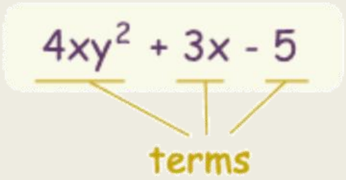
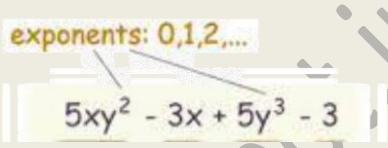
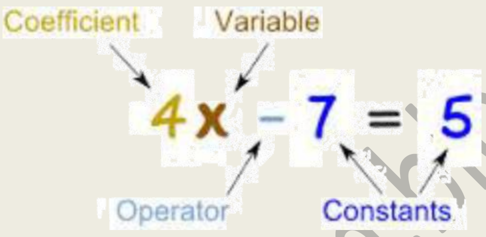
Polynomials

An expression containing variables, constant and any arithmetic operation is called polynomial.

Polynomial comes from *poly*- (meaning "many") and -*nomial* (in this case meaning "term") ... so it says "many terms"





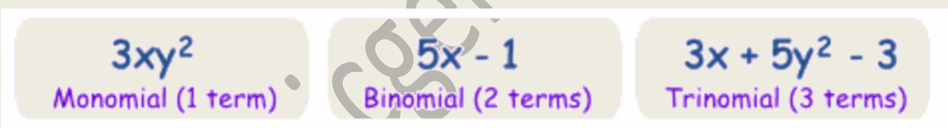


• Polynomials contain three types of terms:-

(1) Monomial: - A polynomial with one term.

(2) Binomial: - A polynomial with two terms.

(3) Trinomial: - A polynomial with three terms.



• Degree of polynomial: - the highest power of the variable in a polynomial is termed as the degree of polynomial.

• Constant polynomial: - A polynomial of degree zero is called constant polynomial.

• Linear polynomial: - A polynomial of degree one.

• E.g.:$-9x + 1$

• Quadratic polynomial: - A polynomial of degree two. E.g.:$-3/2y^2 - 3y + 3$

• Cubic polynomial: - A polynomial of degree three.

• E.g.:$-12x^3 - 4x^2 + 5x + 1$

• Bi – quadratic polynomial: - A polynomial of degree four.

• E.g.:- $10x^4 - 7x^3 + 8x^2 - 12x + 20$

Standard Form

• The Standard Form for writing a polynomial is to put the terms with the highest degree first.

• **Example: Put this in Standard Form: $3x^2 - 7 + 4x^3 + x6$**

The highest degree is 6, so that goes first, then 3, 2 and then the constant last:

$x^6 + 4x^3 + 3x^2 - 7$

Reminder theorem

• Let p(x) be any polynomial of degree greater than or equal to one and let 'a' be any real number. If p(x) is divided by linear polynomial x-a then the reminder is p(a).

• Proof: - Let p(x) be any polynomial of degree greater than or equal to 1. Suppose that when p(x) is divided by x-a, the quotient is q(x) and the remainder is r(x), i.e;

p(x) = (x-a) q(x) +r(x)

Since the degree of (x-a) is 1 and the degree of r(x) is less than the degree of x-a,
the degree of r(x) = 0.

This means that r(x) is a constant .say r.

So, for every value of x, r(x) = r.

Therefore, p(x) = (x-a) q(x) + r

In particular, if x = a, this equation gives us

p (a) =(a-a) q(a) + r

This proves the theorem.

Factor Theorem

Let p(x) be a polynomial of degree (n > 1) and let 'a' be any real number. If p (a) = 0 then (x-a) is a factor of p(x).

PROOF:-By the reminder theorem,

P(x) = (x-a) q(x) + p (a).

1. If p (a) = 0, then p(x) = (x-a) q(x), which shows that (x-a) is a factor of p(x).

2. Since x-a is a factor of p(x),

p (x) = (x-a) g(x) for same polynomial g(x).

In this case, p (a) = (a-a) g (a) =0

ALGEBRIC IDENTITIES

- $(x+y)^2 = x^2 + 2xy + y^2$
- $(x-y)^2 = x^2 - 2xy + y^2$
- $(x+y)(x-y) = x^2 - y^2$
- $(x+a)(x+b) = x^2 + (a+b)x + ab$
- $(x+y+z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$
- $(x+y)^3 = x^3 + y^3 + 3xy(x+y) = x^3 + y^3 + 3x^2y + 3xy^2$
- $(x-y)^3 = x^3 - y^3 - 3xy(x-y) = x^3 - y^3 - 3x^2y + 3xy^2$
- $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$
- $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$