FUNCTIONS:

* Interchanging fns:

Period of

$\dfrac{f(x)}{g(x)} \pm h(x)$ LCM of period of $f(x)$, $g(x)$ and $h(x)$.

But, in case of interchanging functions, we get period by inspection.

$f(x+h) = h(x)$

e.g. $\left|\sin\left(x+\frac{\pi}{2}\right)\right| = |\cos x|$

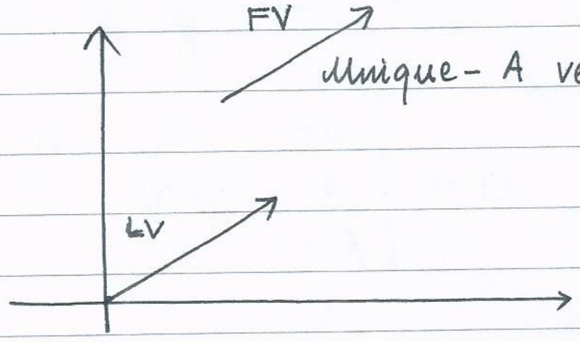
* LIMIT:

Sandwitch Theorem:

$\lim\limits_{x\to a} h(x) \le \lim\limits_{x\to a} f(x) \le \lim\limits_{x\to a} g(x)$.

VECTORS.

→ Unlike Vector - Antiparallel vector.



Unique- A vector is defined by its direction and magnitude.

1) Sum of 2 unit vectors is a unit vector. Find $|\hat{a}-\hat{b}|$

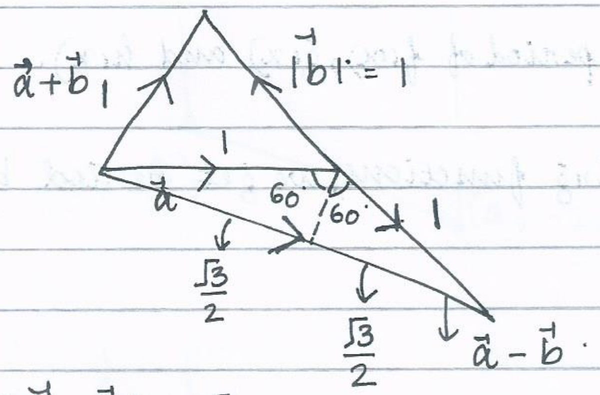
$\hat{a}+\hat{b}=\hat{i}$

$|\vec{a}+\vec{b}|^2 + |\vec{a}-\vec{b}|^2 = 2(|\vec{a}|^2+|\vec{b}|^2)$

$1 + |\vec{a}-\vec{b}|^2 = 2(2)$

$|\vec{a}-\vec{b}| = \sqrt{3}$.

OR.



$|\vec{a}-\vec{b}| = \sqrt{3}$.

2) Points with position vectors $\vec{a}(60\hat{i}+3\hat{j})$, $\vec{b}(40\hat{i}-8\hat{j})$ and $\vec{c}(a\hat{i}-52\hat{j})$ are collinear. Find a.

$AB = \lambda BC$
$\quad = \lambda AC$

$AB = \lambda AC$

$\Rightarrow -20\hat{i}-11\hat{j} = \lambda((a-60)\hat{i}-55\hat{j})$

$\lambda(-55) = -11$

$\lambda = \frac{1}{5}$

$\frac{1}{5}(a-60) = -20$

$a-60 = -\cancel{40}\ 100$

$a = -40$.

3) A vector with components $2p$ and 1 in the rectangular cartesian system. The axes are rotated about the origin in the counterclockwise sense. The components made by the vector with the new axes are $p+1$ and 1. Find p.

Magnitude of vector is the same in both the cases.

$$\therefore \sqrt{4p^2+1} = \sqrt{(p+1)^2+1}$$

$$4p^2+1 = p^2+2p+1+1$$

$$3p^2-2p-1 = 0$$

$$p = -\frac{1}{3}, 1.$$

4) $(\hat{i}+\hat{j}+3\hat{k})x + (3\hat{i}-3\hat{j}+\hat{k})y - (-4\hat{i}+5\hat{j})z = \lambda(x\hat{i}+y\hat{j}+z\hat{k})$

$$\hat{i}((1-\lambda)x+3y-4z) + \hat{j}(x-(\lambda+3)y+5z) + \hat{k}(3x+y-\lambda z) = 0$$

$$(1-\lambda)x + 3y - 4z = 0$$

$$x - (\lambda+3)y + 5z = 0$$

$$3x + y - \lambda z = 0$$

$$\begin{vmatrix} 1-\lambda & 3 & -4 \\ 1 & -(\lambda+3) & 5 \\ 3 & 1 & -\lambda \end{vmatrix} = 0$$

$\lambda = 0$ or $\lambda = -1$.

5. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$

$\vec{b} = 4\hat{i} + 3\hat{j} + 4\hat{k}$

$\vec{c} = \hat{i} + \alpha\hat{j} + \beta\hat{k}$

are linearly dependent and $|c| = \sqrt{3}$. Find α and β.

$\vec{c} = \lambda\vec{a} + y\vec{b}$

$\hat{i} + \alpha\hat{j} + \beta\hat{k} = \lambda(\hat{i} + \hat{j} + \hat{k}) + y(4\hat{i} + 3\hat{j} + 4\hat{k})$

$\lambda + 4y = 1$

$\lambda + 4y = \beta$

$\beta = 1$

$|c| = \sqrt{3}$

$\sqrt{1 + \alpha^2 + \beta^2} = \sqrt{3}$

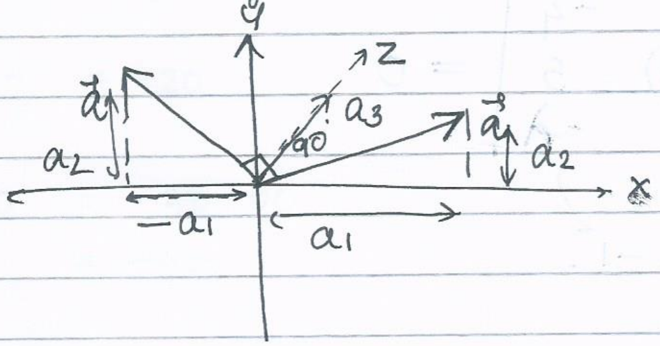
$\alpha^2 = 1$

$\alpha = \pm 1$

$\therefore \alpha = \pm 1$

$\beta = 1$

6. The components of a vector $\vec{a}$ are $\vec{a_1}, \vec{a_2}, \vec{a_3}$. The vector is rotated about z axis by $\frac{\pi}{2}$. Find the components vector in the new system.



$\therefore$ New components are:

$-a_1, a_2, a_3$.

7) $2\vec{p} + \vec{q} = \hat{i} + \hat{j}$

$\vec{p} + 2\vec{q} = \hat{i} - \hat{j}$

Find angle between $\vec{p}$ & $\vec{q}$.

$\vec{q} = \frac{\hat{i}}{3} - \hat{j} \quad \vec{p} = \frac{\hat{i}}{3} + \hat{j}$.

Angle:

$$\cos\theta = \frac{\vec{p} \cdot \vec{q}}{|\vec{p}||\vec{q}|}$$

$$= \frac{-\frac{8}{9}}{\frac{10}{9}}$$

$$= -\frac{4}{5}$$

8) Unit vector makes $45°$ $2\hat{i} + 2\hat{j} - \hat{k}$ and $60°$ with vector $\hat{j} - \hat{k}$.

$\frac{1}{\sqrt{2}} = \frac{2x + 2y - z}{3} \qquad \frac{1}{2} = \frac{y - z}{\sqrt{2}}$

$2x + 2y - z = \frac{3}{\sqrt{2}} \qquad \frac{1}{\sqrt{2}} = y - z$