

22. Vectors and Their Properties

Exercise 22

1. Question

Write down the magnitude of each of the following vectors:

A. $\vec{a} = \hat{i} + 2\hat{j} + 5\hat{k}$

B. $\vec{b} = 5\hat{i} - 4\hat{j} - 3\hat{k}$

C. $\vec{c} = \left(\frac{1}{\sqrt{3}}\hat{i} - \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k} \right)$

D. $\vec{d} = (\sqrt{2}\hat{i} + \sqrt{3}\hat{j} - \sqrt{5}\hat{k})$

Answer

Tip - For any vector $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$ the magnitude $|\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$

A. $\vec{a} = \hat{i} + 2\hat{j} + 5\hat{k}$

$$\therefore |\vec{a}| = \sqrt{1^2 + 2^2 + 5^2}$$

$$= \sqrt{30} \text{ units}$$

B. $\vec{a} = 5\hat{i} - 4\hat{j} - 3\hat{k}$

$$\therefore |\vec{a}| = \sqrt{5^2 + 4^2 + 3^2}$$

$$= 5\sqrt{2} \text{ units}$$

C. $\vec{a} = \frac{1}{\sqrt{3}}\hat{i} - \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$

$$\therefore |\vec{a}| = \sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2}$$

$$= 1 \text{ unit}$$

D. $\vec{a} = \sqrt{2}\hat{i} + \sqrt{3}\hat{j} - \sqrt{5}\hat{k}$

$$\therefore |\vec{a}| = \sqrt{(\sqrt{2})^2 + (\sqrt{3})^2 + (\sqrt{5})^2}$$

$$= \sqrt{10} \text{ units}$$

2. Question

Find a unit vector in the direction of the vector:

A. $(3\hat{i} + 4\hat{j} - 5\hat{k})$

B. $(3\hat{i} - 2\hat{j} + 6\hat{k})$

C. $(\hat{i} + \hat{k})$

$$D. (2\hat{i} + \hat{j} + 2\hat{k})$$

Answer

Tip - For any vector $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$ the unit vector is represented as $\hat{a} = \frac{a_x\hat{i} + a_y\hat{j} + a_z\hat{k}}{\sqrt{a_x^2 + a_y^2 + a_z^2}}$

$$A. \vec{a} = 3\hat{i} + 4\hat{j} - 5\hat{k}$$

$$\begin{aligned}\therefore \hat{a} &= \frac{3\hat{i} + 4\hat{j} - 5\hat{k}}{\sqrt{3^2 + 4^2 + 5^2}} \\ &= \frac{3}{5\sqrt{2}}\hat{i} + \frac{4}{5\sqrt{2}}\hat{j} - \frac{5}{5\sqrt{2}}\hat{k}\end{aligned}$$

$$B. \vec{a} = 3\hat{i} - 2\hat{j} + 6\hat{k}$$

$$\begin{aligned}\therefore \hat{a} &= \frac{3\hat{i} - 2\hat{j} + 6\hat{k}}{\sqrt{3^2 + 2^2 + 6^2}} \\ &= \frac{3}{7}\hat{i} - \frac{2}{7}\hat{j} + \frac{6}{7}\hat{k}\end{aligned}$$

$$C. \vec{a} = \hat{i} + \hat{k}$$

$$\begin{aligned}\therefore \hat{a} &= \frac{\hat{i} + \hat{k}}{\sqrt{1^2 + 1^2}} \\ &= \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{k}\end{aligned}$$

$$D. \vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$$

$$\begin{aligned}\therefore \hat{a} &= \frac{2\hat{i} + \hat{j} + 2\hat{k}}{\sqrt{2^2 + 1^2 + 2^2}} \\ &= \frac{2}{3}\hat{i} + \frac{1}{3}\hat{j} + \frac{2}{3}\hat{k}\end{aligned}$$

3. Question

If $\vec{a} = (2\hat{i} - 4\hat{j} + 5\hat{k})$ then find the value of λ so that $\lambda\vec{a}$ may be a unit vector.

Answer

$$\vec{a} = 2\hat{i} - 4\hat{j} + 5\hat{k}$$

$$\therefore \lambda\vec{a} = 2\lambda\hat{i} - 4\lambda\hat{j} + 5\lambda\hat{k}$$

For a unit vector, its magnitude equals to 1.

Tip - For any vector $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$ the magnitude $|\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$

$$\therefore |\lambda\vec{a}| = \sqrt{(2\lambda)^2 + (4\lambda)^2 + (5\lambda)^2} = 1$$

$$\Rightarrow 45\lambda^2 = 1$$

$$\Rightarrow \lambda^2 = \frac{1}{45} = \frac{1}{(3\sqrt{5})^2}$$

$$\Rightarrow \lambda = \pm \frac{1}{3\sqrt{5}}$$

4. Question

If $\vec{a} = (-\hat{i} + \hat{j} - \hat{k})$ and $\vec{b} = (2\hat{i} - \hat{j} + 2\hat{k})$ then find the unit vector in the direction of $(\vec{a} + \vec{b})$.

Answer

$$\vec{a} = -\hat{i} + \hat{j} - \hat{k}$$

$$\vec{b} = 2\hat{i} - \hat{j} + 2\hat{k}$$

$$\therefore \vec{a} + \vec{b}$$

$$= (-\hat{i} + \hat{j} - \hat{k}) + (2\hat{i} - \hat{j} + 2\hat{k})$$

$$= \hat{i} + \hat{k}$$

Tip - For any vector $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$ the unit vector is represented as $\hat{a} = \frac{a_x\hat{i} + a_y\hat{j} + a_z\hat{k}}{\sqrt{a_x^2 + a_y^2 + a_z^2}}$

$$\therefore \widehat{(\vec{a} + \vec{b})}$$

$$= \frac{\hat{i} + \hat{k}}{\sqrt{1^2 + 1^2}}$$

$$= \frac{1}{\sqrt{2}}(\hat{i} + \hat{k})$$

5. Question

If $\vec{a} = (3\hat{i} + \hat{j} - 5\hat{k})$ and $\vec{b} = (\hat{i} + 2\hat{j} - \hat{k})$ then find a unit vector in the direction of $(\vec{a} - \vec{b})$.

Answer

$$\vec{a} = 3\hat{i} + \hat{j} - 5\hat{k}$$

$$\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$$

$$\therefore \vec{a} - \vec{b}$$

$$= (3\hat{i} + \hat{j} - 5\hat{k}) - (\hat{i} + 2\hat{j} - \hat{k})$$

$$= 2\hat{i} - \hat{j} - 4\hat{k}$$

Tip - For any vector $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$ the unit vector is represented as $\hat{a} = \frac{a_x\hat{i} + a_y\hat{j} + a_z\hat{k}}{\sqrt{a_x^2 + a_y^2 + a_z^2}}$

$$\therefore \widehat{(\vec{a} - \vec{b})}$$

$$= \frac{2\hat{i} - \hat{j} - 4\hat{k}}{\sqrt{2^2 + 1^2 + 4^2}}$$

$$= \frac{1}{\sqrt{21}}(2\hat{i} - \hat{j} - 4\hat{k})$$

6. Question

If $\vec{a} = (\hat{i} + 2\hat{j} - 3\hat{k})$ and $\vec{b} = (2\hat{i} + 4\hat{j} + 9\hat{k})$ then find a unit vector parallel to $(\vec{a} + \vec{b})$.

Answer

$$\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$$

$$\vec{b} = 2\hat{i} + 4\hat{j} + 9\hat{k}$$

$$\therefore \vec{a} + \vec{b}$$

$$= (\hat{i} + 2\hat{j} - 3\hat{k}) + (2\hat{i} + 4\hat{j} + 9\hat{k})$$

$$= 3\hat{i} + 6\hat{j} + 6\hat{k}$$

Tip - For any vector $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$ the unit vector is represented as $\hat{a} = \frac{a_x\hat{i} + a_y\hat{j} + a_z\hat{k}}{\sqrt{a_x^2 + a_y^2 + a_z^2}}$

$$\therefore \widehat{(\vec{a} + \vec{b})}$$

$$= \frac{3\hat{i} + 6\hat{j} + 6\hat{k}}{\sqrt{3^2 + 6^2 + 6^2}}$$

$$= \pm \frac{1}{9}(3\hat{i} + 6\hat{j} + 6\hat{k})$$

$$= \pm \frac{1}{3}(\hat{i} + 2\hat{j} + 2\hat{k})$$

7. Question

Find a vector of magnitude 9 units in the direction of the vector $(-2\hat{i} + \hat{j} + 2\hat{k})$.

Answer

Let λ be an arbitrary constant and the required vector is $-2\lambda\hat{i} + \lambda\hat{j} + 2\lambda\hat{k}$

Tip - For any vector $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$ the magnitude $|\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$

$$\therefore \sqrt{(2\lambda)^2 + (\lambda)^2 + (2\lambda)^2} = 9$$

$$\Rightarrow 3\lambda = 9$$

$$\Rightarrow \lambda = 3$$

The required vector is $-6\hat{i} + 3\hat{j} + 6\hat{k}$

8. Question

Find a vector of magnitude 8 units in the direction of the vector $(5\hat{i} - \hat{j} + 2\hat{k})$.

Answer

Let λ be an arbitrary constant and the required vector is $5\lambda\hat{i} - \lambda\hat{j} + 2\lambda\hat{k}$

Tip - For any vector $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$ the magnitude $|\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$

$$\therefore \sqrt{(5\lambda)^2 + (\lambda)^2 + (2\lambda)^2} = 8$$

$$\Rightarrow \sqrt{30}\lambda = 8$$

$$\Rightarrow \lambda = \frac{8}{\sqrt{30}}$$

The required vector is $\frac{8}{\sqrt{30}}(5\hat{i} - \hat{j} + 2\hat{k})$

9. Question

Find a vector of magnitude 21 units in the direction of the vector $(2\hat{i} - 3\hat{j} + 6\hat{k})$.

Answer

Let λ be an arbitrary constant and the required vector is $2\lambda\hat{i} - 3\lambda\hat{j} + 6\lambda\hat{k}$

Tip - For any vector $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$ the magnitude $|\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$

$$\therefore \sqrt{(2\lambda)^2 + (3\lambda)^2 + (6\lambda)^2} = 21$$

$$\Rightarrow 7\lambda = 21$$

$$\Rightarrow \lambda = 3$$

The required vector is $(6\hat{i} - 9\hat{j} + 18\hat{k})$

10. Question

If $\vec{a} = (\hat{i} - 2\hat{j})$, $\vec{b} = (2\hat{i} - 3\hat{j})$ and $\vec{c} = (2\hat{i} + 3\hat{k})$, find $(\vec{a} + \vec{b} + \vec{c})$.

Answer

$$\vec{a} = \hat{i} - 2\hat{j}$$

$$\vec{b} = 2\hat{i} - 3\hat{j}$$

$$\vec{c} = 2\hat{i} + 3\hat{k}$$

$$\therefore \vec{a} + \vec{b} + \vec{c}$$

$$= (\hat{i} - 2\hat{j}) + (2\hat{i} - 3\hat{j}) + (2\hat{i} + 3\hat{k})$$

$$= 5\hat{i} - 5\hat{j} + 3\hat{k}$$

11. Question

If A(-2, 1, 2) and B(2, -1, 6) are two given points, find a unit vector in the direction of \overline{AB} .

Answer

$$A = (-2, 1, 2)$$

$$B = (2, -1, 6)$$

$$\therefore \overline{AB}$$

$$= \{2 - (-2)\}\hat{i} + \{(-1) - 1\}\hat{j} + \{6 - 2\}\hat{k}$$

$$= 4\hat{i} - 2\hat{j} + 4\hat{k}$$

Tip - For any vector $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$ the unit vector is represented as $\hat{a} = \frac{a_x\hat{i} + a_y\hat{j} + a_z\hat{k}}{\sqrt{a_x^2 + a_y^2 + a_z^2}}$

$$\therefore \widehat{AB}$$

$$= \frac{4\hat{i} - 2\hat{j} + 4\hat{k}}{\sqrt{4^2 + 2^2 + 4^2}}$$

$$= \frac{4}{6}\hat{i} - \frac{2}{6}\hat{j} + \frac{4}{6}\hat{k}$$

$$= \frac{2}{3}\hat{i} - \frac{1}{3}\hat{j} + \frac{2}{3}\hat{k}$$

12. Question

Find the direction ratios and direction cosines of the vector $\vec{a} = (5\hat{i} - 3\hat{j} + 4\hat{k})$.

Answer

$$\vec{a} = 5\hat{i} - 3\hat{j} + 4\hat{k}$$

Tip - For any vector $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$ the direction ratios are represented as (a_x, a_y, a_z) and the direction cosines are given by $\frac{a_x}{\sqrt{a_x^2 + a_y^2 + a_z^2}}, \frac{a_y}{\sqrt{a_x^2 + a_y^2 + a_z^2}}, \frac{a_z}{\sqrt{a_x^2 + a_y^2 + a_z^2}}$

The direction ratios are (5, -3, 4)

The direction cosines are $\frac{5}{\sqrt{5^2 + 3^2 + 4^2}}, \frac{-3}{\sqrt{5^2 + 3^2 + 4^2}}, \frac{4}{\sqrt{5^2 + 3^2 + 4^2}}$

$$= \frac{5}{5\sqrt{2}}, \frac{-3}{5\sqrt{2}}, \frac{4}{5\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}}, \frac{-3}{5\sqrt{2}}, \frac{4}{5\sqrt{2}}$$

13. Question

Find the direction ratios and the direction cosines of the vector joining the points A(2, 1, -2) and B(3, 5, -4).

Answer

$$A = (2, 1, -2)$$

$$B = (3, 5, -4)$$

$$\therefore \vec{AB}$$

$$= \{3 - 2\}\hat{i} + \{5 - 1\}\hat{j} + \{(-4) - (-2)\}\hat{k}$$

$$= \hat{i} + 4\hat{j} - 2\hat{k}$$

Tip - For any vector $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$ the direction ratios are represented as (a_x, a_y, a_z) and the direction cosines are given by $\frac{a_x}{\sqrt{a_x^2 + a_y^2 + a_z^2}}, \frac{a_y}{\sqrt{a_x^2 + a_y^2 + a_z^2}}, \frac{a_z}{\sqrt{a_x^2 + a_y^2 + a_z^2}}$

The direction ratios are (1, 4, -2)

The direction cosines are $\frac{1}{\sqrt{1^2 + 4^2 + 2^2}}, \frac{4}{\sqrt{1^2 + 4^2 + 2^2}}, \frac{-2}{\sqrt{1^2 + 4^2 + 2^2}}$

$$= \frac{1}{\sqrt{21}}, \frac{4}{\sqrt{21}}, \frac{-2}{\sqrt{21}}$$

14. Question

Show that the points A, B and C having position vectors $(\hat{i} + 2\hat{j} + 7\hat{k}), (2\hat{i} + 6\hat{j} + 3\hat{k})$ and $(3\hat{i} + 10\hat{j} - 3\hat{k})$ respectively, are collinear.

Answer

$$A = \hat{i} + 2\hat{j} + 7\hat{k}$$

$$B = 2\hat{i} + 6\hat{j} + 2\hat{k}$$

$$C = 3\hat{i} + 10\hat{j} - 3\hat{k}$$

$$\therefore \overline{AB}$$

$$= (2\hat{i} + 6\hat{j} + 2\hat{k}) - (\hat{i} + 2\hat{j} + 7\hat{k})$$

$$= \hat{i} + 4\hat{j} - 5\hat{k}$$

$$\therefore \overline{BC}$$

$$= (3\hat{i} + 10\hat{j} - 3\hat{k}) - (2\hat{i} + 6\hat{j} + 2\hat{k})$$

$$= \hat{i} + 4\hat{j} - 5\hat{k}$$

$$\overline{AB} = \overline{BC}$$

So, the points A, B and C are collinear.

15. Question

The position vectors of the points A, B and C are $(2\hat{i} + \hat{j} - \hat{k})$, $(3\hat{i} - 2\hat{j} + \hat{k})$ and $(\hat{i} + 4\hat{j} - 3\hat{k})$ respectively.

Show that the points A, B and C are collinear.

Answer

$$A = 2\hat{i} + \hat{j} - \hat{k}$$

$$B = 3\hat{i} - 2\hat{j} + \hat{k}$$

$$C = \hat{i} + 4\hat{j} - 3\hat{k}$$

$$\therefore \overline{AB}$$

$$= (3\hat{i} - 2\hat{j} + \hat{k}) - (2\hat{i} + \hat{j} - \hat{k})$$

$$= \hat{i} - 3\hat{j} + 2\hat{k}$$

$$\therefore \overline{BC}$$

$$= (\hat{i} + 4\hat{j} - 3\hat{k}) - (3\hat{i} - 2\hat{j} + \hat{k})$$

$$= -2\hat{i} + 6\hat{j} - 4\hat{k}$$

$$(-3)\overline{AB} = \overline{BC}$$

So, the points A, B and C are collinear.

16. Question

If the position vectors of the vertices A, B and C of a ΔABC be $(\hat{i} + 2\hat{j} + 3\hat{k})$, $(2\hat{i} + 3\hat{j} + \hat{k})$ and $(3\hat{i} + \hat{j} + 2\hat{k})$ respectively, prove that ΔABC is equilateral.

Answer

$$A = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$B = 2\hat{i} + 3\hat{j} + \hat{k}$$

$$C = 3\hat{i} + \hat{j} + 2\hat{k}$$

$$\therefore \overline{AB}$$

$$= (2\hat{i} + 3\hat{j} + \hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k})$$

$$= \hat{i} + \hat{j} - 2\hat{k}$$

$$\therefore \overrightarrow{BC}$$

$$= (3\hat{i} + \hat{j} + 2\hat{k}) - (2\hat{i} + 3\hat{j} + \hat{k})$$

$$= \hat{i} - 2\hat{j} + \hat{k}$$

$$\therefore \overrightarrow{CA}$$

$$= (\hat{i} + 2\hat{j} + 3\hat{k}) - (3\hat{i} + \hat{j} + 2\hat{k})$$

$$= -2\hat{i} + \hat{j} + \hat{k}$$

Tip - For any vector $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$ the magnitude $|\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$

$$\therefore |\overrightarrow{AB}| = \sqrt{1^2 + 1^2 + 2^2} = \sqrt{6}$$

$$\therefore |\overrightarrow{BC}| = \sqrt{1^2 + 2^2 + 1^2} = \sqrt{6}$$

$$\therefore |\overrightarrow{CA}| = \sqrt{2^2 + 1^2 + 1^2} = \sqrt{6}$$

$$\therefore |\overrightarrow{AB}| = |\overrightarrow{BC}| = |\overrightarrow{CA}|$$

The three sides of the triangle are equal in magnitude, so the triangle is equilateral.

17. Question

Show that the points A, B and C having position vectors $(3\hat{i} - 4\hat{j} - 4\hat{k})$, $(2\hat{i} - \hat{j} + \hat{k})$ and $(\hat{i} - 3\hat{j} - 5\hat{k})$ respectively, form the vertices of a right-angled triangle.

Answer

$$A = 3\hat{i} - 4\hat{j} - 4\hat{k}$$

$$B = 2\hat{i} - \hat{j} + \hat{k}$$

$$C = \hat{i} - 3\hat{j} - 5\hat{k}$$

$$\therefore \overrightarrow{AB}$$

$$= (2\hat{i} - \hat{j} + \hat{k}) - (3\hat{i} - 4\hat{j} - 4\hat{k})$$

$$= -\hat{i} + 3\hat{j} + 5\hat{k}$$

$$\therefore \overrightarrow{BC}$$

$$= (\hat{i} - 3\hat{j} - 5\hat{k}) - (2\hat{i} - \hat{j} + \hat{k})$$

$$= -\hat{i} - 2\hat{j} - 6\hat{k}$$

$$\therefore \overrightarrow{CA}$$

$$= (3\hat{i} - 4\hat{j} - 4\hat{k}) - (\hat{i} - 3\hat{j} - 5\hat{k})$$

$$= 2\hat{i} - \hat{j} + \hat{k}$$

Tip - For any 2 perpendicular vectors \vec{a} & \vec{b} , $\vec{a} \cdot \vec{b} = 0$

$$\therefore \overrightarrow{AB} \cdot \overrightarrow{CA}$$

$$= (-\hat{i} + 3\hat{j} + 5\hat{k}) \cdot (2\hat{i} - \hat{j} + \hat{k})$$

$$= -2 - 3 + 5$$

$$= 0$$

The triangle is right-angled.

18. Question

Using vector method, show that the points A(1, -1, 0), B(4, -3, 1) and C(2, -4, 5) are the vertices of a right-angled triangle.

Answer

$$A = (1, -1, 0)$$

$$B = (4, -3, 1)$$

$$C = (2, -4, 5)$$

$$\therefore \overrightarrow{AB}$$

$$= (4 - 1)\hat{i} + (-3 + 1)\hat{j} + (1 - 0)\hat{k}$$

$$= 3\hat{i} - 2\hat{j} + \hat{k}$$

$$\therefore \overrightarrow{BC}$$

$$= (2 - 4)\hat{i} + (-4 + 3)\hat{j} + (5 - 1)\hat{k}$$

$$= -2\hat{i} - \hat{j} + 4\hat{k}$$

$$\therefore \overrightarrow{CA}$$

$$= (1 - 2)\hat{i} + (-1 + 4)\hat{j} + (0 - 5)\hat{k}$$

$$= -\hat{i} + 3\hat{j} - 5\hat{k}$$

Tip - For any 2 perpendicular vectors \vec{a} & \vec{b} , $\vec{a} \cdot \vec{b} = 0$

$$\therefore \overrightarrow{AB} \cdot \overrightarrow{BC}$$

$$= (3\hat{i} - 2\hat{j} + \hat{k}) \cdot (-2\hat{i} - \hat{j} + 4\hat{k})$$

$$= -6 + 2 + 4$$

$$= 0$$

The triangle is right-angled.

19. Question

Find the position vector of the point which divides the join of the points $(2\vec{a} - 3\vec{b})$ and $(3\vec{a} - 2\vec{b})$ (i) internally and (ii) externally in the ratio 2 : 3.

Answer

$$\vec{A} = 2\vec{a} - 3\vec{b}$$

$$\vec{B} = 3\vec{a} - 2\vec{b}$$

Formula to be used - The point dividing a line joining points a and b in a ratio m:n internally or externally is given by $\frac{m\vec{b} \pm n\vec{a}}{m \pm n}$ respectively.

The position vector of the point dividing the line internally

$$= \frac{2 \times (3\vec{a} - 2\vec{b}) + 3 \times (2\vec{a} - 3\vec{b})}{2 + 3}$$

$$= \frac{12}{5}\hat{a} - \frac{13}{5}\hat{b}$$

The position vector of the point dividing the line externally

$$= \frac{2 \times (3\hat{a} - 2\hat{b}) - 3 \times (2\hat{a} - 3\hat{b})}{2 - 3}$$

$$= -5\hat{b}$$

20. Question

The position vectors of two points A and B are $(2\vec{a} + \vec{b})$ and $(\vec{a} - 3\vec{b})$ respectively. Find the position vector of a point C which divides AB externally in the ratio 1 : 2. Also, show that A is the mid-point of the line segment CB.

Answer

$$\vec{A} = 2\vec{a} + \vec{b}$$

$$\vec{B} = \vec{a} - 3\vec{b}$$

Formula to be used - The point dividing a line joining points a and b in a ratio m:n externally is given by $\frac{mb-na}{m-b}$ respectively.

The position vector of the point C dividing the line externally

$$= \frac{1 \times (\vec{a} - 3\vec{b}) - 2 \times (2\vec{a} + \vec{b})}{2 - 3}$$

$$= 3\vec{a} + 5\vec{b}$$

The midpoint of B and C may be given by

$$\frac{(\vec{a} - 3\vec{b}) + (3\vec{a} + 5\vec{b})}{2}$$

$$= 2\vec{a} + \vec{b} \text{ i.e. point A}$$

A is the midpoint of B and C.

21. Question

Find the position vector of a point R which divides the line joining A(-2, 1, 3) and B(3, 5, -2) in the ratio 2 : 1 (i) internally (ii) externally.

Answer

$$A = (-2, 1, 3)$$

$$B = (3, 5, -2)$$

$$\therefore \vec{OA} = -2\hat{i} + \hat{j} + 3\hat{k}$$

$$\therefore \vec{OB} = 3\hat{i} + 5\hat{j} - 2\hat{k}$$

Formula to be used - The point dividing a line joining points a and b in a ratio m:n internally or externally is given by $\frac{mb+na}{m+n}$ respectively.

The position vector of the point dividing the line internally

$$= \frac{2 \times (-2\hat{i} + \hat{j} + 3\hat{k}) + 1 \times (3\hat{i} + 5\hat{j} - 2\hat{k})}{2 + 1}$$

$$= \frac{4}{3}\hat{i} + \frac{11}{3}\hat{j} - \frac{1}{3}\hat{k}$$

The position vector of the point dividing the line externally

$$\begin{aligned} &= \frac{2 \times (-2\hat{i} + \hat{j} + 3\hat{k}) - 1 \times (3\hat{i} + 5\hat{j} - 2\hat{k})}{2 - 1} \\ &= 8\hat{i} + 9\hat{j} - 7\hat{k} \end{aligned}$$

22. Question

Find the position vector of the mid-point of the vector joining the points $A(3\hat{i} + 2\hat{j} + 6\hat{k})$ and $B(\hat{i} + 4\hat{j} - 2\hat{k})$.

Answer

$$\vec{OA} = 3\hat{i} + 2\hat{j} + 6\hat{k}$$

$$\vec{OB} = \hat{i} + 4\hat{j} - 2\hat{k}$$

Formula to be used - The midpoint of a line joining points a and b is given by $\frac{a+b}{2}$.

The position vector of the midpoint

$$\begin{aligned} &= \frac{(3\hat{i} + 2\hat{j} + 6\hat{k}) + (\hat{i} + 4\hat{j} - 2\hat{k})}{2} \\ &= 2\hat{i} + 3\hat{j} + 2\hat{k} \end{aligned}$$

23. Question

If $\vec{AB} = (2\hat{i} + \hat{j} - 3\hat{k})$ and A(1, 2, -1) is the given point, find the coordinates of B.

Answer

$$A = (1, 2, -1)$$

Let the co-ordinates of point B be (b_1, b_2, b_3)

$$\vec{AB} = 2\hat{i} + \hat{j} - 3\hat{k}$$

$$\Rightarrow [(b_1 - 1)\hat{i} + (b_2 - 2)\hat{j} + (b_3 + 1)\hat{k}] = 2\hat{i} + \hat{j} - 3\hat{k}$$

Comparing the respective co-efficient,

$$b_1 - 1 = 2 \text{ i.e. } b_1 = 3$$

$$b_2 - 2 = 1 \text{ i.e. } b_2 = 3$$

$$b_3 + 1 = -3 \text{ i.e. } b_3 = -4$$

The required co-ordinates of B are (3, 3, -4)

24. Question

Write a unit vector in the direction of \vec{PQ} , where P and Q are the points (1, 3, 0) and (4, 5, 6) respectively.

Answer

$$P = (1, 3, 0)$$

$$Q = (4, 5, 6)$$

$$\therefore \vec{PQ}$$

$$= (4 - 1)\hat{i} + (5 - 3)\hat{j} + (6 - 0)\hat{k}$$

$$= 3\hat{i} + 2\hat{j} + 6\hat{k}$$

Tip - For any vector $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$ the unit vector is represented as $\hat{a} = \frac{a_x\hat{i} + a_y\hat{j} + a_z\hat{k}}{\sqrt{a_x^2 + a_y^2 + a_z^2}}$

$\therefore \widehat{PQ}$

$$= \frac{3\hat{i} + 2\hat{j} + 6\hat{k}}{\sqrt{3^2 + 2^2 + 6^2}}$$

$$= \frac{1}{7}(3\hat{i} + 2\hat{j} + 6\hat{k})$$