JEE-MAINS

2007 Onwards....

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"Fully - Solved"

CONTENTS

Y.R-(2007 — 2016) → Questions of JEE MAINS.

Sets, Relations, Functions

1) If $f(x) + 2f\left(\frac{1}{x}\right) = 3x$, $x \neq 0$ and $S = \{x \in R : f(x) = f(-x)\}$, then S

a) is an empty set b) contains exactly one element (2016)

✓c) contains exactly two elements d) contains more than 2 elements

Sol: $\left.\begin{array}{l} f(x) + 2f\left(\frac{1}{x}\right) = 3x \\ f(1/x) + 2f(x) = \frac{3}{x} \end{array}\right\} \Rightarrow$ Eliminating $f(1/x)$ we get $f(x) = \frac{2}{x} - x$

for finding elements of S: $f(x) = f(-x) \Rightarrow \frac{2}{x} - x = -\frac{2}{x} + x$

$\Rightarrow \frac{2}{x} - x = 0 \Rightarrow x^2 = 2 \Rightarrow x = \pm\sqrt{2}$

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2) For $x \in R$, $x \neq 0$, $x \neq 1$, Let $f_0(x) = \frac{1}{1-x}$ and (2016) online.

$f_{n+1}(x) = f_0(f_n(x))$, $n = 0, 1, 2 \ldots$ Then the value of

$f_{100}(3) + f_1\left(\frac{2}{3}\right) + f_2\left(\frac{3}{2}\right) =$ (a) $\frac{8}{3}$ (b) $\frac{4}{3}$ ✓(c) $\frac{5}{3}$ (d) $\frac{1}{3}$

Sol: $f_{0+1}(x) = f_0(f_0(x)) = \frac{1}{1 - \frac{1}{1-x}} = \frac{x-1}{x}$

$f_2(x) = f_{1+1}(x) = f_0(f_1(x)) = \frac{1}{1 - \frac{x-1}{x}} = x$

$f_3(x) = f_{2+1}(x) = f_0(f_2(x)) = \frac{1}{1-x} = f_0(x)$

$f_4(x) = f_{3+1}(x) = f_0(f_3(x)) = f_0(f_0(x)) = \frac{x-1}{x}$

$\therefore f_0 = f_3 = f_6 \cdots\cdots = \frac{1}{1-x}$; $f_1 = f_4 = f_7 = \cdots = \frac{x-1}{x}$

$f_2 = f_5 = f_8 = \cdots\cdots x$:

$f_{100}(3) = f_{99+1}(3) = \frac{3-1}{3} = \frac{2}{3}$; $f_1\left(\frac{2}{3}\right) = \frac{2/3 - 1}{2/3} = -1/2$

$f_2(3/2) = 3/2$ $\therefore f_{100}(3) + f_1\left(\frac{2}{3}\right) + f_2\left(\frac{3}{2}\right) = 5/3$.

(*) 3) The function $f : R \rightarrow \left[-\frac{1}{2}, \frac{1}{2}\right]$ defined as $f(x) = \frac{x}{1+x^2}$ is

a) injective but not surjective c) Neither surjective nor injective

✓b) surjective but not injective d) invertible (2017) online.

Sol: $f(x_1) = f(x_2)$ (Assume)

$\Rightarrow \frac{x_1}{1+x_1^2} = \frac{x_2}{1+x_2^2}$

$\Rightarrow x_1 x_2^2 + x_1 = x_2 + x_2 x_1^2$

$\Rightarrow x_1 x_2^2 - x_2 x_1^2 = x_2 - x_1$

$\Rightarrow x_1 x_2 (x_2 - x_1) - (x_2 - x_1) = 0$

$\Rightarrow (x_1 - x_2)[x_1 x_2 - 1] = 0$

$\Rightarrow x_1 - x_2$ is not necessarily 0.

Now let $y = \frac{x}{1+x^2} \Rightarrow y + x^2 y - x = 0$

$x = \frac{1 \pm \sqrt{1 - 4y^2}}{2y}$; x exists for $1 - 4y^2 \geq 0$

$\Rightarrow 4(y^2 - 1/4) \leq 0 \Rightarrow y \in [-1/2, 1/2]$

$\therefore$ option (b) as Range = Codomain.

NOTE :- '(*)' marked problems pertain to class 12.

4) Let $f(x) = 2^{10}x+1$, $g(x) = 3^{10}x-1$. If $fog(x) = x$, then $x=$

a) $\frac{3^{10}-1}{3^{10}-2^{-10}}$ (b) $\frac{2^{10}-1}{2^{10}-3^{-10}}$ (c) $\frac{1-3^{-10}}{2^{10}-3^{-10}}$ ✓(d) $\frac{1-2^{-10}}{3^{10}-2^{-10}}$ online (2017)

Sol $f(g(x)) = 2^{10}g(x)+1 = 2^{10}(3^{10}x-1)+1 = 6^{10}x-2^{10}+1 = x$

$\Rightarrow (6^{10}-1)x = 2^{10}-1 \Rightarrow x = \frac{2^{10}-1}{2^{10}3^{10}-1} = \frac{\cancel{2^{10}}(1-2^{-10})}{\cancel{2^{10}}(3^{10}-2^{-10})}$

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(8) 5) The function $f: N \to N$ defined by $f(x) = x-5\left[\frac{x}{5}\right]$ where N is the set of natural nos. and $[x]$ denotes the greatest integer or less than equal to x is (2017)-online.

a) one-one and onto ✓c) neither one-one nor onto

b) onto but not one-one d) one-one but not onto

Sol: $f(5) = 5-5\left[\frac{5}{5}\right] = 0$, $f(10) = 10-5\left[\frac{10}{5}\right] = 0$.

$f(5.5) = 5.5-5\left[\frac{5.5}{5}\right] = 5.5-5 = .5 \notin$ Codomain. So (c).

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6) In a certain town 25% of the families own a phone and 15% own a car, 65% own neither a phone nor a car, and 2000 families own both a car and a phone. (2015) online.

Consider the following statements:

(1) 5% families own both a car and a phone

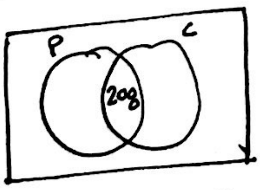
(2) 35% families own either a car or a phone

(3) 40000 families live in the town.

Then (a) only (1) and (2) are correct (c) only (2) and (3) are correct

✓(b) only (1) and (3) are correct (d) only (1), (2), (3) are correct.

Sol:



$n(P\cap C) = 2000$

$n(\overline{P\cup C}) = 65\% \Rightarrow n(P\cup C) = 35\%$

$n(P\cup C) = n(P) + n(C) - n(P\cap C)$

$\Rightarrow 35 = 25+15-n(P\cap C) \Rightarrow n(P\cap C) = 5\%$

$\Rightarrow \frac{5}{100} \times$ Total population $= 2000 \Rightarrow$ Total pop$^{ln} = 40000$. (option b)

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7) Let A and B be two sets containing 2 elements and 4 elements respectively. The no. of subsets of $A\times B$ having 3 or more elements is (2013)

a) 211 b) 256 c) 220 ✓d) 219.

Sol: $A\times B$ will have 8 elements (2×4).

$\therefore$ No. of subsets is ${}^8C_0 + {}^8C_1 + {}^8C_2 + {}^8C_3 + \dots + {}^8C_8 = 2^8 = 256$

$\therefore$ " " " with 3 or more elements $= 256 - ({}^8C_0 + {}^8C_1 + {}^8C_2) = 219$.

8) Let $X = \{1, 2, 3, 4, 5\}$. No. of different ordered pairs (Y, Z) that can be formed such that $Y \subseteq X$, $Z \subseteq X$, and $Y \cap Z$ is empty is (a) 2^5 (b) 5^3 (c) 5^2 (d) 3^5 (2012)

Sol:

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9) Domain of $f(x) = \dfrac{1}{\sqrt{|x| - x}}$ is (2011)

✓a) $(-\infty, 0)$ b) $(-\infty, \infty) - \{0\}$ (c) $(-\infty, \infty)$ (d) $(0, \infty)$

Sol: $|3| - 3 = 0$ $|-3| - (-3) = 3 + 3 = 6 > 0$

$|0| - 0 = 0$, $\therefore$ option a.

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10) Let R be a set of real nos.

Stmt 1: $A = \{(x, y) \in R \times R : y - x$ is integer$\}$ is an equivalence relation on R

Stmt 2: $B = \{(x, y) \in R \times R : x = \alpha y$ for some rational no. $\alpha\}$ is an equivalence relation on R. (2011)

✓(a) stmt 1 is true, stmt 2 is false
(b) stmt 1 is false, stmt 2 is true
(c) stmt 1 is true, stmt 2 is true. stmt 2 is a correct explanation for stmt 1.
(d) stmt 1 is true, stmt 2 is true. stmt 2 is not a correct explanation of stmt 1.

Sol: $(x, x) \in A$ is true — (Reflexive)

If $(x, y) \in A \Rightarrow (y, x) \in A$ — (Symmetric)

$(x, y) \in A \Rightarrow x - y$ is integer

$(y, z) \in A \Rightarrow y - z$ " "

$(x, z) \in A \Rightarrow x - z$ is " (transitive)

$(0, y) \in B$ but $(y, 0) \notin B$ then α is not rational.

So B is false.

option a.

(2011)

8) Consider the following relations:

$R = \{(x, y) \mid x, y$ are real nos. and $x = wy$ for some rational no. w

$S = \{(\frac{m}{n}, \frac{p}{q}) \mid m, n, p, q \in$ Integers such that $n, q \neq 0$ and $qm = np\}$

A-3

a) R is an equivalence relation, but S is not an equivalence relation.
b) Neither R nor S is an equivalence relation.
✓c) S is an equivalence relation, but R is not an equivalence relation.
d) R and S are both equivalence relation.

Sol: we have $(x,x) \in R$ for $\omega = 1$ — Reflexive.
$(0,a) \in R \Rightarrow \omega$ is rational
$(a,0) \in R \Rightarrow \omega$ is not rational
$\Rightarrow$ Not Symmetric $\Rightarrow R$ is not equivalence r/n.

As $\left(\frac{m}{n}, \frac{m}{n}\right) \in S \Rightarrow mn = mn \Rightarrow$ Reflexive.
$\left(\frac{m}{n}, P/q\right) \in S \Rightarrow qm = pn$
$\left(P/q, m/n\right) \in S \Rightarrow np = mq$
$\Rightarrow$ Symmetric.
$\left(\frac{m}{n}, \frac{p}{q}\right) \in S, \left(\frac{p}{q}, \frac{a}{b}\right) \in S \Rightarrow qm = pn$ and $bp = aq$ i.e $\frac{m}{n} = \frac{p}{q}$ and $\frac{p}{q} = \frac{a}{b}$ $\Rightarrow \frac{m}{n} = \frac{a}{b}$
$\therefore \left(\frac{m}{n}, \frac{a}{b}\right) \in S$ (transitive) $\Rightarrow S$ is equivalence.

(2009)

9) If A, B, C are three sets such that $A \cap B = A \cap C$ and $A \cup B = A \cup C$, then (a) $A = C$ ✓(b) $B = C$ (c) $A \cap B = \phi$ (d) $A = B$

Sol: $A \cap B = A \cap C \Rightarrow B \cup (A \cap B) = B \cup (A \cap C)$
$\Rightarrow (B \cup A) \cap (B \cup B) = (B \cup A) \cap (B \cup C)$.
$\Rightarrow (B \cup A) \cap B = (B \cup A) \cap (B \cup C) \Rightarrow B = (A \cup C) \cap (B \cap C)$
$\Rightarrow B = (A \cap B) \cup C = (A \cap C) \cup C = C$

(2009)

10) $\forall x \in R$, $f(x) = x^3 + 5x + 1$ then
a) f is onto but not one-one
✓b) f is one-one and onto
c) f is neither one-one nor onto
d) f is one-one but not onto.

Sol: $f'(x) = 3x^2 + 5 > 0 \; \forall x \in R \Rightarrow f(x)$ is strictly increasing on $x \in R \therefore f(x)$ is one-one & onto.

(2009)

11) Let $f(x) = (x+1)^2 - 1, \; x \geq -1$
Stmt 1: The set $\{x : f(x) = f^{-1}(x)\} = \{0, -1\}$
Stmt 2: f is bijection.
(a) Stmt 1 is true, stmt-2 is true, stmt 2 is not correct expl^n of stmt-1.
✓(b) Stmt 1 is true, stmt-2 is false
(c) Stmt 1 is false, stmt-2 is true.
(d) Stmt 1 is true, stmt-2, stmt-2 is correct explanation of stmt 1.

Sol: $\because f(x) = f^{-1}(x) \Rightarrow f(x) = x \Rightarrow (x+1)^2 - 1 = x \Rightarrow x(x+1) = 0 \Rightarrow x = \{0, -1\}$
As no codomain is specified, we can't say it ONTO. So option (b)

12) Let $f: N \to Y$ be a function defined as $f(x) = 4x+3$ where $Y = \{y \in N : y = 4x+3$ for some $x \in N\}$ such that f is invertible. Its inverse is (2008)

a) $g(y) = 4 + \frac{y+3}{4}$ b) $g(y) = \frac{y-3}{4}$ c) $g(y) = \frac{3y+4}{3}$ d) $g(y) = \frac{y-3}{4}$

Sol: Let $y = f(x) \Rightarrow x = f^{-1}(y) \Rightarrow x = \frac{y-3}{4}$, $f^{-1}(y) = \frac{y-3}{4}$.

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13) Let R be the real line, consider the following subsets of (2008) plane $R \times R$; $S = \{(x,y) : y = x+1, 0 < x < 2\}$ $T = \{(x,y) : x-y$ is integer$\}$ which is true.

a) Both S and T are equivalence relation on R.

b) T is equivalence relation in R, S is not.

c) Neither S nor T is equivalence

d) S is equivalence on R, but T is not.

Sol: Clearly $(x,x) \notin S$ as $x \neq x+1$ not reflexive.

But $(x,x) \in T$ as 0 is integer – reflexive.

$(x,y) \in T \Rightarrow x-y = k$ for $k \in Z$.

$(y,x) \Rightarrow y-x = -k \in Z \Rightarrow (y,x) \in T$ symm.

$(x,y) \in T \Rightarrow x-y = k, \; k \in Z$

$(y,z) \in T \Rightarrow y-z = k_1, \; k_1 \in Z$

$x-z = k+k_1 \in Z \quad \therefore (x,z) \in T.$

$\Rightarrow T$ is equivalence

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14) Set $S = \{1, 2, 3, \ldots, 12\}$ is to be partitioned into 3 sets. $A \cup B \cup C = S$. A, B, C are of equal size. Thus $A \cap B = B \cap C = A \cap C = \Phi$. No. of ways to partition is (2007)

a) $\frac{12!}{(4!)^3}$ b) $\frac{12!}{(4!)^4}$ c) $\frac{12!}{3!(4!)^3}$ d) $\frac{12!}{3!(4!)^4}$

Sol: No. of ways $= {}^{12}C_4 \times {}^{8}C_4 \times {}^{4}C_4 = \frac{12!}{(4!)^3}$

2017 – Set D

1) If S is the set of distinct values of 'b' for which the following system of linear equations $x+y+z=1$; $x+ay+z=1$; $ax+by+z=0$, has no solutions is (i) an empty set (ii) an infinite set (iii) a finite set containing two or more elements 4) a singleton.

Sol: $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a & 1 \\ a & b & 1 \end{vmatrix} = 0 \Rightarrow (a-b) - 1(1-a) + 1(b-a^2) = 0$

$\Rightarrow (1-a)^2 = 0 \Rightarrow a = 1$

Pl omit: ~~If $a=1$~~, ∴ $\left.\begin{matrix} x+y+z=1 \\ by+z=0 \\ x+z=1 \end{matrix}\right\} \Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 0 & b & 1 \\ \phi & 0 & 1 \end{vmatrix} = 0 \Rightarrow \cancel{b}+1-\cancel{b} = 0$ (Not possible)

Thus $a=1$; so $\left.\begin{matrix} x+y+z=1 \\ x+y+z=1 \\ x+by+z=0 \end{matrix}\right\} \Rightarrow$ for $b=1$, the system of equation have no solutions i.e it represents parallel planes.

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2) The following statement $(p \to q) \to [(\sim p \to q) \to q]$ is
i) a tautology (ii) equivalent to $\sim p \to q$ (iii) equivalent to $p \to \sim q$
4) a fallacy.

Sol: $(p \to q) \to ((\sim p \to q) \to q) \Rightarrow (p \to q) \to ((p \vee q) \to q)$

$= (p \to q) \to ((\sim p \wedge \sim q) \vee q) = (p \to q) \to ((\sim p \vee q) \wedge (\sim q \vee q))$

$= (p \to q) \to (\sim p \vee q) = (p \to q) \to (p \to q) = T$

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3) If $5(\tan^2 x - \cos^2 x) = 2\cos 2x + 9$, then value of $\cos 4x$ is
1) $-3/5$ (ii) $1/3$ (iii) $2/9$ (iv) $-7/9$.

Sol: $5(\sec^2 x - 1 - \cos^2 x) = 2(2\cos^2 x - 1) + 9$ let $t = \cos^2 x$

$5\left(\frac{1}{t} - 1 - t\right) = 2(2t-1)+9 \Rightarrow 5(1-t-t^2) = 4t^2 + 7t$

$\Rightarrow 9t^2 + 12t - 5 = 0 \Rightarrow t = 1/3, 5/3 \Rightarrow \cos^2 x = 1/3$

$\cos 2x = 2\cos^2 x - 1 = 2/3 - 1 = -1/3$ $\cos 4x = 2\cos^2 2x - 1 = -7/9$.

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4) For 3 events A, B and C, P(Exactly one of A or B occurs) = P(Exactly one of B or C occurs) = P(Exactly one of C or A occurs) = 1/4 and P(all the three events occur simultaneously) = 1/16. Then the prob that at least one of them occur is
1) 7/32 ii) 7/16 iii) 7/64 iv) 3/16

Sol: $\left.\begin{matrix} P(A \cup B) - P(A \cap B) = 1/4, \\ P(B \cup C) - P(B \cap C) = 1/4 \\ P(C \cup A) - P(C \cap A) = 1/4 \end{matrix}\right\} \Rightarrow \begin{matrix} P(A) + P(B) - 2P(A \cap B) = 1/4 \\ P(B) + P(C) - 2P(B \cap C) = 1/4 \\ P(C) + P(A) - 2P(A \cap C) = 1/4. \end{matrix}$

$\Rightarrow \Sigma P(A) - \Sigma P(A\cap B) = 3/8$

So, $P(A\cup B\cup C) = \Sigma P(A) - \Sigma P(A\cap B) + P(A\cap B\cap C) = \frac{3}{8}+\frac{1}{16} = 7/16$.

5) Let ω be a complex no such that $2\omega+1 = z$ where $z=\sqrt{-3}$. If $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2-1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} = 3K$, then $K =$

(i) $-z$ (2) z (iii) -1 (iv) 1

Sol: $2\omega+1 = \sqrt{3}i \Rightarrow \omega = \frac{\sqrt{3}i-1}{2}$ $\therefore \omega^2 = \frac{-1-\sqrt{3}i}{2}$ Thus $1+\omega+\omega^2=0$.

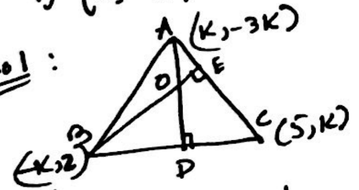
$\therefore \begin{vmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{vmatrix} = \begin{vmatrix} 3 & 1 & 1 \\ 1+\omega+\omega^2 & \omega & \omega^2 \\ 1+\omega^2+\omega & \omega^2 & \omega \end{vmatrix} = 3\begin{vmatrix} 1 & 1 & 1 \\ 0 & \omega & \omega^2 \\ 0 & \omega^2 & \omega \end{vmatrix}$

$= 3(\omega^2-\omega^4) = 3(\omega^2-\omega = 3(-1-2\omega) = -3z$ $\therefore K = -z$.

6) Let K be an integer such that the triangle with vertices $(K,-3K)$, $(5,K)$ and $(-K,2)$ has area 28 sq. units. Then orthocentre of this triangle is at the point

i) $(2,-1/2)$ (ii) $(1,3/4)$ (iii) $(1,-3/4)$ (iv) $(2,1/2)$

Sol:



Now $\frac{1}{2}\left|\begin{vmatrix} K & -3K & 1 \\ -K & 2 & 1 \\ 5 & K & 1 \end{vmatrix}\right| = 28 \Rightarrow K=2$

$\therefore A(2,-6)$, $B(-2,2)$, $C(5,2)$

$m_{AD} = -\frac{1}{m_{BC}} = \frac{-1}{0} = \infty$ $\therefore$ AD: $x=2$.

$m_{BE} = -\frac{1}{m_{AC}} = \frac{-1}{-8/3} = -3/8$; BE: $\frac{y-2}{x+2} = -3/8$

Equating BE, AD we get orthocentre $O = (2, 1/2)$.

7) Twenty metres of wire is available for fencing off a flower bed in the form of a circular sectors. Then the maximum area in (sq-m) of the flower bed is

i) 12.5 (ii) 10 (iii) 25 (iv) 30

Sol:

Length of wire $= r\theta + 2r = 20 \Rightarrow \theta = \frac{2(10-r)}{r}$

Area $A = \frac{\theta r^2}{2} \Rightarrow 10r - r^2$

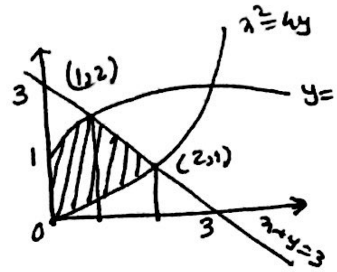
$\frac{dA}{dr} = 0 \Rightarrow 10 - 2r = 0 \Rightarrow r = 5$

Area is max when $r=5$ $\therefore$ Area max $= 25$

8) The area of the region $\{(x,y): x\geq 0,\ x+y\leq 3,\ x^2\leq 4y \text{ and } y\leq 1+\sqrt{x}\}$

i) $59/12$ (ii) $3/2$ (iii) $7/3$ (iv) $5/2$

Sol:



$$A_s = \int_0^1 (1+\sqrt{x})\,dx + \frac{1}{2}(3\times 1) - \int_0^2 \frac{x^2}{4}\,dx$$

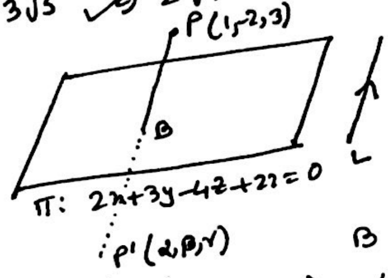
$$= \left[x + \frac{x^{3/2}}{3/2}\right]_0^1 + \frac{3}{2} - \left[\frac{x^3}{12}\right]_0^2$$

$$= 5/2 \text{ sq. units.}$$

9) If the image of point $P(1,-2,3)$ in the plane, $2x+3y-4z+22=0$ measured parallel to the line $\frac{x}{1} = \frac{y}{4} = \frac{z}{5}$, then $PQ =$

1) $3\sqrt{5}$ ✓2) $2\sqrt{42}$ 3) $\sqrt{42}$ 4) $6\sqrt{5}$

Sol:



$\Pi: 2x+3y-4z+22=0$

$L: \frac{x-0}{1} = \frac{y-0}{4} = \frac{z-0}{5}$.

$PP': \frac{x-1}{1} = \frac{y+2}{4} = \frac{z-3}{5} = K$.

Any point on PP': $(K+1, 4K-2, 5K+3) = Q$

Q satisfies the equation of plane.

$\therefore 2(K+1) + 3(4K-2) - 4(5K+3) + 22 = 0 \Rightarrow K = 1$.

$\therefore Q = (2,2,8)$ $\therefore$ $P'(\alpha,\beta,\gamma) = (3,6,13)$ $\Rightarrow |PQ| = 2\sqrt{42}$.

10) If $x \in (0, 1/4)$, the derivative of $\tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^3}\right)$ is $\sqrt{x}\, g(x)$. then $g(x) =$ (i) $\frac{9}{1+9x^3}$ (ii) $\frac{3x\sqrt{x}}{1-9x^3}$ (iii) $\frac{3x}{1-9x^3}$ ✓(iv) $\frac{3}{1+9x^3}$

Sol: $y = \tan^{-1}\left(\frac{6x^{3/2}}{1-(3x^{3/2})^2}\right) \Rightarrow y = \tan^{-1}\left(\frac{2\cdot 3x^{3/2}}{1-(3x^{3/2})^2}\right)$ let $3x^{3/2} = \tan\theta$.

$y = \tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right) \Rightarrow y = \tan^{-1}\tan 2\theta \Rightarrow y = 2\theta$

$\Rightarrow y = 2\tan^{-1} 3x^{3/2} \Rightarrow y' = \cancel{2}\cdot\frac{1}{1+(9x^3)}\cdot\frac{3}{\cancel{2}}\sqrt{x} = \sqrt{x}\times\frac{3}{1+9x^3}$

$\therefore g(x) = \frac{3}{1+9x^3}$

11) If $(2+\sin x)\frac{dy}{dx} + (y+1)\cos x = 0$ and $y(0) = 1$ and $y\left(\frac{\pi}{2}\right) =$

✓1) $1/3$ 2) $-2/3$ 3) $-1/3$ 4) $4/3$

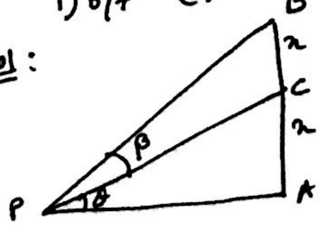
Sol: $\frac{d}{dx}\{(2+\sin x)(y+1)\} = C \Rightarrow (y+1)(2+\sin x) = C$ $\because y(0) = 1$, $\therefore C = 4$.

$\therefore$ At $x = \pi/2$, $y = 1/3$.

12) Let a vertical tower AB have its end A on the level ground. Let C be the midpoint of AB, and P be a point on the ground such that $AP = 2AB$. If $\angle BPC = \beta$, then $\tan\beta =$

1) $6/7$ (ii) $1/4$ ✓(iii) $2/9$ (iv) $4/9$.

Sol:



$\tan(\theta+\beta) = 1/2$

$\tan\theta = 1/4$

$\therefore 1/2 = \frac{\tan\theta + \tan\beta}{1-\tan\theta\cdot\tan\beta}$

$\Rightarrow \tan\beta = 2/9$.