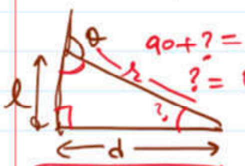
2.1 Magnetic Field due to Straight Wire

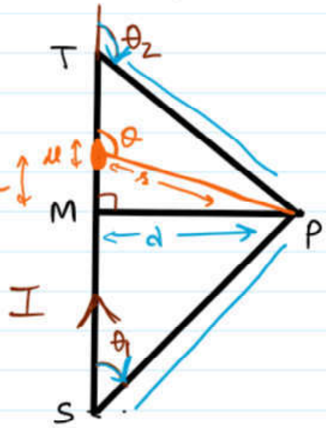
Consider a straight wire carrying current 'I' as shown & we need to find 'B' at P

$\boxed{MP = d}$



$90 + ? = \theta$

$? = \theta - 90^\circ$



As per Right Hand Rule, magnetic Field at P due to 'dl' will be inside the plane of this paper

$\boxed{r^2 = d^2 + l^2}$

$\tan(\theta - 90) = \frac{l}{d}$

$-\tan(90 - \theta) = \frac{l}{d}$

$\boxed{-\cot\theta = \frac{l}{d}}$

$\boxed{l = -d\cot\theta}$ (★)

$dl = (-d)(-\mathrm{cosec}^2\theta)\, d\theta$

$\boxed{dl = d \cdot \mathrm{cosec}^2\theta\, d\theta}$

From Biot-Savart law

$$dB = \frac{\mu_0}{4\pi} \frac{I\, dl \sin\theta}{r^2}$$

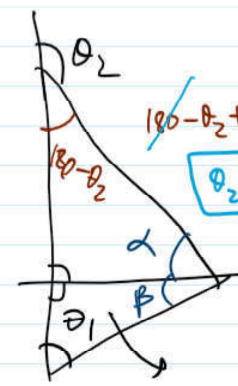
$$dB = \frac{\mu_0}{4\pi} \frac{(I)(d)(\mathrm{cosec}^2\theta)(d\theta)(\sin\theta)}{(d^2 + l^2)}$$

$$dB = \frac{\mu_0}{4\pi} \frac{(I)(d)(\mathrm{cosec}^2\theta)(d\theta)(\sin\theta)}{d^2 + d^2\cot^2\theta}$$

$$dB = \frac{\mu_0}{4\pi} \frac{(I)\cancel{(d)}\cancel{(\mathrm{cosec}^2\theta)}(d\theta)\sin\theta}{d^{\cancel{2}}\cancel{(1+\cot^2\theta)}}$$

$$\int dB = \int \frac{\mu_0 I}{4\pi d} \sin\theta\, d\theta$$

$$B = \frac{\mu_0 I}{4\pi d} \int_{\theta_1}^{\theta_2} \sin\theta\, d\theta$$



$\cancel{180} - \theta_2 + \alpha + 90^\circ = \cancel{180}$

$\boxed{\theta_2 = \alpha + 90}$

$$B = \frac{\mu_0 I}{4\pi d} - (\cos\theta)_{\theta_1}^{\theta_2}$$

$\theta_1 + \beta + 90^\circ = 180^\circ$

$\boxed{\theta_1 = 90 - \beta}$

$$B = \frac{\mu_0 I}{4\pi d} \left[-(\cos\theta_2 - \cos\theta_1)\right]$$

$$\Rightarrow \boxed{B = \frac{\mu_0 I}{4\pi d} (\cos\theta_1 - \cos\theta_2)} \quad (★)$$

In terms of α & β

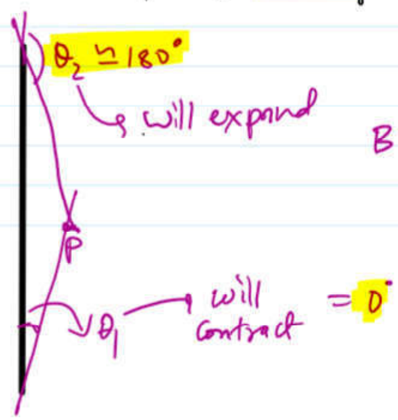
$$B = \frac{\mu_0 I}{\ } \left(\cos(90 - \beta) - \cos(90 + \alpha)\right)$$

In terms of α & β

$$B = \frac{\mu_0 I}{4\pi d}\left(\cos(90-\beta) - \cos(90+\alpha)\right)$$

$$\boxed{B = \frac{\mu_0 I}{4\pi d}\left(\sin\beta + \sin\alpha\right)} \quad ⭑$$

Case 1: Assume wire is of infinite length [Assume 'P' is located very close to wire]

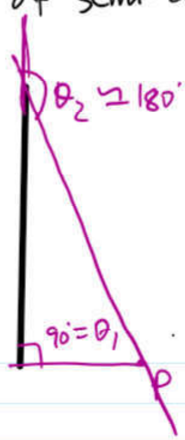


$$B = \frac{\mu_0 I}{4\pi d}\left(\cos\theta_1 - \cos\theta_2\right)$$

$$B = \frac{\mu_0 I}{4\pi d}\left(\cos 0° - \cos 180°\right)$$

$$B = \frac{\mu_0 I}{4\pi d}(2) = \frac{\mu_0 I}{2\pi d}$$

Case 2: Wire is of semi-∞ length [one end known & other end located very far]



$$B = \frac{\mu_0 I}{4\pi d}\left(\cos\theta_1 - \cos\theta_2\right)$$

$$B = \frac{\mu_0 I}{4\pi d}\left(\cos 90° - \cos 180°\right)$$

$$\boxed{B = \frac{\mu_0 I}{4\pi d}}$$