Cylinder Volume Lesson Plan

Concept/principle to be demonstrated:

This lesson will demonstrate the relationship between the diameter of a circle and its circumference, and impact on area. The simplest way to demonstrated understanding is by applying the formulas ($A=\pi r^2$ and $V = Bh$) to solve additional construction related problems using a calculator.

Lesson objectives/Evidence of Learning:

- Distinguishes between area and perimeter of 2-D figures, surface area, and volume of 3-D figures
- Calculates the volume and surface area of spheres, right rectangular prisms, and right circular cylinders
- Applies formula to solve variety of construction problems
- Uses calculator to compute accurately

How this math connects to construction jobs:

Area of circles and volume of a cylinder is used extensively by plumbers and carpenters in construction. This lesson will help students comprehend how areas of circles, and volume of cylinders, are used to determine capacity in everything from indoor pipes to culvert drains.

- **Carpenters** use volume to determine the amount of concrete needed in foundations.
- **Plumbers** use formulas to select the correct pipe size for the required flow.
- **Heating Ventilation and Air Conditioning (HVAC) installers** use volume formulas to ensure correct air movement to and from rooms.
- **Underwater welders** use volume of a cylinder to determine the safe length of time submerged.

Teacher used training aids:

- 10” diameter 3/4” plywood circle (optional)
- 60” cloth tape (optional)
- 2” diameter PVC pipe 12” long w/ end cap
- 4” diameter PVC pipe 12” long w/ end cap
- Approximately 4 cups (1 quart) white sand
- 1 quart measuring cup with spout & handle
- Circle Formulas sheet for each student

Materials needed per student:

- Pencil
- Calculator with √ key & memory +/- functions
- ***Circle and Cylinder Worksheets***

Terms:

- **Area:** The number of square units that covers a shape or figure; $A = \pi r^2$.
- **Base:** The bottom of a plane figure or three-dimensional figure.
- **Circle:** The set of points in a plane that are a fixed distance from a given point, called the center.
- **Circumference:** The distance around a circle; $C= \pi d$
- **Cylinder:** A three-dimensional figure having two parallel bases that are equal circles.
- **Diameter:** The line segment joining two points on a circle and passing through the center of the circle.
- **Pi (π):** The ratio of the circumference of a circle to its diameter; $\pi = 3.141593$
- **Radius:** The distance from the center to a point on a circle; the line segment from the center to a point on a circle.
- **Volume:** A measurement of space, or capacity. It is the product of the base area times the height of the cylinder; $V = Bh$

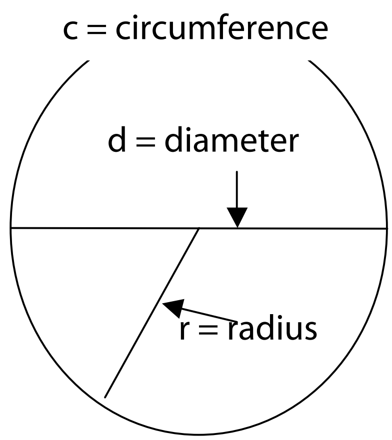
Lesson Introduction:

The perimeter of a circle is called the circumference. The distance to the center of the circle is the radius. The distance across the circle is the diameter. The diameter is twice the radius.

Formulas for the circumference and area of a circle involve **π** (pi). **π** represents the ratio of the circumference of any circle to its diameter, and it is always the same regardless of the size of the circle. **π** is approximately 22/7, or 3.14. Many calculators have a _ key because it is a value that is used so frequently. The circumference of a circle is found by the formula: $\mathbf{C= \pi 2r}$ or $\mathbf{C= \pi d}$. The area of a circle is found by the formula: $\mathbf{A = \pi r^2}$.

Lesson Components:

1. Draw on white board and explain:



2. Use plywood circle prop to show relationship of the diameter (10”) and the circumference (10π=31.415 or 31 7/16”). Have students use calculator to change decimal to fraction.

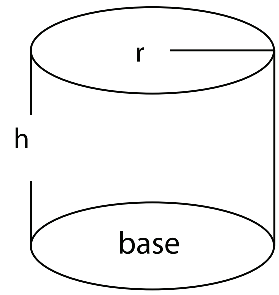
3. Have students use calculator to determine the area: $\pi r^2 = \pi(5 \text{ in.})^2$. Remind students to square the radius before multiplying by π. $\pi(25 \text{ in.}^2) = 78.5 \text{ in.}^2$

4. The cylinder has parallel and equal sized circles as bases. To find the volume of a cylinder, multiple the area of a base by the height of the cylinder:
   $V = Bh$ V = volume   B = area of a base = (πr^2)   h = height of cylinder

5. Since the base of a cylinder is always a circle, substitute the formula for the area of a circle into the formula for the volume (ask teacher which format to use)

   $V=\pi r^2 h$
   $V=\pi \text{ x } r^2 \text{ x } h$
   $V=\pi \cdot r^2 \cdot h$
   $V=\pi(r^2)(h)$

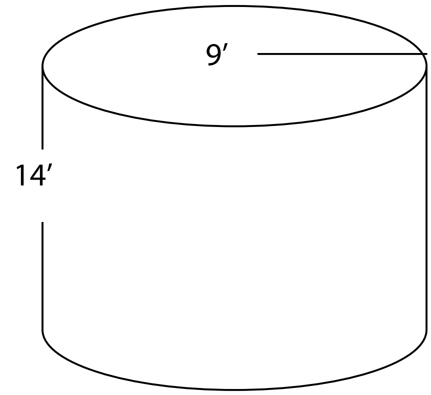
6. Draw on white board and explain:



7. **Demonstration** for determining volume of a cylinder:

   a. Show the 2” diameter PVC pipe.
   b. Measure sand in cup.
   c. Ask the students to guess what the volume will be.
   d. Pour sand into cylinder. Determine how many ml or ounces it holds. (Note: metric is another class.)
   e. Ask the students how much more the 4” cylinder will hold. (Most students will guess twice as much.)
   f. Pour the sand from the 2” into the 4” cylinder.
   g. Show the students how full the larger cylinder is. Ask why it isn’t half full.
      1. Show mathematically why it is 1/4
      2. On board write the volume formula for each cylinder.
      3. $V=\pi r^2 h$ $\quad V=\pi(1^2)(12)$ and $V=\pi(2^2)(12)$
      4. Insert radius values $V=\pi(1)(12)$ and $V=\pi(4)(12)$
      5. Radius of the small cylinder is 1. 1^2 is still 1. Radius of the large cylinder is 2. 2^2 is 4.
      6. Erase or strikeover π and the 12” height of both formulas because they are constants.
      7. The ratio is 1:4. (This is very cool stuff to see.)
      8. Hand out ***Circle and Cylinder Worksheets***

8. Find the volume of this cylinder. r=9’ and h=14’



a. V = 3.14 (9’ x 9’) 14 ft
b. V = 3.14 (81 ft^2) 14 ft
c. V = 3.14 (1134 ft^3)
d. V = 3560.76 ft^3

9. Students can work individually or in teams to solve the ***Circle and Cylinder Worksheets***.

Name ________________________________

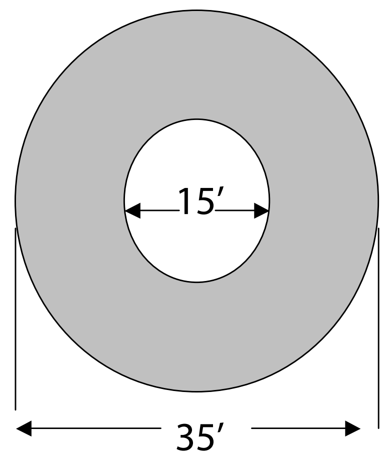
Circle and Cylinder Worksheets

Problem #1

Find the area of a circle with a radius of 15 inches.

Problem #2

Find the area of the ring (the shaded area). Hint: subtract area of the smaller circle from the larger circle area.

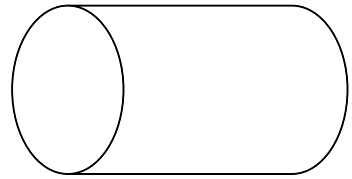


Problem #3

Carpenters are going to use flexible board to form the circles in Problem #2. What is the circumference of each circle?

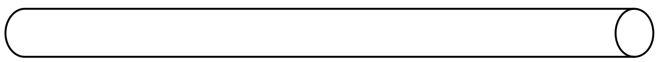
Problem #4

What is the volume of a cylindrical drum used for storing kerosene that has a diameter of 2 ft. 6 in. and is 4 ft. long? (Round to the nearest cubic foot.)



Problem #5

Water pipes with an outside diameter of 1 in. are to be insulated with a thin sheet of foam. Assuming no waste or overlap, how many square inches of foam is needed to cover pipes that are 40 feet long?



Problem #6

A drilled well has a diameter 3 ft. The water standing in the well is 200 ft. deep. How many gallons of water are in the well? (1 ft^3 =7.5 gal)

Circle and Cylinder Worksheets

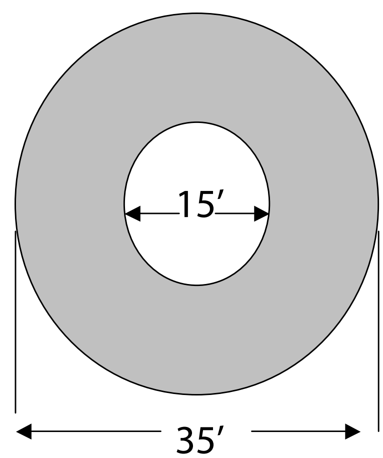
Problem #1

Find the area of a circle with a radius of 15 inches.

$$(7.5\text{ in})\ \pi = 176.71\text{ in}^2$$

Problem #2

Find the area of the ring (the shaded area). Hint: subtract area of the smaller circle from the larger circle area.



$$(17.5\text{ ft})^2\ \pi = 962.1127\text{ ft}^2$$
$$(7.5\text{ ft})^2\ \pi = 176.7146\text{ ft}^2$$

$$962.1127\text{ ft}^2 - 176.7146\text{ ft}^2 = 785.3981\text{ ft}^2$$

Problem #3

Carpenters are going to use flexible board to form the circles in Problem #2. What is the circumference of each circle?

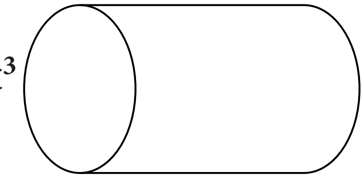
$$15\ \pi = 47.1239\text{ ft.}$$
$$35\ \pi = 109.9557\text{ ft.}$$

Problem #4

KEY

What is the volume of a cylindrical drum used for storing kerosene that has a diameter of 2 ft. 6 in. and is 4 ft. long? (Round to the nearest cubic foot.)

$$(1.25 \text{ ft})^2\ \pi \times 4\text{ft.} = 19.63495 \text{ ft}^3 = 20 \text{ ft}^3$$



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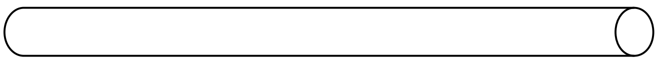
Water pipes with an outside diameter of 1 in. are to be insulated with a thin sheet of foam. Assuming no waste or overlap, how many square inches of foam is needed to cover pipes that are 40 feet long?



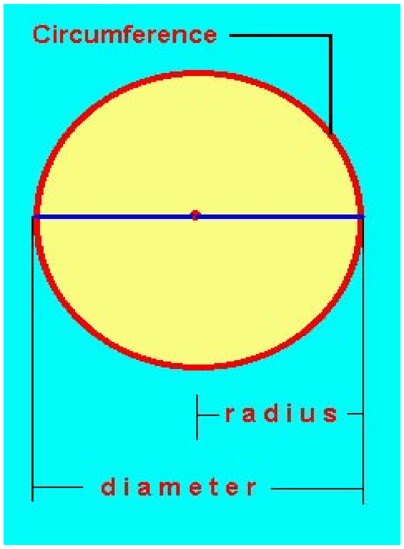
$$1\text{in} \times \pi \times 480 \text{ in} = 1184.353 \text{ in}^2$$

Problem #6

A drilled well has a diameter 3 ft. The water standing in the well is 200 ft. deep. How many gallons of water are in the well? (1 ft^3 =7.5 gal)

$$(1.5 \text{ ft})^2\ \pi \times 200\text{ft.} = 1413.717 \text{ ft}^3 \times 7.5 \text{ gal/ft}^3 = 10602.88 \text{ gal.}$$

Circle Formulas



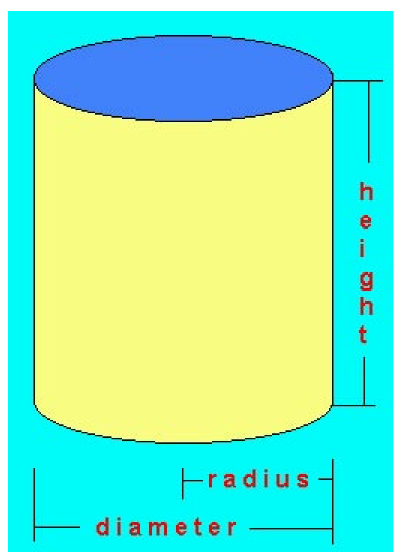
Circumference = $2 \cdot \pi \cdot$ radius =

$\pi \cdot$ diameter

Circle Area = $\pi \cdot r^2$ =

$1/4 \cdot \pi \cdot d^2$

Cylinder Formula



Volume = $\pi \cdot r^2 \cdot$ height =

$1/4 \cdot \pi \cdot d^2 \cdot$ height