

COMBINATIONS

For combinations, **k objects are selected from a set of n objects to produce subsets without ordering.**

Contrasting the previous permutation example with the corresponding combination, the AB and BA subsets are no longer distinct selections;

by eliminating such cases there remain only 10 different possible subsets—AB, AC, AD, AE, BC, BD, BE, CD, CE, and DE.

The number of such subsets is denoted by **nC_k** , read “ **n choose k .**” For combinations, since **k objects have $k!$ arrangements**, there are $k!$ indistinguishable permutations for each choice of k objects;

hence dividing the permutation formula by $k!$ yields the following combination formula:

$$nC_k = \frac{n!}{k!(n-k)!}$$

This is the same as the (n, k) binomial coefficient (see **binomial theorem**). For example, the number of combinations of five objects taken two at a time is

$$\begin{aligned} {}_5C_2 &= \frac{5!}{(2!(5-2)!)} = \frac{5!}{(2!(3)!)} = \frac{(1)(2)(3)(4)(5)}{(1)(2)(1)(2)(3)} \\ &= \frac{120}{12} = 10. \end{aligned}$$

The formulas for **nPk** and **nCk** are called counting formulas

since they can be used to count the number of possible permutations or combinations in a given situation **without having to list them**

