Code No: R22011 **R10** SET - 1

II B. Tech II Semester Supplementary Examinations May/June - 2015
PROBABILITY AND STATISTICS
(Com. to CE, CHEM, PE)

Time: 3 hours Max. Marks: 75

Answer any **FIVE** Questions
All Questions carry **Equal** Marks
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1. a) Define conditional probability and State general multiplication rule of probability.
   b) If A and B be events with $P(A) = 0.6, P(B) = 0.3 \text{ and } P(A \cap B) = 0.2$.
   $(i). P(A/B) \text{ and } P(B/A)$ $(ii). P(A \cup B)$
   $(iii). P(A^c) \text{ and } P(B^c)$ $(iv). P(A^c/B^c) \text{ and } P(B^c/A^c)$.
   c) State and prove Rule of total probability. (5M+5M+5M)

2. a) Define Distribution function for discrete and continuous random variables.
   b) Find the value of k and the distribution function F(x) given the probability density function of a random variable X as: $f(x) = \frac{k}{x^2+1}, \quad -\infty < x < \infty$ (7M+8M)

3. a) If X is a Poisson variate such that $P(X = 2) = 9P(X = 4) + 90P(X = 6)$, find the standard deviation.
   b) Find Moment Generating Function for normal distribution and hence find its mean and variance. (7M+8M)

4. a) If a 1-gallon can of paint covers on the average 513.3 square feet with a standard deviation of 31.5 square feet, what is the probability that the sample mean area covered by a sample of 40 of these 1-gallon cans will be anywhere from 510.0 to 520.0 square feet?
   b) Determine a 95% confidence interval for the mean of a normal distribution with variance $\sigma^2 = 0.25$, using a sample of $n = 100$ values with mean $\bar{x} = 212.3$.
   c) Find the value of $F_{0.95}$ for $\nu_1 = 12$ and $\nu_2 = 15$ degrees of freedom. (5M+5M+5M)

5. a) Explain the test procedure for large sample test concerning one proportion.
   b) A storekeeper wanted to buy a large quantity of bulbs from two brands A and B respectively. He bought 100 bulbs from each brand A and B and found by testing brand A had mean life time of 1120 hrs and the S.D of 75 hrs and brand B had mean life time 1062 hrs and S.D of 82 hrs. Examine whether the difference of means is significant. Use a 0.01 level of significance. (7M+8M)
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6. a) Explain the test procedure for test of equality of two variances.
   b) In a shop study, a set of data was collected to determine whether or not the proportion of defectives produced by workers are the same for the day, evening, or night shift was worked. The data were collected and shown in Table.

Shift	Day	Evening	Night
Defectives	45	55	70
Non defectives	905	890	870

Use a 0.05 level of significance to determine if the proportion of defectives is the same for all three shifts. (7M+8M)

7. The following data shows the values of sample mean ($\bar{x}$) and range (R) for 10 samples for size 6 each. Calculate the values for central line and the control limits for Mean – chart and Range – chart. Draw the control charts and comment on the state of control.

Sample No.	1	2	3	4	5	6	7	8	9	10
Mean ($\bar{x}$)	43	49	37	44	45	37	51	46	43	47
Range (R)	5	6	5	7	7	4	8	6	4	6

(15M)

8. The containers from railway goods wagons are unloaded at a single platform of a railway goods yard. The arrival rate of wagons is 8 wagons per day and service rate is 14 wagons per day. Assuming the arrival rate and service rate to follow Poisson distribution, determine the following:
   i) Utilization of railway goods yard
   ii) Average number of waiting wagons in the queue
   iii) Average number of waiting wagons in the system
   iv) Average waiting time per wagon in the queue
   v) Expected waiting time per wagon in the system (15M)

Note :- The following Statistical tables are required

i) Areas under the Standard Normal Curve from 0 to Z,

ii) Percentile Values t_p for Student's t- distribution with v degrees of freedom,

iii) Percentile Values χ_p^2 for the Chi-Square distribution with v degrees of freedom,

iv) Percentile Values (0.05 levels), $F_{0.05}$ for the F- Distribution and

v) Percentile Values (0.01 levels), $F_{0.01}$ for the F Distribution

vi) Control Chart Constants

Code No: R22011 **R10** SET - 2

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PROBABILITY AND STATISTICS
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Answer any **FIVE** Questions
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1. a) State and prove Baye's theorem
   b) If A and B be independent events with $P(A)=0.3$, $P(B)=0.4$.
   Find $(i).\ P(A\cap B)\ and\ P(A\cup B)$ $(ii).\ P(A/B)\ and\ P(B/A)$
   $(iii).\ P(A^c)\ and\ P(B^c)$ $(iv).\ P(A^c/B^c)\ and\ P(B^c/A^c)$. (7M+8M)

2. a) A discrete random variable X has the following probability distribution

Value of X	0	1	2	3	4	5	6	7	8
$P(X=x)$	k	$3k$	$5k$	$7k$	$9k$	$11k$	$13k$	$15k$	$17k$

   (i) Find the value of 'k' (ii) Find $P(X<3), P(0<X<3), P(X\geq 3)$ [7 M]
   b) Find the value of k and the distribution function F(x) given the probability density function of a random variable X as:
   $$f(x)=\begin{cases} k(3+2x) & if\ 0<x<2 \\ 0 & otherwise \end{cases}$$ (7M+8M)

3. a) Find the mean and variance of the Poisson distribution.
   b) Find the probabilities that a random variable having the normal distribution with $\mu=16.2$ and $\sigma^2=1.5625$ will take on a value
   i) between 13.6 and 18.8 ;
   ii) between 16.5 and 16.7 ;
   iii) greater than 16.8;
   iv) Less than 14.9.
   (7M+8M)

4. a) Find the value of $F_{0.95}$ for $\nu_1=10\ and\ \nu_2=20$ degrees of freedom.
   b) Determine a 99% confidence interval for the mean of a normal distribution with variance σ^2 = 9, using a sample of $n=100$ values with mean $\bar{x}=5$.
   c) Find the value of the finite population correction factor for $n=100\ and\ N=5000$.
   (5M+5M+5M)

5. a) Explain the test procedure for test concerning difference between two proportions.
   b) A manufacturer claims that the average tensile strength of thread A exceed the average tensile strength of thread B by at least 12 kilograms. To test his claim, 50 pieces of each type of thread are tested under similar conditions. Type A thread had an average tensile strength of 86.7 kilograms with known standard deviation of $\sigma_A=6.28$ kilograms, while type B thread had an average tensile strength of 77.8 kilograms with known standard deviation of $\sigma_B=5.61$ kilograms. Test the manufacturers claim at 0.05 level of significance. (7M+8M)
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6. a) Explain the test procedure of χ^2 test for analysis of $r \times c$ table.
   b) The following random samples are measurements of the heat-producing capacity (in millions of calories per ton) of specimens of coal from two mines:
   Mine 1: 8,260 8,130 8,350 8,070 8,340
   Mine 2: 7,950 7,890 7,900 8,140 7,920 7,840
   Use the 0.01 level of significance to test whether the differences between the mean of these two samples is significance. (7M+8M)

7. Samples of 100 tubes are drawn randomly from the output of a process that produces several thousand units daily. Sample items are inspected for quality and defective tubes are rejected. The results of 15 samples are shown below :

Sample No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
No. of Defective tubes	8	10	13	9	8	10	14	6	10	13	18	15	12	14	9

   On the basis of information given above prepare a control chart for fraction defective (p - chart). What conclusion do you draw from the control chart? (15M)

8. A self-service store employs one cashier at its counter. 9 customers arrive on an average every 5 minutes while the cashier can serve 10 customers in 5 minutes. Assuming Poisson distribution for arrival rate and exponential distribution for service rate, find
   i) Average number of customers in the system.
   ii) Average number of customers in queue or average queue length.
   iii) Average time a customer spends in the system.
   iv) Average time a customer waits before being served. (15M)

Note :- The following Statistical tables are required

i) Areas under the Standard Normal Curve from 0 to Z,
ii) Percentile Values t_p for Student's t- distribution with v degrees of freedom,
iii) Percentile Values χ_p^2 for the Chi-Square distribution with v degrees of freedom,
iv) Percentile Values (0.05 levels), $F_{0.05}$ for the F- Distribution and
v) Percentile Values (0.01 levels), $F_{0.01}$ for the F Distribution
vi) Control Chart Constants

R10

SET - 3

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PROBABILITY AND STATISTICS
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Time: 3 hours Max. Marks: 75

Answer any **FIVE** Questions
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1. a) Define probability and write the axioms of probability for a finite sample space.
   b) Given $P(A) = 0.35$, $P(B) = 0.40$ *and* $P(A \cap B) = 0.20$, Find
   (i) $P(A \cup B)$ (ii) $P(\bar{A} \cap B)$ (iii) $P(A \cap \bar{B})$ (iv) $P(\bar{A} \cup \bar{B})$ (v) Are A and B independent?
   c) Two cards are drawn at random from an ordinary deck of 52 playing cards. What is the probability of getting two aces if (i) the first card is replaced before the second card is drawn; (ii) the first card is not replaced before the second card is drawn? (5M+5M+5M)

2. a) Define discrete random variable and discrete probability distribution.
   b) Let X be a continuous random variable with distribution :
   $$f(x)=\begin{cases} k\,x^2 & if\ 0\le x\le 1 \\ 0 & elsewhere \end{cases}$$
   (i) Evaluate k (ii) Find $p(1/4 \le X \le 3/4)$.(iii) Find $p(X > 2/3)$. (7M+8M)

3. a) Find Moment Generating Function for Binomial distribution and hence find its mean and variance.
   b) Find the probabilities that a random variable having the standard normal distribution will take on a value
   i) between 0.87 and 1.28 ;
   ii) between -0.34 and 0.62 ;
   iii) greater than 0.85;
   iv) greater than -0.65. (7M+8M)

4. a) Take 30 slips of paper and label five each and -4,four each and -3,three each -2 and 2, and two each -1,0 and 1.If each slip of paper has the same probability of being drawn , find the probability of getting and find the mean and the variance of this distribution.
   b) Determine a 99% confidence interval for the mean of a normal distribution with variance $\sigma^2 = 9$, using a sample of $n = 100$ values with mean $\bar{x} = 5$. (8M+7M)
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5. a) Explain the test procedure for large sample test concerning mean when σ is known.
   b) A study of TV viewers was conducted to find the opinion about the mega serial 'Ramayana'. If 56% of a sample of 300 viewers from south and 48% of 200 viewers from north preferred the serial, , test the claim at 0.05 level of significance that there is a difference of opinion between south and north. (7M+8M)

6. a) Explain the test procedure for small sample test concerning difference between two means.
   b) Explain procedure for one-way classification of analysis of variance (7M+8M)

7. The following table shows the outer diameter values, 16 samples each measured five times, of cores used for winding of a transparent tape in millimetres. Develop a control chart scheme using both Mean ($\overline{X}$) and Range (R) charts. (15M)

Sample no.	**Sub-Sample observations (in milliliters)**					**Sample no.**	**Sub-Sample observations (in milliliters)**				
	1	**2**	**3**	**4**	**5**		**1**	**2**	**3**	**4**	**5**
1	37	37	35	41	39	**9**	39	37	42	40	38
2	29	28	31	33	31	**10**	35	35	35	36	40
3	33	30	35	42	31	**11**	38	33	32	35	32
4	35	37	33	34	36	**12**	28	30	28	32	31
5	35	35	33	34	33	**13**	31	35	35	35	34
6	33	34	35	33	34	**14**	29	32	34	35	37
7	30	31	32	34	31	**15**	38	35	34	34	34
8	35	40	38	39	39	**16**	31	37	39	44	38

8. a) Explain briefly the main characteristics of Queuing system?
   b) Explain Traffic intensity?
   c) Explain (M/ M/ 1): (∞ / FCFS) Queuing model. (5M+5M+5M)

Note :- The following Statistical tables are required

i) Areas under the Standard Normal Curve from 0 to Z,

ii) Percentile Values t_p for Student's t- distribution with v degrees of freedom,

iii) Percentile Values χ_p^2 for the Chi-Square distribution with v degrees of freedom,

iv) Percentile Values (0.05 levels), $F_{0.05}$ for the F- Distribution and

v) Percentile Values (0.01 levels), $F_{0.01}$ for the F Distribution

vi) Control Chart Constants

Code No: R22011 **R10** SET - 4

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PROBABILITY AND STATISTICS
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Time: 3 hours Max. Marks: 75

Answer any **FIVE** Questions
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1. a) If A and B are any events in then prove that
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
b) If A and B be events with $P(A) = \frac{1}{3}, P(B) = \frac{1}{4}$ *and* $P(A \cup B) = \frac{1}{2}$. Find
i) $P(A/B)$ ii) $P(B/A)$ iii) $P(A \cap B^c)$ iv) $P(A/B^c)$
c) If $P(A) = 0.65$, $P(B) = 0.40$ *and* $P(A \cap B) = 0.24$, are the events A *and* B is independent?
(5M+5M+5M)

2. a) Define continuous random variable and continuous probability distribution.
b) Let X be a continuous random variable with distribution :
$$f(x) = \begin{cases} x & for \quad 0 < x < 1 \\ 2 - x & for \quad 1 \le x < 2 \\ 0 & elsewhere \end{cases}$$
Find (i) $p(0.2 \le X \le 0.8)$ (ii) $p(0.6 \le X \le 1.2)$ (7M+8M)

3. a) Find Moment Generating Function for Poisson distribution and hence find its mean and variance.
b) Find the probabilities that a random variable having the standard normal distribution will take on a value
i) less than 1.75 ;
ii) less than -1.25 ;
iii) greater than 2.06;
iv) greater than -1.82. (7M+8M)

4. a) Find the value of the finite population correction factor for $n = 10$ *and* $N = 1000$.
b) Find the value of $F_{0.99}$ for $\nu_1 = 6$ *and* $\nu_2 = 20$ degrees of freedom.
c) Determine a 95% confidence interval for the mean of a normal distribution with variance $\sigma^2 = 4$, using a sample of $n = 200$ values with mean $\bar{x} = 120$. (5M+5M+5M)

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5. a) Explain the test procedure for large sample test concerning difference between two means.
   b) In a survey of A.C. machines produced by company A it was found that 19 machines were defective in a random sample of 200 while for company B 5 were defective out of 100. At 0.05 level of significance is there reason to believe that there is significance difference in performance of A.C. machines between the two companies A and B. (7M+8M)

6. a) Explain the test procedure for small sample test concerning mean.
   b) Explain procedure for two-way classification of analysis of variance. (7M+8M)

7. During an inspection, 20 of successively selected samples of polished metal sheet, the number of defects observed per sheet is recorded, as shown in the following table. Construct a C-chart for the number of defects. (15M)

Sample no.	**No. of defects**	**Sample no.**	**No. of defects**
1	3	**11**	5
2	0	**12**	2
3	5	**13**	1
4	1	**14**	1
5	2	**15**	2
6	3	**16**	3
7	2	**17**	4
8	4	**18**	0
9	0	**19**	1
10	2	**20**	2

8. a) Derive the average number of customers in the Queue. In (M/M/1) (∞/FCFS) model.
   b) At a public telephone booth the arrivals are on the average 15 per hour. A call on the average takes 3 minutes. If there is just one phone.
   i) What is expected number of callers in the booth at any time?
   ii) For what proportion of time in the booth expected to be idle. (7M+8M)

<u>Note</u> :- The following Statistical tables are required

i) Areas under the Standard Normal Curve from 0 to Z,

ii) Percentile Values t_p for Student's t- distribution with v degrees of freedom,

iii) Percentile Values χ_p^2 for the Chi-Square distribution with v degrees of freedom,

iv) Percentile Values (0.05 levels), $F_{0.05}$ for the F- Distribution and

v) Percentile Values (0.01 levels), $F_{0.01}$ for the F Distribution

vi) Control Chart Constants