REAL NUMBERS

CLASS X

CBSE

First Term

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REAL NUMBERS

The Fundamental Theorem of Arithmetic.

- Any / Every composite number can be written as the product of powers of primes.

This factorisation is unique, apart from the order in which prime factor occurs.

$$32760 = 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7 \times 13$$
$$= 2^3 \times 3^2 \times 5 \times 7 \times 13$$

Example: Consider the number 4^n, where n is a natural number. Check whether there is any value of n for which 4^n ends with the digit 0

Solution $4^n = (2)^{2n}$

So, prime factor in the factorisation of 4^n is 2.

If the number 4^n, were to end with 0, it would be divisible by 5. which is not possible.

There is no natural number n, for which 4^n ends with 0.

PRIME FACTORISATION METHOD

Q. Find the LCM & HCF of 6 and 20, by the prime factorisation method.

Solution

$$6 = 2^1 \times 3^1$$

$$20 = 2^1 \times 2^1 \times 5^1 = 2^2 \times 5^1$$

$$HCF(6, 20) = 2$$

$$LCM(6, 20) = 2^2 \times 3^1 \times 5^1$$

$$= 60$$

HCF = Product of the smallest power of each common prime factor in the numbers.

LCM = Product of the greatest power of each prime factor involved in the numbers.

For any two positive integers a and b,

$$HCF(a, b) \times LCM(a, b) = a \times b.$$

Example:- Given that HCF (306, 657) = 9. find LCM (306, 657).

HCF (306, 657), LCM (306, 657)

$$306 = 2^1 \times 3^1 \times 3^1 \times 17^1 = 2^1 \times 3^2 \times 17^1$$

$$657 = 3^1 \times 3^1 \times 73^1 = 3^2 \times 73^1$$

$$HCF(306, 657) = 9$$

$HCF(306, 657) \times LCM(306, 657) = 306 \times 657$

$\Rightarrow \quad 9 \times LCM = 306 \times 657$

$\Rightarrow \quad LCM = \dfrac{306 \times \cancel{657}^{73}}{\cancel{9}}$

$\Rightarrow \quad LCM = 22,338$

Q. There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes, for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again.

A) 46 min. B) 36 min.

C) 28 min D) 34 min.

LCM (12, 18).

$12 = 2^1 \times 2^1 \times 3^1 = 2^2 \times 3^1$

$18 = 2^1 \times 3^1 \times 3^1 = 2^1 \times 3^2$

$LCM = 2^2 \times 3^2 = 4 \times 9 = 36$ min

B) Correct Option

RATIONAL NUMBERS Any number which can be written in th form $\frac{p}{q}$, p and q are integers.

Rational numbers: $\frac{2}{5}$, 5, 3, $\frac{1}{7}$, $\frac{1}{100}$.

Irrational numbers: $\sqrt{2}$, $\sqrt{3}$, π, $\sqrt{5}$,

$$2+\sqrt{3}$$

THEOREM 1.3 Let p, be a prime number, If p divides a^2, then p divides a, where a, is a positive integer.

Example

$$\frac{144}{6} = \frac{12 \times 12}{6}$$

If 6 divides 144, 6 divides 12.

$$\frac{\cancel{144}^{24}}{\cancel{6}} = 24.$$

$$\frac{12 \times \cancel{12}^{2}}{\cancel{6}} = 12 \times 2 = 24$$

$$\frac{\cancel{12}^{2}}{\cancel{6}} = 2.$$

Theorem 1.4

$\sqrt{2}$ is irrational

Proof: Let us assume that $\sqrt{2}$ is rational,

$\sqrt{2} = \frac{r}{s}$, r and s have a common factor other than 1

Divide $\frac{r}{s}$ by a common factor

$\therefore \sqrt{2} = \frac{a}{b}$, a and b are co-prime.

$b\sqrt{2} = a$

$2b^2 = a^2$ [Squaring on both sides] . — ①

$b^2 = \frac{a^2}{2}$

Therefore, 2 divides a^2,

By Theorem, 1.3., 2 divides a

$\frac{a}{2} = c$

$\therefore a = 2c$, c = integer.

Substituting, $a = 2c$ on ①.

$2b^2 = (2c)^2$

$2b^2 = 4c^2$

$b^2 = 2c^2$

So, $\frac{b^2}{2} = c^2$,

2 divides b^2

By theorem, 1.3., 2 divides b.

But, a and b are co-primes.

This contradicts the result that a and b have prime factor other than 1

Hence, $\sqrt{2}$ is irrational.

REVISITING RATIONAL NUMBERS AND THEIR DECIMAL EXPANSIONS

Theorem 1.5 Let x be a rational number whose decimal expansion terminates.

x can be expressed in th form of $\frac{p}{q}$ where p and q are co-prime, and the prime factorisation of q is of the form $2^m 5^m$, where n, m are non-negative integers

Theorem 1.6 Let $x = \frac{p}{q}$ be a rational number, such that the prime factorisation of q is of the form $2^m 5^m$, where n, m are non-negative integers. Then x has a decimal expansion which terminates.

MULTIPLE CHOICE QUESTIONS

Q.1 Which of the following rational numbers will have a terminating decimal expansion

A) $\frac{23}{2^3 5^3}$ B) $\frac{129}{2^2 5^7 7^5}$

C) $\frac{6}{15}$ D) $\frac{5}{6403}$

A) $\frac{23}{2^3 5^3} = \frac{p}{q}$.

$q = 2^3 5^3$, which is in th form of $2^m 5^n$.

A in the correct answer.

The denominator of B) C) D) cannot be written in the form of $2^m 5^n$.

Q.2 $\frac{7}{80} = ?$

A) 0.875 B) 0.0875

C) 0.00875 D) 0.0785

$$\frac{7}{80} = \frac{7}{8 \times 10} = \frac{7}{2^3 \times 2 \times 5} = \frac{7}{2^4 \times 5}$$

$$= \frac{7 \times 5^3}{2^4 \times 5^4}$$

B) Correct.

$$= \frac{875}{10^4}$$

$$= 0.0875$$

(3) If two positive integers a an b are written as $a = p^3q^2$ and $b = pq^3$; p, q are prime numbers, then HCF (a,b) is A) pq B) pq^2 C) p^3q^3 D) p^3q^3

$a = p^3q^2$

$b = pq^3$

$\therefore$ HCF $= pq^2$ B) Correct Option

(4) If HCF of 20 and 8 is expressed in the form of '$2k - 2$, then the value of k is
A) 6 B) 4 C) 3 D) 2

$20 = 2 \times 2 \times 5 = 2^2 \times 5^1$

$8 = 2 \times 2 \times 2 = 2^3$

HCF $= 2^2 = 4$

$\therefore$ Given, $2k - 2 = 4$

$2k = 4 + 2$

$k = \frac{4+2}{2} = \frac{6}{2} = 3$

C) Correct Option.

(5) Which of the following number is rational.

A) $5-\sqrt{3}$ B) $\frac{\sqrt{4}}{6}$ C) $\sqrt{3}$

D) $\sqrt{2}$

B) $\frac{\sqrt{4}}{6} = \frac{\sqrt{2^2}}{6} = \frac{\cancel{2}^{1}}{\cancel{6}_{3}} = \frac{1}{3}$

(6) Which of the following pair of numbers are co-prime.

A) 8, 6 B) 12, 49

C) 3, 9 D) 15, 21

B) 12, 49 are co-prime

$12 = 1 \times 2 \times 2 \times 3$

$49 = 1 \times 7 \times 7$

12 and 49 do not have any other common prime factor other than 1.

8, 6 = 1 and 2 are the common factor

3, 9 = 1 and 3 are the common factor

15, 21 = 1 and 3 are the common factor.