Taxbuoyancy: If GDP grew by x%, then how much % Income tax collection will grow.

→ Give

$$x^2 + \frac{1}{\sqrt{\cos t}}2x + \frac{1}{\sin t} = 2\sqrt{2}$$

For which value of $t \in \mathbb{R}$ the equation has exactly one solution.

Here $t \neq n\pi$, $t \neq (2n+1)\pi/2$

Since for $n\pi$, $\sin t$ will become '0' & for $t = (2n+1)\pi/2$, $\cos t$ is '0'

In both cases sol is not defined (Here sol = solution)

The given equation is quadratic equation in form

$$ax^2 + bx + c = 0$$

For having exactly one solution it's determinant $b^2 - 4ac = 0 \Rightarrow$

$$\boxed{b^2 = 4ac}$$

Now

Comparing $ax^2 + bx + c$ with $x^2 + \frac{1}{\sqrt{\cos t}}2x + \frac{1}{\sin t} - 2\sqrt{2} = 0$

we can interpret

$$a = 1;\ b = \frac{2}{\sqrt{\cos t}};\ c = \left(\frac{1}{\sin t} - 2\sqrt{2}\right)$$

$\therefore$

$$b^2 = 4ac \Rightarrow \left(\frac{2}{\sqrt{\cos t}}\right)^2 = 4(1)\left(\frac{1}{\sin t} - 2\sqrt{2}\right)$$

$$\Rightarrow \frac{\cancel{4}}{\cos t} = \cancel{4}\left(\frac{1}{\sin t} - 2\sqrt{2}\right) \Rightarrow \frac{1}{\cos t} = \frac{1 - 2\sqrt{2}\sin t}{\sin t} \Rightarrow \sin t = (1 - 2\sqrt{2}\sin t)\cos t$$

$$\sin t = \cos t - 2\sqrt{2}\sin t \cdot \cos t \Rightarrow \sin t - \cos t + 2\sqrt{2}\sin t \cos t = 0$$

$$\Rightarrow \sin t - \cos t = -2\sqrt{2}\sin t \cos t$$

multiply & divide L.H.S by $\sqrt{2}$

$$\cancel{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\sin t - \frac{1}{\sqrt{2}}\cos t\right) = -2\cancel{\sqrt{2}}\sin t \cos t$$

using $\sin \pi/4 = \cos \pi/4 = 1/\sqrt{2}$, we can write

$$\left(\sin t \cdot \cos \pi/4 - \sin \pi/4 \cos t\right) = -2\sin t \cos t$$

using $(\sin(A-B) = \sin A \cos B - \sin B \cos A)$ on L.H.S & $\sin 2\theta = 2\sin\theta \cos\theta$) on R.H.S

we can write

$$\sin(t - \pi/4) = -\sin 2t \quad (\text{also apply } -\sin 2\theta = \sin(-2\theta))$$

we get now

$$\sin(t - \pi/4) = \sin(-2t)$$

Comparing L.H.S & R.H.S

$$t - \pi/4 = -2t \Rightarrow 3t = \pi/4 \Rightarrow \boxed{t = \pi/12}$$

$\therefore t = \pi/12$ for which given equation has exactly one solution