1.MATHEMATICAL PHYSICS

POLYNOMIALS

BESSEL

Eqn : $x^2y''+xy'+(x^2 - n^2)y = 0$

Function : $J_n(x) = \sum_{r=0}^{\infty} \frac{(-1)^r \left(\frac{x}{2}\right)^{n+2r}}{r!\,\overline{|n+r+1}}$ $J_{1/2}(x) = \sqrt{\frac{2}{\Pi x}}\sin x$ $J_{-1/2}(x) = cos x$

Generating function : coefficient of t^n in the expansion of $e^{x/2(t-1/t)} = \sum_{n=-\infty}^{\infty} J_n(x)t^n$

Orthogonality : $\int_0^1 xJ_m(\alpha x)J_n(\beta x)dx = 0$

LEGENDRE

Eqn : $(1 - x^2)y'' - 2xy' + n(n+1)y = 0$

Function= $P_n(x)$ $P_n(x)=0$ for n=odd $p_0(x)=1$,$p_1(x)=x$, $p_2(x)=\frac{1}{2}(3x^2-1)$,

Generating function : $\sum P_n(x)t^n = \left(1-2xt+t^2\right)^{-\frac{1}{2}}$ $p_3(x)= \frac{1}{2}(5x^3-3x)$

Orthogonality: $\int_0^1 p_m(x)p_n(x)dx = \begin{Bmatrix} o \;\; if\ n \neq m \\ \frac{2}{2n+1}\;\; if\ \ n = m \end{Bmatrix}$

Rodrigue's formula: $P_n(x)= \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n$

LAGUERRE

Eqn : $xy''+(1-x)y'+ny = 0$ $L_0(x)=1$, $L_1(x)=-x+1$, $L_2(x) = x^2-4x+2$

Function: $L_n(x)$

Generating function: $\frac{e^{\frac{-xt}{1-t}}}{1-t} = \sum_{n=0}^{\infty}\frac{L_n(x)}{n!}t^n$

Orthogonality: $\int_0^{\infty} e^{-x}L_m(x)L_n(x)dx = \begin{Bmatrix} 0\ if\ m \neq n \\ (n!)^2\ if\ m = n \end{Bmatrix}$

Rodrigue's formula: $L_n(x)= e^x\frac{d^n}{dx^n}[x^n e^x]$

HERMITE

Eqn: $y''-2xy'+2ny=0$ $H_{2n}(0) = (-1)^n\frac{(2n)!}{n!}$ $H_{2n+1}(0)=0$ $H_n(-x)=(-1)^nH_n(x)$

Function= $H_n(x)$ $H_0(x)=1$, $H_1(x)=2x$, $H_2(x)=4x^2-2$

Generating function: $e^{2tx-t^2} = \sum_{n=0}^{\infty}\frac{H_{n(x)}}{n!}t^n$

Orthogonality: $\int_{-\infty}^{\infty} e^{-x^2}H_m(x)H_n(x)dx = \begin{Bmatrix} 0\ if\ m \neq n \\ 2^n n!\sqrt{\pi}\ if\ m = n \end{Bmatrix}$

Rodrigue's formula: $H_n(x)= (-1)^n e^{x^2}\frac{d^n}{dx^n}e^{-x^2}$

DIRAC DELTA FUNCTION

One dimensional dirac delta function is defined as $\delta(x) = 0$ at $x \neq 0$

$$\int_{-\infty}^{\infty}\delta(x)dx = 1$$

$$\int_{-\infty}^{\infty} f(x)\delta(x)dx = f(0)$$

If we shift the origin of the co-ordinate system to x=a then $\int_{-\infty}^{\infty} f(x)\delta(x-a)dx = f(a)$

Eg: $\int_{-\infty}^{\infty} x\delta(x-4)dx = f(4) = 4$

SOME REPRESENTATIONS OF DIRAC DELTA FUNCTION(DDF)

1. $\delta(x) = \lim_{g\to 0} \frac{\sin gx}{\pi x}$ 2. $\delta(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{igx}dg$ 3. $\delta(x)$ =u'(x) where u(x)= $\begin{cases} 0 \; for \; x < 0 \\ 1 \; for \; x > 0 \end{cases}$

4. $\lim_{\sigma\to 0} R_\sigma(x)$ where $R_\sigma(x) = \begin{cases} \frac{1}{2\sigma} \; for -\sigma < x < \sigma \end{cases}$, 0 for $|x| > \sigma$ .this is rectangular function

5. $\delta(x) = \lim_{\sigma\to 0} G_\sigma(x)$ where $G_\sigma(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{igx}dg \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}}$ .This is Gaussian function

PROPERTIES OF DDF

1. $\delta(x-a) = \delta(a-x)$ 2. $f(x)\delta(x-a) = f(a)\delta(x-a)$

3.If c is a real number then $\delta[c(x-a)] = \frac{1}{c}\delta(x-a)$

4. $\delta(x^2-a^2) = \frac{1}{2a}[\delta(x-a)+\delta(x+a)]$ 5. $\int_{-\infty}^{\infty} \delta(x-a)\delta(x-b) = \delta(a-b)$

6.nth derivative of ddf= $\int_{-\infty}^{\infty} \delta^n(x-a)f(x)dx = (-1)^n f^n(a)$

7.Fourier transform of ddf = $\frac{1}{\sqrt{2\pi}}$ 8.Laplace transform of ddf =1

LAPLACE TRANSFORM

Laplace transform of a function F(t) is defined as $L\{F(t)\} = f(s) = \int_0^{\infty} e^{-st}F(t)dt$

$$L\left\{\frac{F(t)}{t}\right\} = \int_s^{\infty} f(s)ds$$

$$L\{t^nF(t)\} = (-1)^n \frac{d^n}{ds^n}[f(s)]$$

Laplace Transform of derivatives -Theorem(Derivative of any order n)

$$L\{F^n(t)\} = s^{n-1}L\{F(t)\} - s^{n-1}F(0) - s^{n-2}F'(0) - F^{n-1}(0)$$

ie $L\{F''(t)\} = s^2L\{F(t)\} - sF(0) - F'(0)$

Shifting Theorems

1.First shifting theorem(shifting on x-axis)

If $L\{F(t)\} = f(s)$ then $L\{e^{at}F(t)\} = f(s-a)$

2. Second shifting theorem(shifting on the t-axis-unit step function)

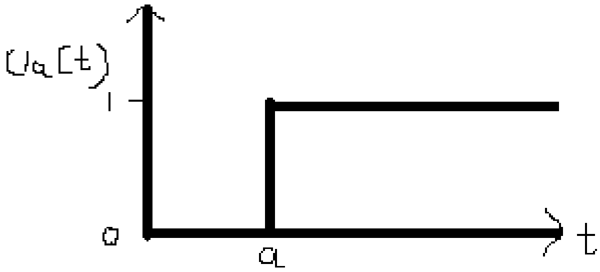
If f(s) is the transform of F(t), then $e^{-as}f(s)$ (a>0) is the transform of the function

$$G(t) = \begin{cases} F(t-a) if \; t > a \\ 0 \; if \; t < a \end{cases}$$

$$L\{G(t)\} = e^{-as}f(s)$$

G(t) can be written as unit step function

$$U_a(t) = \begin{cases} 0 \; if \; t < a \\ 1 \; if \; t > a \end{cases}$$



$$e^{-as} f(s) = L\{F(t-a)U_a(t)\}$$

A rectangular pulse of unit height and width k can be described by F(t)=U(t) - U(t-k)

Eg: Laplace transform of F(t)= $\begin{cases} 1 \;\; if \;\; 0 < t < \pi \\ 0 \;\; if \;\; \pi < t < 2\pi \\ sint \;\; if \; t > 2\pi \end{cases}$

$F(t)=U_0(t) - U_\pi(t) + U_{2\pi}(t)\sin t$

$$L\{F(t)\} = \frac{1}{s} - \frac{e^{-\pi s}}{s} + \frac{e^{-2\pi s}}{s^2+1}$$

Convolution theorem of Laplace transform

If F(t) and G(t) are the inverse transforms of f(s) and g(s), then the inverse transform of the product is the convolution of F(t) and G(t) written as (F*G)(t) and defined by

$$(F*G)(t) = \int_0^t F(t-u)G(u)\,du$$

Corollary: By putting t-u=v in the above eqn we get $(F*G)(t) = -\int_t^0 F(v)G(t-v)\,dv$

$$= \int_0^t G(t-v)F(v)\,dv = (G*F)(t)$$

Transform of periodic functions

Laplace transform of a piecewise continous priodic function F(t) with period T is

$$L\{F(t)\} = \frac{1}{1-e^{-sT}} \int_0^{\frac{T}{2}} e^{-st} F(t)\,dt$$

Inverse Laplace transform

If $L\{F(t)\} = f(s)$ then $F(t) = L^{-1}\{f(s)\}$

Eg: $L\{e^{at}\} = \frac{1}{s-a}$ then $L^{-1}\left\{\frac{1}{s-a}\right\} = e^{at}$

Laplace transform of integral of a function

$$L\left\{\int_0^x f(x)\,dx\right\} = \frac{f(s)}{s}$$

Some Laplace transforms

f(t)	$L\{F(t)\}$
1	$\frac{1}{s}$
t	$\frac{1}{s^2}$

t^n	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
cos at	$\frac{s}{s^2+a^2}$
sinat	$\frac{a}{s^2+a^2}$
coshat	$\frac{s}{s^2-a^2}$
sin hat	$\frac{a}{s^2-a^2}$
t^a	$\frac{\sqrt{(a+1)}}{s^{a+1}}$
$e^{at}\,t^n$	$\frac{n!}{(s-a)^{n+1}}$
$e^{at}\cos\omega t$	$\frac{s-a}{(s-a)^2+\omega^2}$
$E^{at}\sin\omega t$	$\frac{\omega}{(s-a)^2+\omega^2}$

INVERSE LAPLACE TRANSFORMS

f(s)	F(t)
$\frac{1}{s}$	1
$\frac{1}{s^n}$	$\frac{t^{n-1}}{(n-1)!}$
$\frac{1}{s-a}$	e^{at}
$\frac{1}{(s-a)^n}$	$\frac{t^{n-1}e^{at}}{(n-1)!}$
$\frac{1}{(s^2-a^2)}$	$\frac{1}{a}\sinh at$
$\frac{s}{(s^2-a^2)}$	cosh at

Eg: Find $L^{-1}\left\{e^{-3s}\frac{1}{s^3}\right\}$

$$L^{-1}\left\{e^{-as}f(s)\right\}=F(t-a)U_a(t) \qquad t^2=\frac{2!}{s^3}$$

$$=\frac{(t-3)^2}{2!}U_3(t)$$

FOURIER TRANSFORMS

Given a function f(x) , the fourier transform F(ω) of f(x) is given by

$F(\omega)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)e^{-i\omega x}dx$

PROPERTIES

1.Fourier transform of derivative: $F[f'(t)]=i\omega F(\omega)$

2.Fourier transform of integral : $F\left\{\int_0^t f(t)dt\right\}=\frac{1}{i\omega}F(\omega)$

3.Scaling : $F\{f(at)\}=\frac{1}{a}F\left(\frac{\omega}{a}\right)$

4.Shifting translation: $F[f(t+a)]=e^{ia\omega}F(\omega)$ $F[f(t-a)]=e^{-ia\omega}F(\omega)$

5.Exponential multiplication: $F[e^{\alpha t}f(t)]=F(\omega+i\alpha)$

6.Consider the D.E $\frac{d^2\phi}{dx^2}-k^2\phi=f(x)$ .By taking the Fourier transform of the eqn it's

Solution can be written as $\phi(x)=-\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{i\omega x}\frac{F(\omega)}{(\omega^2+k^2)}d\omega$

7.Convolution: $g(t)*h(t)=\int_{-\infty}^{\infty} g(\tau)h(t-\tau)d\tau$

8. Parseval's theorem : $\int_{-\infty}^{\infty}|f(t)|^2 dt=\int_{-\infty}^{\infty}|F(\omega)|^2 d\omega$

FOURIER SERIES

Fourier series and Euler formulae

Let f(x) be a periodic function of period 2π which can be represented by a trigonometric series, then

$$f(x)=a_0+\sum_{n=1}^{\infty}a_n\cos nx+b_n\sin nx$$

$$a_0=\frac{1}{2\pi}\int_{-\pi}^{\pi}f(x)dx \qquad a_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\cos nx dx \qquad b_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\sin nx dx \quad \text{n=1,2,3…}$$

DIFFERENTIAL EQUATIONS

Homogenous equation: $\frac{dy}{dx}+\frac{x-2y}{2x-y}=0$, put y=vx and solve

Non-homogenous equation: $\frac{dy}{dx}=\frac{x-2y+5}{2x+y-1}$,put x=X+h y=Y+k . choose h and k such that ah+bk+c=0

Variable separable: Mdx+Ndy=0 ,bring like variables together and integrate

Linear equation : $\frac{dy}{dx}+py=Q$. Solution is $ye^{\int pdx}=\int Qe^{\int pdx}+c$

Bernoulli's equation: $\frac{dy}{dx}+py=Qy^n$ .To solve this divide the eqn by y^n.Then we get

$y^{-n}\frac{dy}{dx}+py^{1-n}=Q$. Put y^{1-n}=z and reduce it to a linear eqn.

LINEAR DIFFERENTIAL EQUATIONS

An eqn in which the dependent variable and it's derivatives occur only in first degree.

LINEAR DIFFERENTIAL EQUATIONS WITH CONSTANT COEFFICIENTS

$a_0\frac{d^ny}{dx^n}+a_1\frac{d^{n-1}y}{dx^{n-1}}+...+a_{n-1}\frac{dy}{dx}+a_ny=\phi(x)$,where $a_0,a_1,a_2,\dots,a_n$ are constants is the most general form of a linear differential eqn with constant coefficients.

$y = c_1 f_1(x) + ... + c_n f_n(x) + \upsilon$ is the general solution of the above eqn.

$c_1 f_1(x) + ... + c_n f_n(x)$ is called the Complimentary function and υ is called the particular integral. Denoting $\frac{dy}{dx}$ and it's higher powers as D, D^2,D^3 we can write the above eqn as

$a_0 D^n + a_1 D^{n-1} + ... + a_{n-1} D + a_n = f(D)$

To find complementary function(cf)

1.If roots of auxiliary eqn are not equal

Let m_1,m_2 be the roots. If $m_1 \neq m_2$ C.F=$c_1 e^{m_1 x} + c_2 e^{m_1 x}$

If $m_1 = m_2 = m$ C.F= $e^{mx}(c_1 + c_2 x)$

If $m_1, m_2 = \pm$ mi C.F= $c_1 \cos mx + c_2 \sin mx$

If $m_1, m_2 = r \pm$ mi C.F= $e^r (c_1 \cos mx + c_2 \sin mx)$

To find particular integral

1. P.I for $\frac{1}{f(D^2)} \sin ax = \frac{1}{f(a^2)} \sin ax$ if $f(-a^2) \neq 0$
2. P.I for $\frac{1}{f(D^2)} cosax = \frac{1}{f(-a^2)} \cos ax$ if $f(-a^2) \neq 0$

If $f(a^2) = 0$ this method fails

Eg: $\frac{1}{D^2 + a^2} \sin ax = \frac{1}{D^2 + a^2} I.P$ of $e^{iax} = \frac{1}{(D+ia)(D-ia)} e^{iax} = \frac{1}{2ia} \frac{1}{(D-ia)} e^{iax}$

$$= \frac{1}{2ia} xe^{iax} = \frac{-i}{2a} x(cosx + i \sin x)$$

i.e I.P(Imaginary Part) of $e^{iax} = \frac{-x}{2a} cosax$

Similarly $\frac{1}{D^2 + a^2} \cos ax = \frac{x}{2a} \sin ax$

Particular integrals for special cases

1. When $\frac{1}{f(D)} e^{ax} = \frac{1}{f(a)} e^{ax}$ when $f(a) \neq 0$

   If $f(a) = 0$ then $\frac{1}{f(D)} e^{ax} = \frac{x}{\phi(a)} e^{ax}$
2. When $\phi(x) = x^m$, m being a positive integer expand $\frac{1}{f(D)}$ ie $\{f(D)\}^{-1}$ in ascending integral powers of D
3. When $\phi(x) = e^{ax} V$, where V is any function $\frac{1}{f(D)} e^{ax} V = e^{ax} \frac{1}{f(D+a)} V$

COMPLEX NUMBERS

A complex analytic function z=x+iy

Cauchy-Reimann equations: $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial v}{\partial y} = -\frac{\partial v}{\partial x}$

Laplace's equations: $\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$ and $\nabla^2 v = \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}$

MILAN THOMPSN METHOD

Let z=u+iv be an analytic function. If u is given to find f(z) find $U_x'(z,0)$ and $U_y'(z,0)$

$f(z)=\int U_x^{'}(z,0)-iU_y^{'}(z,0)$

CAUCHY'S INTEGRAL THEOREM: $\int_c f(z)dz=0$ if $f(z)$ is analytic

CAUCHY'S INTEGRAL FORMULA: $f(z_0)=\frac{1}{2\pi i}\int_c \frac{f(z)}{z-z_0}dz$

DERIVATIVES OF AN ANALYTIC FUNCTION IN GENERAL $f^n(z_0)=\frac{n!}{2\pi i}\int_c \frac{f(z)}{(z-z_0)^{n+1}}dz$

FINDING RESIDUE OF AN ANALYTIC FUNCTION

1.Residue at a simple pole: $\operatorname{Re}s_{z=0} f(z)=\lim_{z\to z_0}(z-z_0)\, fz$

2.Residue at a pole of order m>1 : $\operatorname{Re}s_{z=z_0} f(z)=\frac{1}{(m-1)!}\lim_{z\to z_0}\left\{\frac{d^{m-1}}{dz^{m-1}}\left\{(z-z_0)^m f(z)\right\}\right\}$

3.If $f(z)$ has a simple pole at z= z_0 we may take $f(z)=\frac{p(z)}{q(z)}$

$$\operatorname{Re}s_{z=0} f(z)=\operatorname{Re}s_{z=0}\frac{p(z)}{q(z)}=\frac{p(z_0)}{q^{'}(z_0)}$$

4.CAUCHY RESIDUE THEOREM: $\int_c f(z)dz=2\pi i\sum_{k=1}^{n}\operatorname{Re}sf(z)$

$=2\pi i$ (sum of residues at all singularities)

LAURENT SERIES: $f(z)=\sum_{n=0}^{\infty}a_n(z-z_0)^n+\sum_{n=1}^{\infty}\frac{b_n}{(z-z_0)^n}$

TAYLOR SERIES : $f(z)=\sum_{m=0}^{\infty}\frac{f^m(z_0)}{m!}(z-z_0)^m$

To solve problems like $\frac{x\sin ax}{x^2+k^2}$ take $f(z)=\frac{ze^{iaz}}{z^2+k^2}$ and take the imaginary part

If integration around the unit circle $\int_0^{2\pi} f(\sin\theta,\cos\theta)$ take $z=e^{i\theta}$, $\sin\theta=\frac{z-\frac{1}{z}}{2i}$, $\cos\theta=\frac{z+\frac{1}{z}}{2}$

<u>JORDAN'S LEMMA</u>

If $f(z)$ is analytic except at a finite number of singularities and if $f(z)\to 0$ uniformly as $z\to\infty$,then $\lim_{R\to\infty}\int_C e^{imz} f(z)dz=0, m\rangle 0$ where Γ denotes the semi circle, $|z|=R, I(z)\rangle 0$

MATRICES

If there are three matrices A,B,C the order of multiplication is from left $(A\times B)C$

Inverse of a 2×2 matrix can be found by interchanging diagonal elements and changing the sign of non-diagonal elements.

Diagonalization of a matrix by similarity transformation

Let A be the matrix

Using $A-\lambda I=0$,find eigen values of A

Create a matrix S with the eigen vectors of A

Find the inverse of s

$S^{-1}AS$ is the diagonal matrix similar to A

VECTOR ANALYSIS

Gradient of a scalar field: $\nabla = i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\frac{d\phi}{ds}$

Directional derivative: $\frac{d\phi}{ds} = \nabla\phi.a$

Divergence of a vector point function: $divf = \nabla.f = i.\frac{\partial f}{\partial x} + j.\frac{\partial f}{\partial y} + k.\frac{\partial f}{\partial z}$

A vector is solenoidal if $divf = 0$

Curl of a vector point function: $\nabla f = \begin{pmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f_1 & f_2 & f_3 \end{pmatrix}$

$Curlf = 0$, the vector is irrotational.Then it will form a conservative field.

Identities

div(ϕ f)= ϕ divf+f.(grad ϕ)

Curl(ϕ F)= ϕ curl F+(grad ϕ) $\times$ F

div(f $\times$ g)=g.curl f - f.curl g

curl(f $\times$ g)=(g.∇)f – (f.∇)g + f divg –g div f

$\nabla\times\nabla\phi = 0 \qquad Curl(r^n\vec{r}) = 0 \qquad div\left(r^n\vec{r}\right) = (n+3)r^n \qquad \nabla r^n = nr^{n-2}\vec{r}$

$\nabla.r^n = (n+2)r^{n-1} \qquad \nabla^2 r^n = n(n+1)r^{n-2}$

Gradient,divergence & curl in polar co-ordinates

$$\nabla = \frac{\partial}{\partial r}\vec{r} + \frac{1}{r}\frac{\partial}{\partial\theta}\vec{\theta} + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\vec{\phi}$$

$$\nabla. = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}$$

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2}$$

$$\nabla\times V = \frac{1}{r^2\sin^2\theta}\begin{pmatrix} \vec{r} & r\vec{\theta} & r\sin\theta\vec{\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ V_r & rV_\theta & r\sin\theta V_\phi \end{pmatrix}$$

Gradient,divergence &curl in cylindrical co-ordinates

$$\nabla = \frac{\partial}{\partial r}\vec{r} + \frac{1}{r}\frac{\partial}{\partial\theta}\vec{\theta} + \frac{\partial}{\partial z}\vec{z}$$

$$\nabla. = \frac{1}{r}\frac{\partial}{\partial r}\left(r\ \right) + \frac{1}{r}\frac{\partial}{\partial\theta} + \frac{\partial}{\partial z}$$

$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\ \frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} + \frac{\partial^2}{\partial z^2} \qquad x = r\cos\theta\sin\phi, y = r\sin\theta\sin\phi, z = z$$

$$\nabla\times V = \frac{1}{r}\begin{pmatrix} \vec{r} & r\vec{\theta} & \vec{z} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial z} \\ V_r & rV_\theta & V_z \end{pmatrix}$$

GAUSS THEOREM : $\iint_s F.nds = \iiint_v \nabla.Fdv$

STOKE'S THEOREM : $\iint_S (\nabla \times F).nds = \oint F.dr$

GREEN'S THEOREM: $\iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) = \oint_C (Mdx + Ndy)$

Scalar or dot product: $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

Projection of a vector $\vec{a}$ along $\vec{b}$: $\frac{\vec{a}.\vec{b}}{|\vec{b}|}$

Vector product:

$$\vec{a} \times \vec{b} = |\vec{a}||\vec{b}|\sin\theta$$

Unit vectors perpendicular to $\vec{a}$ and $\vec{b}$ = $|\vec{a}||\vec{b}|\sin\theta$

If $\vec{a}$ and $\vec{b}$ are collinear $|\vec{a}| \times |\vec{b}| = 0$

$|\vec{a} \times \vec{b}|$ is the area of the parallelogram. $\vec{a} \times \vec{b}$ = $\begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

Scalar triple product : $(\vec{a} \times \vec{b}).\vec{c} = \vec{a}.(\vec{b} \times \vec{c}) = [\vec{a}\vec{b}\vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$ It is the volume of a parallelepiped

Vector triple product: $\vec{a} \times (\vec{b} \times \vec{c})$=$(\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c}$

Vector product of four vectors: $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = [\vec{a}\vec{b}\vec{d}]\vec{c} - [\vec{a}\vec{b}\vec{c}]\vec{d}$

RANDOM VARIABLE

Binomial probability distribution

p(x)=