Maths-I

Chapter 1 : Mathematical Logic

Q1 Using truth table prove the following logical equivalence.

a) $p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$

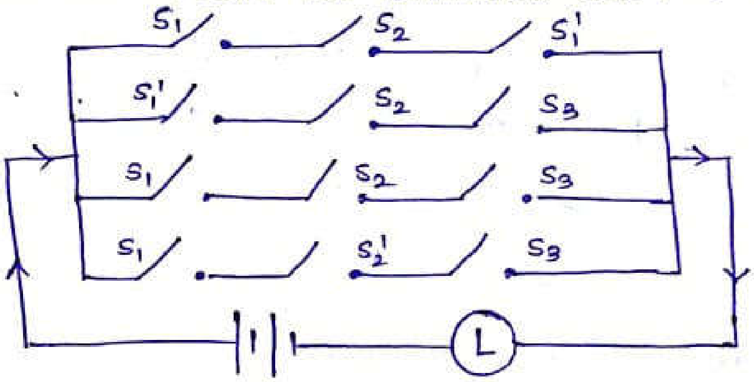
b) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

Q2 Construct the switching circuit of the following statement.

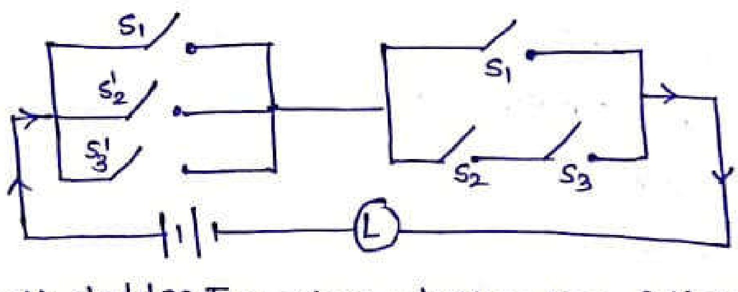
a) $(p \wedge \sim q \wedge r) \vee [p \wedge (\sim q \vee -r)]$

b) $[(p \wedge r) \vee (\sim q \wedge \sim r)] \wedge (\sim p \wedge \sim r)$

Q3 Give alternative arrangement for the following circuit so that new ~~switches~~ circuit has minimum switches only.



Q4 Represent the following switching circuit in symbolic form & construct its switching table. Write your conclusion from switching table.



Q5 Using truth tables Examine whether the following statement patterns are tautology, contradiction or contigency.

a) $(p \wedge \sim q) \longleftrightarrow (p \rightarrow q)$

b) $(p \wedge q) \vee (p \wedge r)$

Q6 Write dual i) $(p \vee q) \wedge T$

Chapter 2 : Matrices

Q1 find the inverse of $A = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$ by using elementary row transformations only.

Q2 find the inverse of $A = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$ by adjoint method.

Q3 Solve the following equation by method of reduction.

$$x+3y+3z = 12$$
$$x+4y+4z = 15$$
$$x+3y+4z = 13$$

Q4 Solve the following equations by the method of inversion.

$$x-y+z = 4$$
$$2x+y-3z = 0$$
$$x+y+z = 2$$

Q5 State whether the following matrix is invertible or not.

a] $\begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$ b] $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ c] $\begin{bmatrix} 1 & 3 & 2 \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}$

Chapter 3 : Trigonometric functions :

Q1 find the general solutions of equations.

a] $\cos x = -\frac{1}{2}$ b] $\sin x = \frac{-1}{\sqrt{2}}$ c] $\cot x = -\sqrt{3}$

Q2 find the general solution of

a] $\sec^2 2x = 1 - \tan 2x$

b] $\sin x + \sin 3x + \sin 5x = 0$

c] $\cos 3x = \cos 2x$

d] $\cos x - \sin x = -1$

e] $\cot x + \tan x = 2\operatorname{cosec} x$

f] $\sin x = \tan x$

g] $\sqrt{3}\cos x - \sin x = 1$

Q3 Prove that $\sin^{-1}\left(\frac{3}{5}\right) + \cos^{-1}\left(\frac{12}{13}\right) = \sin^{-1}\left(\frac{56}{65}\right)$

Q4 Prove that $\tan^{-1}\left[\sqrt{\frac{1-\cos x}{1+\cos x}}\right] = \frac{x}{2}$

Q5 Prove that $\cos^{-1}\left(\frac{3}{5}\right) + \cos^{-1}\left(\frac{4}{5}\right) = \frac{\pi}{2}$

Q6 show that in ΔABC $b\cos^2\left(\frac{C}{2}\right) + c\cos^2\left(\frac{B}{2}\right) = s$

Chapter 4: Pair of Straight Lines

Q1 find the separate equations of lines represented by

a] $3x^2 - 7xy + 4y^2 = 0$

b] $8(x-1)^2 + 10(x-1)(y-3) + 3(y-3)^2 = 0$

Q2 find p and q if the following equation represents a pair of ~~lines~~ perpendicular lines $2x^2 + 4xy - py^2 + 4x + qy + 1 = 0$

Q3 Show that $x^2 - 6xy + 5y^2 + 10x - 14y + 9 = 0$ represents a pair of lines. find the acute angle between them. Also find the point of intersection of lines.

Q4 find the value of k if the slope of one of the lines given by $4x^2 + kxy + y^2 = 0$ is four times the other.

Q5 find the value of k if $2x + y = 0$ is one of the lines given by $3x^2 + kxy + 2y^2 = 0$

chapter 5 : Vectors

Q1 Prove that three vectors $\bar{a}, \bar{b}, \bar{c}$ are coplanar if & only if there exist a non-linear combination $x\bar{a} + y\bar{b} + z\bar{c} = 0$

Q2 show that the points $A(2,-1,0)$ $B(-3,0,4)$ $C(-1,-1,4)$ & $D(0,-5,2)$ are non-coplanar.

Q3 If the vectors $-3i + 4j - 2k$, $i + 2k$ and $i - pj$ are coplanar then find the value of p.

Q4 If $\bar{a}$ $\bar{b}$ $\bar{c}$ are the positions vectors of the points A, B, C respectively and $2\bar{a} + 3\bar{b} - 5\bar{c} = 0$ then find the ratio in which the point C divides the line segment AB.

Q5 Show that the three points A $(1,-2,3)$ B $(2,3,-4)$ and C $(0,-7,10)$ are collinear.

Q6 If A $(5,1,p)$ B $(1,q,p)$ & C $(1,-2,3)$ are vertices of triangle and G $(r,-\frac{4}{3},\frac{1}{3})$ is its centroid then find the values of p, q, r.

Q7 Using vector method, find the incenter of the triangle whose vertices are. A $(0,3,0)$ B $(0,0,4)$ and C $(0,3,4)$.

Chapter 6 : Three Dimensional Geometry.

Q1 If a line makes angles $90°, 135°, 45°$ with X, Y, Z axes respectively then find its direct cosines.

Q2 If a line makes angles α, β, γ with co-ordinate axes Prove that i) $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

ii) $\cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$

Q3 find the values of λ for which the points $(6,-1,2)$ $(8,-7,\lambda)$ and $(5,2,4)$ are collinear.

Q4 find the angle between the lines whose direction ratios are $5, 12, -13$ and $3, -4, 5$.

Q5 find the direction cosines of the lines which is perpendicular to the lines. with direction ratios $-1, 2, 2$ and $0, 2, 1$.

Q6 If a line has direction ratios $4, -12, 18$ then find direction cosines.

Chapter 7: Line:

Q1 find the vector equation of a line passing through a point $2\hat{i}+\hat{j}-3\hat{k}$ and perpendiculars to the vectors $\hat{i}+\hat{j}+\hat{k}$ and $\hat{i}+2\hat{j}-\hat{k}$.

Q2 Find the vector equation of the line passing through a point with position vector $2\hat{i}+\hat{j}-\hat{k}$ and parallel to the line joining the points $-\hat{i}+\hat{j}+4\hat{k}$ and $\hat{i}+2\hat{j}+2\hat{k}$. Also find the cartesian form of this equation.

Q3 find the equation of the line passing through the point $(3,1,2)$ and perpendicular to the lines $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$ and $\frac{x}{-3} = \frac{y}{2} = \frac{z}{5}$.

Q4 find the shortest distance between the lines

$\bar{r} = (2\hat{i}-\hat{j}) + \lambda(2\hat{i}+\hat{j}-3\hat{k})$ &

$\bar{r} = (\hat{i}-\hat{j}+2\hat{k}) + \mu(2\hat{i}+\hat{j}-5\hat{k})$

Q5. find the foot of perpendicular drawn from the point $2\hat{i}-\hat{j}+5\hat{k}$ to the line $\bar{r} = (11\hat{i}-2\hat{j}-8\hat{k}) + \lambda(10\hat{i}-4\hat{j}-11\hat{k})$ Also find the length of perpendicular.

Q6 find the length of perpendicular from $(2,-3,1)$ to the line $\frac{x+1}{2} = \frac{y-3}{3} = \frac{z+2}{-1}$

Q7 find the vector & cartesian equation of line that passes through i) origin & $(5,-2,3)$ ii) $(3,-2,-5)$ & $(3,-2,6)$

chapter 8 : Plane

Q1 find the vector equation of the plane which is at a distance of 6 units from the origin and which is normal to the vector $2\hat{i}-\hat{j}+2\hat{k}$.

Q2 find the cartesian form of the equation of the plane $\bar{r} = (s+t)\hat{i} + (2+t)\hat{j} + (3s+2t)\hat{k}$ where 's' & 't' are parameters.

Q3 find the vector equation of the plane passing through the points $A(1,1,2)$ $B(0,2,3)$ $C(4,5,6)$ Also find the cartesian equation of the plane.

Q4 find the distance of the point $2\hat{i}+\hat{j}+\hat{k}$ from the plane $\bar{r}(\hat{i}+2\hat{j}+4\hat{k}) = 13$

Q5 find the angle between the line $\bar{r} = (\hat{i}+2\hat{j}+\hat{k}) + \lambda(\hat{i}+\hat{j}+\hat{k})$ and the plane $\bar{r}(2\hat{i}-\hat{j}+\hat{k}) = 5$

chapter 9 : Linear Programming

Q1 Minimize $z = 8x+10y$ subject to $2x+y \geq 7, 2x+3y \geq 15$ $y \geq 2,\ x \geq 0,\ y \geq 0$.

Q2 Maximize $z = 11x+8y$ subject to $x \leq 4, y \leq 6, x+y \leq 6$ $x \geq 0, y \geq 0$

Q3 Maximize $z = x_1+x_2$ subject to $x_1+x_2 \leq 10,\ 3x_2-2x_1 \leq 15\ \ x_1 \leq 6\ \ x_1, x_2 \geq 0$