flows in the secondary coil *S* in the same direction as that at the *time of make of the primary circuit*.

iv) If the current through the primary coil *P* is decreased with the help of the rheostat (R_h), the induced current flows in the secondary coil *S* in the same direction as that at the *time of break of the primary circuit*.

v) The deflection in the galvanometer becomes larger if the cylindrical support of iron (a ferromagnetic material) is used in this setup.

Explanation: We know that the current flowing through a coil produces a magnetic field, direction of which may be given by *Right Hand Thumb rule*.

i) When the circuit is just closed from the open position with the help of tapping key (*k*), the current rises from zero to some steady state value, during which the flux produced by the coil also changes from zero to some steady state value. This changing flux is also associated with the secondary coil in the vicinity which induces an emf in it, so the galvanometer shows a momentary deflection and dies as soon as the current become steady in the circuit.

ii) When the circuit is made open from the closed position with the help of tapping key (*k*), the current decays from its steady state value to zero, during which the flux produced by the coil also changes from its steady state value to zero value. This changing flux is also associated with the secondary coil in the vicinity which induces and emf in it but reversed polarity, so the galvanometer shows a momentary deflection but in opposite direction to earlier and dies as soon as the current reaches zero in the circuit.

iii) When the current is increased in the circuit with the help of rheostat (R_h), the flux produced by the coil increases. This increasing flux causes the induced emf and induced current in the circuit and galvanometer shows a deflection in the same direction as that at the *time of make of the primary circuit*.

iv) When the current is decreased in the circuit with the help of rheostat (R_h), the flux produced by the coil reduces. This reducing flux causes the induced emf and induced current in the circuit and galvanometer shows a deflection in the same direction as that at the *time of break of the primary circuit*.

v) If an iron core is used as the support structure, the flux produced by the same current in the coil increases several thousand times due to relative permeability of iron, and hence the change in flux also becomes large on variation of current through the primary coil. So, the induced emf and induced current also become large in the secondary winding resulting in larger deflection in the galvanometer.

We may easily conclude from the above experiments that:

i) *Whenever the magnetic flux linked* (*flux linkages*) *with a closed circuit change, an induced emf and hence an induced current is setup in the closed circuit*.

ii) *The higher the rate of change of magnetic flux linked* (*flux linkages*) *with the closed circuit, the greater is the induced emf and induced current in the closed circuit*.

6.5 **Laws of Electromagnetic Induction:** *Michael Faraday* gave the basic law for *electromagnetic induction* which gives the magnitude of induced emf, and Lenz's law gives the direction of induced emf.

Faraday's Law of Electromagnetic Induction and ***Lenz's Law***, collectively, are known as ***Laws of Electromagnetic Induction***.

***a*) Faraday's Law of Electromagnetic Induction:**

First Law: *Whenever the magnetic flux linked* (*flux linkages*) *with a closed circuit change, an emf* (*and hence a current*) *is induced in the closed circuit, which lasts as long as the change in flux is there. This phenomenon is known as electromagnetic induction*.

Second Law: *The magnitude of induced emf in the circuit is directly proportional to the rate of change of the flux linked with the circuit.*

i.e. $$e \propto \frac{d\phi}{dt} \quad (6.7)$$

If there are *N* number of turns in the coil, the magnitude of the induced emf may be given as:

$$E \propto N \times \frac{d\phi}{dt} \quad (6.8)$$

***b*) Lenz's Law:** This law states that, "*the direction of induced current is in such a way, that it always opposes the cause due to which it is being induced*". So, *the tendency of the induced current is to oppose the rate of change of magnetic flux linked with the coil* (*flux linkages of the coil*).

So, $$E = -N \times \frac{d\phi}{dt} = -\frac{d(N\phi)}{dt} = -\frac{d\lambda}{dt} \quad (6.9)$$

Where, (–) ve sign shows that, *the electromagnetic quantity induced opposes the cause due to which it is being induced*. **The term $\lambda = N\phi$, is known as *flux linkages of the coil* having *N* Turns**.

If magnetic flux is in *Weber* and the time is in *Seconds*, the induced emf is in *Volts*.

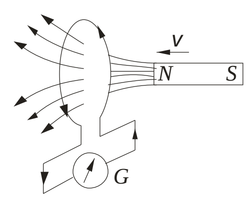
If the magnetic flux linked with a coil changes from its initial value ϕ_1 to a final value ϕ_2 in time *t*, the average induced emf in the coil may be given as:

$$E = -N \times \frac{(\phi_2 - \phi_1)}{t} \quad (6.9)$$

Where, (–) ve sign shows that, *the electromagnetic quantity induced opposes the cause due to which it is being induced*.

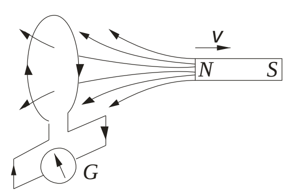
6.6 Explanation of Lenz's Law: *Lenz's law* is another form of *law of conservation of energy, i.e. energy can neither be created nor destroyed, although it may be converted from one form to another*.

i) Now consider the setup, shown in the Fig. 6.7, for electromechanical energy conversion, where a north pole is coming towards a coil and the mechanical energy of the magnet is being converted into the electrical energy and is being dissipated in the coil and the galvanometer. When magnet moves towards the coil, the flux linked with the coil goes on increasing and increasing continuously, so the current induced in the coil may produce an equal amount of the flux in the opposite direction, so as to oppose this increasing flux. This may happen only when the induced current is in counter-clockwise direction in the coil, as shown in the figure. The reader may verify it himself, with the help of *Right Hand Thumb Rule*, that the face of the coil facing the magnet becomes a *North pole*. So, the magnet and the coil are repelling each other and we have to do some extra work to bring the magnet near to the coil. This work done on the magnet, to provide mechanical energy to the magnet, is being converted into the electrical energy due to this repulsion / opposition. And, the electrical energy so produced is being dissipated in the coil and the galvanometer, *and hence the law of conservation of energy holds good*.



North Pole Coming Towards the Coil
Fig. 6.7

ii) Now consider the setup, shown in the Fig. 6.8, for electromechanical energy conversion, where a north pole is going away from a coil and the mechanical energy of the magnet is being converted into the electrical energy and is being dissipated in the coil and the galvanometer. When magnet moves away from the coil, the flux linked with the coil goes on decreasing and decreasing continuously, so the current induced in the coil must produce an equal amount of the flux in the opposite direction so as to oppose this decreasing flux. This may happen only when the induced current is in clockwise direction in the



North Pole Going Away from the Coil
Fig. 6.8

coil, as shown in the figure. The reader may verify it himself, with the help of *Right Hand Thumb Rule*, that the face of the coil facing the magnet becomes a *South pole*. So, the magnet and the coil are attracting each other and we have to do some extra work to move the magnet away from the coil. This work done on the magnet, to provide mechanical energy to the magnet, is being converted into the electrical energy due to this repulsion / opposition. And, the electrical energy so produced is being dissipated in the coil and the galvanometer, *and hence the law of conservation of energy holds good*.

So, *we may conclude here that the* ***Lenz's Law is nothing but a consequence of Law of Conservation of Energy***.

Problem 6.1: *A rectangular loop of area* 20 *cm* × 30 *cm is placed in a magnetic field of strength* 0.3 *T with its plane: i) normal to the magnetic field, ii) inclined* 30° *to the magnetic field, iii) parallel to the magnetic field. Determine the value of flux linked with the coil in each case.*

Solution: $A = 20 \times 30\ \text{cm}^2$, $B = 0.3$ T, $\theta_1 = 0°$, $\theta_2 = (90° - 30°) = 60°$, $\theta_3 = 90°$

The flux linked with the coil in first case, when the plane of the coil is normal to magnetic field *i.e.* the angle between the magnetic field and normal to the plane of coil is $\theta_1 = 0°$, may be given as:

$$\phi_1 = B\,A \cos\theta_1 = 0.3 \times 0.20 \times 0.30 \times \cos 0° = 0.018 \text{ Wb} = 18 \text{ mWb}$$

The flux linked with the coil in second case, when the plane of the coil is inclined at an angle of 30° to magnetic field *i.e.* the angle between the magnetic field and the normal to the plane of coil is $\theta_2 = (90° - 30°) = 60°$, may be given as:

$$\phi_2 = B\,A \cos\theta_2 = 0.3 \times 0.20 \times 0.30 \times \cos 60° = 0.018 \times \frac{1}{2} = 0.009 \text{ Wb} = 9 \text{ mWb}$$

The flux linked with the coil in third case, when the plane of coil is parallel to magnetic field *i.e.* the angle between the magnetic field and the normal to the plane of coil is $\theta_3 = 90°$, may be given as:

$$\phi_3 = B\,A \cos\theta_3 = 0.3 \times 0.20 \times 0.30 \times \cos 90° = 0 \text{ (Zero)}$$

Problem 6.2: *A small piece of metal conductor is dragged across the gap between the pole pieces of a magnet in* 0.5 *sec. The magnetic flux between the two pole pieces is* 0.8 *mWb. Determine the average induced emf in the conductor.* [NCERT]

Solution: $dt = 0.5$ sec, $d\phi = 0.8$ mWb

The average induced emf in the conductor may be given as:

$$E = -N \times \frac{d\phi}{dt} = -1 \times \frac{0.8 \times 10^{-3}}{0.5} = -1.6 \text{ mV}$$

Where (–)ve sign indicates the opposition of the cause due to which the emf is being induced.

Problem 6.3: *The magnetic flux through a coil perpendicular to the plane is varying according to the relation:* $\phi = (5\,t^3 + 4\,t^2 + 2\,t - 5)$ *Wb.*

Determine the induced current through the coil at t = 2 *sec, if the resistance of the coil is* 5 Ω. [Punjab 1997-98]

Solution: $\phi = (5\,t^3 + 4\,t^2 + 2\,t - 5)$ Wb, $t = 2$ sec, $R = 5\ \Omega$

The induced emf in the coil may be given as:

$$E = -N \times \frac{d\phi}{dt} = -1 \times \frac{d}{dt}\,(5\,t^3 + 4\,t^2 + 2\,t - 5)$$

The induced current in the coil may be given as:

$$I = \frac{E}{R} = -\frac{d}{dt}(5t^3 + 4t^2 + 2t - 5) \times \frac{1}{5} = -(15t^2 + 8t + 2) \times \frac{1}{5}$$

So, the induced current in the coil at $t = 2$ sec may be given as:

$$I_{(t = 2\,\mathrm{sec})} = -(15t^2 + 8t + 2) \times \frac{1}{5} = -[15 \times (2)^2 + 8 \times (2) + 2] \times \frac{1}{5} = -15.6\ \mathrm{A}$$

Where (–)ve sign indicates the opposition of the cause due to which the current is being induced.

Problem 6.4: *A square loop of side* 10 *cm and a resistance of* 0.7 Ω *is placed vertically in the east-west plane. A uniform magnetic field of* 0.1 *T is setup across the plane in the north-east direction. The magnetic field is decreased to zero in* 0.7 *sec at a steady rate. Determine the magnitudes of induced emf and induced current during this time interval.* [NCERT]

Solution: $a = 10$ cm (vertically in east-west plane), $R = 0.7\ \Omega$,

$B_1 = 0.1$ T (North-East direction), $B_2 = 0$ T, $dt = 0.7$ sec

Since the square loop is in east-west plane and the uniform magnetic field is setup across the plane in north-east direction, so the angle between the magnetic field and the normal to the loop may be given as: $\theta = 45°$

and, hence the flux linked initially with the square loop may be given as:

$$\phi_1 = B\,A \cos\theta = 0.1 \times (0.1 \times 0.1) \times \cos 45° = 0.707\ \mathrm{mWb}$$

Now, the magnitude of the induced emf may be given as:

$$E = -N \times \frac{d\phi}{dt} = -N \times \frac{(\phi_2 - \phi_1)}{dt} = -1 \times \frac{(0 - 0.707 \times 10^{-3})}{0.7} = 1.01 \times 10^{-3}\ \mathrm{V} = 1.01\ \mathrm{mV}$$

So, the magnitude of the induced current may be given as:

$$I = \frac{E}{R} = \frac{1.01 \times 10^{-3}}{0.7} = 1.443 \times 10^{-3}\ \mathrm{A} = 1.443\ \mathrm{mA}$$

Problem 6.5: *A coil, with* 10 Ω *resistance, has* 1000 *Turns and a flux of* 0.55 *mWb is crossing it at a certain point of time. If the flux decreases to* 0.05 *mWb in a duration of* 0.1 *sec, determine the induced emf and the total amount of charge that has flown through the coil in this time.*

Solution: $R = 10\ \Omega$, $N = 1000$ Turns, $\phi_1 = 0.55$ mWb, $\phi_2 = 0.05$ mWb, $dt = 0.1$ sec

The induced emf in the coil may be given as:

$$E = -N \times \frac{d\phi}{dt} = -N \times \frac{(\phi_2 - \phi_1)}{dt} = -1000 \times \frac{(0.05 - 0.55) \times 10^{-3}}{0.1} = 5\ \mathrm{V}$$

The charge flown through the coil in this duration may be given as:

$$q = I \times dt = \frac{E}{R} \times dt = \frac{5}{10} \times 0.1 = 0.05\ \mathrm{C}$$

Problem 6.6: *A coil, with an average diameter of* 0.02 *m, is placed perpendicular to a uniform magnetic field of* 6000 *T. If the induced emf is* 11 *V, when the magnetic field is changed to* 1000 *T in* 4 *sec, determine the number of turns in the coil.* [CBSE 1993-94]

Solution: $d = 0.02$ m, $\theta = 0°$, $B_1 = 6000$ T, $E = 11$ V, $B_2 = 1000$ T, $dt = 4$ sec

The initial and final flux passing through the coil may respectively be given as:

$$\phi_1 = B_1 A \cos\theta \qquad \text{and,} \qquad \phi_2 = B_2 A \cos\theta$$

The expression for the induced emf in the N-turn coil may be given as:

$$E = -N \times \frac{d\phi}{dt} = -N \times \frac{(\phi_2 - \phi_1)}{dt}$$

$$= -N \times \frac{(B_2 A\cos\theta - B_1 A\cos\theta)}{dt} = -N \times \frac{(B_2 - B_1) \times A\cos\theta}{dt}$$

or, $$N = \frac{E \times dt}{(B_2 - B_1) \times A\cos\theta} = \frac{E \times dt}{(B_2 - B_1) \times (\pi d^2/4) \times \cos\theta}$$

$$= \frac{11 \times 4 \times 4}{(6000 - 1000) \times \pi \times (0.02)^2 \cos 0°} = 28.011 \approx 28 \text{ Turns}$$

Problem 6.7: *An* 88 *cm long wire is bent into a circular loop and is placed perpendicular to the magnetic field of flux density* 2.5 *T. The loop is changed into a square of side* 22 *cm within a time period of* 0.5 *sec and simultaneously the flux density is increased to* 3 *T. Determine the value of induced emf.* [Himachal 1989-90]

Solution: l = 88 cm (circle), $\theta = 0°$, B_1 = 2.5 T, a = 22 cm (square), dt = 0.5 sec, B_2 = 3 T

Area of the circular loop and square loop may respectively be given as:

$$A_{circle} = \pi r^2 = \pi \times \left(\frac{l}{2\pi}\right)^2 = \pi \times \left(\frac{0.88}{2\pi}\right)^2 = 0.0616 \text{ m}^2$$

and, $A_{square} = 0.22 \times 0.22 = 0.0484 \text{ m}^2$

The initial and final flux passing through the loop may respectively be given as:

$$\phi_1 = B_1 A_1 \cos\theta = 2.5 \times 0.0616 \times \cos 0° = 0.154 \text{ Wb}$$

and, $\phi_2 = B_2 A_2 \cos\theta = 3 \times 0.0484 \times \cos 0° = 0.1452 \text{ Wb}$

So, the induced emf in the loop may be given as:

$$E = -N \times \frac{d\phi}{dt} = -N \times \frac{(\phi_2 - \phi_1)}{dt} = -1 \times \frac{(0.1452 - 0.154)}{0.5} = 0.0176 \text{ V} = 17.6 \text{ mV}$$

Problem 6.8: *A coil of mean area* 500 *cm*^{2} *and having* 1000 *Turns is held perpendicular to a uniform magnetic field of* 0.4 *Gauss. The coil is turned through* 180° *in* 0.1 *sec. Determine the average induced emf in the coil.* [MNREC 1987]

Solution: A = 500 cm^2, N = 1000, $\theta_1 = 0°$, B = 0.4 G, $\theta_2 = 180°$, dt = 0.1 sec

The initial and final flux linkages of the coil may respectively be given as:

$$\lambda_1 = N\phi_1 = N B A \cos\theta_1 = 1000 \times 0.4 \times 10^{-4} \times 500 \times 10^{-4} \times \cos 0° = 2 \times 10^{-3} \text{ Wb}$$

and, $\lambda_2 = N\phi_2 = N B A \cos\theta_2 = 1000 \times 0.4 \times 10^{-4} \times 500 \times 10^{-4} \times \cos 180° = -2 \times 10^{-3} \text{ Wb}$

The average induced emf in the coil may be given as:

$$E = -\frac{d\lambda}{dt} = -\frac{(\lambda_2 - \lambda_1)}{dt} = -\frac{(-2-2) \times 10^{-3}}{0.1} = 0.04 \text{ V} = 40 \text{ mV}$$

Problem 6.9: *A circular coil of radius* 10 *cm,* 500 *Turns and resistance of* 2 Ω *is placed with its plane perpendicular to the horizontal component of the earth's magnetic field. It is rotated about its* *vertical diameter through* 180° *in* 0.25 *sec. Determine the magnitude of average induced emf and average induced current in the coil. Horizontal component of the earth's magnetic field at the place is given as* 3×10^{-5} *T.* [NCERT]

Solution: $r = 10$ cm, $N = 500$ Turns, $R = 2\ \Omega$, $\theta_1 = 0°$, $\theta_2 = 180°$,

$dt = 0.25$ sec $B_H = 3 \times 10^{-5}$ T

The initial and final flux linkages of the coil may respectively be given as:

$$\lambda_1 = N\,\phi_1 = N\,B\,A \cos\theta_1 = 500 \times 3 \times 10^{-5} \times \pi \times (0.10)^2 \times \cos 0° = 4.712 \times 10^{-4}\ \text{Wb}$$

and, $$\lambda_2 = N\,\phi_2 = N\,B\,A \cos\theta_2 = 500 \times 3 \times 10^{-5} \times \pi \times (0.10)^2 \times \cos 180° = -4.712 \times 10^{-4}\ \text{Wb}$$

The magnitude of average induced emf in the coil may be given as:

$$E = -\frac{d\lambda}{dt} = -\frac{(\lambda_2 - \lambda_1)}{dt} = -\frac{(-4.712 - 4.712)\times 10^{-4}}{0.25} = 3.77\ \text{mV}$$

The magnitude of average induced current in the coil may be given as:

$$I = \frac{E}{R} = \frac{3.77}{2} = 1.885\ \text{mA}$$

Problem 6.10: *A coil of cross sectional area A lies in a uniform magnetic field B with its plane perpendicular to the magnetic field. The normal to the coil makes an angle of* 0° *with the magnetic field. The coil rotates at uniform rate to complete one rotation in time T. Determine the average induced emf in the coil during the interval when the coil rotates: i) from* 0° *to* 90°, *ii) from* 90° *to* 180°, *iii) from* 180° *to* 270°, *iv) from* 270° *to* 360°. *Also discuss about the direction of current in two halves of the rotation of the coil.*

Solution: $\theta_1 = 0°$, $\theta_2 = 90°$, $\theta_3 = 180°$, $\theta_4 = 270°$, $\theta_1 = 360° = 0°$

For rotation of the coil from 0° to 90°: $dt = \frac{T}{4}$

The initial and final flux passing through the coil may respectively be given as:

$$\phi_1 = B\,A \cos\theta_1 = B\,A \cos 0° = B\,A$$

and, $$\phi_2 = B\,A \cos\theta_2 = B\,A \cos 90° = 0$$

The magnitude of average induced emf in the coil may be given as:

$$E = -N \times \frac{d\phi}{dt} = -N \times \frac{(\phi_2 - \phi_1)}{dt} = -1 \times \frac{(0 - B\,A)}{(T/4)} = \frac{4\,B\,A}{T}$$

For rotation of the coil from 90° to 180°: $dt = \frac{T}{4}$

The initial and final flux passing through the coil may respectively be given as:

$$\phi_1 = B\,A \cos\theta_1 = B\,A \cos 90° = 0$$

and, $$\phi_2 = B\,A \cos\theta_2 = B\,A \cos 180° = -B\,A$$

The magnitude of average induced emf in the coil may be given as:

$$E = -N \times \frac{d\phi}{dt} = -N \times \frac{(\phi_2 - \phi_1)}{dt} = -1 \times \frac{(-B\,A - 0)}{(T/4)} = \frac{4\,B\,A}{T}$$

For rotation of the coil from 180° to 270°: $dt = \dfrac{T}{4}$

The initial and final flux passing through the coil may respectively be given as:

$$\phi_1 = B\,A \cos\theta_1 = B\,A \cos 180° = -\,B\,A$$

and, $$\phi_2 = B\,A \cos\theta_2 = B\,A \cos 270° = 0$$

The magnitude of average induced emf in the coil may be given as:

$$E = -\,N \times \frac{d\phi}{dt} = -\,N \times \frac{(\phi_2 - \phi_1)}{dt} = -\,1 \times \frac{[0-(-B\,A)]}{(T/4)} = -\,\frac{4\,B\,A}{T}$$

For rotation of the coil from 270° to 360°: $dt = \dfrac{T}{4}$

The initial and final flux passing through the coil may respectively be given as:

$$\phi_1 = B\,A \cos\theta_1 = B\,A \cos 270° = 0$$

and, $$\phi_2 = B\,A \cos\theta_2 = B\,A \cos 360° = B\,A$$

The magnitude of average induced emf in the coil may be given as:

$$E = -\,N \times \frac{d\phi}{dt} = -\,N \times \frac{(\phi_2 - \phi_1)}{dt} = -\,1 \times \frac{(B\,A-0)}{(T/4)} = -\,\frac{4\,B\,A}{T}$$

Since the induced emf is negative in second half of the rotation, so the current will reverse its direction after first half of the rotation and the induced emf and induced current in the coil is alternating in nature.

Problem 6.11: *A conducting circular loop is placed in a uniform transverse magnetic field of* 0.02 *T. The radius of the loop begins to decrease, somehow, at a constant rate of* 1 *mm/sec. Determine the induced emf in the loop at the instant when the radius is* 2 *cm.*

Solution: $\theta = 0°$, $B = 0.02$ T, $\dfrac{dr}{dt} = -\,1$ mm/sec, $r = 2$ cm

The magnetic flux linked with the loop of radius r may be given as:

$$\phi = B\,A \cos\theta = B \times \pi\, r^2 \times \cos 0° = \pi\, r^2\, B$$

So, the induced emf may be given as:

$$E = -\,N \times \frac{d\phi}{dt} = -\,1 \times \frac{d}{dt}\,(\pi\, r^2\, B) = -\,2\,\pi\, r\, B \times \frac{dr}{dt}$$

$$= -\,2\,\pi \times 0.02 \times 0.02 \times (-\,0.001) = 2.513 \times 10^{-6}\ \text{V} = 2.513\ \mu\text{V}$$

Problem 6.12: *Determine the magnetic flux linked with a rectangular coil of size* 6 *cm* × 8 *cm placed at right angle to a magnetic field of* 0.5 *Wbm*$^{-2}$.

Solution: $A = 6 \times 8$ cm^2, $\theta = 0°$, $B = 0.5$ T

The magnetic flux linked with the rectangular coil may be given as:

$$\phi = B\,A \cos\theta = 0.5 \times 6 \times 8 \times 10^{-4} \times \cos 0° = 2.4 \times 10^{-3}\ \text{Wb} = 2.4\ \text{mWb}$$

Problem 6.13: *A square coil, of side* 20 *cm and having* 600 *Turns, is placed with its plane inclined at* 30° *to a uniform magnetic field of* 4.5×10^{-4} *T. Determine the flux linkages of the coil.*

Solution: $a = 20$ cm, $N = 600$ Turns, $\theta = (90° - 30°) = 60°$, $B = 4.5 \times 10^{-4}$ T

The flux linkages of the coil may be given as:

$$\lambda = N \times \phi = N \times B\,A \cos\theta = 600 \times 4.5 \times 10^{-4} \times (0.20)^2 \times \cos 60°$$

$$= 5.4 \times 10^{-3} \text{ Wb} = 5.4 \text{ mWb}$$

Problem 6.14: *The magnetic flux threading a coil changes from* 12 *mWb to* 6 *mWb in* 0.01 *sec. Determine the induced emf in the coil.* [CBSE 1989-90]

Solution: $\phi_1 = 12$ mWb, $\phi_2 = 6$ mWb, $dt = 0.01$ sec

The magnitude of average induced emf in the coil may be given as:

$$E = -N \times \frac{d\phi}{dt} = -N \times \frac{(\phi_2 - \phi_1)}{dt} = -1 \times \frac{(6-12)\times 10^{-3}}{0.01} = 0.6 \text{ V}$$

Problem 6.15: *A magnetic field of flux density* 1 *Tesla acts normal to an* 80 *Turns coil of area* 0.01 *m*^{2}*. Determine the induced emf in the coil, if this coil is removed from the magnetic filed in* 0.1 *sec.* [Haryana 2001-02]

Solution: $B_1 = 1$ T, $\theta = 0°$, $N = 80$ Turns, $A = 0.01$ m^2, $dt = 0.\ 1$ sec, $B_2 = 0$

The initial and final flux linkages of the coil may respectively be given as:

$$\lambda_1 = N\,\phi_1 = N\,B_1\,A \cos\theta = 80 \times 1 \times 0.01 \times \cos 0° = 0.8 \text{ Wb}$$

$$\lambda_2 = N\,\phi_2 = N\,B_2\,A \cos\theta = N \times 0 \times A \cos\theta = 0$$

The induced emf in the coil may be given as:

$$E = -\frac{d\lambda}{dt} = -\frac{(\lambda_2 - \lambda_1)}{dt} = -\frac{(0-0.8)}{0.1} = 8 \text{ V}$$

Problem 6.16: *A* 70 *Turns coil with average diameter of* 0.02 *m is placed perpendicular to a magnetic field of* 9000 *T. If the magnetic field is changed to* 6000 *T in* 3 *sec, determine the magnitude of induced emf in the coil.* [CBSE 1993-94]

Solution: $N = 70$ Turns, $d = 0.02$ m, $\theta = 0°$, $B_1 = 9000$ T, $B_2 = 6000$ T, $dt = 3$ sec

The induced emf in the coil may be given as:

$$E = -N \times \frac{d\phi}{dt} = -N \times \frac{(\phi_2 - \phi_1)}{dt} = -N \times \frac{(B_2\,A\cos\theta - B_1\,A\cos\theta)}{dt}$$

$$= -N \times \frac{(B_2 - B_1)\times[\pi(d)^2/4]\times\cos\theta}{dt}$$

$$= -70 \times \frac{(6000-9000)\times\pi\times(0.02)^2\times\cos 0°}{3\times 4} = 21.99 \text{ V}$$

Problem 6.17: *A magnetic field of flux density* 10 *T acts normal to a* 50 *Turns coil of* 100 *cm*^{2} *area. Determine the induced emf in the coil, if the coil is removed from the field in* $\frac{1}{20}$ *sec.*

Solution: $B_1 = 10$ T, $\theta = 0°$, $N = 50$ Turns, $A = 100$ cm^2, $dt = \frac{1}{20}$ sec, $B_2 = 0$ T

The induced emf in the coil may be given as:

$$E = -N \times \frac{d\phi}{dt} = -N \times \frac{(\phi_2 - \phi_1)}{dt} = -N \times \frac{(B_2 A \cos\theta - B_1 A \cos\theta)}{dt}$$

or, $$E = -N \times \frac{(B_2 - B_1) \times A \cos\theta}{dt}$$

$$= -50 \times \frac{(0 - 10) \times 100 \times 10^{-4} \times \cos 0°}{(1/20)} = 100 \text{ V}$$

Problem 6.18: *A coil has* 400 *Turns and an area of* 500 *cm*^{2}*. It is placed at right angles to a magnetic field of density* 5×10^{-5} *Tesla. The coil is rotated through* 180° *in* 0.2 *sec. Determine the average induced emf in the coil.*

Solution: $N = 400$ Turns, $A = 500$ cm^2, $\theta_1 = 0°$, $B = 5 \times 10^{-5}$ T, $\theta_2 = 180°$, $dt = 0.2$ sec

The average induced emf in the coil may be given as:

$$E = -N \times \frac{d\phi}{dt} = -N \times \frac{(\phi_2 - \phi_1)}{dt} = -N \times \frac{(B A \cos\theta_2 - B A \cos\theta_1)}{dt}$$

$$= -N \times \frac{(\cos\theta_2 - \cos\theta_1) \times B A}{dt} = -400 \times \frac{(\cos 180° - \cos 0°) \times 5 \times 10^{-5} \times 500 \times 10^{-4}}{0.2}$$

$$= 0.01 \text{ V} = 10 \text{ mV}$$

Problem 6.19: *A coil, of area* 0.04 *m*^{2} *having* 1000 *Turns is suspended perpendicular to a magnetic field of density* 5×10^{-5} *T. It is rotated through* 90° *in* 0.2 *sec. Determine the average induced emf in the coil.*

Solution: $A = 0.04$ m^2, $N = 1000$ Turns, $\theta_1 = 0°$, $B = 5 \times 10^{-5}$ T, $\theta_2 = 90°$, $dt = 0.2$ sec

The average induced emf in the coil may be given as:

$$E = -N \times \frac{d\phi}{dt} = -N \times \frac{(\phi_2 - \phi_1)}{dt} = -N \times \frac{(B A \cos\theta_2 - B A \cos\theta_1)}{dt}$$

$$= -N \times \frac{(\cos\theta_2 - \cos\theta_1) \times B A}{dt} = -1000 \times \frac{(\cos 90° - \cos 0°) \times 5 \times 10^{-5} \times 0.04}{0.2}$$

$$= 0.01 \text{ V} = 10 \text{ mV}$$

Problem 6.20: *A* 40 *cm long wire is bent into a rectangular loop* 15 *cm* × 5 *cm and placed perpendicular to a magnetic field of flux density* 0.8 *T. The loop is changed into a square of side* 10 *cm within* 1 *sec and simultaneously the flux density is increased to* 1.4 *T. Determine the value of induced emf in the coil.*

Solution: $l = 40$ cm, $A_1 = 15 \times 5$ cm^2, $\theta = 0°$, $B_1 = 0.8$ T, $A_2 = 10 \times 10$ cm^2, $dt = 1$ sec

$B_2 = 1.4$ T

The initial and final flux passing through the loop may respectively be given as:

$$\phi_1 = B_1 A_1 \cos\theta = 0.8 \times 15 \times 5 \times 10^{-4} \times \cos 0° = 0.006 \text{ Wb}$$

and, $$\phi_2 = B_2 A_2 \cos\theta = 1.4 \times 10 \times 10 \times 10^{-4} \times \cos 0° = 0.014 \text{ Wb}$$

So, the induced emf in the loop may be given as:

$$E = -N \times \frac{d\phi}{dt} = -N \times \frac{(\phi_2 - \phi_1)}{dt} = -1 \times \frac{(0.014 - 0.006)}{1} = -0.008 \text{ V} = -8 \text{ mV}$$

Where (–)ve sign indicates the opposition of the cause due to which the emf is being induced.

Problem 6.21: *An air cored solenoid of length* 50 *cm and area of cross section* 28 cm^2 *has* 200 *Turns and carries a current of* 5 *A. The current decreases to zero within a time interval of* 1 *milli sec, when the current is switched OFF. Determine the average emf induced across the ends of the open switch in the circuit.*

Solution: l = 50 cm, A = 28 cm^2, N = 200 Turns, I_1 = 5 A, dt = 1 m-sec, I_2 = 0

The magnetic field created by the solenoid may be given as: $B = \mu_0 \, n \, I = \frac{\mu_0 N I}{l}$

The initial and final flux linkages of the solenoid may be given as:

$$\lambda_1 = N \times \phi = N \times B\,A = N \times \frac{\mu_0 N I_1}{l} \times A = 200 \times \frac{4\pi \times 10^{-7} \times 200 \times 5}{0.50} \times 28 \times 10^{-4}$$

$$= 1.407 \times 10^{-3} \text{ Wb-Turns}$$

$$\lambda_2 = N \times \phi = N \times B\,A = N \times \frac{\mu_0 N I_2}{l} \times A = 0$$

So, the induced emf across the open switch may be given as:

$$E = -\frac{d\lambda}{dt} = -\frac{(\lambda_2 - \lambda_1)}{dt} = -\frac{(0 - 1.407 \times 10^{-3})}{1 \times 10^{-3}} = 1.407 \text{ V}$$

Problem 6.22: *A closed coil consists of* 500 *Turns on a rectangular frame of area* 4 cm^2 *and has a resistance of* 50 Ω. *It is kept with its plane perpendicular to a uniform magnetic field of* 0.2 *Tesla. Determine the amount of charge flowing through the coil when it is turned over* (*rotated through* 180°). *Will the induced emf depend on the speed at which the coil is rotated?*

Solution: N = 500 Turns, A = 4 cm^2, R = 50 Ω, θ_1 = 0°, B = 0.2 T, θ_2 = 180°

The initial and final flux linkages of the coil may be given as:

$$\lambda_1 = N\,\phi = N\,B\,A \cos\theta_1 = N\,B\,A \cos 0° = N\,B\,A$$

and, $$\lambda_2 = N\,\phi = N\,B\,A \cos\theta_2 = N\,B\,A \cos 180° = -N\,B\,A$$

The induced emf in the coil may be given as:

$$E = -\frac{d\lambda}{dt} = -\frac{(\lambda_2 - \lambda_1)}{dt} = -\frac{(-N\,B\,A - N\,B\,A)}{dt} = \frac{2\,N\,B\,A}{dt}$$

The amount of charge flowing through the coil may be given as:

$$q = I \times dt = \frac{E}{R} \times dt = \frac{2\,N\,B\,A}{dt} \times \frac{1}{R} \times dt = \frac{2\,N\,B\,A}{R} \qquad (6.10)$$

$$= \frac{2 \times 500 \times 0.2 \times 4 \times 10^{-4}}{50} = 1.6 \times 10^{-3} \text{ C}$$

The reader may observe from equation (6.10) that the amount of charge flowing through the coil is independent of the time taken in the rotation, so it does not depend on the speed of rotation of the coil.

Problem 6.23: *The magnetic flux through a coil perpendicular to its plane and directed into the plane of the paper is varying according to the relation* $\phi = (5\,t^2 + 10\,t + 5)$ *milli-Wb. Determine the emf induced in the coil at* $t = 5$ *sec.*

Solution: $\phi = (5\,t^2 + 10\,t + 5)$ m-Wb, $t = 5$ sec

The induced emf in the coil may be given as:

$$E = -N \times \frac{d\phi}{dt} = -1 \times \frac{d}{dt}(5\,t^2 + 10\,t + 5) \times 10^{-3} = -(10\,t + 10) \times 10^{-3}$$

$$= -(10 \times 5 + 10) \times 10^{-3} = -60 \times 10^{-3}\ \text{V} = -60\ \text{mV}$$

Where (–)ve sign indicates the opposition of the cause due to which the emf is being induced.

6.7 Dynamic (Motional) EMF from Faraday's Law: *The emf induced across the ends of a conductor due to its motion (dynamics) in a magnetic field is known as* ***Dynamic / Motional Emf****.*

Consider a conductor *PQ* of length *l* free to move on *U* shaped conducting rails situated in a uniform and time dependent magnetic field *B*, directed normally into the plane of paper. The conductor *PQ* is moved towards left with a speed v, so that the area of the loop *PQRS* continuously decreases. This results in the decrement of the magnetic flux linked with the closed loop, and hence an emf is being induced in the loop *PQRS*, due to which an induced current flows in the loop *PQRS*. The direction of the induced current may be determined using ***Fleming's Right hand Rule***.

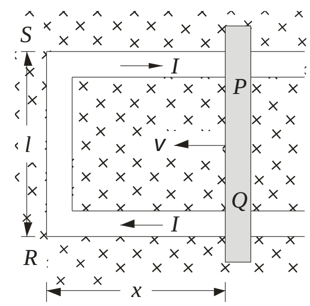


Fig. 6.9

Let us assume that at a certain instant (*t*) the length of the loop (*PS*) inside the magnetic field is *x*. So, the magnetic flux linked with the loop *PQRS* at this instant of time may be given as:

$$\phi = B\,A = B \times l\,x$$

The induced emf in the loop, according to *Faraday's law of electromagnetic induction*, may be given as:

$$E = -\frac{d\phi}{dt} = -\frac{d}{dt}(B\,l\,x) = -B\,l\,\frac{dx}{dt} = B\,l\,v \qquad (6.11)$$

Where, $\frac{dx}{dt} = -v$, (–)ve sign indicates that the velocity v is in a direction tending to decrease the area of the loop inside the magnetic field. *The induced emf* ($B\,l\,v$) *is known as dynamic / motional emf* because *this emf is induced due to the motion of a conductor inside the magnetic field*.

Fleming's Right Hand Rule: This rule gives the direction of induced emf and hence direction of induced current in a conductor, moving perpendicular to a magnetic field inside the magnetic field.

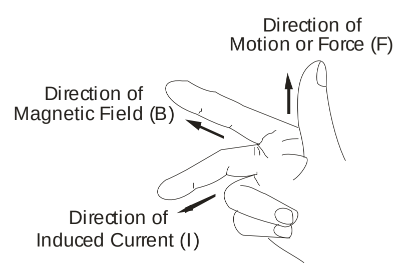


Fig. 6.10

If a conductor is moving in the perpendicular direction to that of the magnetic field, an emf and hence a current is being induced in the conductor according to *Faraday's law of electromagnetic induction*, the direction of which may be given according to the ***Fleming's Right Hand Rule***, we call it ***F.B.I.*** (*easy to remember: Federal Bureau of Investigation*) as shown in the Fig. 6.10, *F* = Force, *B* = Magnetic Flux Density, *I* = Current. *If the pointing finger is showing the direction of magnetic field and the thumb is showing the direction of applied force or motion of the conductor, the direction of induced emf and induced current may be given by the middle finger.*

This is generating action, as mechanical energy of the moving conductor is being converted into electrical energy. So, ***Fleming's Right Hand Rule*** **is for Generator**, while ***Fleming's Left Hand Rule*** **is for Motor**.

In general ***Fleming's Right Hand Rule*** is known as the ***Generator / Generating Rule***.

6.8 **Dynamic (Motional) EMF from Lorentz Force and Energy Consideration:** A conductor has a large number of free electrons. When it moves in a magnetic field, its free electrons also moves with the conductor. We know that the moving charge in a magnetic field experiences the *Lorentz Force*, the direction of which may be given by *Fleming's Left Hand Rule*. Now consider the Fig. 6.11, where a conductor *PQ* of length l is free to move on *U* shaped conducting rails situated in a uniform and time dependent magnetic field B, directed normally into the plane of paper. The conductor *PQ* is moving towards the left on *U* shaped conducting rails with a velocity v, so the equivalent current (I_e) due to these moving electrons is in the direction towards right. The reader may verify by applying *Fleming's Left Hand Rule* that the free electrons of the conductor *PQ* will experience a force in the direction *QP*. The magnitude of the force experienced by the free electrons of the conductor *PQ* may be given as:

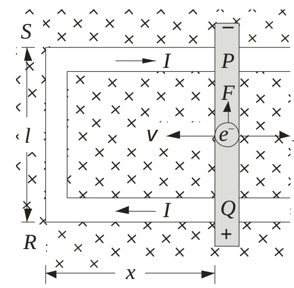


Fig. 6.11

$$F_m = q\,v\,B \tag{6.12}$$

The free electrons of the conductor *PQ* will start to accumulate at the end *P* under the influence of this *Lorentz force*. So, there is a negative potential at the end *P* and a positive potential at the end *Q*. An electric field is setup inside the conductor *PQ* directed from *Q* to *P*. The electrons accumulated at the end *P* will experience a force F_e, due to this electric field towards the end *Q*, which may be given as:

$$F_e = q\left|\vec{E}\right| \tag{6.13}$$

The electrons continue to accumulate at the end *P* until both these forces become equal for the equilibrium condition. So, the electric field at the equilibrium condition may be given by the relationship:

$$F_e = F_m \qquad \text{or,} \qquad q\left|\vec{E}\right| = q\,v\,B$$

or, $$\left|\vec{E}\right| = v\,B \tag{6.14}$$

The potential difference developed across the ends *P* and *Q* may be given as:

$$V = \left|\vec{E}\right| \times l = B\,l\,v \tag{6.15}$$

So, the induced emf across the points *P* and *Q* may be given as:

$$E = V = B\,l\,v \tag{6.16}$$

So, a current (I) starts to flow in the loop *PQRS* in the clock-wise direction under the influence of this induced emf.

The induced emf ($B\ l\ v$) *is known as dynamic / motional emf* because *this emf is induced due to the motion of a conductor inside the magnetic field.*

Current Induced in the Loop: Let R be the resistance of the complete loop *PQRS*, as shown in the Fig. 6.11. Then, the current flowing through the loop under the influence of the induced emf (E) may be given as:

$$I = \frac{E}{R} = \frac{B\,l\,v}{R} \tag{6.17}$$

Force experienced by the Movable Arm: Since the current carrying arm (*PQ*) is placed inside a magnetic field, it will experience a force, which may be given as:

$$F = B\,I\,l \sin\theta = B \times \frac{B\,l\,v}{R} \times l \times \sin 90°$$

or, $$F = \frac{B^2\,l^2\,v}{R} \qquad (6.18)$$

This force (*F*) experienced by the conductor due to magnetic field is in the direction opposite to that of the direction of movement of the conductor *PQ*, the reader may verify it by applying *Fleming's Left Hand Rule*. So, the conductor *PQ* must be pulled with a force *F*, given by the equation (6.18), in order to move it at a constant velocity towards left.

Power Delivered by the External Force (*F*): The mechanical power supplied by the external force (*F*) to maintain the motion of the conductor *PQ* may be given as:

$$P_m = F \times v = \frac{B^2\,l^2\,v}{R} \times v = \frac{B^2\,l^2\,v^2}{R} = \frac{E^2}{R} = P_E \qquad (6.19.1)$$

or, P_m (Mechanical Power Supplied) = P_E (Electrical Power Generated) (6.19.2)

Power Dissipated as Joules / Heating Loss (P_{Cu}): The electrical power dissipated as joules / heating loss in the loop *PQRS* may be given as:

$$P_{Cu} = I^2 R = \left(\frac{B\,l\,v}{R}\right)^2 \times R = \frac{B^2\,l^2\,v^2}{R} = P_E = P_m \qquad (6.19.3)$$

So, we may conclude here that the mechanical power (P_m) supplied by the force (*F*) is being converted in the electrical power (P_E) and then it is being lost as heat (P_{Cu}) in the resistance of the loop *PQRS, i.e.* the law of conservation of the energy holds good for this system. The electrical equivalent of this system may be drawn as shown in the Fig. 6.12.

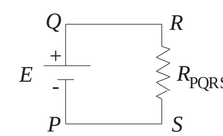


Fig. 6.12

6.9 Relation between Induced Charge and the Change in Magnetic Flux: The magnitude of induced emf in a conductor placed in a changing magnetic flux, according to ***Faraday's Law of Electromagnetic Induction***, may be given as:

$$E = \frac{d\,\phi}{d\,t} \qquad (6.20)$$

The current flowing through a conductor having a resistance *R* due to this emf (*E*) may be given as:

$$I = \frac{E}{R} = \frac{d\,\phi}{d\,t} \times \frac{1}{R} \qquad \text{or,} \qquad \frac{d\,q}{d\,t} = \frac{d\,\phi}{d\,t} \times \frac{1}{R}$$

or, $$dq = \frac{d\,\phi}{R} = \frac{\text{Net Change in Magnetic Flux}}{\text{Resistance of the conductor}} \qquad (6.21)$$

So, *the charge transferred depends only on the net change in magnetic flux and the resistance of the conductor and is independent of the rate of change of magnetic flux.*

Problem 6.24: *An aircraft with a wing span of* 40 *m flies with a velocity of* 1080 *km/hr in the west to east direction at a constant altitude* (*height*) *in the northern hemisphere, where the vertical component of earth's magnetic field is* 1.75×10^{-5} *T. Determine the induced emf across the farthest ends of the wings due to the vertical component of earth's magnetic field.* [NCERT]

Solution: $l = 40$ m, $v = 1080$ km/hr $= 1080 \times \frac{5}{18} = 300$ m/sec, $B_V = 1.75 \times 10^{-5}$ T

The induced emf due to vertical component of earth's magnetic field across the farthest ends of the wings may be given as:

$$E = B_V\, l\, v \sin\theta = 1.75 \times 10^{-5} \times 40 \times 300 \times \sin 90° = 0.21 \text{ V}$$

Problem 6.25: *A jet plane is travelling west at a velocity of* 450 *m/sec. If the horizontal component of earth's magnetic field at this place is* 4×10^{-4} *Tesla and the angle of dip is* 30°, *determine the emf induced across the farthest ends of the wings of jet plane having a span of* 30 *m.* [CBSE 2007-08]

Solution: $v = 450$ m/sec, $B_H = 4 \times 10^{-4}$ T, $\delta = 30°$, $l = 30$ m

The earth's magnetic field may be given as:

$$B = \frac{B_H}{\cos\delta}$$

The induced emf due to earth's magnetic field may be given as:

$$E = B\, l\, v \sin\delta = \frac{B_H}{\cos\delta} \times l\, v \sin\delta = B_H\, l\, v \tan\delta$$

$$= 4 \times 10^{-4} \times 30 \times 450 \times \tan 30° = 3.118 \text{ V}$$

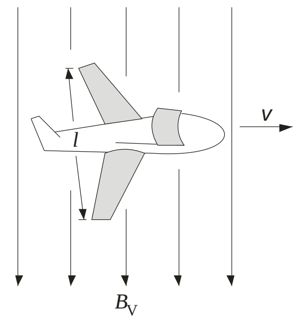


Fig. 6.13

Alternatively: The jet plane is cutting the vertical component of earth's magnetic field only, at an angle of 90°. The vertical component of earth's magnetic field may be given as:

$$B_V = B_H \tan\delta$$

So, the induced emf due to earth's magnetic field across the farthest ends of the wings may be given as:

$$E = B_V\, l\, v \sin\theta = B_H \tan\delta \times l\, v \sin 90° = B_H\, l\, v \tan\delta$$

$$= 4 \times 10^{-4} \times 30 \times 450 \times \tan 30° = 3.118 \text{ V}$$

Problem 6.26: *A railway track running north-south has two parallel rails* 1 *m apart. Determine the value of induced emf across the rails, when a train passes at a speed of* 90 *km/hr. The horizontal component of earth's magnetic field at that place is* 3×10^{-5} *T and angle of dip is* 60°.

[Haryana 2000-01]

Solution: $l = 1$ m, $v = 90$ km/hr $= 90 \times \frac{5}{18} = 25$ m/sec, $B_H = 3 \times 10^{-5}$ T, $\delta = 60°$

The train is cutting the vertical component of earth's magnetic field only, at an angle of 90°. The vertical component of earth's magnetic field may be given as:

$$B_V = B_H \tan\delta$$

So, the induced emf due to earth's magnetic field across the two rails may be given as:

$$E = B_V\, l\, v \sin\theta = B_H \tan\delta \times l\, v \sin 90° = B_H\, l\, v \tan\delta$$

$$= 3 \times 10^{-5} \times 1 \times 25 \times \tan 60° = 1.299 \times 10^{-3} \text{ V} = 1.299 \text{ mV}$$

Problem 6.27: *A conductor of length* 1 *m falls freely under the gravity from a height of* 10 *m, so that it cuts the magnetic lines of force of the horizontal component of earth's magnetic field of* 3×10^{-5} *Tesla. Determine the emf induced in the conductor.*

Solution: $l = 1$ m, $h = 10$ m, $B_H = 3 \times 10^{-5}$ T

The velocity of the conductor, when it hits the ground, may be given by the relationship:

$$v^2 = u^2 + 2\,g\,h$$

or, $$v = \sqrt{u^2 + 2\,g\,h} = \sqrt{(0)^2 + 2 \times 9.81 \times 10} = 14 \text{ m/sec}$$

So, the induced emf due to earth's magnetic field across the conductor, when it hits the ground, may be given as:

$$E = B_H\, l\, v \sin\theta = 3 \times 10^{-5} \times 1 \times 14 \times \sin 90° = 0.42 \times 10^{-3} \text{ V} = 0.42 \text{ mV}$$

Problem 6.28: *Twelve wires of equal length* (10 *cm each*) *are connected in the form of a Skelton-cube. i*) *If the cube is moving with a velocity of* 5 *m/sec in the direction of a magnetic field of* 0.05 *Tesla, determine the emf induced in each arm of the cube. ii*) *If the cube moves perpendicular to the field, what will be the induced emf in each arm?*

Solution: l = 10 cm, v = 5 m/sec, B = 0.05 T

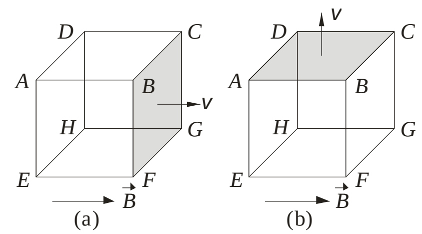


Fig. 6.14

The Cube is Moving in the Direction of Magnetic Field: Since the velocity of the cube is parallel to the magnetic field, so there will be no induced emf in any of the arm of the cube.

The Cube is Moving Perpendicular to the Direction of Magnetic Field: The arms *AD*, *BC*, *FG* and *EH* are perpendicular to both *B* and v, so the induced emf will be there only in these branches, and may be given as:

$$E = B\, l\, v \sin 90° = 0.05 \times 0.10 \times 5 \times 1 = 0.025 \text{ V} = 25 \text{ mV}$$

Problem 6.29: *A conducting rod PQ is shown in the Fig.* 6.15 *in contact with metal rails PS and RQ which are* 25 *cm apart in a uniform magnetic field of flux density of* 0.4 *Tesla acting perpendicular into the plane of the paper. The ends of rails R and S are connected through a* 5 Ω *resistor. Determine the direction and magnitude of induced emf and induced current, if the rod PQ moves to the left with a velocity of* 5 *m/sec. If the rod moves to the right with the same speed, determine the new direction and magnitude of the induced current.* [ISCE 1994-95]

Solution: l = 25 cm, B = 0.4 T, R = 5 Ω, v = 5 m/sec

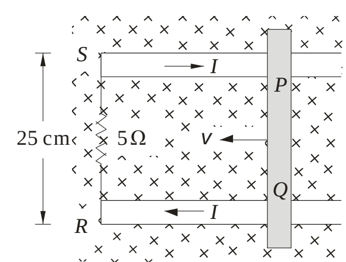


Fig. 6.15

The magnitude of induced emf may be given as:

$$E = B\, l\, v \sin\theta = 0.4 \times 0.25 \times 5 \times \sin 90° = 0.5 \text{ V}$$

So, the current flowing through the 5 Ω resistor may be given as:

$$I = \frac{E}{R} = \frac{0.5}{5} = 0.1 \text{ A}$$

The direction of the induced emf and the induced current may be given by the ***Fleming's Right Hand Rule***, and is clock-wise in the loop *PQRS* as shown in the figure for left side movement of the rod *PQ*. So, the induced current flows from the end *R* to the end *S* in the 5 Ω resistor.

If the rod moves towards right the magnitude of the induced emf and the induced current remains same but the direction will become reverse, *i.e.* counter-clock-wise in the loop *PQRS*, the reader may verify it applying ***Fleming's Right Hand Rule***. So, the induced current flows from the end *S* to the end *R* in the 5 Ω resistor.

Problem 6.30: *A metallic rod of length L is rotated at an angular speed* ω, *about its one end, normal to a uniform magnetic field B. Derive the expression for the: i*) *induced emf in the rod, ii*) *induced current, iii*) *heat dissipation, if the resistance of the rod is R.* [CBSE 2007-08]

Solution: The flux swept by the rod in one revolution may be given as:

$$d\,\phi = B \times A_{circle} = B \times \pi\, L^2$$

The time taken by the rod to complete one revolution may be given as:

$$d\,t = \frac{1}{f} = \frac{1}{(\omega / 2\pi)} = \frac{2\pi}{\omega}$$

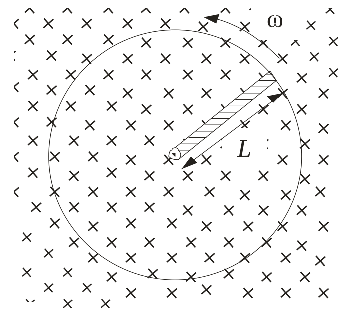


Fig. 6.16

So, the induced emf may be given as:

$$E = \frac{d\,\phi}{d\,t} = \frac{B \times \pi L^2}{(2\pi / \omega)} = \frac{1}{2}\; B\, L^2\, \omega$$

The induced current in the rod may be given as:

$$I = \frac{E}{R} = \frac{B L^2\, \omega}{2R}$$

The heat dissipated in time *t* may be given as:

$$H = P \times t = \frac{E^2\, t}{R} = \frac{B^2\, L^4\, \omega^2\, t}{4R}$$

Problem 6.31: *A metal disc of radius R rotates with an angular speed* ω *about an axis perpendicular to its plane passing through its center in a magnetic field* $\vec{B}$ *acting perpendicular to the plane of the disc. Determine the induced emf between the rim and the axis of the disc.*

Solution: The flux swept by the disc in one revolution may be given as:

$$d\,\phi = B \times A_{disc} = B \times \pi\, R^{\,2}$$

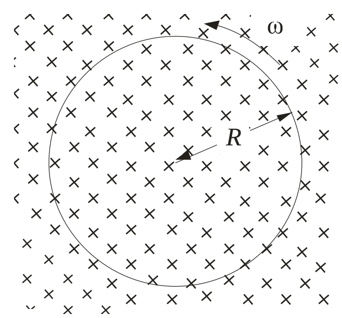


Fig. 6.16

The time taken by the disc to complete one revolution may be given as:

$$d\,t = \frac{1}{f} = \frac{1}{(\omega / 2\pi)} = \frac{2\pi}{\omega}$$

So, the induced emf may be given as:

$$E = \frac{d\,\phi}{d\,t} = \frac{B \times \pi R^{\,2}}{(2\pi / \omega)} = \frac{1}{2}\; B\, R^{\,2}\, \omega$$

Problem 6.32: *A wheel with* 10 *metallic spokes each* 0.5 *m long is rotated with a speed of* 120 *rpm in a plane normal to the horizontal component of earth's magnetic field* B_H = 0.4 *Gauss. Determine the emf induced between the axel and the rim of the wheel.* [NCERT]

Solution: n = 10 spokes, l = 0.5 m, N = 120 rpm, B_H = 0.4 Gauss

The flux swept by each spoke in one revolution may be given as:

$$d\,\phi = B_H \times \pi\, l^{\,2} = 0.4 \times 10^{-4} \times \pi \times (0.5)^2 = 3.14 \times 10^{-5}\ \text{Wb}$$

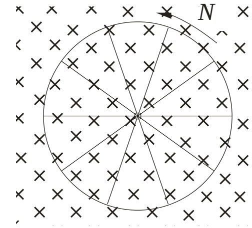


Fig. 6.17

Speed of the rotation of the spokes may be given as:

$$N = 120\ \text{rpm} = \frac{120}{60} = 2\ \text{rps}$$

So, the time taken by each spoke to complete one rotation may be given as:

$$d\,t = \frac{1}{N} = \frac{1}{2} = 0.5 \text{ sec}$$

The emf induced across each spoke may be given as:

$$e = \frac{d\,\phi}{d\,t} = \frac{3.14\times10^{-5}}{0.5} = 6.28 \times 10^{-5} \text{ V} = 62.8 \text{ μV}$$

As, all the ten spokes are connected as one end at the axel and one end at the rim, so they are connected in parallel. Hence the emf across the axel and the rim may be given as:

$$E = e = 62.8 \text{ μV}$$

Problem 6.33: *A wheel with some metallic spokes each* 1.2 *m long is rotated with a certain speed in a plane normal to the magnetic field of intensity* 0.5 *Gauss. If the induced emf between the axel and the rim of the wheel is* 10 *mV, determine the speed of rotation of the wheel.*

Solution: l = 1.2 m, B = 0.5 Gauss, E = 10 mV

Since, all the spokes are connected as one end at the axel and one end at the rim, so they are connected in parallel. Hence, the emf across each individual spoke may be given as:

$$e = E = 10 \text{ mV}$$

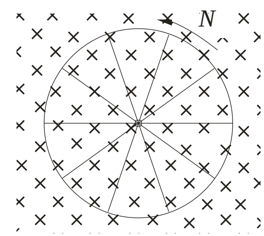


Fig. 6.18

The flux swept by each spoke in one revolution may be given as:

$$d\,\phi = B \times \pi\, l^2 = 0.5 \times 10^{-4} \times \pi \times (1.2)^2 = 2.262 \times 10^{-4} \text{ Wb}$$

Let speed of the rotation of the spokes may be given as:

$$N \text{ rpm} = \frac{N}{60} \text{ rps}$$

So, the time taken by each spoke to complete one rotation may be given as:

$$d\,t = \frac{1}{(N/60)} = \frac{60}{N} \text{ sec}$$

The emf induced across each spoke may be given by the relationship:

$$e = 10 \times 10^{-3} = \frac{d\,\phi}{d\,t} = \frac{2.262\times10^{-4}}{(60/N)} = \frac{2.262\times10^{-4}\times N}{60}$$

or, $$N = \frac{10\times10^{-3}\times60}{2.262\times10^{-4}} = 2652.52 \text{ rpm}$$

Problem 6.34: *A metallic rod of* 1 *m length is rotated with a frequency of* 50 *rps, with one end hinged at the center, about an axis passing through its center. A constant and uniform magnetic field of* 1 *T is present parallel to the axis of rotation of the rod. Determine the induced emf across the ends of the rod.* [NCERT]

Solution: l = 1 m, f = 50 rps, B = 1 T

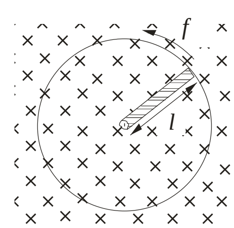


Fig. 6.19

The flux swept by the rod in one revolution may be given as:

$$d\,\phi = B \times A_{circle} = B \times \pi\, l^2 = 1 \times \pi \times (1)^2 = \pi \text{ Wb}$$

The time taken by the rod to complete one revolution may be given as:

$$d\,t = \frac{1}{f} = \frac{1}{50} = 0.02 \text{ sec}$$

So, the induced emf may be given as:

$$E = \frac{d\,\phi}{d\,t} = \frac{\pi}{0.02} = 157.07 \text{ V}$$

Problem 6.35: *A circular copper disc* 10 *cm in radius rotates at* 20 π *rad/sec about an axis through its center and perpendicular to the disc. If a uniform magnetic field of* 0.2 *Tesla is applied perpendicular to the rotating disc, determine: i) the induced emf across the axis of the disc and the rim, ii) the induced current, if the resistance of the disc is* 2 Ω. [CBSE 2000-01]

Solution: $r = 10$ cm, $\omega = 20\,\pi$ rad/sec, $B = 0.2$ T, $R = 2\,\Omega$

The flux swept by the disc in one revolution may be given as:

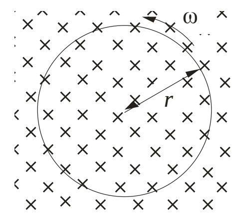


Fig. 6.20

$$d\,\phi = B \times A_{disc} = B \times \pi\, r^2 = 0.2 \times \pi \times (0.10)^2 = 6.283 \times 10^{-3} \text{ Wb}$$

The time taken by the rod to complete one revolution may be given as:

$$d\,t = \frac{\theta}{\omega} = \frac{2\pi}{20\pi} = 0.1 \text{ sec}$$

So, the induced emf may be given as:

$$E = \frac{d\,\phi}{d\,t} = \frac{6.283\times10^{-3}}{0.1} = 62.83 \times 10^{-3} \text{ V} = 62.83 \text{ mV}$$

The induced current may be given as:

$$I = \frac{E}{R} = \frac{62.83}{2} = 31.415 \text{ mA}$$

Problem 6.36: *A* 0.5 *m long metal rod PQ completes the circuit as shown in the Fig.* 6.21. *The plane of the circuit is perpendicular to the magnetic field of flux density* 0.15 *T. If the resistance of the total circuit is* 3 Ω, *determine the force required to move the rod in the direction as indicated with a constant speed of* 2 *m/sec.* [CBSE 2005-06]

Solution: $l = 0.5$ m, $B = 0.15$ T, $R = 3\,\Omega$, $v = 2$ m/sec

If a conductor moves inside a magnetic field, there will be an induced emf and an induced current (as the circuit is closed) in the conductor. The force required to move this current carrying conductor inside the magnetic field is equal to the force experienced by this current carrying conductor. So, the required force may be given as:

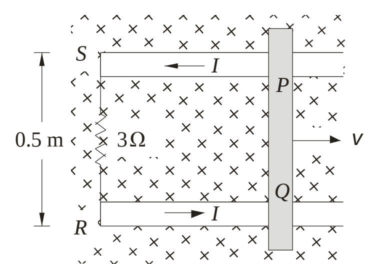


Fig. 6.21

$$F = B\,I\,l \sin\theta = B \times \frac{E}{R} \times l \sin\theta$$

$$= B \times \frac{B\,l\,v \sin\theta}{R} \times l \sin\theta = \frac{B^2 l^2 v \sin^2\theta}{R} = \frac{(0.15)^2 \times (0.5)^2 \times 2 \times \sin^2 90°}{3}$$

$$= 3.75 \times 10^{-3} \text{ N}$$

Problem 6.37: *A straight conductor of length* 1 *m moves with* 2 *m/sec at right angles to both, its length and a uniform magnetic field of strength* 10^4 *Gauss. Determine the value of induced emf in the conductor.* [Punjab 1995-96]

Solution: $l = 1$ m, $v = 2$ m/sec, $\theta = 90°$, $B = 10^4$ Gauss $= 1$ T

The induced emf in the conductor may be given as:

$E = B\,l\,v \sin\theta = 1 \times 1 \times 2 \times \sin 90° = 2$ V

Problem 6.38: *If a* 10 *m long metallic bar moves in a direction at right angle to a magnetic field with a speed of* 5 *m/sec,* 25 *V emf is induced in the bar. Determine the strength of the magnetic field in tesla.* [Punjab 1998-99]

Solution: $l = 10$ m, $\theta = 90°$, $v = 5$ m/sec, $E = 25$ V

The expression for the induced emf in a long metallic bar may be given as:

$$E = B\,l\,v \sin\theta$$

So, $$B = \frac{E}{l\,v \sin\theta} = \frac{25}{10 \times 5 \times \sin 90°} = 0.5 \text{ Tesla}$$

Problem 6.39: *A* 0.4 *m long straight conductor is moved in a magnetic field of induction* 0.9 *Tesla with a velocity of* 7 *m/sec. Determine the maximum induced emf in this conductor.* [Roorkee 1982]

Solution: $l = 0.4$ m, $B = 0.9$ T, $v = 7$ m/sec

The maximum value of emf will be induced in this conductor when it moves at right angle to both, its length as well as the magnetic field. So, the maximum value of the induced emf in this conductor may be given as:

$$E = B\,l\,v \sin 90° = 0.9 \times 0.4 \times 7 \times 1 = 2.52 \text{ V}$$

Problem 6.40: *A* 1 *km long horizontal telephone wire is lying east-west in earth's magnetic field. It falls freely to the ground from a height of* 10 *m. Determine the emf induced in the wire on striking the ground. The horizontal component of earth's magnetic field at this place is given as* 0.32 *Gauss.*

Solution: $l = 1$ Km $= 10^3$ m, $\theta = 90°$, $h = 10$ m, $B = 0.32$ Gauss

The velocity of the conductor, when it hits the ground, may be given by the relationship:

$$v^2 = u^2 + 2\,g\,h$$

or, $$v = \sqrt{u^2 + 2\,g\,h} = \sqrt{(0)^2 + 2 \times 9.81 \times 10} = 14 \text{ m/sec}$$

So, the induced emf due to earth's magnetic field across the conductor, when it hits the ground, may be given as:

$$E = B_H\,l\,v \sin\theta = 0.32 \times 10^{-4} \times 10^3 \times 14 \times \sin 90° = 0.448 \text{ V}$$

Problem 6.41: *A* 24 *cm long horizontal wire falls in the magnetic field of flux density* 0.8 *Tesla. Determine the emf induced in it at* t = 3 *sec, after it was dropped to fall. Suppose the wire moves perpendicular to its length as well as to the magnetic field. Given* $g = 9.8$ *m/sec*^{2}.

Solution: $l = 24$ cm, $B = 0.8$ T, $t = 3$ sec, $\theta = 90°$, $g = 9.8$ m/sec^2

The velocity of the wire at $t = 3$ sec may be given as:

$$v = u + g\,t = 0 + 9.8 \times 3 = 29.4 \text{ m/sec}$$

So, the induced emf due to the magnetic field across the conductor, at $t = 3$ sec, may be given as:

$$E = B\,l\,v \sin\theta = 0.8 \times 0.24 \times 29.4 \times \sin 90° = 5.6448 \text{ V}$$

Problem 6.42: *Two rails of a railway track insulated from each other and the ground are connected to a milli-voltmeter. Determine the reading of the voltmeter, when a train travels at a speed of* 180 *km/hr along the track. Given that the vertical component of earth's magnetic field is* 0.2×10^{-4} *Tesla and the rails are separated by* 1 *m.* [IIT 1981]

Solution: $v = 180 \text{ km/hr} = 180 \times \frac{5}{18} = 50 \text{ m/sec}$, $B_V = 0.2 \times 10^{-4}$ T, $l = 1$ m, $\theta = 90°$

The train is cutting the vertical component of earth's magnetic field only, at an angle of 90°. So, the induced emf due to earth's magnetic field across the two rails may be given as:

$$E = B_V\, l\, v \sin\theta = 0.2 \times 10^{-4} \times 1 \times 50 \times \sin 90° = 1 \times 10^{-3} \text{ V} = 1 \text{ mV}$$

Problem 6.43: *A train is running due north with a constant speed of* 90 *km/hr on a horizontal track. If the vertical component of earth's magnetic field is* 3×10^{-5} *Tesla, determine the induced emf across the axel of the train of length* 1.25 *m.* [Haryana 2001-02]

Solution: $v = 90 \text{ km/hr} = 90 \times \frac{5}{18} = 25 \text{ m/sec}$, $B_V = 3 \times 10^{-5}$ T, $l = 1.25$ m, $\theta = 90°$

The train is cutting the vertical component of earth's magnetic field only, at an angle of 90°. So, the induced emf due to earth's magnetic field across the two rails may be given as:

$$E = B_V\, l\, v \sin\theta = 3 \times 10^{-5} \times 1.25 \times 25 \times \sin 90° = 0.9375 \times 10^{-3} \text{ V} = 0.9375 \text{ mV}$$

Problem 6.44: *A jet plane is moving at a speed of* 1000 *km/hr. Determine the potential difference across the ends of its wings* 20 *m long. Given total intensity of earth's magnetic field is* 3.5×10^{-4} *Tesla and the angle of dip at this place is* 30°.

Solution: $v = 1000 \text{ km/hr} = 1000 \times \frac{5}{18} = 277.778 \text{ m/sec}$, $l = 20$ m, $B = 3.5 \times 10^{-4}$ T, $\delta = 30°$

The jet plane is cutting the vertical component of earth's magnetic field only, at an angle of 90°. The vertical component of earth's magnetic field may be given as:

$$B_V = B \sin\delta$$

So, the induced emf due to earth's magnetic field across the farthest ends of the wings may be given as:

$$E = B_V\, l\, v \sin\theta = B \sin\delta \times l\, v \sin 90° = B\, l\, v \sin\delta$$

$$= 3.5 \times 10^{-4} \times 20 \times 277.778 \times \sin 30° = 0.972 \text{ V}$$

Problem 6.45: *A straight rod* 2 *m long is placed in an airplane in the east-west direction. The airplane lifts itself in the upward direction at a speed of* 36 *km/hr. Determine the induced emf across the two ends of the rod, if the vertical component of the earth's magnetic field is* $\frac{1}{4\sqrt{3}}$ *Gauss and the angle of dip is* 30° *at this place.*

Solution: $l = 2$ m, $v = 36 \text{ km/hr} = 36 \times \frac{5}{18} = 10 \text{ m/sec}$, $B_V = \frac{1}{4\sqrt{3}}$ Gauss, $\delta = 30°$

When airplane is lifting itself vertically upwards, the horizontal component of earth's magnetic field associated with the rod is changing, and the rod is moving at right angle (90°) to it being in east-west direction.

The horizontal component of earth's magnetic field may be given as:

$$B_H = \frac{B_V}{\tan\delta} = \frac{1}{4\sqrt{3}} \times \frac{1}{\tan 30°} = 0.25 \text{ Gauss}$$

So, the induced emf due to earth's magnetic field across the ends of the rod may be given as:

$$E = B_H\, l\, v \sin\theta = 0.25 \times 10^{-4} \times 2 \times 10 \times \sin 90° = 0.5 \times 10^{-3} \text{ V} = 0.5 \text{ mV}$$

Problem 6.46: *A metal wheel with* 1 *m long spokes is rotated in a magnetic field of flux density* 2×10^{-4} *Tesla normal to the plane of the wheel. An emf of* $\pi \times 10^{-2}$ *V is induced between the rim and the axel. Determine the rate of the rotation of the wheel.*

Solution: $l = 1$ m, $B = 2 \times 10^{-4}$ Tesla, $E = \pi \times 10^{-2}$ V

The expression for the emf induced may be given as:

$$E = \frac{d\phi}{dt} = \frac{\text{Flux Swept in One Rotation}}{\text{Time Taken in One Rotation}}$$

The flux swept in one rotation may be given as:

$$d\phi = B \times A = B \times \pi l^2$$

If the speed of rotation is N rpm, the time taken by the wheel in one rotation may be given as:

$$dt = \frac{1}{f} = \frac{1}{(N/60)} = \frac{60}{N}$$

So, $$E = \frac{B \times \pi l^2}{(60/N)} = \frac{B \times \pi l^2 \times N}{60}$$

or, $$N = \frac{60E}{B \times \pi l^2} = \frac{60 \times \pi \times 10^{-2}}{2 \times 10^{-4} \times \pi \times (1)^2} = 3000 \text{ rpm} = \frac{3000}{60} = 50\text{rps}$$

Problem 6.47: *A fan blade of length* 2*a rotates with frequency f Hz perpendicular to a magnetic field B. Determine the potential difference between the center and the end of the blade.*

Solution: $l = 2a$, Frequency = f, Magnetic flux density = B

The flux swept by the blade in one rotation may be given as:

$$d\phi = BA = B \times \pi (2a)^2 = 4\pi a^2 B$$

The time taken by the fan to complete one rotation may be given as:

$$dt = \frac{1}{f} \text{ sec}$$

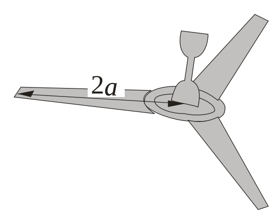


Fig. 6.22

So, the induced emf across the blade of the fan may be given as:

$$E = \frac{d\phi}{dt} = \frac{4\pi a^2 B}{(1/f)} = 4\pi a^2 B f \text{ Volts}$$

Problem 6.48: *In a ceiling fan the blades are rotating in a circle of radius* 0.5 *m. If the fan is rotating at* 20 *rps and the vertical component of the earth's magnetic field is* 0.8 *Gauss, determine the induced emf between the ends of each blade.*

Solution: $r = 0.5$ m, $f = 20$ rps, $B_V = 0.8$ Gauss

The flux swept by the fan blade in one rotation may be given as:

$$d\phi = BA = B \times \pi (r)^2 = 0.8 \times 10^{-4} \times \pi \times (0.5)^2 = 6.283 \times 10^{-5} \text{ Wb}$$

The time taken by the blade to complete one rotation may be given as:

$$dt = \frac{1}{f} = \frac{1}{20} = 0.05 \text{ sec}$$

So, the induced emf across the blade of the fan may be given as:

$$E = \frac{d\phi}{dt} = \frac{6.283\times10^{-5}}{0.05} = 1.2566 \times 10^{-3} \text{ Volts} = 1.2566 \text{ mV}$$

Problem 6.49: *A metal disc of radius* 200 *cm is rotated at a constant angular speed of* 60 *rad/sec in a plane at right angle to an external magnetic field of intensity* 0.05 *Tesla. Determine the emf induced between the center and a point on the rim.* [Punjab 1990-91]

Solution: $r = 200$ cm, $\omega = 60$ rad/sec, $B = 0.05$ T

The flux swept by the disc in one rotation may be given as:

$$d\phi = B\,A = B \times \pi\,(r)^2 = 0.05 \times \pi \times (2)^2 = 0.2\,\pi \text{ Wb}$$

The time taken by the metal disc to complete one rotation may be given as:

$$d\,t = \frac{\theta}{\omega} = \frac{2\pi}{60} = \frac{\pi}{30} \text{ sec}$$

The emf induced between the center of the disc and it's rim may be given as:

$$E = \frac{d\phi}{dt} = \frac{0.2\,\pi}{(\pi/30)} = \frac{0.2\,\pi\times30}{\pi} = 6 \text{ V}$$

Problem 6.50: *A copper disc of radius* 10 *cm is placed with its plane normal to a uniform magnetic field and it completes* 1200 *rotations per minute* (*rpm*)*. If induced emf between the center and the edge of the disc is* 6.284 *mV, determine the intensity of the magnetic field.*

Solution: $r = 10$ cm, $N = 1200 \text{ rpm} = \frac{1200}{60} = 20$ rps, $E = 6.284$ mV

If the magnetic field is $\vec{B}$, the flux swept by the disc in one rotation may be given as:

$$d\phi = B\,A = B \times \pi\,(r)^2 = B \times \pi \times (0.10)^2 = 0.01\,\pi\,B \text{ Wb}$$

The time taken by the disc to complete one rotation may be given as:

$$d\,t = \frac{1}{f} = \frac{1}{20} = 0.05 \text{ sec}$$

The emf induced between the center of the disc and it's rim may be given by the relationship:

$$E = \frac{d\phi}{dt} = \frac{0.01\times\pi\,B}{0.05} = 6.284 \times 10^{-3} \text{ V}$$

or, $$B = \frac{6.284\times10^{-3}\times0.05}{0.01\times\pi} = 0.01 \text{ T} = 100 \text{ Gauss}$$

Problem 6.51: *A gramophone disc of brass having diameter* 30 *cm rotates horizontally at a rate of* $\frac{100}{3}$ *rpm. If the vertical component of earth's magnetic field is* 100 *Gauss, determine the induced emf between the center and the rim of the disc.*

Solution: $d = 30$ cm, $N = \frac{100}{3} \text{ rpm} = \frac{100}{3\times60} = \frac{5}{9}$ rps, $B = 100 \text{ G} = 0.01$ T

The flux swept by the disc in one rotation may be given as:

$$d\phi = B\,A = B \times \frac{\pi(d)^2}{4} = 0.01 \times \frac{\pi\times(0.30)^2}{4} = 7.069 \times 10^{-4} \text{ Wb}$$

The time taken by the disc to complete one rotation may be given as:

$$d\,t = \frac{1}{f} = \frac{1}{(5/9)} = \frac{9}{5} = 1.8 \text{ sec}$$

The emf induced between the center of the disc and it's rim may be given as:

$$E = \frac{d\,\phi}{d\,t} = \frac{7.069 \times 10^{-4}}{1.8} = 3.927 \times 10^{-4} \text{ V} = 0.3927 \text{ mV}$$

6.10 Methods of Generating Induced EMF: An induced emf may be produced by changing the magnetic flux linked with a closed loop / circuit. The magnetic flux (ϕ) linked with the coil of area A may be given as:

$$\phi = B\,A \cos\theta \qquad (6.22)$$

The flux, linked with the loop, given by above equation may be changed in three different ways:

i) *By changing the magnetic field B.*

ii) *By changing the area of the coil A.*

iii) *By changing the relative orientation* (θ) *between the magnetic field* (B) *and the area of the coil* (A).

i*) Induced EMF by Changing the Magnetic Field (*B*):** We have already gone through the exercise in article 6.4 during the experiments by *Michael Faraday* to understand how an induced emf is being produced in a coil on changing the magnetic flux through it by: *a*) moving a magnet towards a stationary coil, *b*) moving a coil towards a stationary magnet, *c*) varying current in a neighboring coil to change the magnetic field. The direction of induced emf may be given by ***Fleming's Right Hand Rule and ***Lenz's Law***.

***ii*) Induced EMF by Changing the Area of the coil (*A*):** We have already gone through the exercise in article 6.7 and 6.8 that how an induced emf is being produced in a coil on changing the area of the coil, while placed in a constant uniform magnetic field. The magnitude of the induced emf may be given by the relationship:

$$E = B\,l\,v \qquad (6.23)$$

The direction of induced emf may be given by ***Fleming's Right Hand Rule*** and ***Lenz's Law***.

***iii*) Induced EMF by Changing the Relative Orientation (θ) of the Magnetic Field (*B*) and the Area of the Coil (*A*):** Consider a coil *PQRS* having *N* turns, as shown in the Fig. 6.23 (a), free to rotate in a constant uniform magnetic field ($\vec{B}$). The axis of the rotation of the coil is perpendicular to the magnetic field. The flux passing through the coil, when it's normal makes an angle θ with the magnetic field as shown in the Fig. 6.23 (b), may be given as:

$$\phi = B\,A \cos\theta \qquad (6.24)$$

Where, *A* is the area of the coil *PQRS*.

If the coil rotates about its axis at a constant angular speed ω and turns through an angle θ in time *t*, then:

$$\theta = \omega\,t$$

So, $$\phi = B\,A \cos\omega\,t \qquad (6.25)$$

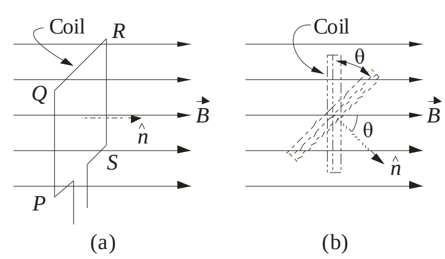


Fig. 6.23

As, the coil is rotating continuously at a constant angular speed ω, the flux linked with the coil is changing continuously (in a sinusoidal manner), and an emf will get induced across the rotating coil, which may be given as:

$$E = -N \times \frac{d\phi}{dt} = -N \times \frac{d}{dt}(B\,A \cos \omega\, t) = N\,B\,A\,\omega \sin \omega\, t \qquad (6.26.1)$$

or, $E = E_0 \sin \omega\, t = E_0 \sin 2\,\pi\, f\, t$ (6.26.2)

Where, $E_0 = N\,B\,A\,\omega$ (6.27)

and, f = frequency of the rotation of the coil = $\frac{2\pi}{\omega}$ Hz (6.28)

So, the induced emf, in a rotating coil with a constant angular speed inside a constant uniform magnetic field, varies in sinusoidal manner with time t.

The induced emf (E) in the rotating coil:

1) If, $\omega\, t = 0°$

   i.e. the plane of the coil is perpendicular to the magnetic field.

   $(\sin \omega\, t) = 0$

   Emf will have minimum value:

   $E = 0$

2) If, $\omega\, t = 90°$

   i.e. the plane of the coil is parallel to the magnetic field.

   $(\sin \omega\, t) = 1$

   Emf will have positive maximum value:

   $E = +\,N\,B\,A\,\omega$

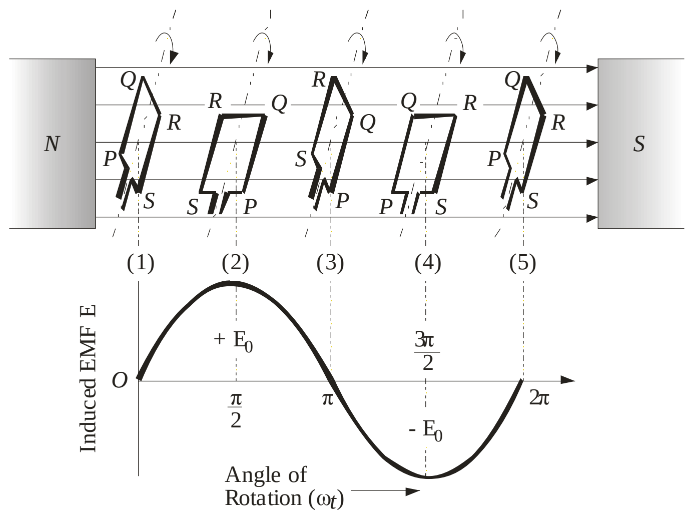


Fig. 6.24

3) If, $\omega\, t = 180°$

   i.e. the plane of the coil is perpendicular to the magnetic field.

   $(\sin \omega\, t) = 0$ so, Emf will again have minimum value: $E = 0$

4) If, $\omega\, t = 270°$

   i.e. the plane of the coil is anti-parallel to the magnetic field.

   $(\sin \omega\, t) = -1$ so, Emf will have negative maximum value: $E = -\,N\,B\,A\,\omega$

5) If, $\omega\, t = 360°$

   i.e. the plane of the coil is perpendicular to the magnetic field.

   $(\sin \omega\, t) = 0$ so, Emf will again have minimum value: $E = 0$

The pattern of changing of induced emf *w.r.t.* the orientation of the coil is shown in the Fig. 6.24, which indicates that the induced emf across the coil is ***alternating in nature*** and has a ***sinusoidal pattern***. As the coil rotates continuously in the same manner, the same cycle of changes repeats again and again over the time period at constant time intervals.

Since, the induced emf in a coil rotating at a constant angular speed inside a constant and uniform magnetic field is ***alternating and sinusoidal***, it is the basic principle of the dynamo and alternator being used for commercial generation of electricity.

Problem 6.52: *A circular coil of area* 300 cm^2 *and having* 25 *turns rotates about its vertical diameter with a constant angular speed of* 40 sec^{-1} *in a uniform horizontal magnetic field of magnitude* 0.05 *T. Determine the value of maximum induced emf in the coil.* [NCERT]

Solution: $A = 300\ \text{cm}^2$, $N = 25$ Turns, $f = 40\ \text{sec}^{-1} = 40$ Hz, $B = 0.05$ T

The maximum value of induced emf in a coil rotating at a constant angular speed inside a uniform magnetic field may be given as:

$$E_0 = \pm N\,B\,A\,\omega = \pm N\,B\,A \times (2\,\pi\,f)$$

$$= \pm\, 25 \times 0.05 \times 300 \times 10^{-4} \times (2\,\pi \times 40) = \pm\, 9.425\ \text{V}$$

Problem 6.53: *A rectangular coil of size* 100 *cm* × 50 *cm and having* 10 *turns is rotated at* 50 *rps inside a magnetic field of strength* 0.5 *T. Determine the peak value of the voltage generated across the ends of the coil.*

Solution: $A = 100 \times 50\ \text{cm}^2$, $N = 10$ Turns, $f = 50$ rps $= 50$ Hz, $B = 0.5$ T

The peak value of the voltage generated across the ends of a coil rotating at a constant angular speed inside a uniform magnetic field may be given as:

$$E_0 = \pm N\,B\,A\,\omega = \pm N\,B\,A \times (2\,\pi\,f)$$

$$= \pm\, 10 \times 0.5 \times (100 \times 50 \times 10^{-4}) \times (2\,\pi \times 50) = \pm\, 785.398\ \text{V}$$

Problem 6.54: *A coil of* 500 *turns, having an area of* 50 cm^2 *and a resistance of* 5 Ω, *is rotating in a uniform magnetic field of strength* 0.14 *Tesla at a constant angular speed of* 150 *rad/sec. The induced emf across the coil is applied to an external resistance of* 10 Ω. *Determine the value of peak current flowing through the external resistance.*

Solution: $N = 500$ Turns, $A = 50\ \text{cm}^2$, $R_{coil} = 5\ \Omega$, $B = 0.14$ T, $\omega = 150$ rad/s, $R_{ext} = 10\ \Omega$

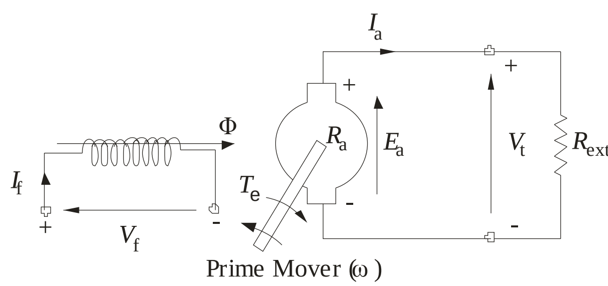


Fig. 6.25

The peak value of the induced emf across the ends of a coil rotating at a constant angular speed inside a uniform magnetic field may be given as:

$$E_0 = \pm N\,B\,A\,\omega$$

$$= \pm\, 500 \times 0.14 \times 50 \times 10^{-4} \times 150$$

$$= \pm\, 52.5\ \text{V}$$

So, the peak value of current flowing through the external resistor may be given as:

$$I_0 = \frac{E_0}{R_a + R_{ext}} = \pm\,\frac{52.5}{5+10} = \pm\, 3.5\ \text{A}$$

Problem 6.55: *A rectangular coil, of size* 0.2 *m* × 0.1 *m having* 2000 *turns, is rotating in a uniform magnetic field about an axis parallel to its length and perpendicular to the direction of magnetic field having a strength of* 0.02 Wb/m^2. *The speed of rotation of the coil is* 4200 *rpm. Determine: i) the maximum value of the induced emf in the coil, ii) the instantaneous value of induced emf when the plane of the coil has rotated through an angle of* 30° *from the initial position.*

Solution: $A = 0.2 \times 0.1\ \text{m}^2$, $N = 2000$ Turns, $B = 0.02$ T, $n = 4200$ rpm, $\theta = 30°$

The maximum value of induced emf in a coil rotating at a constant angular speed inside a uniform magnetic field may be given as:

$$E_0 = \pm N\,B\,A\,\omega = \pm N\,B\,A\,(2\,\pi\,f) = \pm N\,B\,A \times 2\,\pi \times \frac{n}{60}$$

$$= \pm\, 2000 \times 0.02 \times 0.2 \times 0.1 \times 2\,\pi \times \frac{4200}{60} = \pm\, 351.858 \text{ V}$$

The instantaneous value of the induced emf, when the coil has been rotated through an angle of 30°, may be given as:

$$e = E_0 \sin \omega\, t = E_0 \sin \theta = 351.858 \times \sin 30° = 175.929 \text{ V}$$

Problem 6.56: *A rectangular coil, of size* 15 *cm* × 40 *cm having* 200 *turns, is rotating in a uniform magnetic field at a constant angular speed of* 50 *rps about an axis perpendicular to the direction of magnetic field having a strength of* 0.08 *T. Determine the instantaneous value of the induced emf when the plane of the coil makes an angle with the magnetic lines of: i*) 0°*, ii*) 60°*, iii*) 90°*.*

Solution: $A = 15 \times 40$ cm^2, $N = 200$ Turns, $n = 50$ rps, $B = 0.08$ T,

$\theta_1 = (90° - 0°) = 90°$, $\theta_2 = (90° - 60°) = 30°$, $\theta_3 = (90° - 90°) = 0°$

The angular speed of the rotation of the coil may be given as:

$$\omega = 2\,\pi\,f = 2\,\pi \times 50 = 100\,\pi \text{ rad/sec}$$

The maximum value of induced emf in a coil rotating at a constant angular speed inside a uniform magnetic field may be given as:

$$E_0 = \pm N\,B\,A\,\omega = \pm N\,B\,A \times (2\,\pi\,f)$$

$$= \pm\, 200 \times 0.08 \times 15 \times 40 \times 10^{-4} \times (2\,\pi \times 50) = \pm\, 301.593 \text{ V}$$

The instantaneous value of the induced emf at three instants may respectively be given as:

$$e_1 = E_0 \sin \theta_1 = 301.593 \times \sin 90° = 301.593 \text{ V}$$

$$e_2 = E_0 \sin \theta_2 = 301.593 \times \sin 30° = 150.797 \text{ V}$$

$$e_3 = E_0 \sin \theta_3 = 301.593 \times \sin 0° = 0$$

Problem 6.57: *A closely wound rectangular coil of* 200 *turns and size* 30 *cm* × 10 *cm is rotating in a magnetic field of induction* 0.005 *Tesla, with a frequency of revolution* 1800 *rpm about an axis normal to the field. Determine the maximum value of the induced emf.*

Solution: $N = 200$ Turns, $A = 30 \times 10$ cm^2, $B = 0.005$ T, $n = 1800$ rpm

The maximum value of induced emf in a coil rotating at a constant angular speed inside a uniform magnetic field may be given as:

$$E_0 = \pm N\,B\,A\,\omega = \pm N\,B\,A\,(2\,\pi\,f) = \pm N\,B\,A \times \left(\frac{2\pi n}{60}\right)$$

$$= \pm\, 200 \times 0.005 \times 30 \times 10 \times 10^{-4} \times \left(\frac{2\pi \times 1800}{60}\right) = \pm\, 5.655 \text{ V}$$

Problem 6.58: *A rectangular coil of dimensions* 10 *cm* × 50 *cm consists of* 2000 *turns and rotates about an axis parallel to its longer side with a speed of* 2100 *rpm inside a field of strength* 0.1 *T normal to the axis of rotation. Determine: i*) *the maximum value of induced emf in the coil, ii*) *the instantaneous value of induced emf when the coil is oriented at an angle of* 60° *to the magnetic field.*

Solution: $A = 10 \times 50$ cm^2, $N = 2000$ Turns, $n = 2100$ rpm, $B = 0.1$ T, $\theta = (90° - 60°) = 30°$

The maximum value of induced emf in a coil rotating at a constant angular speed inside a uniform magnetic field may be given as:

$$E_0 = \pm N\,B\,A\,\omega = \pm N\,B\,A\,(2\,\pi\,f) = \pm N\,B\,A \times \left(\frac{2\pi n}{60}\right)$$

$$= \pm\,2000 \times 0.1 \times 10 \times 50 \times 10^{-4} \times \left(\frac{2\pi \times 2100}{60}\right) = \pm\,2199.11\text{ V}$$

The instantaneous value of the induced emf may be given as:

$$e = E_0 \sin \omega\, t = E_0 \sin \theta = 2199.11 \times \sin 30° = 1099.56\text{ V}$$

Problem 6.59: *The armature coil of a generator has* 20 *turns and its area is* 0.127 m^2. *Determine the speed, at which it must be rotated inside a magnetic field of* 0.2 *T, in order to induce an emf across the armature terminals having a peak value of* 160 *V.*

Solution: $N = 20$ Turns, $A = 0.127\text{ m}^2$, $B = 0.2$ T, $E_0 = 160$ V

The expression for the maximum value of induced emf in a coil rotating at a constant angular speed inside a uniform magnetic field may be given as:

$$E_0 = \pm N\,B\,A\,\omega = \pm N\,B\,A\,(2\,\pi\,f) = \pm N\,B\,A \times \left(\frac{2\pi n}{60}\right)$$

or, $$n = \frac{60 \times E}{2\pi N B A} = \frac{60 \times 160}{2\pi \times 20 \times 0.2 \times 0.127} = 3007.65\text{ rpm}$$

$$= \frac{3007.65}{60} = 50.13\text{ rps}$$

Problem 6.60: *A* 50 *turn coil of area* 500 cm^2 *is rotating at a rate of* 50 *rps perpendicular to a magnetic field of* 0.5 *Tesla. Determine the maximum value of induced emf in the coil.*

Solution: $N = 50$ Turns, $A = 500\text{ cm}^2$, $f = 50$ rps, $B = 0.5$ T

The maximum value of induced emf in a coil rotating at a constant angular speed inside a uniform magnetic field may be given as:

$$E_0 = \pm N\,B\,A\,\omega = \pm N\,B\,A\,(2\,\pi\,f) = \pm\,50 \times 0.5 \times 500 \times 10^{-4} \times (2\,\pi \times 50) = \pm\,392.7\text{ V}$$

Problem 6.61: *Determine the maximum emf induced in a coil of* 100 *turns and* 0.01 m^2 *area rotating at the rate of* 50 *rps about an axis perpendicular to a uniform magnetic field of* 0.05 *Tesla. If the resistance of the coil is* 30 Ω, *determine the maximum power generated by the rotating coil.*

Solution: $N = 100$ Turns, $A = 0.01\text{ m}^2$, $f = 50$ rps, $B = 0.05$ T, $R_{coil} = 30\ \Omega$

The maximum value of induced emf in a coil rotating at a constant angular speed inside a uniform magnetic field may be given as:

$$E_0 = \pm N\,B\,A\,\omega = \pm N\,B\,A\,(2\,\pi\,f) = \pm\,100 \times 0.05 \times 0.01 \times (2\,\pi \times 50) = \pm\,15.708\text{ V}$$

The value of maximum power generated by the rotating coil may be given as:

$$P_0 = \frac{E_0^2}{R_{coil}} = \frac{(15.708)^2}{30} = 8.224\text{ W}$$

6.11 Eddy Currents: We know very well, till this point of our discussion, that an emf is being induced in a coil / conductor, whenever there is a change in the magnetic flux linked with the coil / conductor. This induced emf lasts as long as the change in magnetic field is there.

Now, consider a conducting sheet or cube or a solid of any shape which is associated with the changing magnetic field. This changing magnetic field associated with this conducting sheet / cube / body will induce an emf in the conducting body. As, the conducting body is quite large and has no defined paths for the flow of current, the induced emf causes the flow of currents inside the conducting body along irregular but closed paths and in the form of eddies / whirlpools in a plane perpendicular to the direction of the magnetic field.

The induced currents inside a conducting body and flowing along the closed random paths within the conducting body, associated with the changing magnetic field, are known as ***Eddy Currents****.*

The eddy currents, so induced, also obey the ***Lenz's Law*** (another form of ***law of conservation of energy***) and oppose the basic cause due to which they are being induced, *i.e.* the change in magnetic field will be opposed by the eddy currents.

The eddy currents, so induced, flow internally inside the solid conducting body along the random but closed paths. So, flow of these eddy currents causes the ***Joules Heating*** **(*I^2R*)** ***Losses*** inside the solid's conducting body and are known as ***Eddy Current Losses***. The conducting body, exposed to changing magnetic fields, starts to heat up due to these ***Eddy Current Losses***.

Experiments to Demonstrate the Eddy Currents:

Experiment Number 1: Take a plane conducting sheet and make a pendulum with the help of a suspending thread, as shown in the Fig. 6.26. This pendulum can be set to oscillate (to and fro) in a magnetic field created by two poles of a strong electromagnet facing each other as shown in the figure. When pendulum is made to swing without the magnetic field, *i.e.* the electromagnet is turned OFF, it continues to swing freely for a longer duration. But, when the electromagnet is turned ON, the pendulum comes to standstill quickly. This happens due to the induced eddy currents in the conducting sheet, which are opposing the cause of induction of the eddy currents, *i.e.* change in magnetic field. The induced eddy currents flow counter-clock-wise as the plate swings into magnetic field and clock-wise as the plate swings out of the magnetic field to oppose the change in magnetic field, the reader may verify himself by applying ***Lenz's Law*** and ***Clock Rule***.

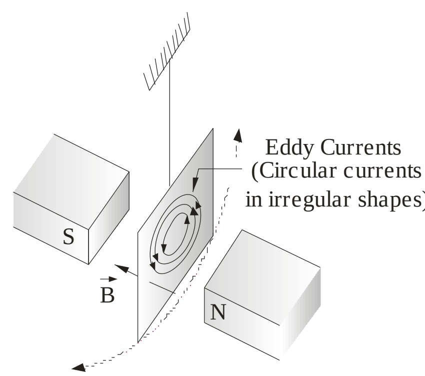


Fig. 6.26

Experiment Number 2: Take a plane conducting sheet, but with narrow slots cut in it, and make a pendulum with the help of a suspending thread, as shown in the Fig. 6.27. This pendulum can be set to oscillate (to and fro) in a magnetic field created by two poles of a strong electromagnet facing each other as shown in the figure. When pendulum is made to swing without the magnetic field, *i.e.* the electromagnet is turned OFF, it continues to swing freely for a longer duration. But, when the electromagnet is turned ON, the pendulum comes to standstill quickly, but this plate swings for longer duration than the plate without slots. This happens due to the fact that induced eddy currents has to follow a longer path in the conducting sheet, which offers more resistance to flow of eddy currents. Hence, smaller eddy currents are being induced in the slotted sheet than that of the sheet without slots. So, opposition to the oscillations became very small. The induced eddy currents flow counter-clock-wise as the plate swings into magnetic field and clock-wise as the plate swings out of the magnetic field to oppose the change in magnetic field, the reader may verify himself by applying ***Lenz's Law*** and ***Clock Rule***.

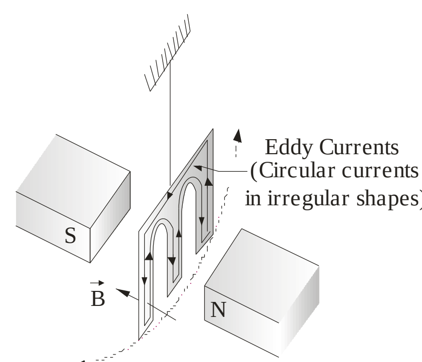


Fig. 6.27

Experiment Number 3: Take a cylindrical electromagnet which may be fed by an A.C. source and place a light metallic disc over it, as shown in the Fig. 6.28. As soon as the switch *S* is closed, an A.C. current sets up in the coil, which in turn sets up an alternating flux in the cylindrical electromagnet as well as inside the light metallic disc. The eddy currents are being induced in the light magnetic disc due to this alternating flux. These induced eddy currents tends to oppose the basic cause due to which they are being induced, *i.e.* the change in the magnetic flux linked with the light magnetic disc. So, if the top face of the electromagnet is a South pole at any instant of time, the bottom face of the light metallic disc will acquire the South polarity due to these eddy currents, resulting in a repulsive force. So, the light metallic disc is thrown up as soon as the current is switched ON in the electromagnet with the help of switch *S*.

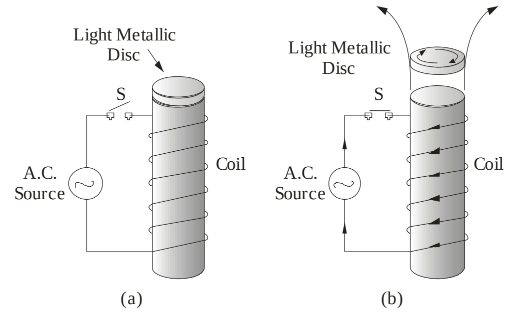


Fig. 6.28

Undesired Effects of Eddy Currents: As soon as the magnetic flux linked with the iron bodies of the transformer cores or the cylindrical cores of the motors changes due to the alternating nature of the flux or the rotation of the rotor's iron body, the eddy currents are being induced in the iron cores. These induced eddy currents in the transformer cores or the cores of the stator / rotor iron body of the generators / motors cause eddy current losses, which appears as heat in the iron cores of electrical machines, which lowers the efficiency of the electrical machines. If these eddy current losses are not within the permissible limits, the insulation of the windings embedded in the iron core may get damaged and the electrical machine may get unusable in a short time period than expected. We must take care to minimize these losses at the time of designing of the electrical machine in order to ensure the proper working and longer life of the electrical machine. Each 10° C rise in working temperature of the electrical machine will reduce the life of an electrical machine by 50%. The eddy current losses (P_e) in the iron core may be given as:

$$P_e \propto f^2 (\phi_m)^2 \qquad (6.29)$$

Where, f = frequency of the changing (alternating) flux

and, ϕ_m = Maximum / peak value of the magnetic flux

Minimization of Eddy Currents and Eddy Current Losses: The eddy currents will get reduced, if the resistance offered by the iron body to the flow of eddy currents will increase. This may be done by the use of laminated punching / stampings, as shown in the Fig. 6.29 (b), in place of a solid iron body shown in the Fig. 6.29 (a). The laminated punching / stampings have layers of insulation material (varnish) over them, so the eddy currents of individual stampings remain confined to flow within the same sheet (stamping) only, as shown in the Fig. 6.29 (b). The effective area of cross section of each laminated stamping is quite small as compared to that of the solid iron body, so the resistance offered by each laminated stamping to the flow of current is quite high and hence induced eddy currents will reduce significantly by use of laminated punching / stampings in place of a solid iron body.

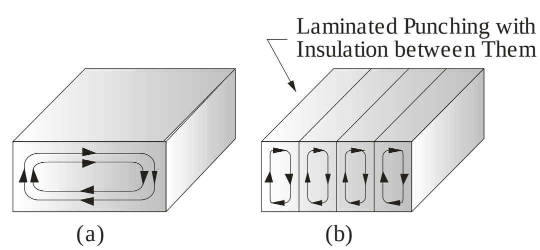


Fig. 6.29

Applications of Induced Eddy Currents: Although, eddy currents are undesirable, but we can take some benefits of eddy currents and their characteristic behavior. Eddy currents are very useful in following devices, due to their peculiar behavior.

1) **Induction Furnace:** If a metal specimen is placed in a strong and rapidly changing magnetic field (generated by a very high frequency A.C. source), very large amount of heat is produced due to large induced eddy currents in the metal specimen as ($P_e \propto f^2$). This heat produced by induced eddy

currents is sufficient to even melt the metal specimen. This process is frequently used in extraction of some metals from their ores in metal industries with the device named as "***Induction Furnace***" according to its function.

2) **Electromagnetic Damping:** Since the induced eddy currents opposes the basic cause (*i.e.* the change in magnetic field) due to which they are being induced, so they may be better used for damping (*to die out*) any unwanted oscillations inside the magnetic field. When a current is passed through a galvanometer (ammeter / voltmeter), its coil gets deflected from its rest position, such that the deflection is proportional to the quantity of current flowing through it. But the pointer oscillates about its steady state value before coming to rest in the final position. As, the coil oscillates in the magnetic field, induced currents is set in the coil and in the core on which the coil is wound, and by virtue of their nature they opposes the oscillations of the coil before coming to rest at its final value. So, the oscillations of the coil are damped (*died out*). This process is known as "***Electromagnetic Damping***". With a careful designing the galvanometer can be made *dead beat*, *i.e.* the coil does not oscillates at all, it gets deflected under the influence of the quantity to be measured and just comes to rest at its final position without any oscillations.

3) **Electric Brakes:** If a strong magnetic field is applied to the rotating drum attached to the wheel, the strong eddy currents induce in the drum and exert a torque on the drum in opposite direction to that of the motion in order to stop the train effectively and quickly. ***The electric braking is much more efficient than the mechanical breaking***.

4) **Speedometers:** A magnet attached to the wheel rotates at the same speed (rpm) as that of the wheel placed inside the aluminum drum to make a speedometer. The aluminum drum is carefully pivoted and held in position by a hair spring. The rotating magnet induces eddy currents in the aluminum drum proportional to the speed of rotating magnet, and the aluminum drum gets deflected due to opposition of these induced eddy currents by an angle depending on the speed of rotation of magnet to give a deflection on the display of the speedometer.

5) **Induction Motor:** A rotating magnetic field is induced in a 3-phase induction motor and two rotating magnetic fields in a single phase induction motor but in opposite directions to each other. This rotating magnetic field produces the induced currents in rotor windings and induced eddy currents in the rotor body. These induced currents, by opposition of the relative speed between the rotating magnetic field and rotor, produces the required torque to rotate the rotor of the motor at certain speed to do some useful mechanical work.

6) **Electromagnetic Shielding:** The property of the eddy currents to provide opposition to the cause due to which they are being induced may be used for *electromagnetic shielding*. If a magnetic field is suddenly switched on, the large eddy currents will be induced in the conducting metallic sheet, as shown in the Fig. 6.29. The sudden change in the magnetic field is only partially detected at points (say *P*) on the other side of the sheet, as the induced currents are in such a direction so as to oppose the change in magnetic field to weaken the building up magnetic flux of the field. So, at the time of switching on the magnetic field an equal and opposite magnetic field is built up by the induced currents.

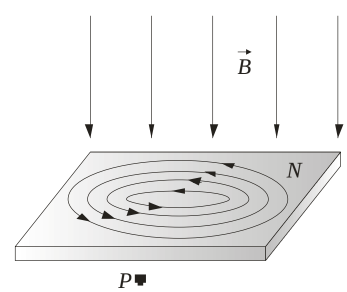


Fig. 6.30

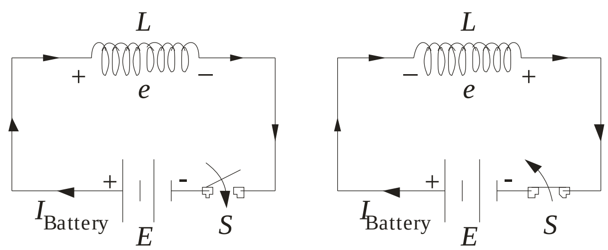
7) **Inducto-thermy:** The eddy currents may be used to heat localized tissues of the human body during medical treatment, known as ***Inducto-thermy***.

8) **Energy Meters:** In old (mechanically rotating disc) type of energy meters, the aluminum disc rotates inside the magnetic field due to the interaction between induced currents produced due to two magnetic fields, one field energized by the load current and another field energized by the load voltage.

6.12 Self Induction: A current flowing through a coil creates its own magnetic field associated with the coil itself. If somehow, the strength of current flowing through the coil changes, the flux associated with the coil also changes. This changing flux, associated with the coil, will induce an emf in the coil itself according to *Faradays Law of Electromagnetic Induction*. This is known as self induction in the coil, since the emf induced in the coil is result of the changing flux, due to changing current, produced due to the coil itself.

So, "*the phenomenon* of *production of the induced emf in a coil itself, when a changing current flows through it, is known as* ***Self Induction***".

A battery connected across a coil (known as ***Inductor, L***) through a tapping switch is shown in the Fig. 6.31 (a) and (b). When the tapping switch (S) is closed at an instant of time (t_1), the current (I) through the inductor (L) rises from zero to its steady state value, *i.e.* the current flowing through the inductor (L) is changing rapidly from zero to its steady state value in a short duration. An emf will be induced in the inductor during this short duration of time as shown in the Fig. 6.30 (a). The polarity of the induced emf is also shown in the figure, which indicates that the emf so induced (***Back Emf***) opposes the rate of change of the current flowing through the inductor.



(a) (b)

Fig. 6.31

Now consider the Fig. 6.31 (b), when the tapping switch (S) is opened at another instant of time (t_2), the current (I) through the inductor (L) falls from its steady state value to zero, *i.e.* the current flowing through the inductor (L) is changing rapidly from its steady state value to zero in a short duration. Again an emf will be induced in the inductor during this short duration of time as shown in the Fig. 6.31 (b). The polarity of the induced emf is also shown in the figure, which indicates that the emf so induced opposes the rate of change of the current flowing through the inductor.

Coefficient of Self Induction (L): The magnetic flux linkages of the coil (*Inductor*), having N turns, at any instant of the time may be given as:

$$\lambda \propto N\,\phi \propto I \qquad (6.30.1)$$

or,
$$\lambda = L\,I \qquad (6.30.2)$$

Where, L = ***Coefficient of Self Inductance*** or simply ***Inductance of the Coil / Inductor***.

If the current is changing during any duration of time, the rate of change of the flux linkages associated with the coil (*Inductor*) may be given as:

$$\frac{d\lambda}{dt} = L \times \frac{dI}{dt} \qquad (6.31)$$

The emf induced due to this changing flux associated with the coil (*Inductor*) may be given according to *Faraday's Law of Electromagnetic Induction* as:

$$E = -N \times \frac{d\phi}{dt} = -\frac{d\lambda}{dt} = -L \times \frac{dI}{dt}$$

or,
$$E = -N \times \frac{d\phi}{dt} = -L \times \frac{dI}{dt} \qquad (6.32)$$

Where, L = ***Coefficient of Self Inductance*** or simply ***Inductance of the Coil / Inductor***.

And, the (–)ve sign is due to *Lenz's law*, indicating the opposition of the basic cause due to which the emf is being induced.

Consider the equation (6.30.2), if $I = 1$ Amp, then $\lambda = L$.

So, "*the self inductance of a coil / inductor may be defined as the flux linkages of the coil, when a unit current flows through the coil / inductor*".

Consider the equation (6.32), if $\dfrac{dI}{dt} = 1$ Amp/sec, then $E = -L$

So, "*the self inductance of a coil / inductor may be defined as the emf induced in the coil, when the current flowing through the coil changes at a unit rate*".

S.I. Unit of Self Inductance: The reader may observe from equation (6.32) that:

$$L = \frac{E}{(dI/dt)} = \frac{\text{V}}{\text{A/sec}} = \text{V sec A}^{-1} = \Omega\text{-sec} = \text{henery (H)}$$

The self inductance of a coil is said to be ***One Henery,*** *if an emf of one volt is induced in the coil when the current through the coil changes at a unit rate.*

The reader may also observe form the equation (6.30.2) that:

$$L = \frac{\lambda}{I} = \frac{N\phi}{I} = \frac{\text{Wb}\times\text{Turns}}{\text{Amp}} = \text{Wb-Turns A}^{-1} \text{ or Wb A}^{-1}$$

6.13 Self Inductance of A Long Solenoid: Consider a long solenoid of length l and radius r such that $r \ll l$, and having n turns per unit length. If a current I flows through the coil of the solenoid, the magnetic field inside the solenoid is almost uniform and may be given as:

$$B = \mu_0 n I \tag{6.33}$$

The magnetic flux linked with each turn of the solenoid may be given as:

$$\phi = B A = \mu_0 n I A \tag{6.34}$$

The flux linkages of the solenoid, having turns $N = n\,l$, may be given as:

$$\lambda = N\phi = n\,l\,\phi = n\,l \times \mu_0 n I A = \mu_0 n^2 I A l \tag{6.35}$$

So, the self inductance of the solenoid may be given as:

$$L = \frac{\lambda}{I} = \frac{\mu_0 n^2 I A l}{I} = \frac{\mu_0 (N/l)^2 I A l}{I} = \frac{\mu_0 N^2 A}{l} \tag{6.36}$$

If the coil is wound over a magnetic material core having a relative permeability of μ_r, the self inductance of the solenoid may be given as:

$$L = \frac{\mu_0 \mu_r N^2 A}{l} \tag{6.37}$$

The reader may observe from above equation that: $L \propto \mu_r N^2 A$

So, the self inductance depends upon the relative permeability of the material of the core, number of turns in the coil and the area of cross section of the turns or core of the solenoid.

Alternatively: Ampere circuital law may be written as:

$$H\,l = N I \qquad \text{or,} \qquad \frac{B}{\mu_0 \mu_r}\, l = N I$$

$$\text{or,} \qquad \frac{\phi}{\mu_0 \mu_r A}\, l = N I \qquad \text{or,} \qquad \phi = \frac{\mu_0 \mu_r N A I}{l} \tag{6.38}$$

The induced emf across the coil may be given as:

$$E = -N \times \frac{d\phi}{dt} = -N \times \frac{d}{dt}\left(\frac{\mu_0 \mu_r N A I}{l}\right)$$

$$= -\frac{\mu_0 \mu_r N^2 A}{l} \times \frac{dI}{dt} = -L \times \frac{dI}{dt}$$

So, $$L = \frac{\mu_0 \mu_r N^2 A}{l} \qquad (6.39)$$

6.14 Phenomenon Associated with Self Inductance: Various peculiar phenomenon are associated with the self inductance of any circuit or element, due to which the performance parameters of a circuit changes / gets affected. Some of them are given below.

1. **Sparking during Making and Breaking of a Circuit:** The making and breaking of a circuit with the help of a switch is very sudden / quick process. When the switch is turned ON or turned OFF, a self induced emf sets up in the circuit, due to self inductance of the circuit, which opposes the change in current which is rising or falling very sharply. This induced emf acts at the small closing or opening gap of the switch and causes the breakdown of air. This results in sparking at the contacts of the switch during making or breaking of an electrical circuit.

2. **Non- Inductive Winding:** In standard resistance boxes and post office boxes, different resistance coils have to be used which must be standard and pure resistances (non-inductive resistances). For making a pure resistance, it must be ensured that the self inductance of the resistance must be zero. So, the wire, forming the standard resistance, must be doubled over itself and then wound in the form of a coil over a bobbin. Due to the doubling of the wire, as shown in the Fig. 6.32, the current in two halves of the wire flows in opposite directions. The inductive effects of the two halves of the coil, being in opposite directions, cancel each other. So, the net self inductance of the resistive coil is zero. Such a winding of the coil is known as ***non-inductive winding*** and can form a pure standard resistance for resistance boxes and post office boxes.

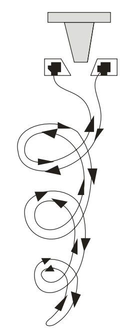


Fig. 6.32

3. **Electromagnetic Damping:** Since the induced eddy currents opposes the basic cause (*i.e.* the change in magnetic field) due to which they are being induced, so they may be better used for damping (*to die out*) any unwanted oscillations inside the magnetic field. When a current is passed through a galvanometer (ammeter / voltmeter), its coil gets deflected from its rest position such that the deflection is proportional to the quantity of current flowing through it. But the pointer oscillates about its steady state value before coming to rest in the final position. As, the coil oscillates in the magnetic field, induced currents is set in the coil and in the core on which the coil is wound, and by virtue of their nature they opposes the oscillations of the coil before coming to rest at its final value. So, the oscillations of the coil are damped (*died out*). This process is known as "***Electromagnetic Damping***". By careful designing a galvanometer can be made *dead beat*, *i.e.* the coil does not oscillates at all, it gets deflected under the influence of the quantity to be measured and just comes to rest at its final position without any oscillations.

6.15 Mutual Induction: A current flowing through a primary coil (P) creates a magnetic flux. If a secondary coil (S) is placed in vicinity of the primary coil (P), the flux produced by the primary coil (P) links both the coils. Now, if the strength of current flowing through the primary coil (P) changes, the flux linked with both the coil also change. This changing flux, associated with the secondary coil (S), will induce an emf in the secondary coil (S) according to *Faradays Law of Electromagnetic Induction*. This is known as mutual induction, since the emf induced in the secondary coil (S) is due to the changing flux produced by the primary coil (P), because of the changing current in primary coil (P).

So, "*the phenomenon* of *production of the induced emf in one coil due to the change in current of its neighboring coil, when a changing current flows through the neighboring coil, is known as* ***Mutual Induction***".

Two magnetically coupled coils (*primary* and *secondary*) placed in close vicinity are shown in the Fig. 6.33. A battery is connected across the primary coil through a tapping switch (k) and the secondary coil is shorted through the galvanometer. When the tapping switch (k) is closed at an instant of time (t_1), the current (I) through the primary coil (P) rises from zero to its steady state value, *i.e.* the current flowing through the primary coil (P) is changing rapidly from zero to its steady state value in a short duration. As a result of this changing current and the changing flux produced by the primary coil, an emf will be induced in the secondary coil (S) during this short duration of time and can be detected by the deflection in galvanometer due to the current flowing through the galvanometer due to this induced emf. The polarity of the induced emf in secondary coil is such that it opposes the rate of change of the current flowing through the primary coil.

Now, if the tapping switch (k) is opened at another instant of time (t_2), the current (I) through the primary coil (P) falls from its steady state value to zero, *i.e.* the current flowing through the primary coil (P) is changing rapidly from its steady state value to zero in a short duration. An emf will again be induced in the secondary coil (S) during this short duration of time. The polarity of the induced emf in secondary coil is such that it opposes the rate of change of the current flowing through the primary coil.

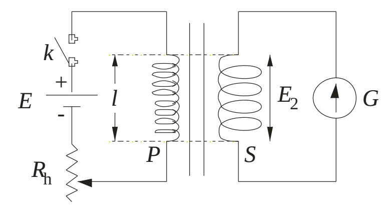


Fig. 6.33

Coefficient of Mutual Induction (M): The magnetic flux linkages of the secondary coil (S) at any instant of time may be given as:

$$\lambda_2 = N_2\,\phi \propto I_1 \tag{6.40.1}$$

$$\lambda_2 = M_{12}\,I_1 \tag{6.40.2}$$

If the current is changing during any duration of time, the rate of change of the flux linkages associated with the secondary coil (S) may be given as:

$$\frac{d\lambda_2}{dt} = N_2\,\frac{d\phi}{dt} = M_{12} \times \frac{dI_1}{dt} \tag{6.41}$$

The emf induced in secondary coil (S) due to these changing flux linkages associated with the secondary coil may be given according to *Faraday's Law of Electromagnetic Induction* as:

$$E_2 = -\frac{d\lambda_2}{dt} = -N_2 \times \frac{d\phi}{dt} = -M_{12}\,\frac{dI_1}{dt}$$

or, $$E_2 = -N_2 \times \frac{d\phi}{dt} = -M_{12}\,\frac{dI_1}{dt} \tag{6.42}$$

Where, M_{12} = ***Coefficient of Mutual Inductance*** or simply ***Mutual Inductance of the Combination***.

The magnetic field produced by the primary coil may be given as:

$$B = \mu_0\,n_1\,I_1 \tag{6.43}$$

The magnetic flux linked with each turn of the secondary coil may be given as:

$$\phi = B\,A_2 = \mu_0\,n_1\,I_1\,A_2 \tag{6.44}$$

The flux linkages of the secondary coil, having turns $N_2 = n_2\,l$, may be given as:

$$\lambda_2 = N_2\,\phi = n_2\,l\,\phi = n_2\,l \times \mu_0\,n_1\,I_1\,A_2 = \mu_0\,n_1\,n_2\,I_1\,A_2\,l \tag{6.45}$$

So, the mutual inductance of the secondary coil may be given as:

$$M_{12} = \frac{\lambda_2}{I_1} = \frac{\mu_0\, n_1\, n_2\, I_1\, A_2\, l}{I_1} = \frac{\mu_0\,(N_1/l)(N_2/l)\, I_1\, A_2\, l}{I_1} = \frac{\mu_0\, N_1\, N_2\, A_2}{l} \qquad (6.46)$$

If two coils are wound over the same core having a cross sectional area A, the mutual inductance of two coils may be given as:

$$M_{12} = \frac{\mu_0\, N_1\, N_2\, A}{l} \qquad (6.47)$$

If two coils are wound over a magnetic material core having a relative permeability of μ_r and cross sectional area A, the mutual inductance of two coils may be given as:

$$M_{12} = \frac{\mu_0\, \mu_r\, N_1\, N_2\, A}{l} \qquad (6.48)$$

The reader may observe from above equation that: $M \propto \mu_r\, N_1\, N_2\, A$

So, the mutual inductance between two coils depends upon the relative permeability of the material of the core, number of turns in each coil and the area of cross section of the turns or core.

Always remember that the coefficient of mutual inductance between two coils is:

$$M_{12} = M_{21} = M$$

Alternatively: Ampere circuital law for two coils wound on the same core of cross sectional area A may be written as:

$$H\, l = N_1\, I_1 \qquad \text{or,} \qquad \frac{B}{\mu_0\, \mu_r}\, l = N_1\, I_1$$

$$\text{or,} \qquad \frac{\phi}{\mu_0\, \mu_r\, A}\, l = N_1\, I_1 \qquad \text{or,} \qquad \phi = \frac{\mu_0\, \mu_r\, N_1\, A\, I_1}{l} \qquad (6.49)$$

The induced emf across the secondary coil may be given as:

$$E_2 = -N_2 \times \frac{d\phi}{dt} = -N_2 \times \frac{d}{dt}\left(\frac{\mu_0\, \mu_r\, N_1\, A\, I_1}{l}\right) = -\frac{\mu_0\, \mu_r\, N_1\, N_2\, A}{l} \times \frac{d I_1}{dt} = -M_{12} \times \frac{d I_1}{dt}$$

$$\text{So,} \qquad M_{12} = \frac{\mu_0\, \mu_r\, N_1\, N_2\, A}{l} \qquad (6.50)$$

Coefficient of Coupling: *The coefficient of coupling of two coils gives a measure of the manner in which two coils are coupled together.* If L_1 and L_2 are the self inductances of two individual coils and M is the mutual inductance between two coils, then the coefficient of coupling between two coils may be given as:

$$K = \frac{M}{\sqrt{L_1\, L_2}} \qquad (6.51)$$

The coefficient of coupling is a unit less quantity and its value lies between zero and one, i.e. $0 < K < 1$.

If two coils are coupled perfectly together, *i.e.* all the flux produced by primary coil is linked with the secondary coil, the value of mutual inductance (M) is maximum and the coefficient of coupling is one, *i.e.* M = maximum and $K = 1$.

On the other hand, if two coils are not coupled at all, *i.e.* the flux produced by primary coil is not linked with the secondary coil, the value of mutual inductance (M) is minimum and the coefficient of coupling is zero, *i.e.* $M = 0$ and $K = 0$.

6.16 Grouping of Inductors: The inductors (coils) may be connected in series / parallel or a combination of series / parallel as similar to that of capacitors and resistors. Let us examine the equivalent expressions for their combinations.

Inductors Connected in Series: If the inductors are connected one end to another then to another and so on, such that the current carrying by all the inductors is identical, they are said to be series connected. The inductors may be connected in series in two different ways as given below.

i) Let us first examine the two inductors connected in series when both the inductors are carrying the same current in the same direction as shown in the Fig. 6.34.

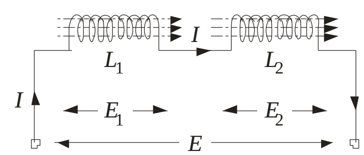


Fig. 6.34

The reader may observe that the two induced emf's across the two inductors are in series.

So, $$E = E_1 + E_2 \qquad (6.52)$$

If the rate of change of the current flowing through two inductors is $\frac{dI}{dt}$, the emf's induced in two inductors may respectively be given as:

$$E_1 = -L_1 \frac{dI}{dt} - M_{12} \frac{dI}{dt} = -L_1 \frac{dI}{dt} - M \frac{dI}{dt} \qquad (6.53)$$

and, $$E_2 = -L_2 \frac{dI}{dt} - M_{21} \frac{dI}{dt} = -L_2 \frac{dI}{dt} - M \frac{dI}{dt} \qquad (6.54)$$

Now, $$E = E_1 + E_2 = -L_1 \frac{dI}{dt} - M \frac{dI}{dt} - L_2 \frac{dI}{dt} - M \frac{dI}{dt}$$

or, $$-L_{eq} \frac{dI}{dt} = -(L_1 + L_2 + 2M) \frac{dI}{dt}$$

So, $$L_{eq} = (L_1 + L_2 + 2M) \qquad (6.55)$$

If two coils are far apart and have no mutual coupling, then $M = 0$ and the equivalent inductance of two series connected inductors may be given as:

$$L_{eq} = (L_1 + L_2) \qquad (6.56)$$

So, the general expression for *N* series connected inductors, having no mutual coupling at all, may be given as:

$$L_{eq} = (L_1 + L_2 + L_3 + \ldots\ldots + L_N) \qquad (6.57)$$

ii) Now, let us examine the two inductors connected in series when both the inductors are carrying the same current in opposite directions as shown in the Fig. 6.35.

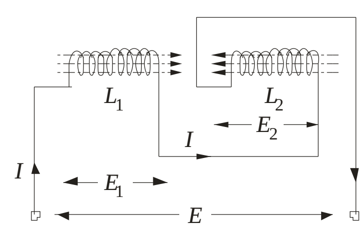


Fig. 6.35

The reader may observe that the two induced emf's across the two inductors are in series.

So, $$E = E_1 + E_2 \qquad (6.58)$$

If the rate of change of the current flowing through two inductors is $\frac{dI}{dt}$, the emf's induced in two inductors may respectively be given as:

$$E_1 = -L_1 \frac{dI}{dt} + M_{12} \frac{dI}{dt} = -L_1 \frac{dI}{dt} + M \frac{dI}{dt} \quad (6.59)$$

and, $$E_2 = -L_2 \frac{dI}{dt} + M_{21} \frac{dI}{dt} = -L_2 \frac{dI}{dt} + M \frac{dI}{dt} \quad (6.60)$$

Now, $$E = E_1 + E_2 = -L_1 \frac{dI}{dt} + M \frac{dI}{dt} - L_2 \frac{dI}{dt} + M \frac{dI}{dt}$$

or, $$-L_{eq} \frac{dI}{dt} = -(L_1 + L_2 - 2M) \frac{dI}{dt}$$

So, $$L_{eq} = (L_1 + L_2 - 2M) \quad (6.61)$$

If two coils are far apart and have no mutual coupling, then $M = 0$ and the equivalent inductance of two inductors connected in series may be given as:

$$L_{eq} = (L_1 + L_2) \quad (6.62)$$

So, the general expression for equivalent inductance of N inductors connected in series, having no mutual coupling at all, may be given as:

$$L_{eq} = (L_1 + L_2 + L_3 + \ldots\ldots + L_N) \quad (6.63)$$

Inductors Connected in Parallel: If the inductors are connected across same pair of points, such that the potential difference across two inductors is equal, they are said to be connected in parallel.

Consider two inductors connected in parallel as shown in the Fig. 6.36, the potential difference across two inductors is E and the current flowing through two inductors are I_1 and I_2 respectively, such that:

$$I = I_1 + I_2 \quad (6.64)$$

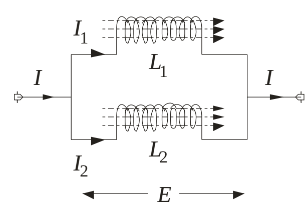


Fig. 6.36

Differentiating above equation *w.r.t.* the time t:

$$\frac{dI}{dt} = \frac{dI_1}{dt} + \frac{dI_2}{dt} \quad (6.65)$$

If the rate of change of current through two inductors are $\frac{dI_1}{dt}$ and $\frac{dI_2}{dt}$ respectively, the emf induced across two inductors may respectively be given as:

$$E_1 = E = -L_1 \frac{dI_1}{dt} - M_{12} \frac{dI_2}{dt} = -L_1 \frac{dI_1}{dt} - M \frac{dI_2}{dt} \quad (6.66)$$

and, $$E_2 = E = -L_2 \frac{dI_2}{dt} - M_{21} \frac{dI_1}{dt} = -L_2 \frac{dI_2}{dt} - M \frac{dI_1}{dt} \quad (6.67)$$

Equating equations (6.66) and (6.67):

$$-L_1 \frac{dI_1}{dt} - M \frac{dI_2}{dt} = -L_2 \frac{dI_2}{dt} - M \frac{dI_1}{dt}$$

or, $$(L_1 - M) \frac{dI_1}{dt} = (L_2 - M) \frac{dI_2}{dt} \quad (6.68)$$

Now reconsider the equation (6.66):

$$E = -L_1 \frac{dI_1}{dt} - M \frac{dI_2}{dt} = -L_1 \frac{dI_1}{dt} - M \times \frac{(L_1 - M)}{(L_2 - M)} \times \frac{dI_1}{dt}$$

$$= -\frac{L_1(L_2 - M) + M(L_1 - M)}{(L_2 - M)} \times \frac{dI_1}{dt} = -\frac{L_1 L_2 - L_1 M + L_1 M - M^2}{(L_2 - M)} \times \frac{dI_1}{dt}$$

or, $$E = -\frac{L_1 L_2 - M^2}{(L_2 - M)} \times \frac{dI_1}{dt}$$

or, $$\frac{dI_1}{dt} = -\frac{E}{\left[\dfrac{L_1 L_2 - M^2}{(L_2 - M)}\right]} \tag{6.69}$$

Similarly reconsider the equation (6.67):

$$E = -L_2 \frac{dI_2}{dt} - M \frac{dI_1}{dt} = -L_2 \frac{dI_2}{dt} - M \times \frac{(L_2 - M)}{(L_1 - M)} \times \frac{dI_2}{dt}$$

$$= -\frac{L_2(L_1 - M) + M(L_2 - M)}{(L_1 - M)} \times \frac{dI_2}{dt} = -\frac{L_1 L_2 - L_2 M + L_2 M - M^2}{(L_1 - M)} \times \frac{dI_2}{dt}$$

$$= -\frac{L_1 L_2 - M^2}{(L_1 - M)} \times \frac{dI_2}{dt}$$

or, $$\frac{dI_2}{dt} = -\frac{E}{\left[\dfrac{L_1 L_2 - M^2}{(L_1 - M)}\right]} \tag{6.70}$$

Putting equation (6.69) and (6.70) in equation (6.65):

$$\frac{dI}{dt} = \frac{dI_1}{dt} + \frac{dI_2}{dt} = -\frac{E}{\left[\dfrac{L_1 L_2 - M^2}{(L_2 - M)}\right]} - \frac{E}{\left[\dfrac{L_1 L_2 - M^2}{(L_1 - M)}\right]}$$

or, $$-\frac{E}{L_{eq}} = -E \times \left[\frac{(L_2 - M)}{(L_1 L_2 - M^2)} + \frac{(L_1 - M)}{(L_1 L_2 - M^2)}\right]$$

or, $$\frac{1}{L_{eq}} = \frac{(L_1 + L_2 - 2M)}{(L_1 L_2 - M^2)}$$

or, $$L_{eq} = \frac{(L_1 L_2 - M^2)}{(L_1 + L_2 - 2M)} \tag{6.71.1}$$

If two inductors are carrying the currents in opposite direction, the equivalent inductance may be given as:

$$L_{eq} = \frac{(L_1 L_2 - M^2)}{(L_1 + L_2 + 2M)} \tag{6.71.2}$$

If two coils are far apart and have no mutual coupling, then $M = 0$ and the equivalent inductance of two inductors connected in parallel may be given as:

$$L_{eq} = \frac{L_1 L_2}{L_1 + L_2} \quad (6.72.1)$$

or, $$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} \quad (6.72.2)$$

So, the general expression for equivalent inductance of N inductors connected in parallel, having no mutual coupling at all, may be given as:

$$\frac{1}{L_{eq}} = \left(\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + ... + \frac{1}{L_N}\right) \quad (6.73)$$

Problem 6.62: *Determine the self inductance of a coil, due to which a magnetic flux of* 40 *milli Weber is produced due to a current of* 2 *A flowing through it.* [CBSE 2001-02]

Solution: ϕ = 40 mWb, I = 2 A

The flux produced by a current carrying coil may be given as:

$$\phi = L\, I$$

So, $$L = \frac{\phi}{I} = \frac{40 \times 10^{-3}}{2} = 20 \times 10^{-3} \text{ H} = 20 \text{ mH}$$

Problem 6.63: *A* 200 *turn coil, having a self inductance of* 20 *mH, carries a current of* 4 *mA. Determine the magnetic flux linked with each turn of the coil.*

Solution: N = 200 Turns, L = 20 mH, I = 4 mA

The total flux produced by the coil may be given as:

$$\phi = L\, I$$

So, the flux linked with each turn of the coil may be given as:

$$\phi_{Turn} = \frac{L\, I}{N} = \frac{20 \times 10^{-3} \times 4 \times 10^{-3}}{200} = 4 \times 10^{-7} \text{ Wb/turn}$$

Problem 6.64: *If a rate of change of current of 4 A/sec induces an emf of* 10 *mV in a solenoid, determine the self inductance of the solenoid.* [CBSE 1995-96]

Solution: $\frac{d\,I}{d\,t}$ = 4 A/sec, E = 10 mV

The expression for the induced emf in a solenoid due to changing current may be given as:

$$E = -L \times \frac{d\,I}{d\,t}$$ (negative sign shows opposition only)

or, $$L = \frac{E}{(d\,I / d\,t)} = \frac{10 \times 10^{-3}}{4} = 2.5 \times 10^{-3} \text{ H} = 2.5 \text{ mH}$$

Problem 6.65: *A* 12 *V battery is connected to a* 6 Ω, 10 *H coil through a switch drives a constant current through the circuit. The switch is suddenly opened. If it takes* 1 *milli sec to open the switch, determine the average emf induced across the switch.*

Solution: V = 12 V, R = 6 Ω, L = 10 H, $d\,t$ = 1 ms

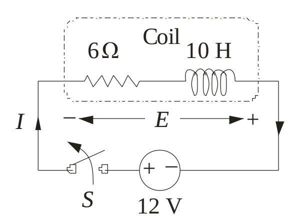


Fig. 6.37

The induced emf across the coil will appear across the switch when current interrupts from its steady state value to zero.

The value of steady state current through the circuit may be given as:

$$I = \frac{V}{R} = \frac{12}{6} = 2 \text{ A}$$

(As inductor behaves as short circuit for steady state current, i.e. $\frac{dI}{dt} = 0$)

So, the change in current in the coil during the time dt may be given as:

$$dI = I_{final} - I_{initial} = 0 - 2 = -2 \text{ A}$$

So, the induced emf across the switch may be given as:

$$E = -L \times \frac{dI}{dt} = -10 \times \left(-\frac{2}{1\times10^{-3}}\right) = 20{,}000 \text{ V} = 20 \text{ kV}$$

Problem 6.65: *An inductor of* 5 *H self inductance carries a steady current of* 2 *A. How can a self induced emf of value* 50 *V be made to appear across the inductor?* [Punjab 2000-01]

Solution: $L = 5$ H, $I = 2$ A, $E = 50$ V

The expression for the induced emf in a solenoid due to changing current may be given as:

$$E = -L \times \frac{dI}{dt} \qquad \text{(negative sign shows opposition only)}$$

or, $$dt = \frac{L \times dI}{E} = \frac{5\times(2-0)}{50} = 0.2 \text{ sec}$$

Hence, a self induced emf may be made to appear across this inductor, if the current of 2 A is interrupted within 0.2 sec.

Problem 6.66: *Determine the self inductance of an air cored solenoid having length of* 50 *cm, radius of* 2 *cm and* 500 *Turns.*

Solution: $l = 50$ cm, $r = 2$ cm, $N = 500$ Turns

The self inductance of the solenoid may be given as:

$$L = \frac{\mu_0 \mu_r N^2 A}{l} = \frac{\mu_0 \mu_r N^2 (\pi r^2)}{l} = \frac{4\pi\times10^{-7}\times1\times(500)^2\times\pi\times(0.02)^2}{0.50}$$

$$= 7.896 \times 10^{-4} \text{ H} = 0.7896 \text{ mH}$$

Problem 6.67: *An air cored solenoid with* 30 *cm length, area of cross section* 25 *cm*^{2} *and* 500 *turns, carries a current of* 2.5 *A. The current through the solenoid is suddenly switched OFF in a short duration of time of* 1 *milli-sec. Determine the average back emf induced across the ends of the open switch in the circuit.* [NCERT]

Solution: $l = 30$ cm, $A = 25$ cm^2, $N = 500$ Turns, $I = 2.5$ A, $dt = 1$ ms

The self inductance of the solenoid may be given as:

$$L = \frac{\mu_0 \mu_r N^2 A}{l} = \frac{4\pi\times10^{-7}\times1\times(500)^2\times25\times10^{-4}}{0.30}$$

$$= 2.618 \times 10^{-3} \text{ H} = 2.618 \text{ mH}$$

The induced emf in the solenoid due to changing current will appear across the open switch and may be given as:

$$E = -L \times \frac{d\,I}{d\,t} = -2.618 \times 10^{-3} \times \frac{(0-2.5)}{1\times10^{-3}} = 6.545 \text{ V}$$

Problem 6.68: *A large circular coil of radius R, and a small circular coil of radius r are put in vicinity of each other. If the coefficient of mutual induction for this pair is 1 mH, determine the flux linked with the larger coil when a current of* 0.5 *A flows through the smaller coil.*

If the current in the smaller coil suddenly falls to zero, what would be its effect on the larger coil? [CBSE 2007-08]

Solution: $r_{large\ coil} = R$, $r_{small\ coil} = r$, $M = 1$ mH, $I_1 = 0.5$ A

The flux linked with the larger coil may be given as:

$$\phi_{large\ coil} = M\,I_1 = 1 \times 10^{-3} \times 0.5 = 0.5 \times 10^{-3} \text{ Wb} = 0.5 \text{ mWb}$$

If the current in the smaller coil suddenly falls to zero, the flux linked with the larger coil changes and an induced emf will be produced across the larger coil given by the relationship:

$$E_{large\ coil} = -M\,\frac{d\,I_1}{d\,t} = -1 \times 10^{-3} \times \frac{0.5}{d\,t} = \frac{0.5}{d\,t} \text{ mV}$$

Problem 6.69: *Determine the mutual inductance of a pair of coils, if a current change of* 6 *A in one coil causes the flux in the second coil of* 2000 *turns to change by* 1.2 *mWb/turn.*

Solution: $d\,I_1 = 6$ A, $N_2 = 2000$ Turns, $(d\,\phi_2)_{per\ turn} = 1.2$ mWb/turn

The total flux linked with the second coil may be given as:

$$\phi_2 = M\,I_1$$

or, $$d\,\phi_2 = M\,d\,I_1$$

The flux linked with the individual turn of the second coil may be given as:

$$(d\,\phi_2)_{per\ turn} = \frac{d\,\phi_2}{N_2} = \frac{M\,d\,I_1}{N_2}$$

or, $$M = \frac{N_2 \times (d\phi_2)_{\text{per turn}}}{d\,I_1} = \frac{2000 \times 1.2 \times 10^{-3}}{6} = 0.4 \text{ H}$$

Problem 6.70: *An emf of* 0.5 *V is developed in a secondary coil, when the current in primary coil changes from* 5 *A to* 2 *A in* 300 *milli-seconds. Determine the mutual inductance of two coils.* [ISCE 1992-93]

Solution: $E = 0.5$ V, $d\,I_1 = 2 - 5 = -3$ A, $t = 300$ milli-seconds

The expression for the induced emf in secondary coil due to change in the current of primary coil may be given as:

$$E_2 = -M \times \frac{d\,I_1}{d\,t}$$ (negative sign shows opposition only)

or, $$M = -\frac{E}{(d\,I_1 / d\,t)} = -\frac{0.5 \times 300 \times 10^{-3}}{(-3)} = 0.05 \text{ H} = 50 \text{ mH}$$

Problem 6.71: *If the current in the primary circuit of a pair of coils changes from* 5 *A to* 1 *A in* 0.02 *sec, determine: i) the induced emf in the secondary coil if the mutual inductance between the two coils is* 0.5 *H, ii) the change of flux per turn in the secondary, if it has* 200 *Turns.*

Solution: $d\,I_1 = 1 - 5 = -4$ A, $t = 0.02$ sec, $M = 0.5$ H, $N_2 = 200$ Turns

The expression for the induced emf in secondary coil due to change in the current of primary coil may be given as:

$$E_2 = -M \times \frac{d\,I_1}{d\,t} \qquad \text{(negative sign shows opposition only)}$$

$$= -0.5 \times \left(-\frac{4}{0.02}\right) = 100 \text{ V}$$

The total flux linked with the second coil may be given as:

$$\phi_2 = M\,I_1$$

or, $$d\,\phi_2 = M\,d\,I_1$$

The change in flux linked with the individual turn of the secondary coil may be given as:

$$(d\,\phi_2)_{per\ turn} = \frac{d\,\phi_2}{N_2} = \frac{M\,d\,I_1}{N_2} = \frac{0.5\times(-4)}{200} = -0.01 \text{ Wb} = -10 \text{ mWb}$$

Problem 6.72: *An air cored solenoid of* 50 *cm length and* 2 *cm in radius has* 500 *turns. Another coil of* 50 *turns is also wound over the first coil. Determine: i) the mutual inductance of two coils, ii) induced emf in the second coil when the current in the primary coil changes from* 0 *A to* 5 *A in* 0.02 *sec.*

Solution: $l = 50$ cm, $r = 2$ cm, $N_1 = 500$ turns, $N_2 = 50$ Turns, $d\,I_1 = 5 - 0 = 5$ A

$d\,t = 0.02$ sec

The mutual inductance of two coils may be given as:

$$M = \frac{\mu_0\,\mu_r\,N_1\,N_2\,A}{l} = \frac{4\pi\times10^{-7}\times1\times500\times50\times\pi\times(0.02^2)}{0.50} = 7.896\times10^{-5} \text{ H} = 78.96\ \mu\text{H}$$

The induced emf in secondary coil due to change in the current of primary coil may be given as:

$$E_2 = -M \times \frac{d\,I_1}{d\,t} \qquad \text{(negative sign shows opposition only)}$$

$$= -78.96\times10^{-6}\times\frac{5}{0.02} = -0.01974 \text{ V} = -19.74 \text{ mV}$$

Problem 6.73: *A solenoid coil has* 50 *turns per cm along its length and a cross sectional area of* $4\times10^{-4}\ m^2$. *Another coil of* 200 *turns is also wound on the same solenoid. If two coils are electrically insulated from each other, determine the coefficient of mutual inductance between the two coils. Given that* $\mu_0 = 4\,\pi\times10^{-7}\ NA^{-2}$. [ISCE 1997-98]

Solution: $n_1 = 50$ Turns/cm $= 5000$ Turns/m, $A = 4\times10^{-4}$ m^2, $N_2 = 200$ Turns

The mutual inductance of two coils may be given as:

$$M = \frac{\mu_0\,\mu_r\,N_1\,N_2\,A}{l} = \mu_0\,\mu_r\,n_1\,N_2\,A$$

$$= 4\,\pi\times10^{-7}\times1\times5000\times200\times4\times10^{-4} = 5.027\times10^{-4} \text{ H} = 502.7\ \mu\text{H}$$

Problem 6.74: *A solenoid with a length* 50 *cm has* 20 *turns per cm and a cross sectional area of* 40 cm^2 *and is completely surrounded by another co-axial solenoid of same length but area of cross section of* 25 cm^2 *with* 25 *turns per cm. Determine the mutual inductance of the system.* [NCERT]

Solution: $l_1 = l_2 = l = 50$ cm, $n_1 = 20$ Turns/cm = 2000 Turns/m, $A_1 = 40\ \text{cm}^2$, $A_2 = 25\ \text{cm}^2$

$n_2 = 25$ Turns/cm = 2500 Turns/m

The mutual inductance of two coils may be given as:

$$M = \frac{\mu_0\, \mu_r\, N_1\, N_2\, A_2}{l} = \mu_0\, \mu_r\, n_1\, (n_2 \times l)\, A_2$$

$$= 4\,\pi \times 10^{-7} \times 1 \times 2000 \times 2500 \times 0.5 \times 25 \times 10^{-4} = 7.85 \times 10^{-3}\ \text{H} = 7.85\ \text{mH}$$

Problem 6.75: a) *A toroidal solenoid with an air core has an average radius of* 15 *cm, area of cross section* 12 cm^2 *and* 1200 *turns. Obtain the self inductance of the toroid. Ignore the field variation along the cross-section of the toroid.*

b) *A second coil of* 300 *turns is wound closely on the same toroid. If the current in the primary coil is increased from* 0 *A to* 2 *A in* 0.05 *sec, obtain the induced emf in the secondary coil.* [NCERT]

Solution: $r = 15$ cm, $A = 12\ \text{cm}^2$, $N_1 = 1200$ Turns, $N_2 = 300$ Turns, $d\,I = 2 - 0 = 2$ A

$d\,t = 0.05$ sec

The self inductance of the toroid may be given as:

$$L = \frac{\mu_0\, \mu_r\, N^2\, A}{l} = \frac{\mu_0\, \mu_r\, N^2\, A}{2\,\pi\, r} = \frac{4\pi \times 10^{-7} \times 1 \times (1200)^2 \times 12 \times 10^{-4}}{2\,\pi \times 0.15}$$

$$= 2.304 \times 10^{-3}\ \text{H} = 2.304\ \text{mH}$$

The mutual inductance of two coils wound on the same toroid may be given as:

$$M = \frac{\mu_0\, \mu_r\, N_1\, N_2\, A}{l} = \frac{\mu_0\, \mu_r\, N_1\, N_2\, A}{2\,\pi\, r} = \frac{4\pi \times 10^{-7} \times 1200 \times 300 \times 12 \times 10^{-4}}{2\,\pi \times 0.15}$$

$$= 5.76 \times 10^{-4}\ \text{H} = 576\ \mu\text{H}$$

So, the induced emf in the secondary coil may be given as:

$$E_2 = -\,M \times \frac{d\,I_1}{d\,t} \qquad \text{(negative sign shows opposition only)}$$

$$= -\,576 \times 10^{-6} \times \frac{2}{0.05} = -\,0.02304\ \text{V} = -\,23.04\ \text{mV}$$

Problem 6.76: *A short solenoid of length* 4 *cm, radius* 2 *cm and number of turns* 100 *lying inside on the axis of a long solenoid of length* 80 *cm and number of turns* 1500, *as shown in the Fig.* 6.38. *Determine the flux through the long solenoid, if a current of* 3 *A flows through the short solenoid. Also determine the mutual inductance of two solenoids.* [NCERT]

Solution: $l_1 = 80$ cm, $N_1 = 1500$ Turns, $l_2 = 4$ cm, $r_2 = 2$ cm, $N_2 = 100$ Turns, $I_2 = 3$ A

Since the smaller solenoid is lying inside the larger solenoid, so the flux produced by one solenoid is completely associated with the second solenoid.

The magnetic field inside the larger solenoid may be given as:

$$B_1 = \mu_0\, \mu_r\, n_1\, I_1 = \frac{\mu_0\, \mu_r\, N_1\, I_1}{l_1}$$

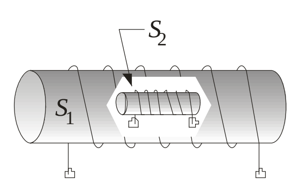


Fig. 6.38

The flux linkages of the smaller solenoid may be given as:

$$\lambda_2 = N_2\, \phi_2 = N_2\, B_1\, A_2 = N_2 \times \frac{\mu_0\, \mu_r\, N_1\, I_1}{l_1} \times A_2 = M_{21}\, I_1$$

So, the mutual inductance of two solenoids may be given as:

$$M_{21} = \frac{\mu_0\, \mu_r\, N_1\, N_2\, A_2}{l_1} = \frac{\mu_0\, \mu_r\, N_1\, N_2\, (\pi r_2^2)}{l_1} = \frac{4\pi \times 10^{-7} \times 1 \times 1500 \times 100 \times \pi \times (0.02)^2}{0.80}$$

$$= 2.96 \times 10^{-4} \text{ H} = 296\ \mu\text{H}$$

The total flux linkages of the longer solenoid due to current in the smaller solenoid may be given as:

$$\lambda_1 = M_{12} \times I_2 = M_{21} \times I_2 = 296 \times 10^{-6} \times 3 = 8.88 \times 10^{-4} \text{ Wb} = 0.888 \text{ mWb}$$

Problem 6.77: *Three inductors are connected as shown in the Fig.* 6.39. *Determine the equivalent inductance of the system across the points A and B.* [Punjab 1992-93]

Solution: $L_1 = 0.75$ H, $L_2 = L_3 = 0.5$ H

The equivalent inductance across the points A and B may be given as:

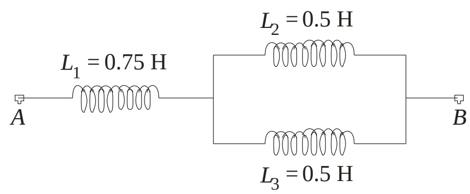


Fig. 6.39

$$L_{AB} = L_1 \text{ (series) } [L_2 \text{ (parallel) } L_3]$$

$$= L_1 + \frac{L_2\, L_3}{L_2 + L_3} = 0.75 + \frac{0.5 \times 0.5}{0.5 + 0.5}$$

$$= 1.0 \text{ H}$$

Problem 6.78: *A magnetic flux of* 5 μ*Wb is linked with a coil, when a current of* 1 *mA flows through it. Determine the self inductance of the coil* [Haryana 1999-2000, CBSE 1992-93]

Solution: $\phi = 5\ \mu$Wb, $I = 1$ mA

The expression for the flux linked with a coil may be given as:

$$\phi = L\, I$$

So, $$L = \frac{\phi}{I} = \frac{5 \times 10^{-6}}{1 \times 10^{-3}} = 5 \times 10^{-3} \text{ H} = 5 \text{ mH}$$

Problem 6.79: *Determine the induced emf in a coil of* 10 *H inductance in which the current changes from* 8 *A to* 3 *A in* 0.2 *sec.*

Solution: $L = 10$ H, $d\, I = 3 - 8 = -5$ A, $d\, t = 0.2$ sec

The induced emf due to changing current may be given as:

$$E = -L \times \frac{d\, I}{d\, t} = -10 \times \left(-\frac{5}{0.2}\right) = 250 \text{ V}$$

Problem 6.80: *A magnetic flux of* 0.8 *mWb is linked with each turn of a* 200 *turns coil, when there is an electric current of* 4 *A is flowing through it. Determine the self inductance of the coil.*

Solution: $\phi = 0.8$ mWb/turn, $N = 200$ Turns, $I = 4$ A

The expression for the flux linkages of the coil may be given as:

$$\lambda = N\, \phi = L\, I$$

or, $$L = \frac{N\phi}{I} = \frac{200 \times 0.8 \times 10^{-3}}{4} = 0.04 \text{ H} = 40 \text{ mH}$$

Problem 6.81: *The self inductance of an inductor (coil) having 100 turns is 20 mH. Determine the magnetic flux through the cross section of the coil corresponding to a current of 4 mA. Also, determine the total flux linkages of the inductor.* [CBSE 1999-2000]

Solution: N = 100 Turns, L = 20 mH, I = 4 mA

The flux linkages of the cross section of the inductor (coil) may be given as:

$$\lambda = N\,\phi = L\,I$$

or, $$\phi = \frac{L\,I}{N} = \frac{20 \times 10^{-3} \times 4 \times 10^{-3}}{100} = 8 \times 10^{-7} \text{ Wb}$$

Total flux linkages of the coil may be given as:

$$\lambda = N\,\phi = 100 \times 8 \times 10^{-7} = 8 \times 10^{-5} \text{ Wb}$$

Problem 6.82: *A coil of inductance 0.5 H is connected to a 18 V battery. Determine the rate of growth of current.*

Solution: L = 0.5 H, E = 18 V

The emf induced across the inductor will be same as that of the emf of battery during the growth/decay of current. The expression for the induced emf across an inductor may be given as:

$$E = -L \times \frac{d\,I}{d\,t} \qquad \text{(negative sign shows opposition only)}$$

So, $$\frac{d\,I}{d\,t} = \frac{E}{L} = \frac{18}{0.5} = 36 \text{ A/sec}$$

Problem 6.83: *An average emf of 25 V is induced in an inductor when the current in it is changed from 2.5 A in one direction to the same value in opposite direction in 0.1 sec. Determine the self inductance of the inductor.*

Solution: E = 25 V, $d\,I$ = 2.5 – (– 2.5) = 5 A, $d\,t$ = 0.1 sec

The expression for the induced emf across an inductor may be given as:

$$E = -L \times \frac{d\,I}{d\,t} \qquad \text{(negative sign shows opposition only)}$$

or, $$L = \frac{E}{(d\,I/d\,t)} = \frac{25 \times 0.1}{5} = 0.5 \text{ H}$$

Problem 6.84: *A inductor (coil) has a self inductance of 10 mH. Determine the maximum magnitude of the induced emf in the inductor, when a current i = 0.1 sin 200t Amp is flowing through it.*

Solution: L = 10 mH, i = 0.1 sin 200t Amp

The expression for the induced emf across an inductor may be given as:

$$e = -L \times \frac{d\,I}{d\,t} = -10 \times 10^{-3} \times \frac{d}{d\,t}(0.1 \sin 200t)$$

$$= -10 \times 10^{-3} \times 0.1 \times 200 \times \cos 200t = -0.2 \cos 200t$$

So, the maximum magnitude of the induced emf, $E = 0.2$ V

Problem 6.85: *Determine the self inductance of a solenoid of length* 40 *cm, area of cross section* 20 *cm*^{2} *and total number of turns* 800.

Solution: $l = 40$ cm, $A = 20$ cm^2, $N = 800$ Turns

The self inductance of the solenoid may be given as:

$$L = \frac{\mu_0 \mu_r N^2 A}{l} = \frac{4\pi\times10^{-7}\times1\times(800)^2\times20\times10^{-4}}{0.40} = 4.02 \times 10^{-3} \text{ H} = 4.02 \text{ mH}$$

Problem 6.86: *The current in a solenoid of* 240 *turns having a length of* 12 *cm and a radius of* 2 *cm changes at the rate of* 0.8 *Amp/sec. Determine the induced emf in the solenoid.*

Solution: $N = 240$ Turns, $l = 12$ cm, $r = 2$ cm, $\frac{dI}{dt} = 0.8$ A/sec

The induced emf across the solenoid may be given as:

$$E = -L \times \frac{dI}{dt} \quad \text{(negative sign shows opposition only)}$$

$$= -\frac{\mu_0 \mu_r N^2 A}{l}\times\frac{dI}{dt} = -\frac{4\pi\times10^{-7}\times1\times(240)^2\times\pi\times(0.02)^2}{0.12}\times 0.8$$

$$= -6.06 \times 10^{-4} \text{ V}$$

Problem 6.87: *Determine the mutual inductance between two coils when a current of* 2 *A changes to 6 A within* 2 *sec and induces an emf of* 20 *mV in the secondary coil.* [Punjab 1998-99]

Solution: $dI = (6 - 2) = 4$ A, $t = 2$ sec, $E = 20$ mV

The expression for the induced emf in secondary coil due to change of current in primary coil may be given as:

$$E = -M \times \frac{dI}{dt} \quad \text{(negative sign shows opposition only)}$$

or, $$M = \frac{E}{(dI/dt)} = \frac{20\times10^{-3}\times2}{4} = 0.01 \text{ H} = 10 \text{ mH}$$

Problem 6.88: *The mutual induction between two coils is* 2.5 *H. If the current in one coil is changed with a rate of* 2 *Amp/sec, determine the induced emf in another coil.*

Solution: $M = 2.5$ H, $\frac{dI}{dt} = 2$ A/sec

The induced emf in secondary coil due to change of current in primary coil may be given as:

$$E = -M \times \frac{dI}{dt} \quad \text{(negative sign shows opposition only)}$$

$$= -2.5 \times 2 = -5 \text{ V}$$

Problem 6.89: *In the spark coil of a car, an emf of* 40 *kV is induced in the secondary when the primary current changes from* 4 *A to* 0 *A in* 10 µ*sec. Determine the mutual inductance between the primary and secondary windings of the spark coil.*

Solution: $E = 40$ kV, $dI = (0 - 4) = -4$ A, $dt = 10$ µsec

The expression for the induced emf in secondary coil due to change of current in primary coil may be given as:

$$E = -M \times \frac{d\,I}{d\,t} \qquad \text{(negative sign shows opposition only)}$$

or, $$M = \frac{E}{(d\,I/d\,t)} = \frac{40\times10^{3}\times10\times10^{-6}}{(-4)} = 0.1 \text{ H} = 100 \text{ mH}$$

Problem 6.90: *If the current in the primary coil of a pair of coils changes from* 10 *A to* 0 *A in* 0.1 *sec, determine: i) the induced emf in the secondary coil, if the mutual inductance between two coils is* 2 *H, ii) the change of flux per turn in the secondary coil if it has* 500 *turns.*

Solution: $d\,I = (0 - 10)$ A = – 10 A, $d\,t = 0.1$ sec, $M = 2$ H, $N = 500$ Turns

The induced emf in secondary coil due to change of current in primary coil may be given as:

$$E = -M \times \frac{d\,I}{d\,t} \qquad \text{(negative sign shows opposition only)}$$

$$= -2 \times \left(-\frac{10}{0.1}\right) = 200 \text{ V}$$

The change in flux linked with the secondary coil may be given as:

$$N \times \Delta\,\phi_{per\ turn} = \phi_2 - \phi_1 = M\,I_2 - M\,I_1 = M \times (I_2 - I_1)$$

$$\Delta\,\phi_{per\ turn} = \frac{M \times d\,I}{N} = \frac{2\times(-10)}{500} = -0.04 \text{ Wb/turn} = -40 \text{ mWb/turn}$$

Problem 6.91: *A conducting wire of* 100 *turns is wound over* 1 *cm as a secondary coil, near the center of a solenoid of* 100 *cm length and* 2 *cm radius having* 1000 *turns. Determine the mutual inductance between two coils.*

Solution: $N_2 = 100$ Turns, $l_2 = 1$ cm, $l_1 = 100$ cm, $r = 2$ cm, $N_1 = 1000$ Turns

The mutual inductance between the two coils may be given as:

$$M = \frac{\mu_0\,\mu_r\,N_1\,N_2\,A_2}{l_1} = \frac{4\pi\times10^{-7}\times1\times1000\times100\times\pi\times(0.02)^2}{100\times10^{-2}}$$

$$= 0.1579 \times 10^{-3} \text{ H} = 0.1579 \text{ mH} = 157.9\ \mu\text{H}$$

Problem 6.92: *A solenoid has* 2000 *turns wound over a length of* 0.3 *m. The cross sectional area of the solenoid is* 1.2×10^{-3} m^2*. A secondary coil of* 300 *turns is also wound closely around its central section. If a current of* 2 *A is reversed in* 0.25 *sec, determine the induced emf in secondary coil.*

Solution: $N_1 = 2000$ Turns, $l_1 = 0.3$ m, $A = 1.2 \times 10^{-3}$ m^2, $N_2 = 300$ Turns,

$d\,I = 2 - (-2) = 4$ A, $d\,t = 0.25$ sec

The mutual inductance between the two coils may be given as:

$$M = \frac{\mu_0\,\mu_r\,N_1\,N_2\,A_2}{l_1} = \frac{4\pi\times10^{-7}\times1\times2000\times300\times1.2\times10^{-3}}{0.3}$$

$$= 3.016 \times 10^{-3} \text{ H} = 3.016 \text{ mH}$$

The induced emf in secondary coil due to change of current in primary coil may be given as:

$$E = -M \times \frac{dI}{dt} \quad \text{(negative sign shows opposition only)}$$

or, $$E = -3.016 \times 10^{-3} \times \left(\frac{4}{0.25}\right) = -0.04826 \text{ V} = 48.26 \text{ mV}$$

Problem 6.93: *Determine the mutual inductance between two coils, if a current of* 10 *A in the primary coil changes the flux by* 5 *mWb per turn in the secondary coil of* 200 *turns. Also, determine the induced emf across the ends of the secondary coil in* 0.5 *sec.*

Solution: $I_1 = 10$ A, $\Delta \phi_{per\ turn} = 5$ mWb/turn, $N_2 = 200$ Turns, $dt = 0.5$ sec

The flux linkages of two mutually coupled coils may be given as:

$$\lambda = N_2 \, \Delta \phi_{per\ turn} = M I_1$$

or, $$M = \frac{N_2 \, \Delta\phi_{per\ turn}}{I_1} = \frac{200 \times 5 \times 10^{-3}}{10} = 0.1 \text{ H} = 100 \text{ mH}$$

The induced emf in secondary coil due to change of current in primary coil may be given as:

$$E = -M \times \frac{dI_1}{dt} \quad \text{(negative sign shows opposition only)}$$

$$= -0.1 \times \left(\frac{10}{0.5}\right) = -2 \text{ V}$$

SHORT ANSWER TYPE QUESTIONS FOR EXERCISE

1. Define: *i*) *electromagnetic induction*, *ii*) *Fraday's laws of electromagnetic induction*, *iii*) *dynamically induced emf*, *iv*) *statically induced emf*.
2. Define: *i*) *Fleming's left hand rule*, *ii*) *Fleming's right hand rule*, *iii*) *Right hand thumb rule*. Also, state which *Fleming's law* is for *generating action* and which one is for *motoring action*?
3. Derive the expression for *induced emf* in a conductor from *Lorentz force* and *energy considerations*.
4. Show one complete cycle of alternating emf, *w.r.t. the angle of rotating coil*, induced in the rotating coil inside a magnetic field with the help of suitable diagrams.
5. Explain *eddy currents* and the location and pattern of induced eddy currents. Also give the beneficial aspects and applications of eddy currents and the method to reduce the eddy current losses. How the eddy currents are related to the frequency of the magnetic field / supply?
6. Define self induction in a coil and name the induced emf in the coil during self induction. Also give the phenomenon associated with the self inductance of an electrical circuit. Determine the coefficient of self inductance for a coil.
7. Define mutual induction between two coils. Determine the coefficient of mutual inductance and coefficient of coupling between two coils. Name at least one electrical machine, which works on the principal of mutual induction.
8. Predict the direction of induced currents in the loops A and B in the same plane, where the current I in the wire is increasing steadily, shown in the Fig. 6.40 (a).
9. Predict the direction of induced currents in the loops A and B in the same plane, where the current I in the wire is decreasing steadily, shown in the Fig. 6.40 (a).

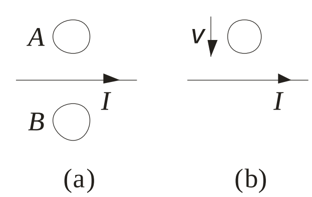


Fig. 6.40

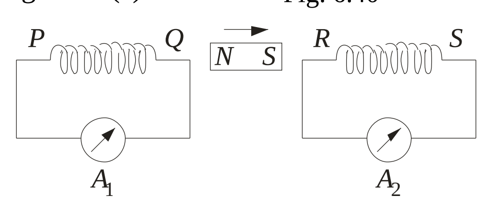


Fig. 6.41

10. Predict the direction of induced current in the metal ring when the ring is moved towards the straight current (I) carrying conductor with a constant speed v, as shown in the Fig. 6.40 (b).

11. A bar magnet is moved in the direction indicated by the arrow, in the Fig. 6.41, between two coils *PQ* and *RS*. Predict the directions of induced current in each coil.

12. The closed loop (*PQRS*) of a conducting wire is moved into a uniform magnetic field at right angles to the plane of the paper, as shown in the Fig. 6.42. Predict the direction of the induced current in the loop (*PQRS*).

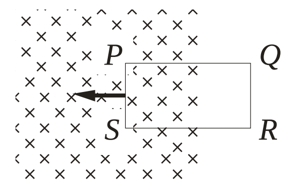


Fig. 6.42

13. Define and explain the term wattles current with the help of a neat phasor diagram.

14. Two bar magnet are quickly moved towards a metallic loop connected across a capacitor '*C*', as shown in the Fig. 6.43. Predict the polarity of emf across the capacitor due to induced current.

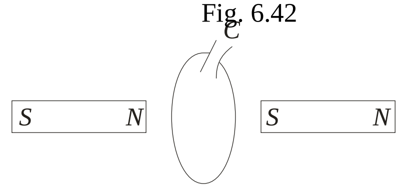


Fig. 6.43

VERY SHORT ANSWER TYPE QUESTIONS

1. Predict the direction of induced currents in the loops *A* and *B* in the same plane, where the current *I* in the wire is increasing steadily, as shown in the Fig. 6.40 (a). [CBSE, D 2012]

2. Predict the direction of induced currents in the loops *A* and *B* in the same plane, where the current *I* in the wire is decreasing steadily, as shown in the Fig. 6.40 (a). [CBSE, D 2012]

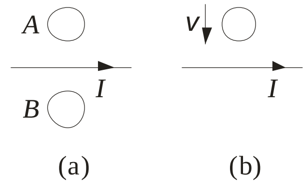


Fig. 6.40

3. Predict the direction of induced current in the metal ring when the ring is moved towards the straight current (*I*) carrying conductor with a constant speed *v*, as shown in the Fig. 6.40 (b). [CBSE, D 2012]

4. A bar magnet is moved in the direction indicated by the arrow, in the Fig. 6.41, between two coils *PQ* and *RS*. Predict the directions of induced current in each coil. [CBSE, AI 2012]

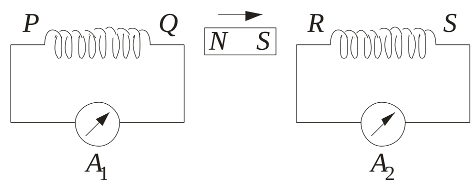


Fig. 6.41

5. The closed loop (*PQRS*) of a conducting wire is moved into a uniform magnetic field at right angles to the plane of the paper, as shown in the Fig. 6.42. Predict the direction of the induced current in the loop (*PQRS*). [CBSE, F 2012]

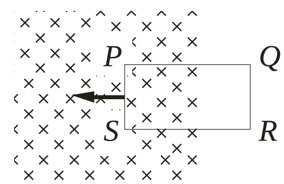


Fig. 6.42

6. Define and explain the term wattles current with the help of a neat phasor diagram. [CBSE, D 2011]

7. Two bar magnet are quickly moved towards a metallic loop connected across a capacitor '*C*', as shown in the Fig.6.43. Predict the polarity of emf across the capacitor due to induced current. [CBSE, AI 2011]

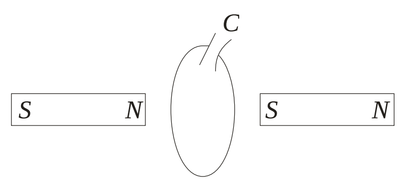


Fig. 6.43

8. Two bar magnet are quickly moved towards a metallic loop connected across a capacitor '*C*', as shown in the Fig.6.45. Predict the polarity of emf across the capacitor due to induced current. [CBSE, AI 2011]