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Sector 15, Noida

CLASS 12 - MATHEMATICS

Paper 4

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

- All the questions are compulsory.
- The question paper consists of 36 questions divided into 4 sections A, B, C, and D. Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 6 questions of 4 marks each. Section D comprises of 4 questions of 6 marks each.
- There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 2 marks each, two questions of 4 marks each, and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
- Use of calculators is not permitted.

Section A

1. Let for any matrix M ,M^{-1} exist. Which of the following is not true. **[1]**

   a) none of these
   b) $(M^{-1})^{-1} = M$
   c) $(M^{-1})^2 = (M^2)^{-1}$
   d) $(M^{-1})^{-1} = (M^{-1})^1$

2. The roots of the equation $\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & 2x & 5x^2 \end{vmatrix} = 0$ are **[1]**

   a) –1, – 2
   b) –1 , 2
   c) 1 , –2
   d) 1 , 2

3. Let f (x + y) = f(x) + f(y) $\forall$ x, y $\in \mathbf{R}$. Suppose that f (6) = 5 and f ‘ (0) = 1, then f ‘ (6) is equal to **[1]**

   a) 1
   b) 30
   c) None of these
   d) 25

4. $\frac{d}{dx}(\tan^{-1}(\sec x + \tan x)$ is equal to **[1]**

   a) $-\frac{1}{2}$
   b) $\frac{1}{2}$
   c) $\frac{1}{2\sec x(\sec x + \tan x)}$
   d) None of these

5. Solution of $\frac{dy}{dx} = 1 + x + y + xy$ is **[1]**

   a) $\log|1+y| = 2x + \frac{x^2}{2} + C$
   b) $\log|1+y| = x + \frac{x^2}{2} + C$
   c) None of these
   d) $\log|1+y| = x + \frac{x^2}{2} + Cy$

6. If $3\sin^{-1}\left(\frac{2x}{1+x^2}\right) - 4\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) + 2\tan^{-1}\left(\frac{2x}{1-x^2}\right) = \frac{\pi}{3}$. Then, x = **[1]**

a) $\frac{1}{\sqrt{3}}$

b) $\frac{1}{\sqrt{2}}$

c) 2

d) 1

7. If A and B are two events such that P(A) = $\frac{1}{4}$, P(B) = $\frac{1}{2}$ and $P(A \cap B) = \frac{1}{8}$, Find P(not A and not B). **[1]**

a) $\frac{1}{8}$

b) $\frac{2}{5}$

c) $\frac{3}{5}$

d) $\frac{3}{8}$

8. $\int_0^2 [2x]$ is equal to , where [.] denotes the Greatest Integer Function **[1]**

a) 2

b) 0

c) 3

d) 4

9. Find the Cartesian equation of the plane $\vec{r}.\left(2\hat{i} + 3\hat{j} - 4\hat{k}\right) = 1$ **[1]**

a) 2x + 3y – 4 z = 3

b) 2x + 3y – 4 z = 2

c) 2x + 3y – 4 z = 4

d) 2x + 3y – 4 z = 1

10. Find $\left|\vec{a} \times \vec{b}\right|$, if $\vec{a} = 3\hat{i} + \hat{j} + 2\hat{k}\ and\ \vec{b} = 2\hat{i} - 2\hat{j} + 4\hat{k}$ **[1]**

a) $8\sqrt{3}$

b) $19\sqrt{3}$

c) $19\sqrt{5}$

d) $17\sqrt{2}$

11. Fill in the blanks: **[1]**

The set of second elements of all ordered pairs in R, i.e. {y : (x, y) ∈ R} is called the ________ of relation R.

12. Fill in the blanks: **[1]**

The probabiltiy of drawing two clubs from a well shuffled pack of 52 cards is ________.

13. Fill in the blanks: **[1]**

The product of any matrix by the scalar ________ is the null matrix.

14. Fill in the blanks: **[1]**

The value of $\int \frac{dx}{\sqrt{16-9x^2}}$

OR

Fill in the blanks:

The indefinite integral of $2x^3 + 4$ is ________.

15. Fill in the blanks: **[1]**

The linear inequalities or restrictions on the variables of an LPP are called ________

OR

Fill in the blanks:

A linear function Z = px + qy (p and q are constants) which has to be maximised or minimised, is called an _______ function.

16. If the points (2, - 3), (λ, -1) and (0, 4) are collinear, find the value of λ. **[1]**

17. If a line makes angles 90°, 60° and θ with X, Y and Z-axes respectively, where θ is acute angle, **[1]**

then find θ.

18. If $\int_0^1 (3x^2 + 2x + k)\, dx = 0$, then find the value of k. **[1]**

OR

Evaluate $\int \tan^2 x \sec^4 x dx$

19. If $P(x) = a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^{2n}$ be a polynomial in $x \in R$ with $0 < a_1 < a_2 \ldots < a_n$, then show that P(x) has a minimum at x = 0 only **[1]**

20. If $\vec{a} = 3\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} - 4\hat{j} - 3\hat{k}$, find $|\vec{a} - 2\vec{b}|$. **[1]**

Section B

21. f: R $\rightarrow$ R be defined as f(x) = 3x check whether the function is one - one ,onto or other. **[2]**

22. If $x\sqrt{1+y} + y\sqrt{1+x} = 0$ for -1<x<1 prove that $\frac{dy}{dx} = -\frac{1}{(1+x)^2}$ **[2]**

OR

Show that the function defined by $g(x) = x - [x]$ is discontinuous at all integral points. Here [x] denotes the greatest integer less than or equal to x.

23. Vectors $\vec{a}$ and $\vec{b}$ be such that $|\vec{a}| = 3$, and $\left|\vec{b}\right| = \frac{\sqrt{2}}{3}$, then $\vec{a} \times \vec{b}$ is a unit vector. Find angle between $\vec{a}$ and $\vec{b}$. **[2]**

24. Find the equation of the tangent and normal to the given curve at the indicated point: **[2]**
$y = x^4 - 6x^3 + 13x^2 - 10x + 5$ at (1, 3)

OR

x and y are the sides of two square such that $y = x - x^2$ Find the rate of change of the area of second square with respect to the area of first square.

25. If the lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-5}$ are perpendicular, find the value of 'k'. **[2]**

26. Determine P(E|F): A dice is thrown three times.E : 4 appears on the third toss, F : 6 and 5 appears respectively on first two tosses. **[2]**

Section C

27. Prove that: $\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right), x \in [0, 1]$ **[4]**

28. Find the value of K so that function f is continuous at the indicated point: **[4]**
$f(x) = \begin{cases} Kx + 1 \ if \ x \leq \pi \\ \cos x \ if \ x > \pi \end{cases}$ at $x = \pi$

OR

If x = cos t(3 - 2 $\cos^2$ t) and y = sin t(3 - 2 $\sin^2$ t), then find the value of $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$.

29. Two biased dice are thrown together. For the first die $P(6) = \frac{1}{2}$, the other scores being equally likely while for the second die, $P(1) = \frac{2}{5}$ and the other scores are equally likely. Find the probability distribution of ‘the number of one seen’. **[4]**

30. Maximise Z = 3x + 4y, subject to the constraints: $x + y \leqslant 1, x \geqslant 0, y \geqslant 0$. **[4]**

31. Solve the differential equation $\frac{dy}{dx} + y\cot x = 2\cos x$, given that y = 0, when $x = \frac{\pi}{2}$. **[4]**

OR

Solve the following differential equation.
$\frac{dy}{dx} + y = \cos x - \sin x$

32. Evaluate $\int \frac{x^2+3x-1}{(x+1)^2}$dx **[4]**

Section D

33. For what values of a and b, the following system of equations is consistent? **[6]**

x + y + z = 6

2x + 5y + az = b

x + 2y + 3z = 14

OR

Given the matrices

A = $\begin{bmatrix} 2 & 1 & 1 \\ 3 & -1 & 0 \\ 0 & 2 & 4 \end{bmatrix}$, B = $\begin{bmatrix} 9 & 7 & -1 \\ 3 & 5 & 4 \\ 2 & 1 & 6 \end{bmatrix}$ and C = $\begin{bmatrix} 2 & -4 & 3 \\ 1 & -1 & 0 \\ 9 & 4 & 5 \end{bmatrix}$

Verify that (A + B) + C = A + (B + C).

34. An open box with a square base is to be made out of a given quantity of cardboard of area C^2 sq units. Show that the maximum volume of box is $\frac{C^3}{6\sqrt{3}}$ cu units. **[6]**

35. Find the area of the region enclosed by the parabola $y^2 = x$ and the line x + y = 2. **[6]**

OR

Find the area of region bounded by the triangle whose vertices are (-1, 1), (0, 5) and (3, 2) using integration.

36. Find the equation of the plane which contains two parallel lines $\frac{x-4}{1} = \frac{y-3}{-4} = \frac{z-2}{5}$ and $\frac{x-3}{1} = \frac{y+2}{-4} = \frac{z}{5}$. **[6]**