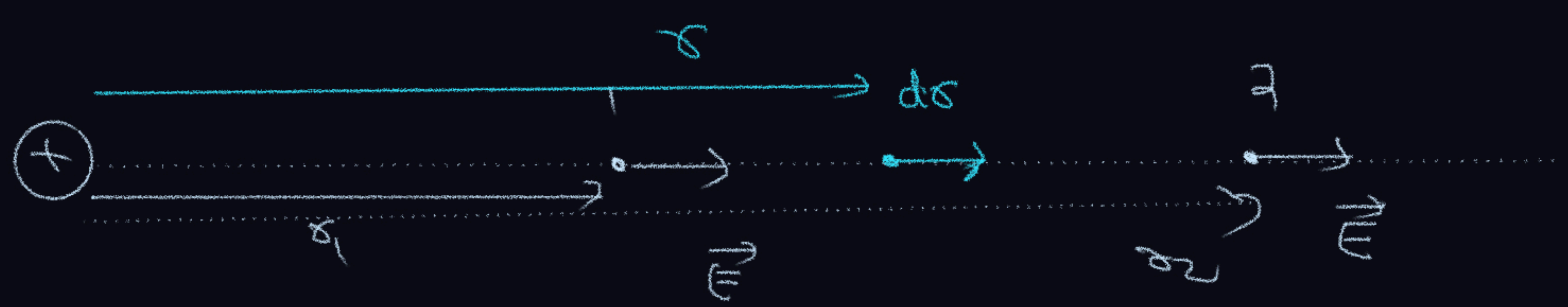
Electric Potential

$$\boxed{W_{conserv} = -\Delta PE}$$

$$\boxed{U = \frac{kq}{r} + U_\infty}$$



$$dW = \vec{E}\cdot d\vec{r} = E\,dr$$

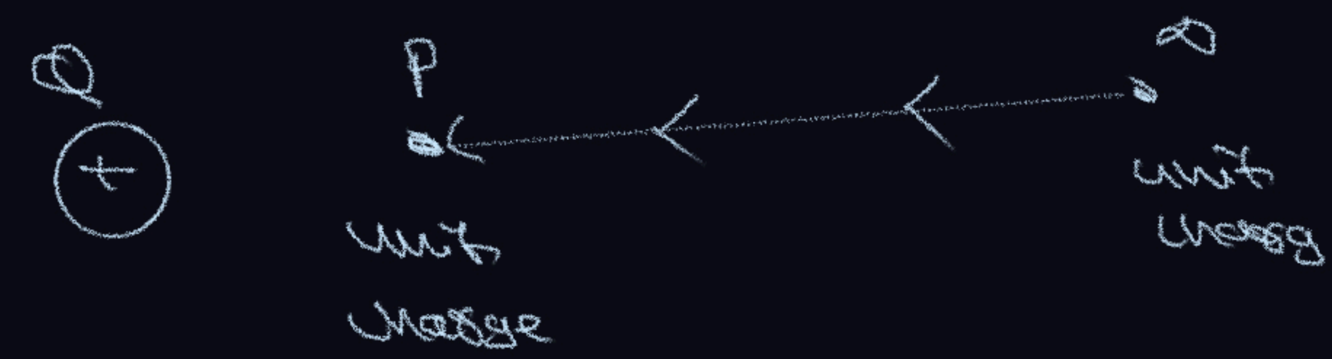
$$W = \int E\,dr = \int \frac{kq}{r^2}\,dr = kq \left. -\frac{1}{r}\right|_{r_1}^{r_2} = kq\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$W_{12} = -\Delta U = -(U_f - U_i) = -(U_2 - U_1) = U_1 - U_2$$

$$U_1 = W_{12} + U_2 = kq\left(\frac{1}{r_1} - \frac{1}{r_2}\right) + U_2$$

$W_{consr} = -\Delta U$

If we move charge slowly ($\Rightarrow KE = 0$)



$W_{ext} + W_{consr} = \Delta KE$

$W_{ext} = -W_{consr}$

$$\boxed{W_{ext} = \Delta U}$$

$W_{consr} = -\Delta U$

$W_{ext} = \Delta U$

$$W_{ext,\ \infty \to P} = U_f - U_i = U_P - U_\infty$$

Ext

$$W_{\infty \to P} = \Delta U = U_f - U_i$$

$$W_{\infty \to P} = U_P - U_\infty = U_P$$

$$\boxed{U_P = W_{ext,\ \infty \to P} = \text{energy spent by ext force to construct system}}$$

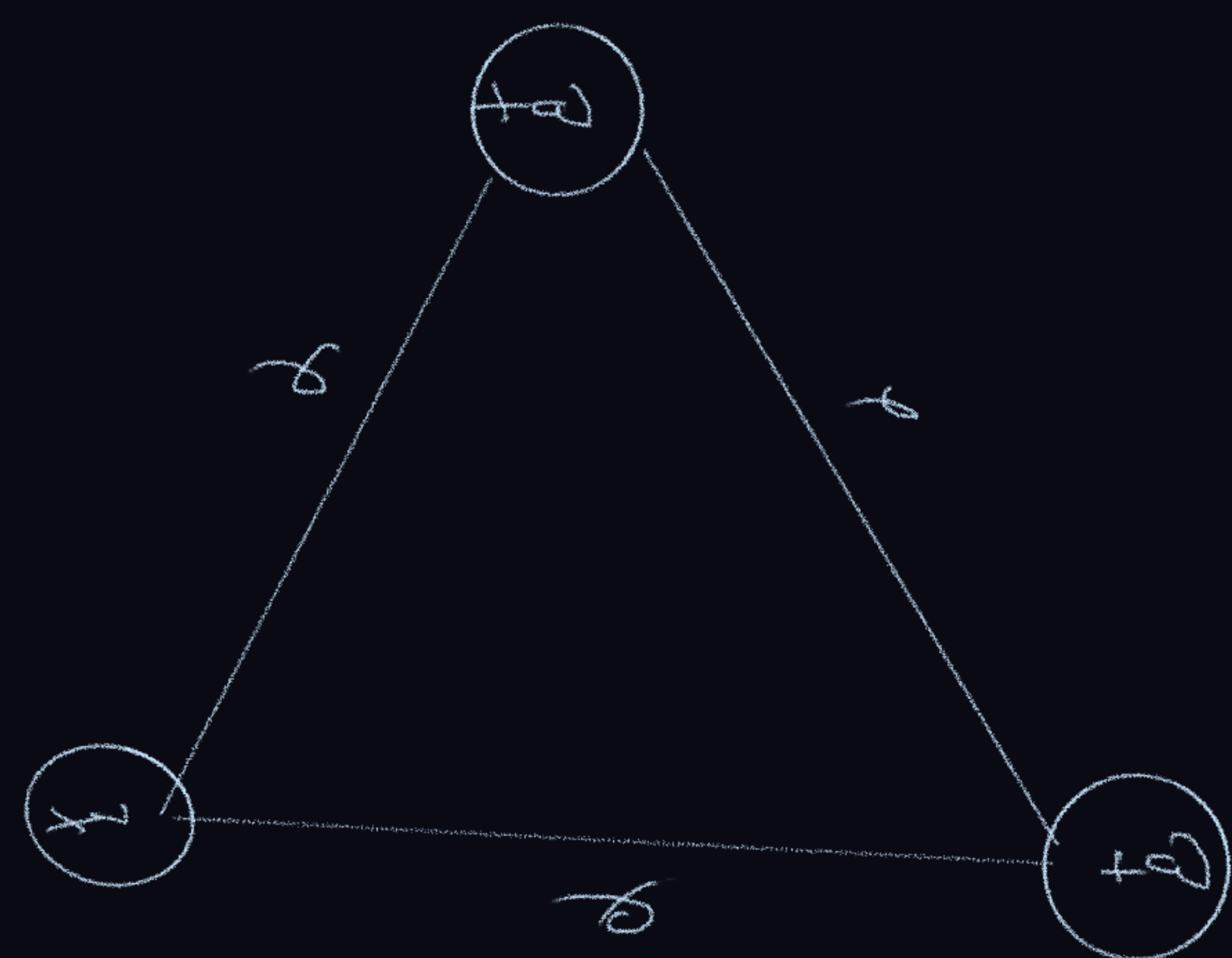
$$\boxed{V_P = W_{\infty \to P}}$$

Potential energy and potential at a point

Force vs feild

Pot energy vs potential

P.E of system = potential at that point



potential energy of system?

$$W_{q_2} = q_2 V_1 = q_2 \frac{kq_1}{r_1}$$

$$W_{q_3} = q_3(V_p) = q_3(V_1 + V_2)$$

$$= q_3 V_1 + q_3 V_2$$

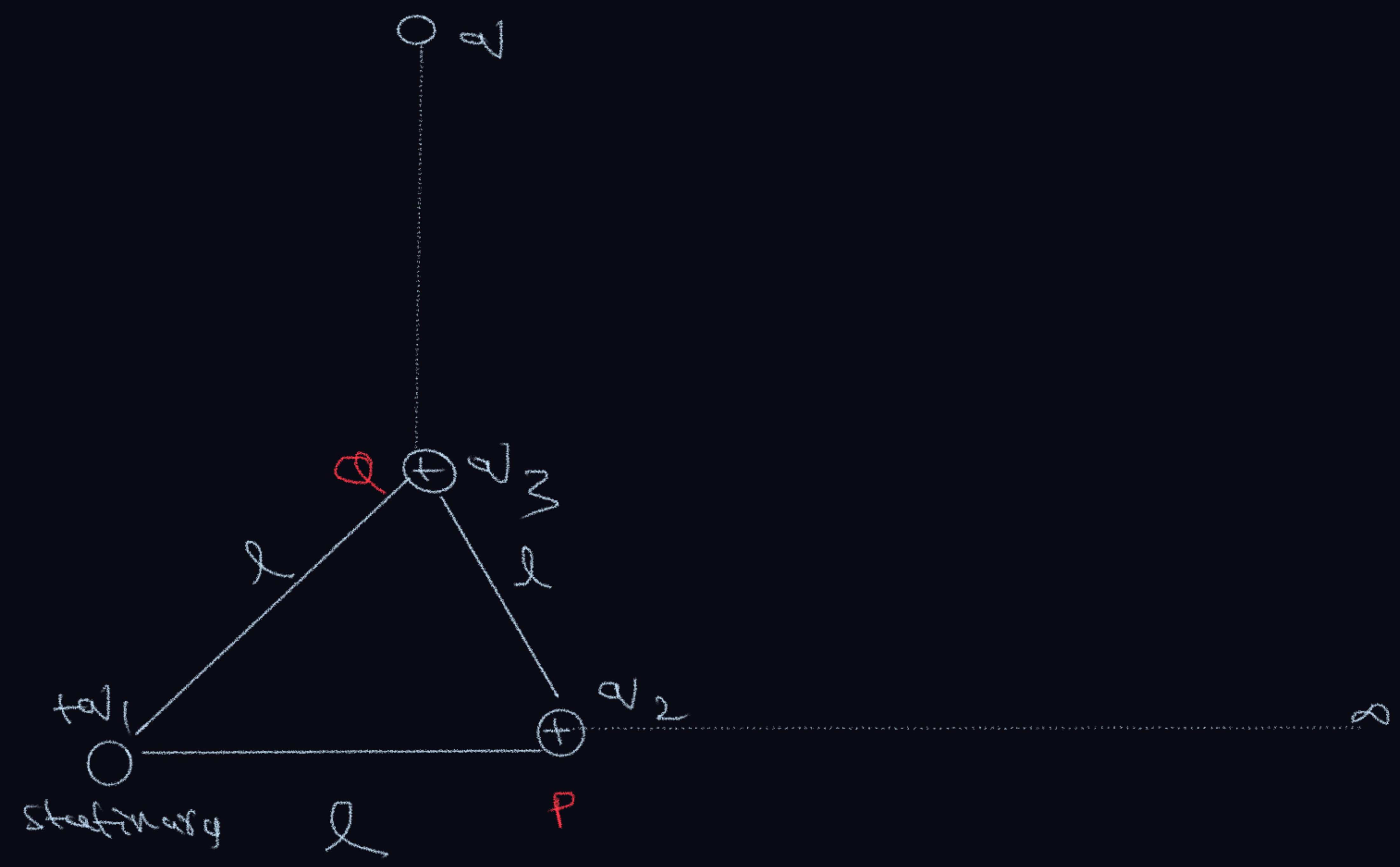
$$= q_3 \frac{kq_1}{r} + q_3 \frac{kq_2}{r}$$

$$W = q\Delta V$$

$$W = (V - V_\infty)q$$

$$W = Vq = \frac{kq^2}{r}$$

$$W = \frac{kq_1q_2}{r} + \frac{kq_2q_3}{r} + \frac{kq_1q_3}{r}$$



$$W_{\infty-P} = q_3 V_Q$$

$$V_Q = \frac{kq_1}{l} + \frac{kq_2}{l}$$

$$W_{\infty P} = q_3 V_Q = q_3\left(\frac{kq_1}{l} + \frac{kq_2}{l}\right)$$

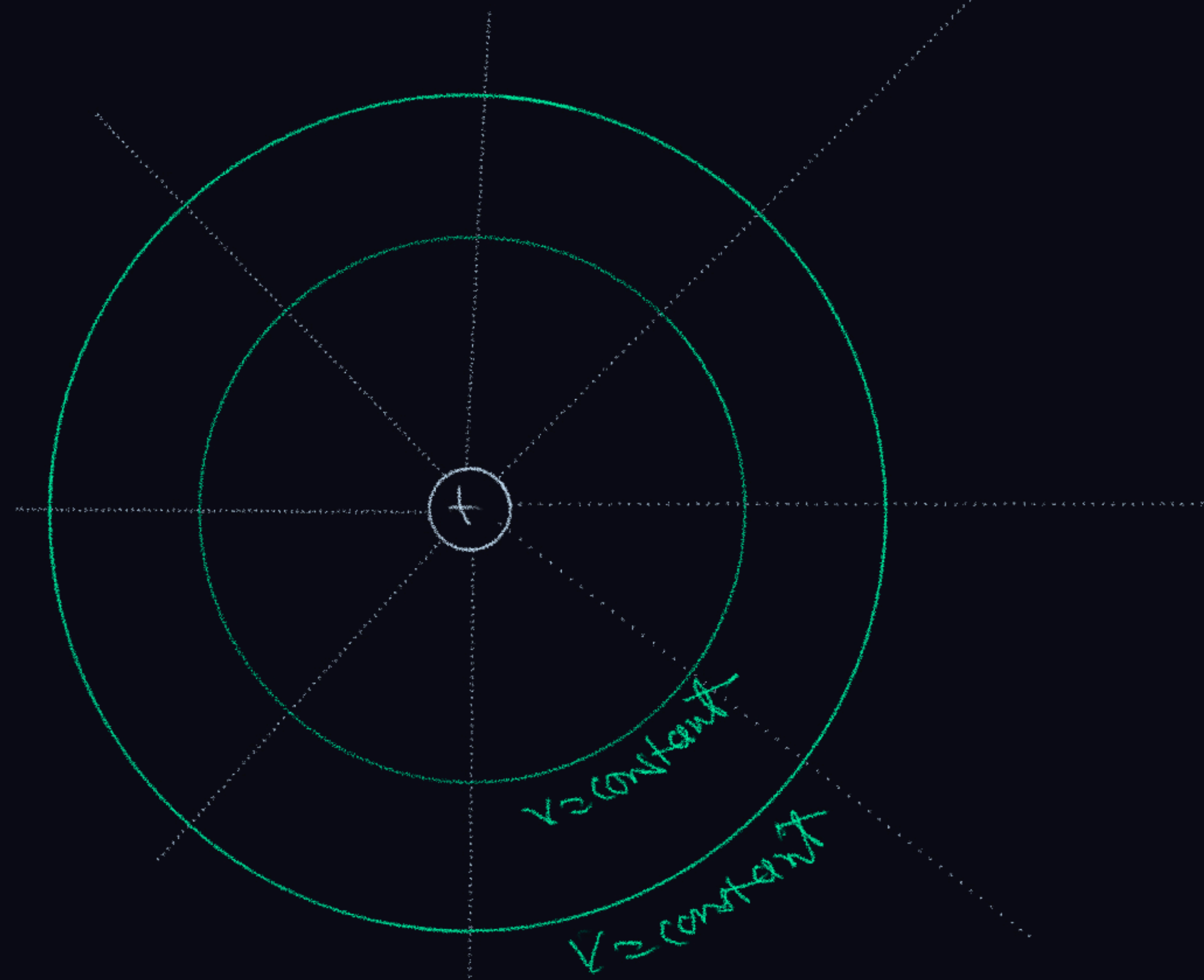
$$W_{total} = \frac{kq_1q_2}{l} + \frac{k\,q_1q_3}{l} + \frac{kq_2q_3}{l}$$

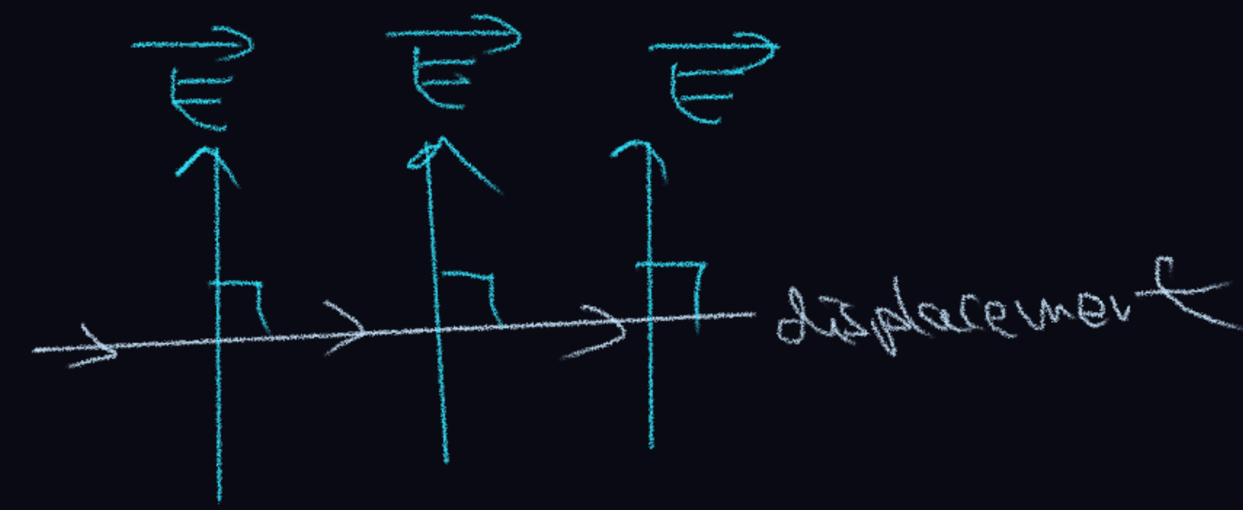
$W_{\infty-P} = V_P$ (unit charge)

$W_{\infty-P} = q_2 V_P$ ('q_1' charge)

$$W_{\infty P} = q_2 \frac{kq_1}{l}$$

Equipotential Surface



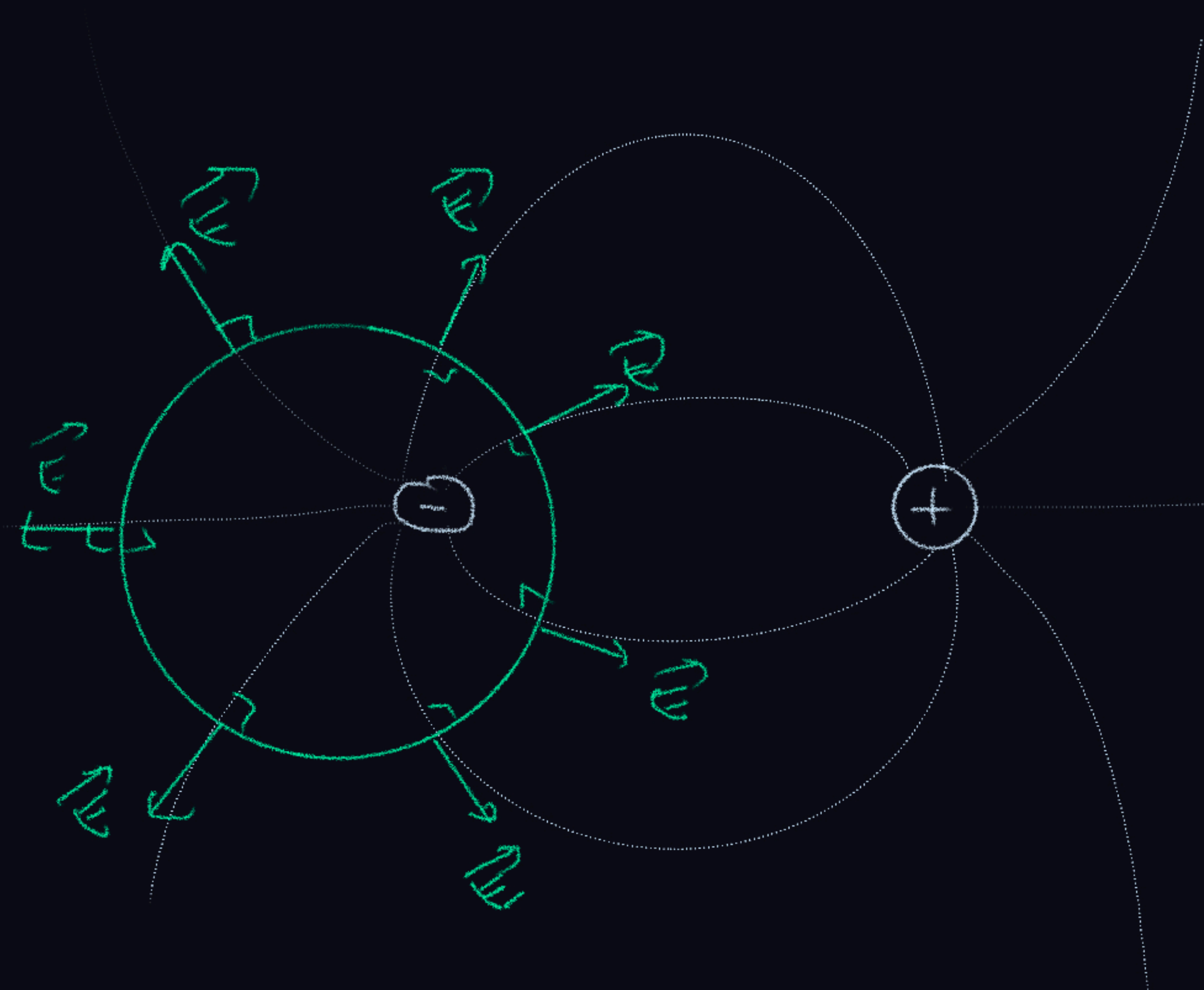


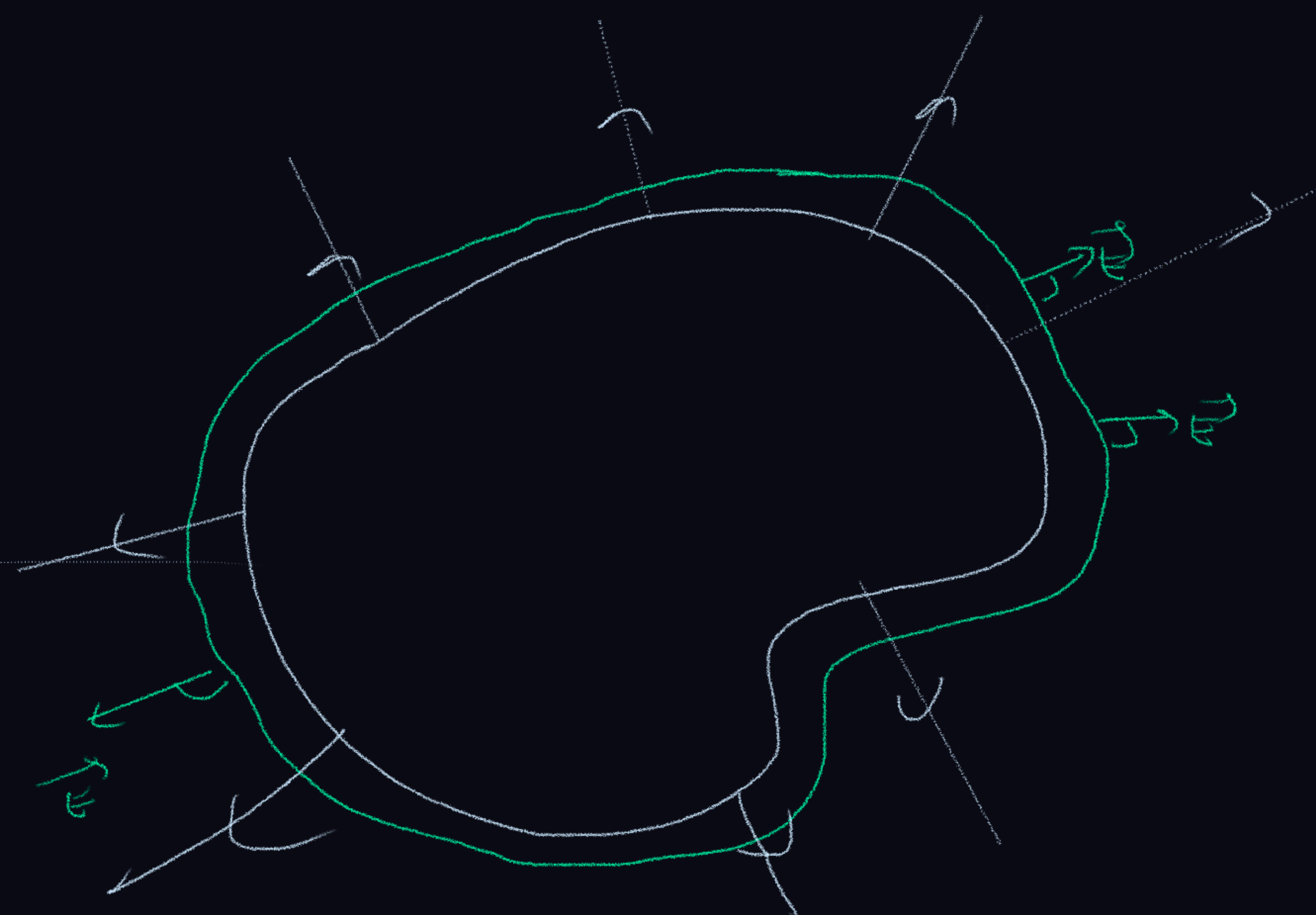
$W_{conserv} = 0$

$-\Delta V = 0$

$V = const$

(Equipotential surface)





finding Feild from potential

$$\boxed{F = -\frac{\partial V}{\partial r}}$$

$$F_x = -\frac{\partial V}{\partial x} \qquad F_z = -\frac{\partial V}{\partial z}$$

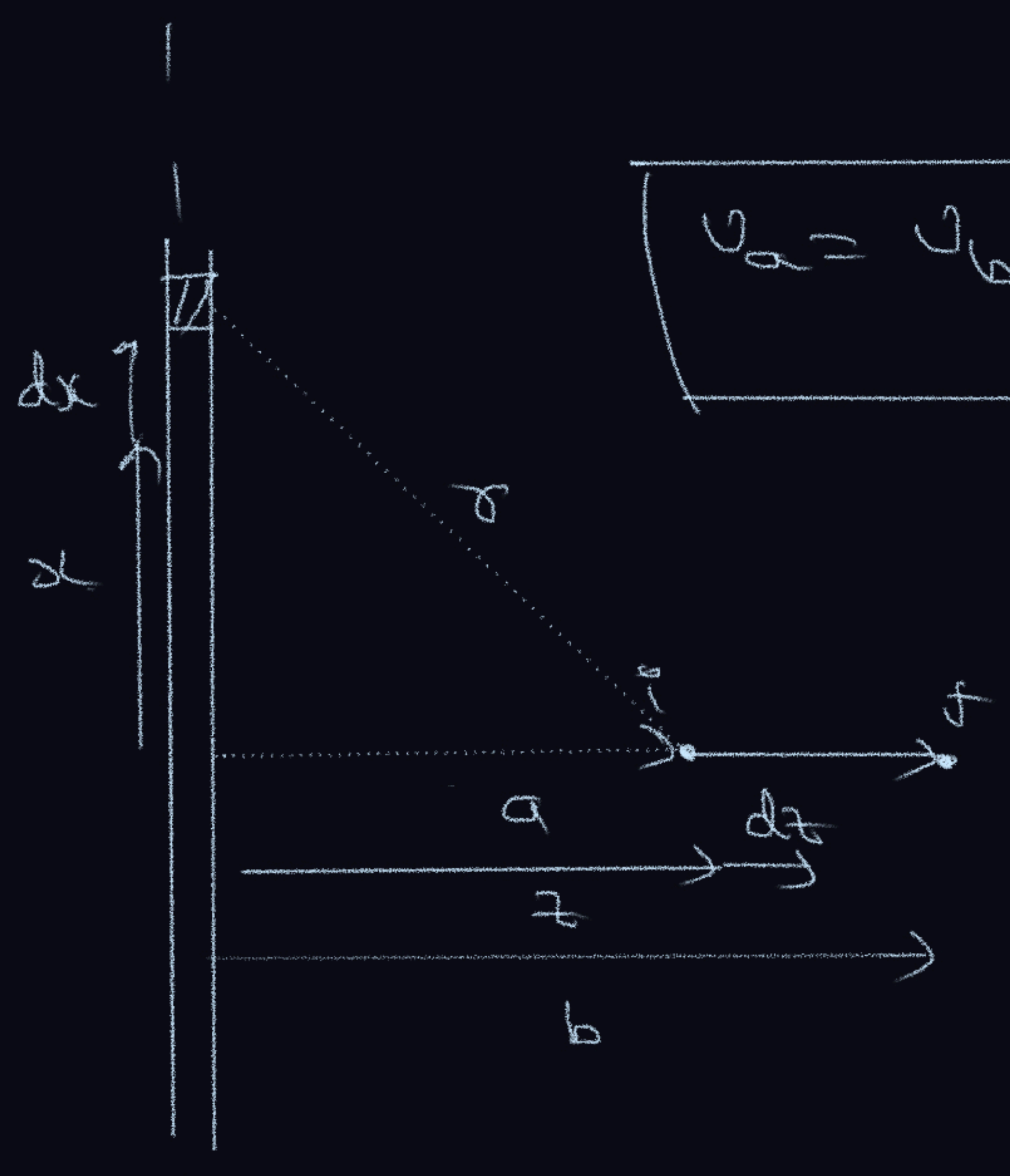
$$F_y = -\frac{\partial V}{\partial y}$$

The potential at any point is given by $V = x(y^2 - 4x^2)$,

Calculate the carterian components of the electric feild at the point.

$$\vec{F} = \frac{-\partial V}{\partial x}\hat{i} + \frac{-\partial V}{\partial y}\hat{j} = -(y^2 - (2x^2))\hat{i} - (2xy)\hat{j}$$

Potential due to uniform line of Charge



$$V = \int dV = \int \frac{k\,dq}{r}$$

$$\boxed{V_a = V_b + k\lambda \log\left(\frac{b}{a}\right)}$$

$$\int dV = \int \frac{k\,dq}{r} = \int \frac{k\lambda\,dx}{\sqrt{x^2+d^2}} = k\lambda \int \frac{dx}{\sqrt{x^2+d^2}}$$

$$= \frac{k\lambda}{d} \log\left(x + \sqrt{x^2+d^2}\right)$$

$$= \frac{k\lambda}{d} \log \frac{L+\sqrt{L^2+d^2}}{-L+\sqrt{L^2+d^2}} = \infty \text{ (not defined)}$$

∴ potential is not defined

$$W = -\Delta U$$

$$\int \frac{k\lambda}{z}\,dz = V_i - V_f$$

$$k\lambda \log z \Big|_a^b = V_i - V_f$$

$$\int \frac{dx}{\sqrt{x^2+a^2}} = \frac{1}{a}\int \frac{d\left(\frac{x}{a}\right)}{\sqrt{\left(\frac{x}{a}\right)^2+1}} = \frac{1}{a}\int \frac{dy}{\sqrt{y^2+1}}$$

$$= \frac{1}{a} \log\left(y + \sqrt{y^2+1}\right)$$

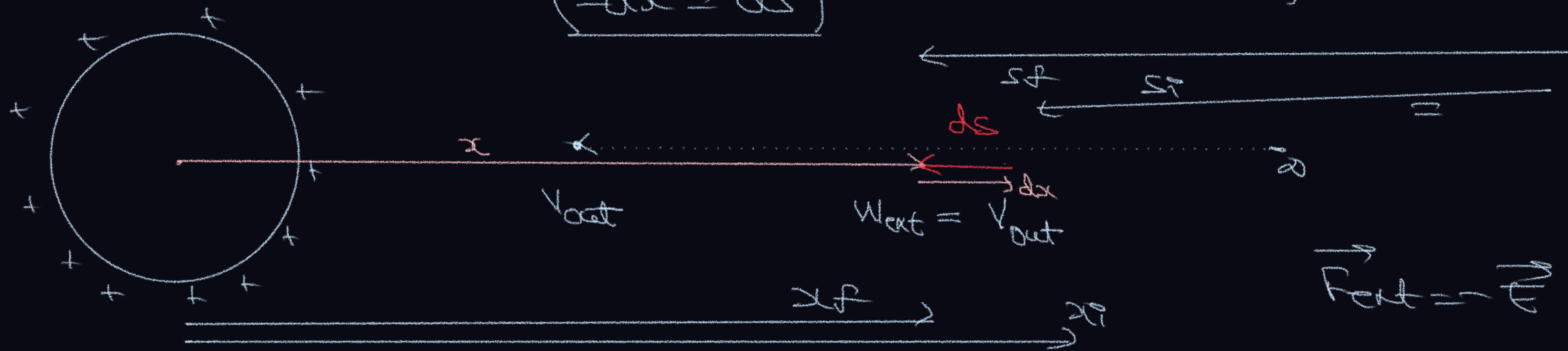
$ds = s_f - s_i = +ve$ $\quad |ds| = ds$ $\quad d\vec{s} = |ds|(-\hat{i}) = ds(-\hat{i})$

$dx = x_f - x_i = -ve$ $\quad |dx| = -dx$ $\quad d\vec{x} = |dx|\,\hat{i} = (-dx)\,\hat{i} = ds\,\hat{i}$

σ, R

$|dx| = |ds|$

$\boxed{-dx = ds}$ $\quad \boxed{d\vec{s} = -d\vec{x}}$



Hollow sphere

$\vec{F}_{ext} = -\vec{E}$

(If moved slowly)

$dW = \vec{F}_{ext} \cdot d\vec{s}$

$dW = -\vec{E} \cdot d\vec{s} = +E\,ds = -E\,dx$

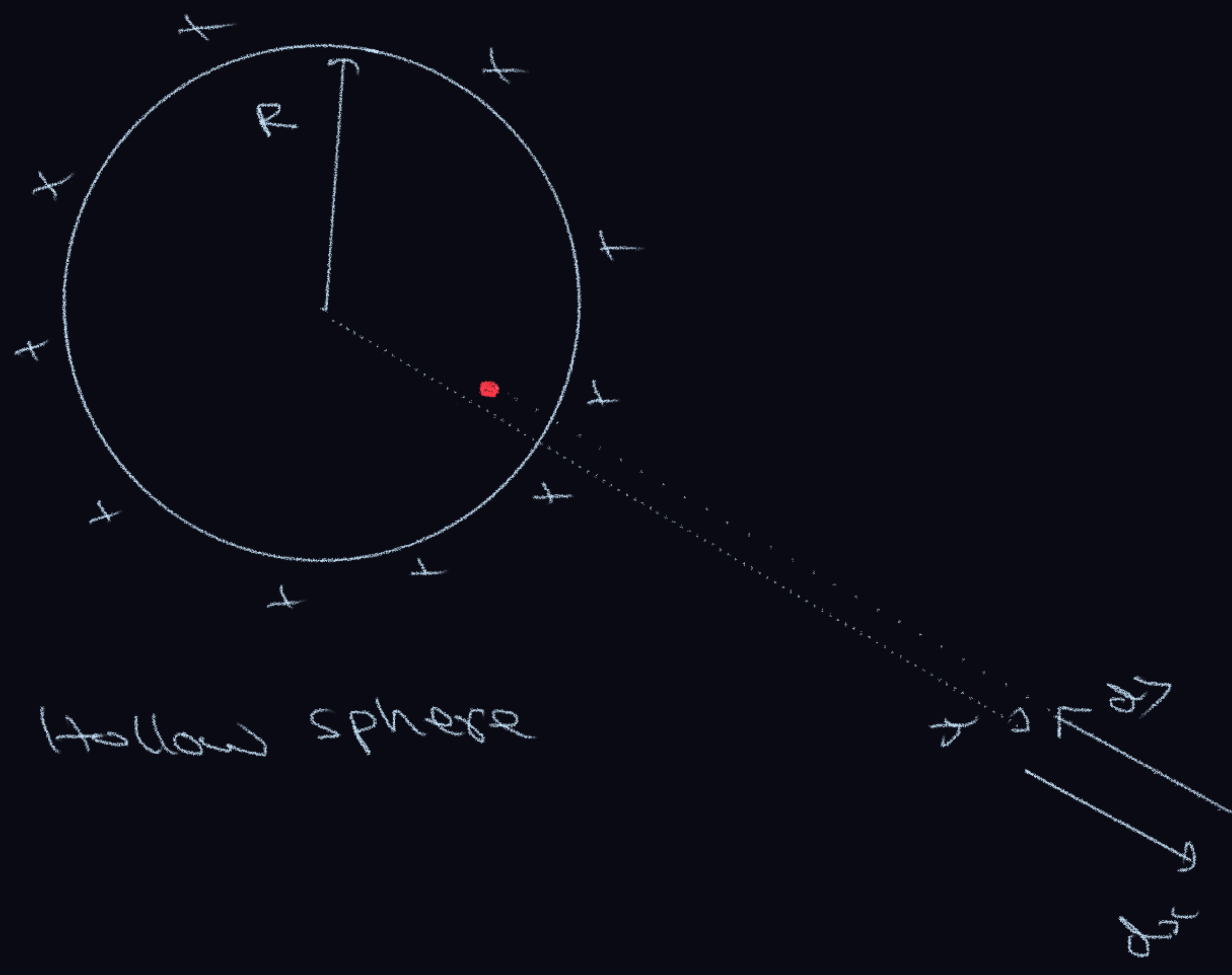
$dW = -\vec{E} \cdot (-d\vec{x}) = \vec{E} \cdot d\vec{x} = |E||dx|\cos(0)$

$= |E|\,|dx| = E(-dx) = -E\,dx$

$$W_{\infty \to P} = \int dW = \int -E\,dx = \int -\frac{kq}{x^2}\,dx = +\frac{kq}{x}\Big|_{\infty}^{P} = \frac{kq}{r} - \frac{kq}{\infty}$$

$$\boxed{W_{\substack{\infty \to P \\ ext}} = \frac{kq}{r} = V_P}$$

Potential inside a hollow sphere



Hollow sphere

$$W_{ext\,\infty \to P} = \Delta U = U_f - U_i$$

$$W_{e\,\infty \to P} = U - U_\infty$$

$$\boxed{U = U_\infty + W_{ext\,\infty \to P}}$$

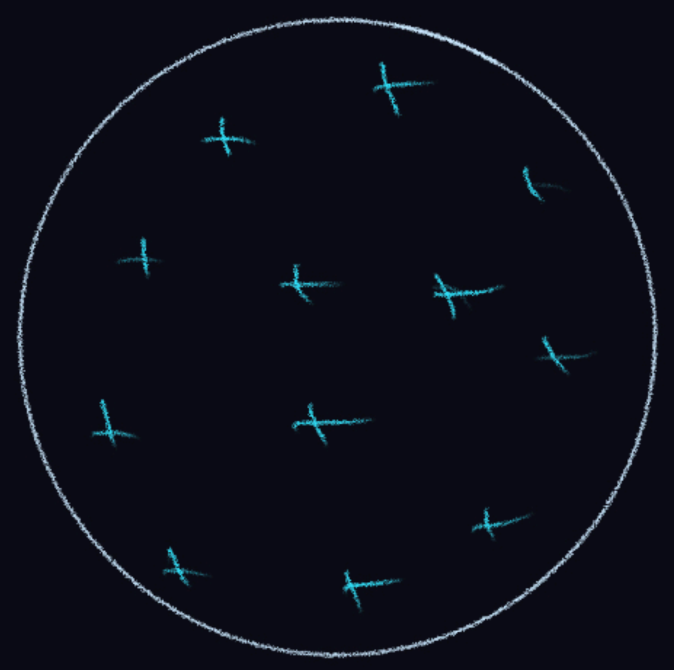
$$V = V_\infty + \int (-\vec{E}) \cdot (d\vec{s}) = \int E\, ds = \int E\,(-dr) = -\int_\infty^{P} E\, dr = -kQ \int \frac{dr}{r^2}$$

$$V = \frac{kQ}{R}$$

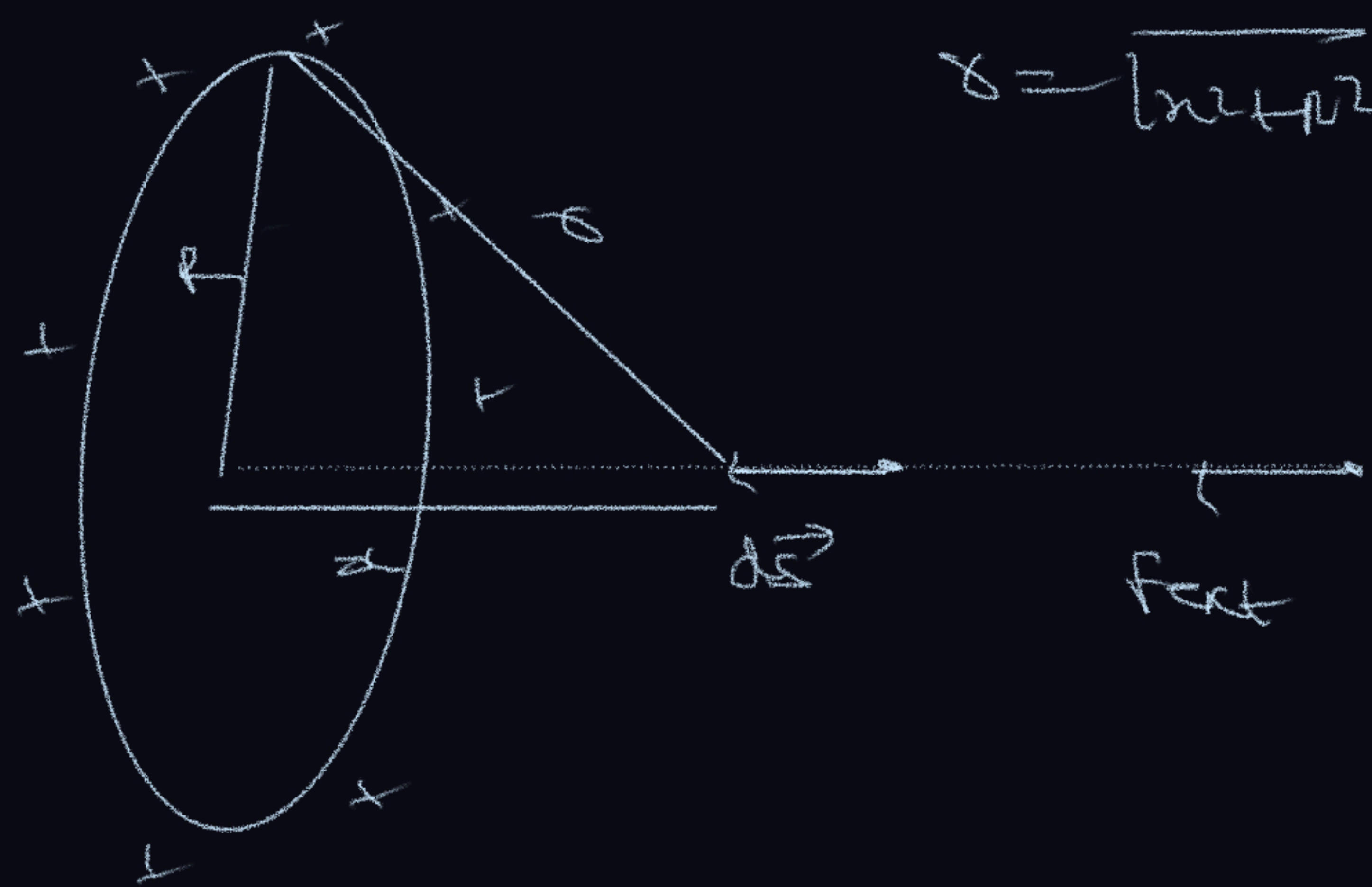
$$W_{\infty P} = W_{\infty\, surf} + W_{surf \to P}$$

$$W_{\infty \to P} = \frac{kQ}{R} + 0$$

ρ, R



solid sphere



$$r = \sqrt{x^2+R^2}$$

$$\vec{E} = kQ \frac{x}{(x^2+R^2)^{3/2}}$$

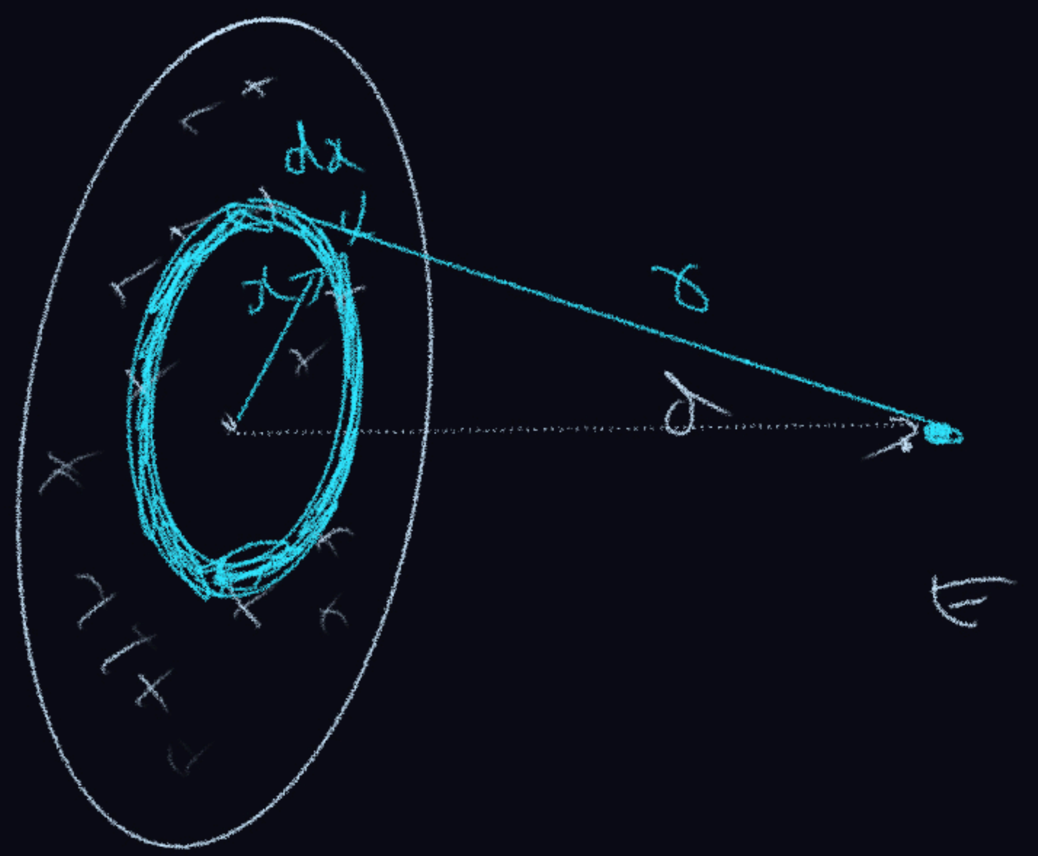
$$W = \int \frac{kQ}{2} \frac{2\,x\,dx}{(x^2+R^2)^{3/2}} = \frac{kQ}{2}\int \frac{dy}{y^{3/2}} = \frac{kQ}{2} \frac{-x}{\sqrt{x^2+R^2}}$$

$$W_{ext} = \frac{kQ}{\sqrt{R^2+x^2}} = \Delta V = V - V_\infty$$

$$V = V_\infty + \frac{kQ}{r}$$

$$\boxed{V = \frac{kQ}{r}}$$

Potential of a disc



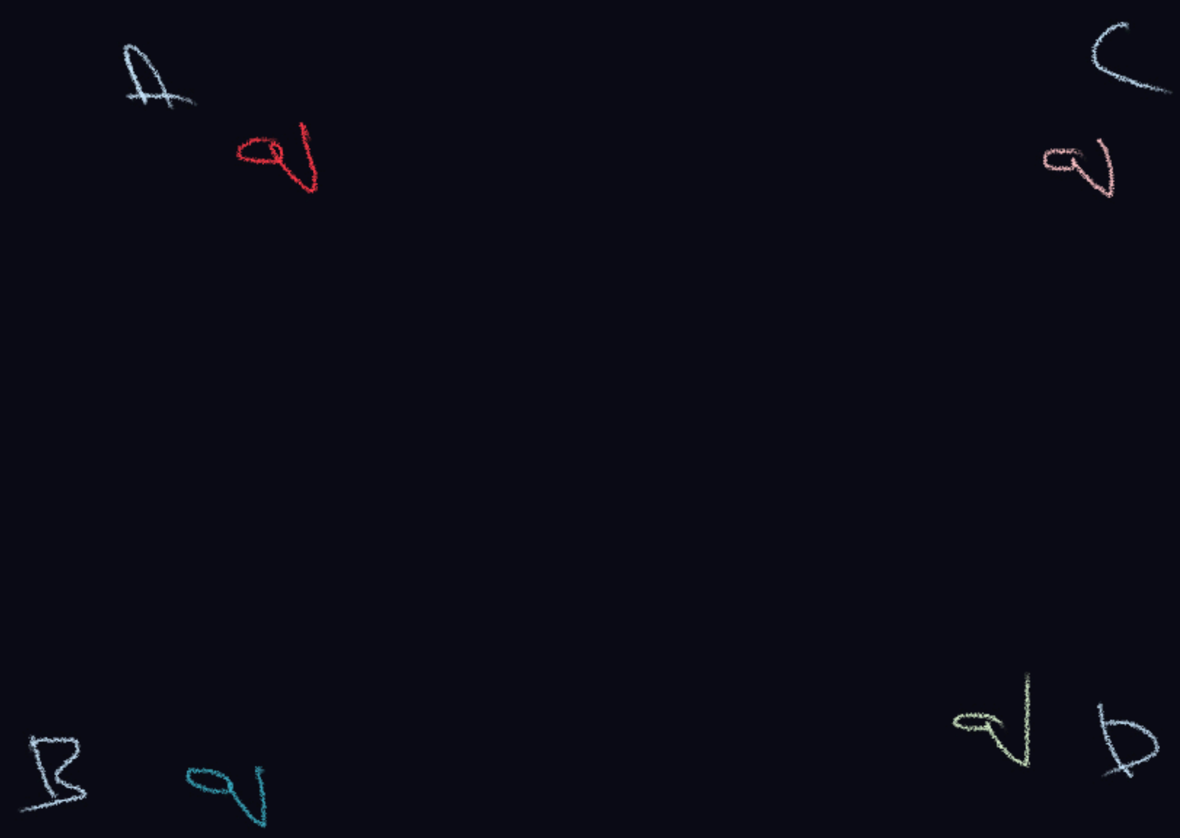
$$dV = \frac{k\,dq}{r} = \frac{k(\sigma 2\pi x\,dx)}{\sqrt{x^2+d^2}}$$

$$dq = \sigma(2\pi x\,dx)$$

$$V = 2\pi\sigma k \int \frac{x\,dx}{\sqrt{x^2+d^2}}$$

$$V = \pi\sigma k \int \frac{d(x^2+d^2)}{\sqrt{x^2+d^2}} = \pi k\sigma \int \frac{dz}{\sqrt{z}}$$

$$V = 2\pi k\sigma \sqrt{x^2+d^2}\Big|_0^R = \frac{\sigma}{2\epsilon_0}\left(\sqrt{R^2+d^2} - d\right) = \frac{\sigma}{2\epsilon_0}(r-d)$$



$$= \left(\frac{kq^2}{a}\right) + \left(\frac{kq^2}{a} + \frac{kq^2}{\sqrt{2}a}\right)$$

$$+ \left(\frac{kq}{a} + \frac{kq}{a} + \frac{kq}{\sqrt{2}a}\right)$$

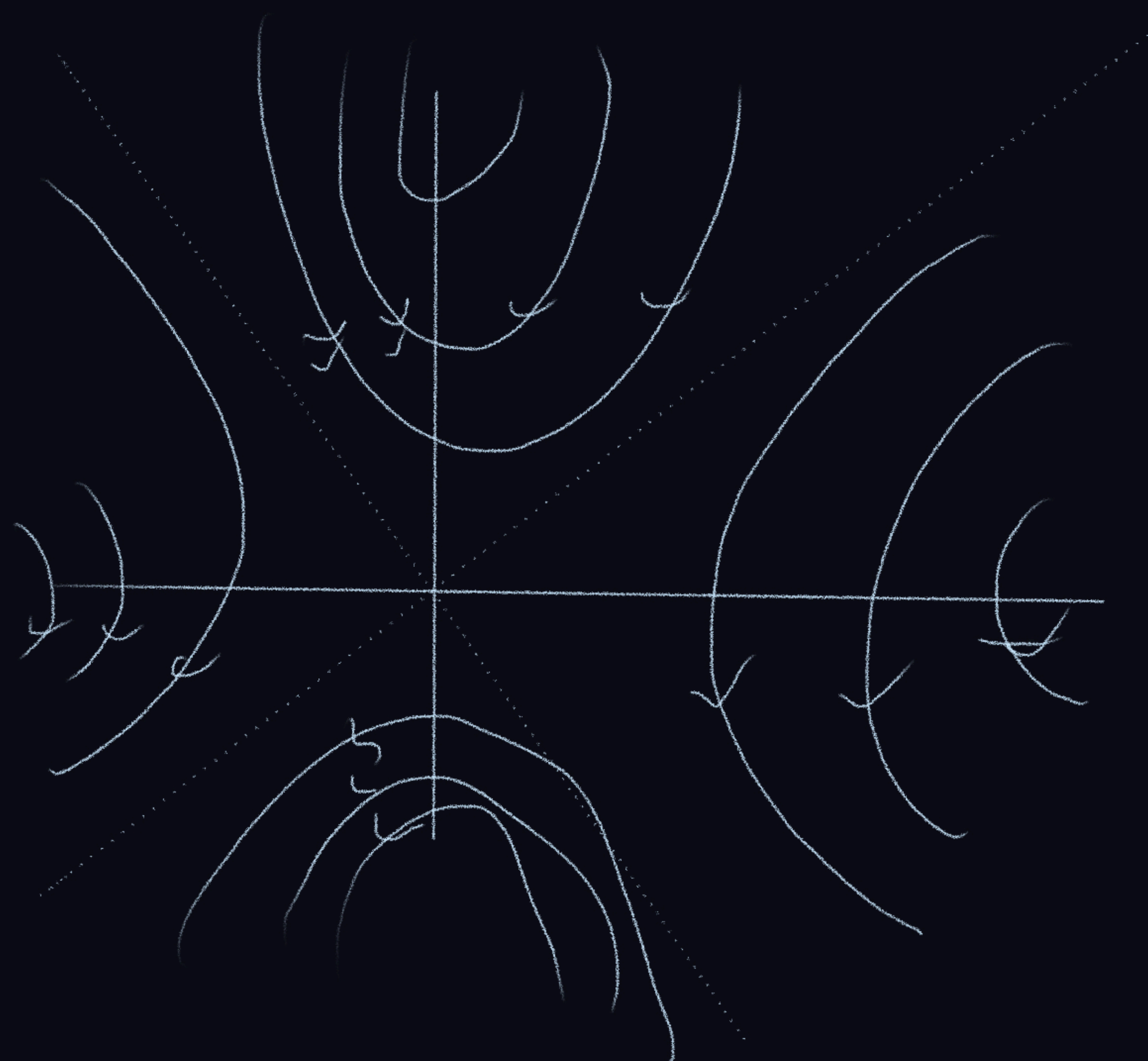
$$U = 2\,\frac{kq^2}{a}\left(2 + \frac{1}{\sqrt{2}}\right)$$

$$U = \frac{1}{2}\,\frac{q^2}{\epsilon_0 a}\left(2 + \frac{1}{\sqrt{2}}\right)$$

energy:-

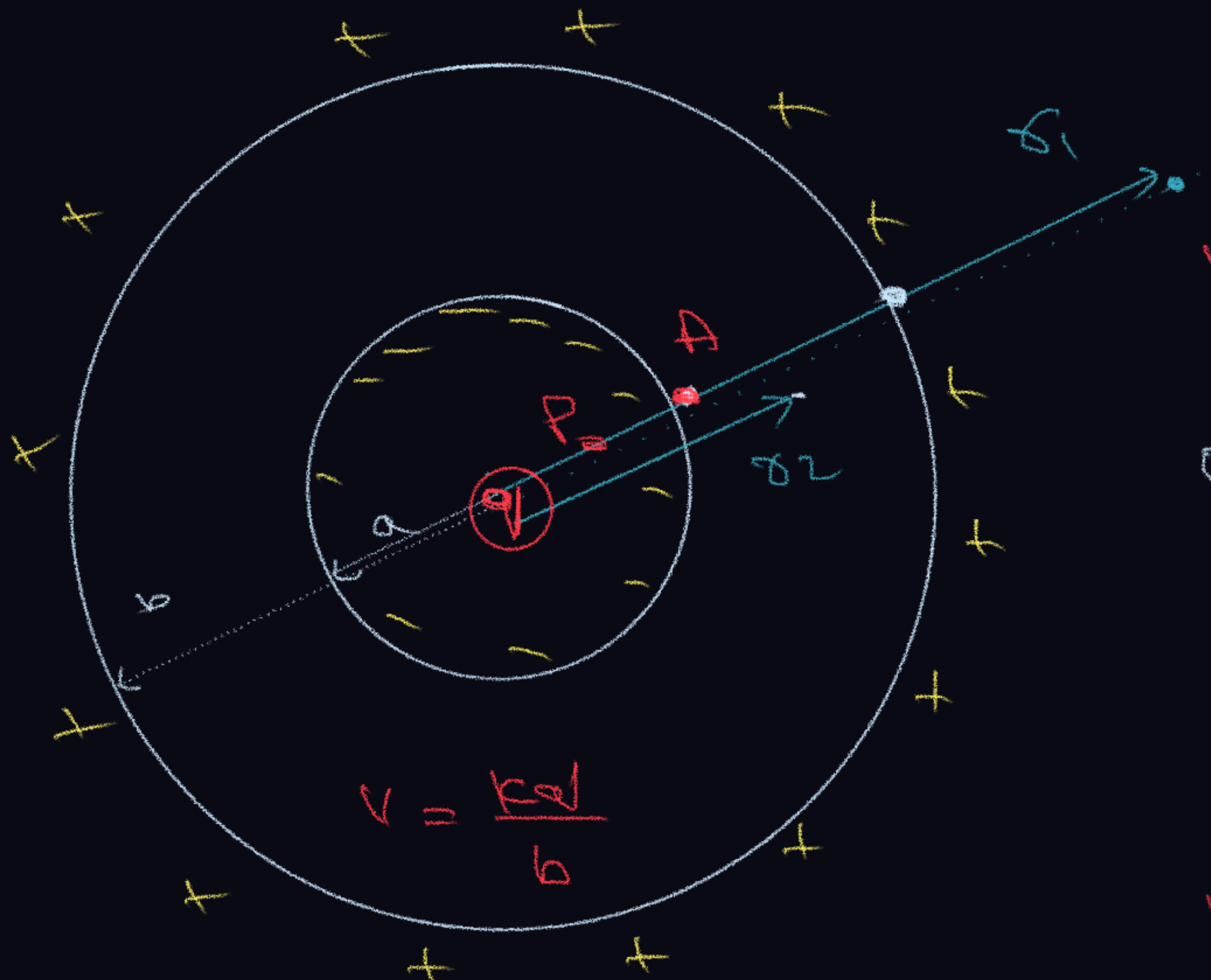
$$\boxed{1\ eV = 1.6 \times 10^{-19}\ J}$$

$$\boxed{1\ MeV = 10^{6}\ eV} = 10^{6} \times 1.6 \times 10^{-19}\ J$$



$y^2 - x^2 = c$

$y^2 = x^2$



$$V = \frac{kq}{r} + V_\infty = \frac{kq}{r}$$

$$E = \frac{kq}{r^2}$$

$$W_{A-P} = \int \vec{E} \cdot d\vec{r} = -\int \frac{kq}{r^2}\, dr$$

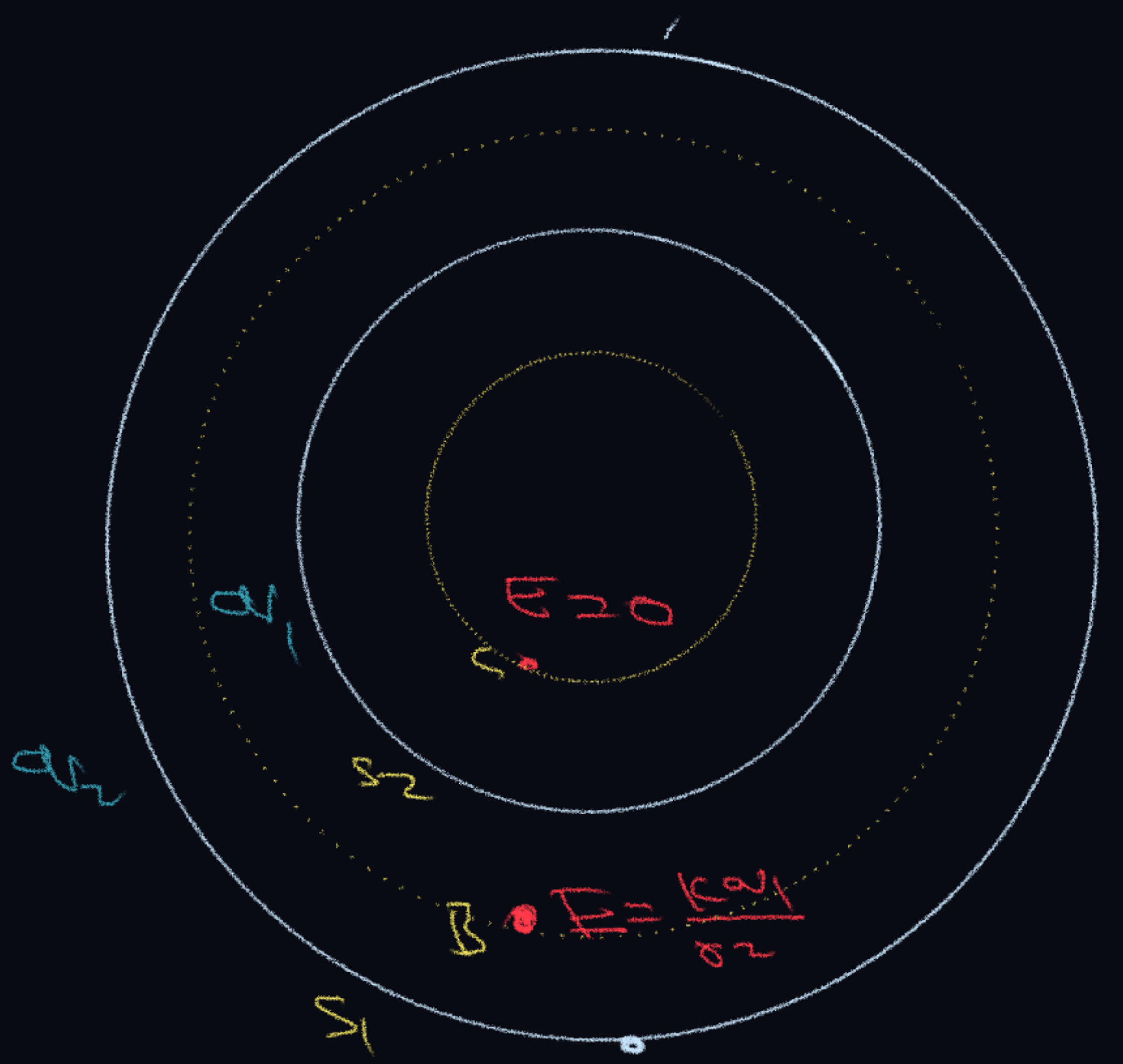
$$W_{\infty-P} = W_{\infty-A} + W_{A-P}$$

$$= \frac{kq}{b} + \int_a^r \frac{kq}{r^2}\, dr$$

$$W_{\infty-P} = \frac{kq}{b} + kq\left(\frac{1}{r} - \frac{1}{a}\right) \qquad (r \leq a)$$

$$V_P = \frac{kq}{r} + \frac{kq}{b} - \frac{kq}{a}$$

$r_1 < r < r_2$



$$\boxed{V_C = V_{S_2}}$$

$$W_{\infty-B} = W_{\infty-S_1} + W_{S_1-B}$$

$$W_{\infty-B} = (V_{S_1} - V_\infty) + W_{S_1-B}$$

$$W_{\infty-B} = \left(\frac{k(q_1+q_2)}{r_2} - 0\right) + W_{S_1-B}$$

$$W_{S_1-B} = \frac{kq_1}{r}\Bigg|_{r_2}^{r} = kq_1\left(\frac{1}{r} - \frac{1}{r_2}\right)$$

A $\quad E = \frac{k(q_1+q_1)}{r^2} \qquad V = \frac{k(q_1+q_2)}{r}$

$$W_{\infty-B} = \frac{k(q_1+q_2)}{r_2} + \frac{kq_1}{r} - \frac{kq_1}{r_2}$$

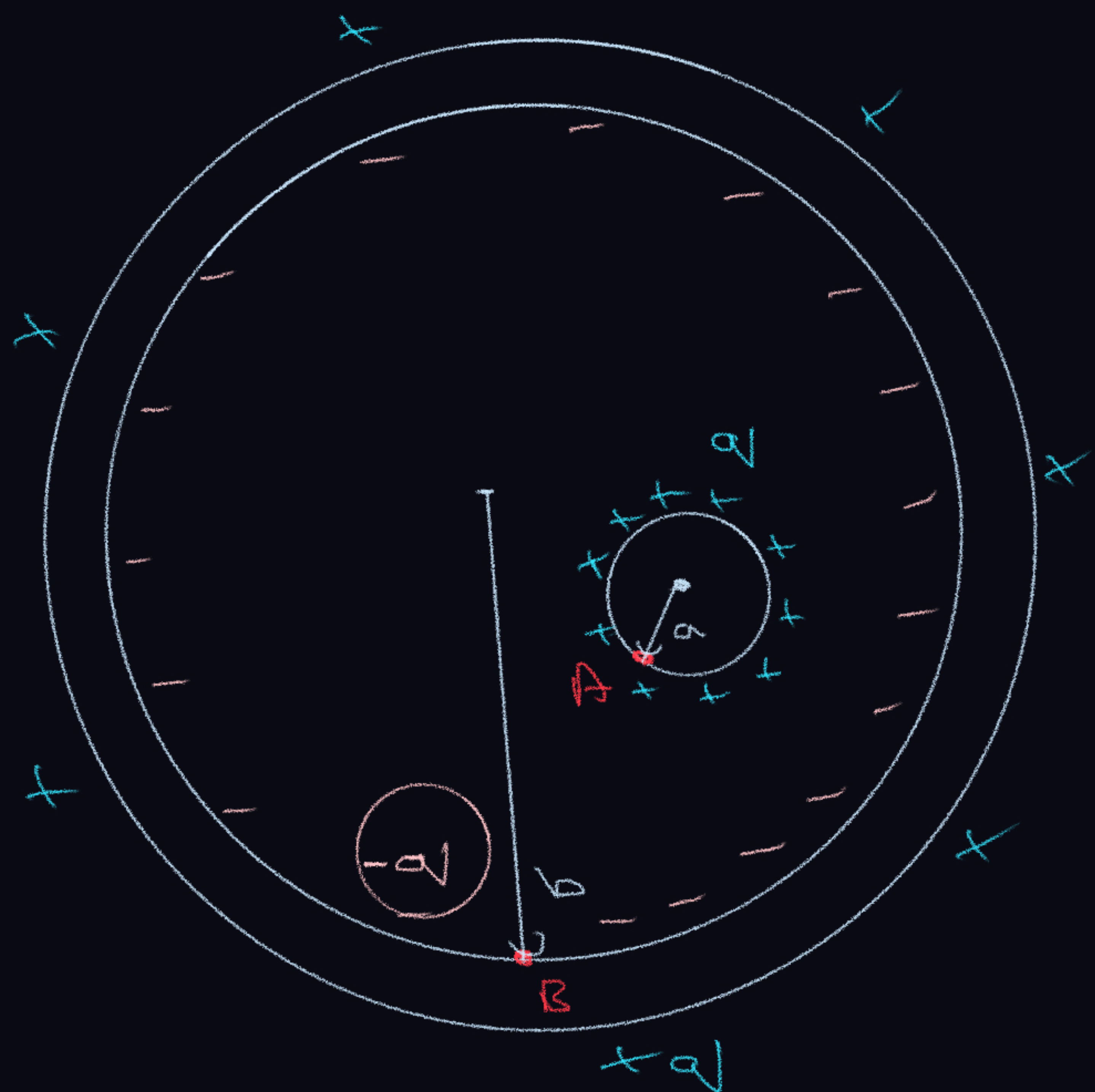
$$V_{S_2} = W_{\infty-S_2} = W_{\infty-S_1} + W_{S_1-S_2}$$

$$= V_{S_1} + \frac{kq_1}{r}\Bigg|_{r_2}^{r_1} = V_{S_1} + kq_1\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$V_{S_2} = \frac{k(q_1+q_2)}{r_2} + \frac{kq_1}{r_1} - \frac{kq_1}{r_2}$$

$$V_B - V_\infty = W_{\infty-B} \text{ (ext)}$$

$$V_B = W_{\infty-B} = \frac{k(q_1+q_2)}{r_2} + \frac{kq_1}{r} - \frac{kq_1}{r_2}$$



$$\omega_{\infty-A} = \omega_{\infty-B} + \omega_{B-A}$$

$$\omega_{BA} = 0$$

∞

$$V_B = \frac{kq}{b}$$

$$V_B = V_A$$

$$\therefore \omega_{\infty-A} = \omega_{\infty-B}$$

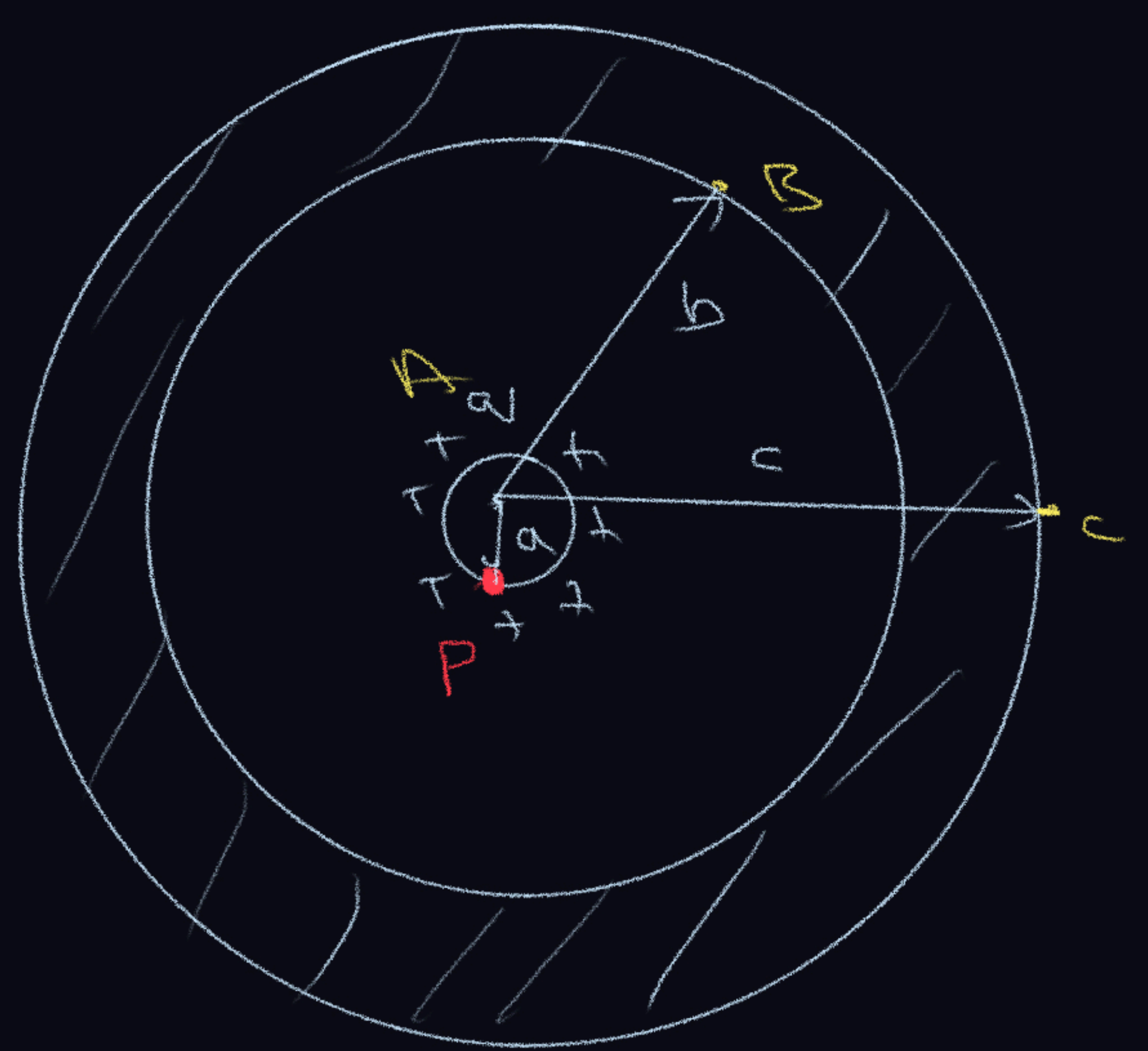
$$V_A = \frac{kq}{a}$$

$$V_A = V_B = \omega_{\infty-B} = \frac{kq}{b}$$

$$V = \frac{kq}{a}$$

$$V_A = \frac{kq}{a}\,\frac{a}{b} = V\frac{a}{b}$$

$V_p = ?$



conducting shell

$$V_p = W_{\infty - c} + W_{c-B} + W_{B-A}$$

$$= \frac{kq}{c} + 0 + \left.\frac{kq}{r}\right|_{b\,(\text{init})}^{a\,(\text{final})}$$

$$V_p = \frac{kq}{c} + 0 + \frac{kq}{a} - \frac{kq}{b}$$

$$W_{\infty-R/2} = W_{R-R/2} + W_{\infty-R}$$

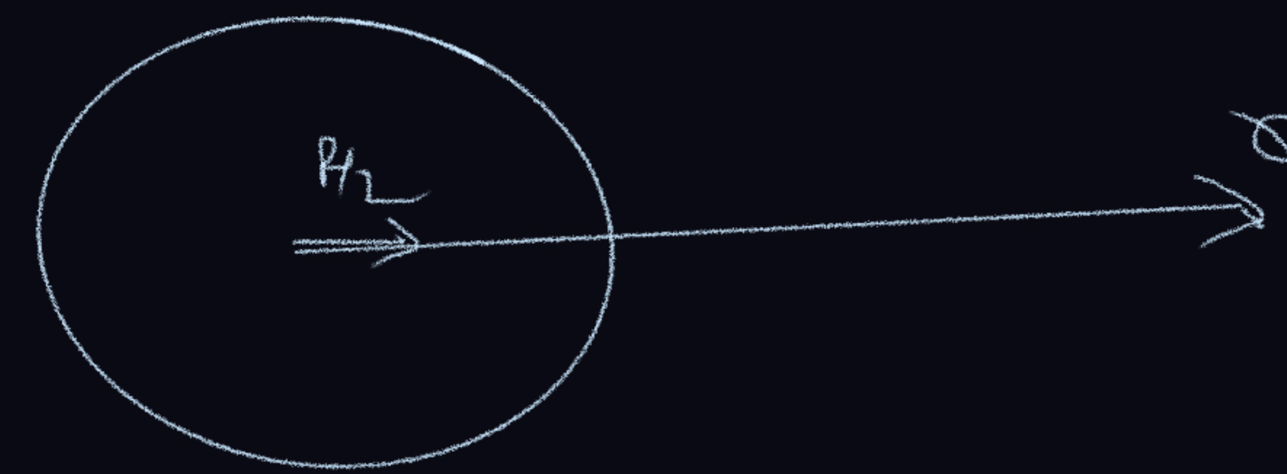
$$= \left(\frac{kQ}{R^3}\right) \frac{r^2}{2}\Bigg|_{R/2}^{R} + \frac{kQ}{r}\Bigg|_{\infty}^{R}$$

$$= \frac{kQ}{2R^3}\left(R^2 - \frac{R^2}{4}\right) + \frac{kQ}{R}$$

$$= \frac{kQ}{2R}\left(1 - \frac{1}{4}\right) + \frac{kQ}{R}$$

$$V_p = \frac{11}{8}\frac{kQ}{R} = \frac{1}{2}mv^2$$

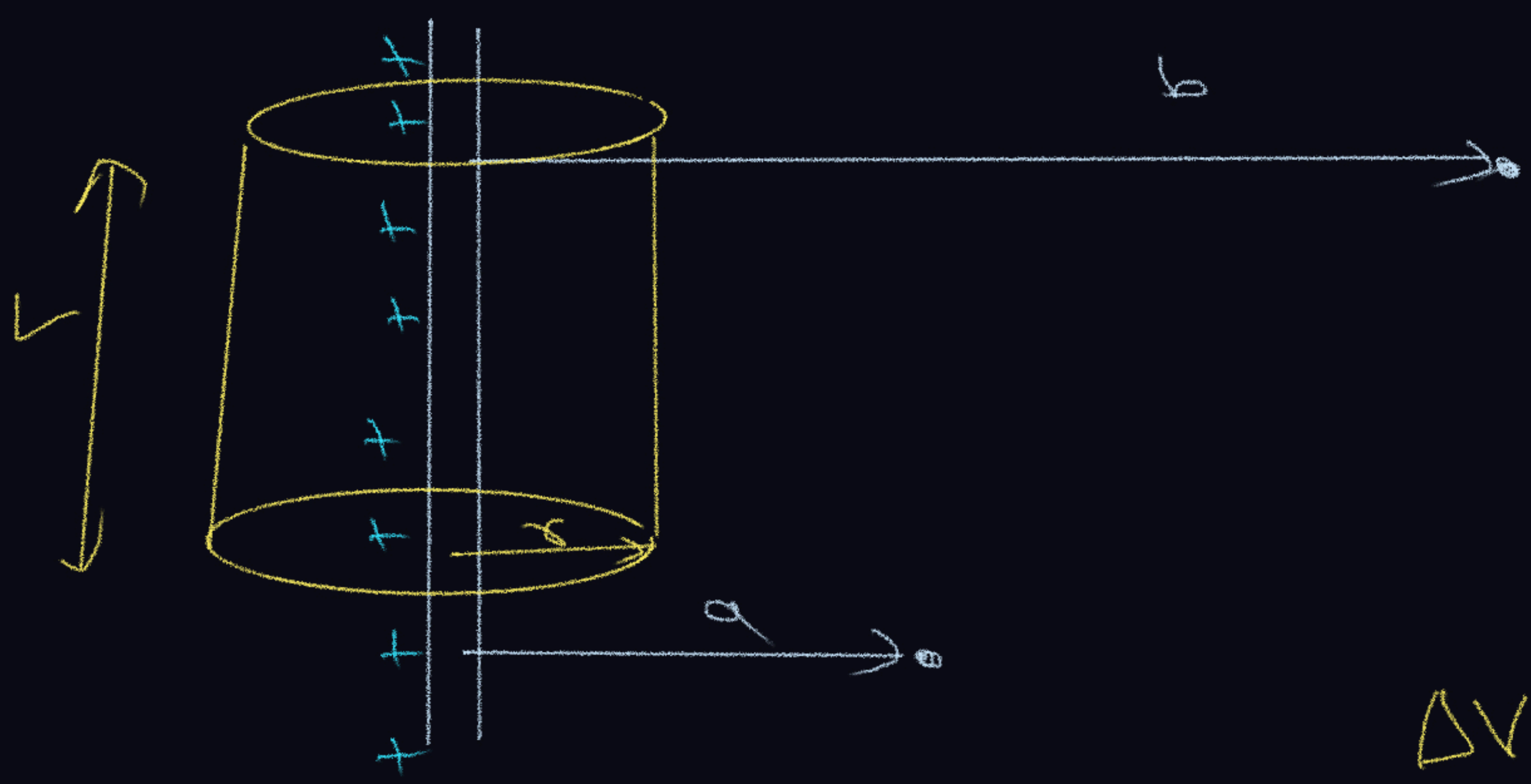
$$v = \sqrt{\frac{11}{4}\frac{kQ}{mR}}$$



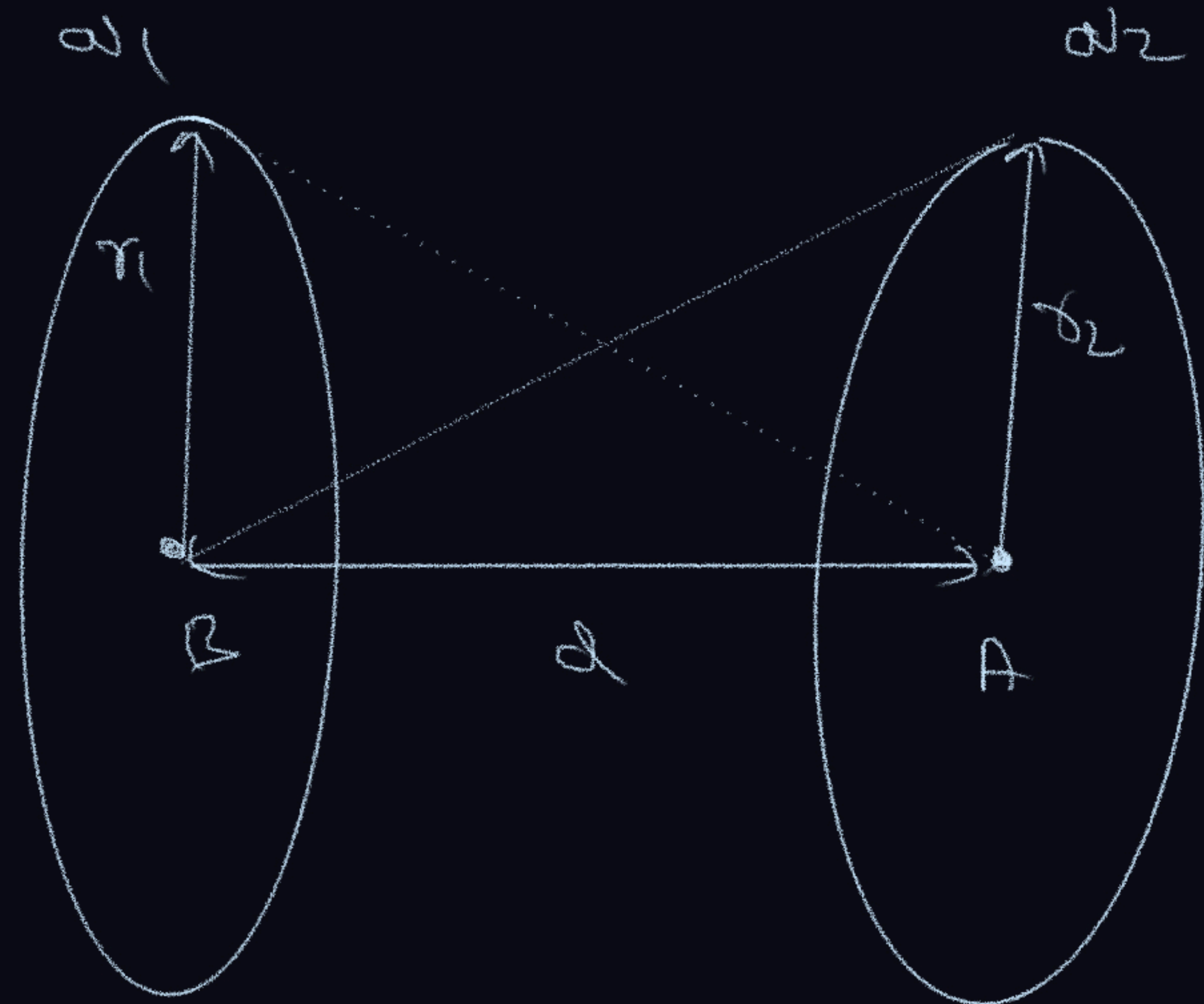
$W = \Delta KE$

$\Delta V = \frac{1}{2}mv^2$

$$\frac{11}{8}\frac{kQ}{R} - \frac{kQ}{r} = \frac{1}{2}mv^2$$



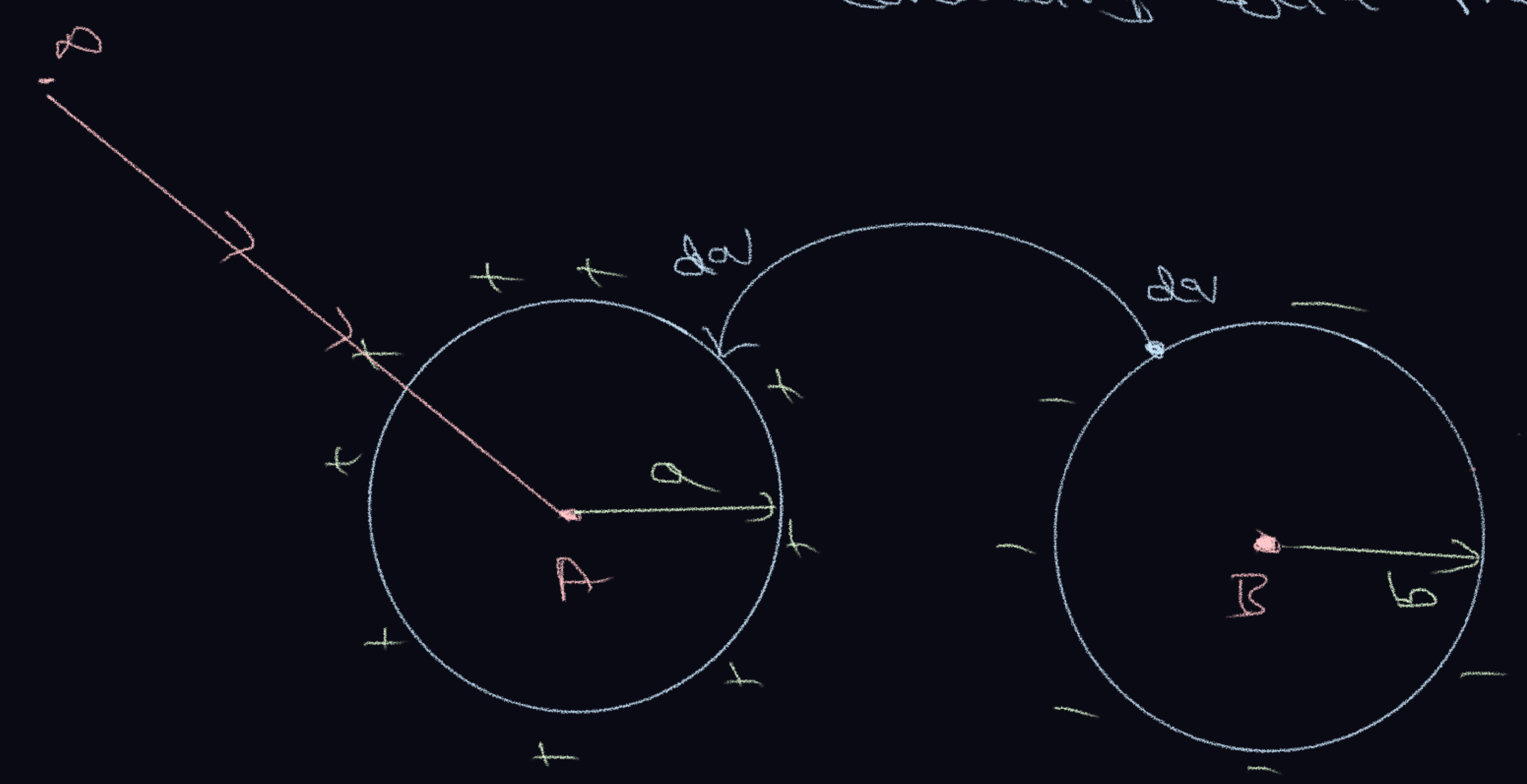
$$\Delta V = 2k \ln\left(\frac{b}{a}\right)$$



$$V_A = k\left(\frac{q_2}{r_2} + \frac{q_1}{\sqrt{r_1^2+d^2}}\right)$$

$$V_B = k\left(\frac{q_1}{r_1} + \frac{q_2}{\sqrt{r_2^2+d^2}}\right)$$

two conducting solid spheres (R)



bring together $= W_{sphere A}$

$= q_A V_A = q_A\left(\dfrac{-kq_B}{l}\right)$

How much is energy needed to charge spheres to +Q and −Q

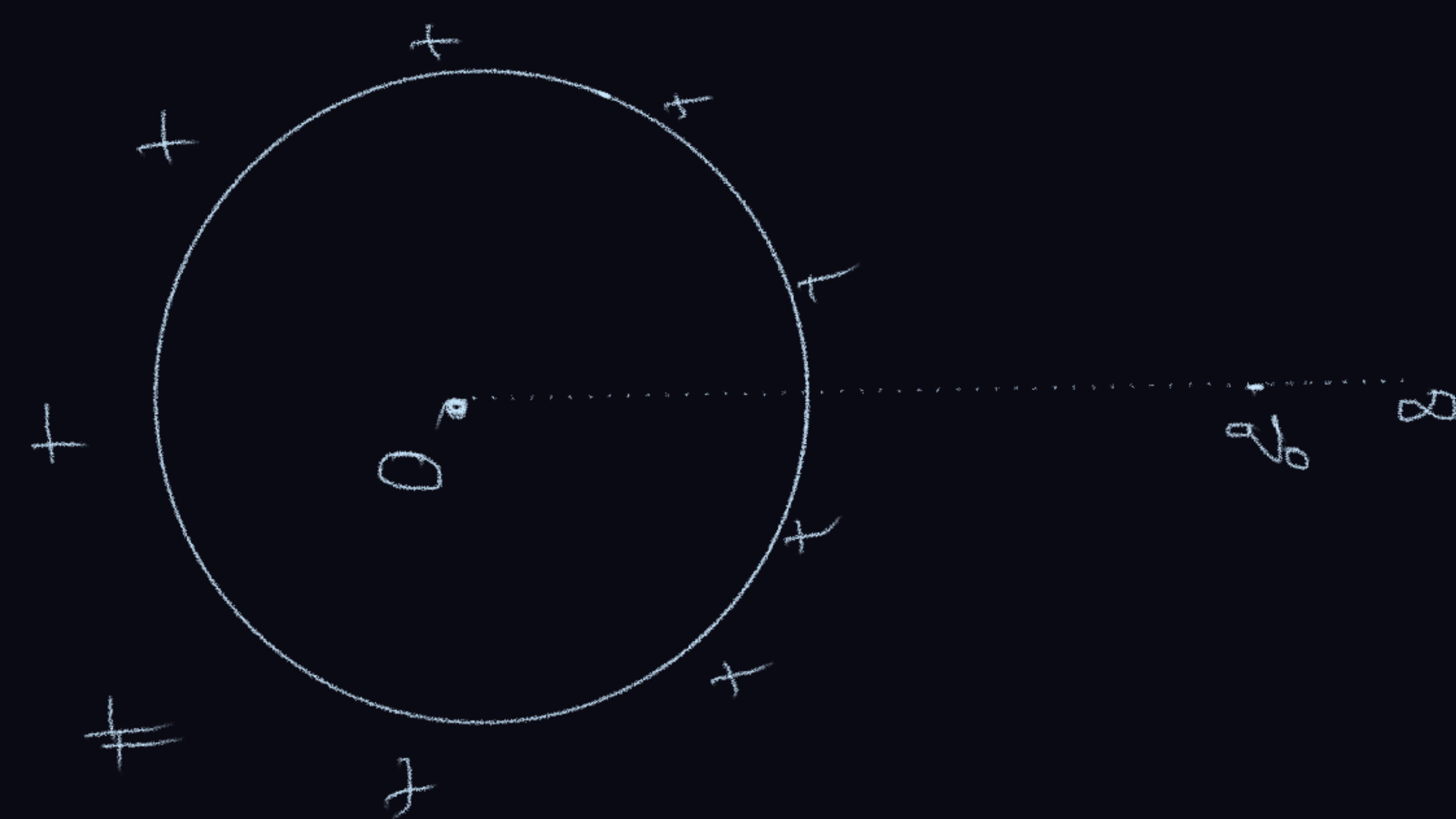
total work $= \dfrac{kq_A^2}{a} + \left(\dfrac{-kq_B^2}{b}\right) + \left(\dfrac{-kq_A q_B}{l}\right)$

Energy $= \dfrac{kQ^2}{2a}$ +

Energy $= \dfrac{-kQ^2}{2b}$ + bring them together.

$V_A = \dfrac{-kQ}{b}$ (due to −ve sphere)

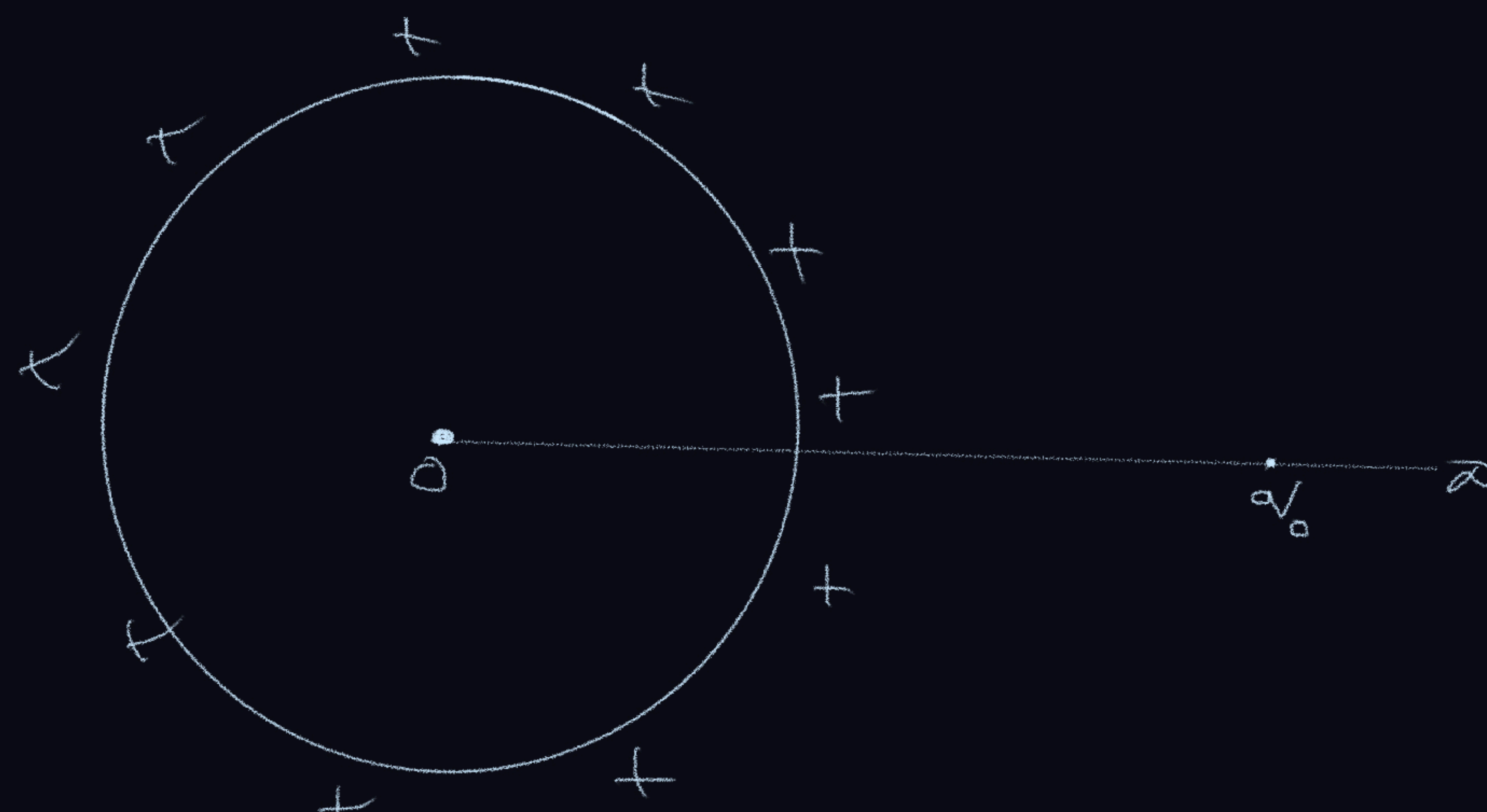
W on sphere A $= q_A V_A$



Work = work to construct sphere with Q + $W_{\infty \to 0}$ on q_0 charge

$$W_1 = \frac{kQ^2}{2R_1} + \frac{kQ}{R_1} q_0$$

$$W_2 = \frac{kQ^2}{2R_2} + \frac{kQ}{R_2} q_0$$



∴ Energy difference = $W_1 - W_2$