EXTRA QUESTIONS

CIRCLES

If two intersecting chords of a circle make equal angles with the diameter passing through their point of intersection, prove that the chords are equal.

Solution :

Given : AB and CD are two chords of a circle, with centre O intersecting at a point E. PQ is a diameter through E, such that $\angle AEQ = \angle DEQ$

To prove: AB = CD.

Construction: Draw perpendiculars OL and OM on chords AB and CD respectively.

Proof:

$\angle LOE = 180^\circ - 90^\circ - \angle LEO$

$= 90^\circ - \angle LEO$ (Angle sum property of a triangle)

$= 90^\circ - \angle AEQ = 90^\circ - \angle DEQ$

$= 90^\circ - \angle MEO = \angle MOE$

In triangles OLE and OME,

$\angle LEO = \angle MEO$ (Why ?)

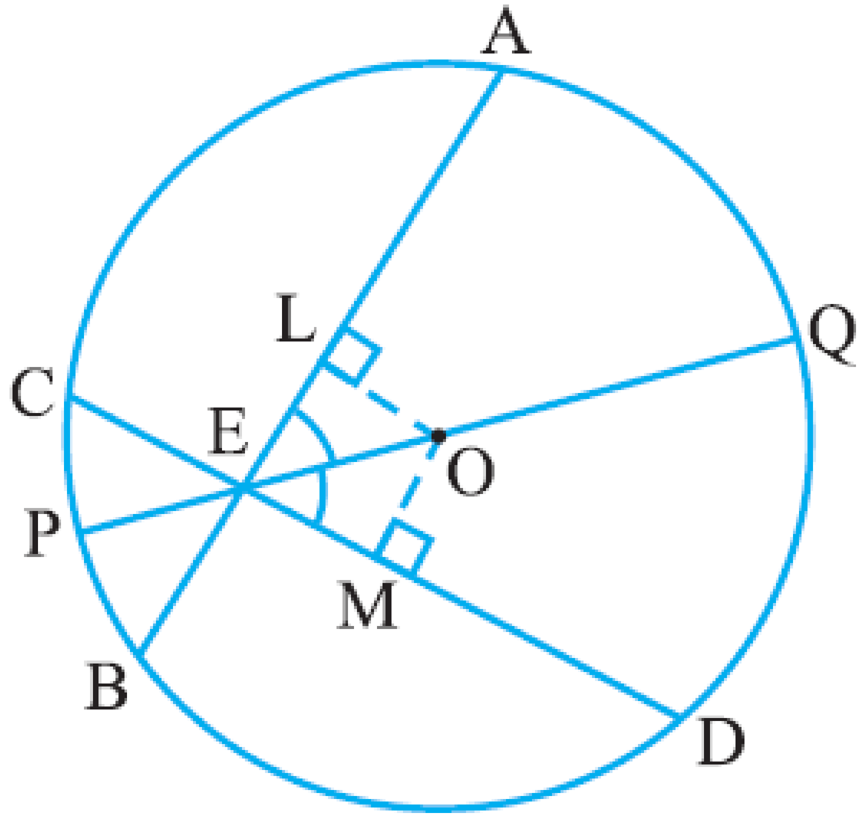
$\angle LOE = \angle MOE$ (Proved above)

EO = EO (Common)

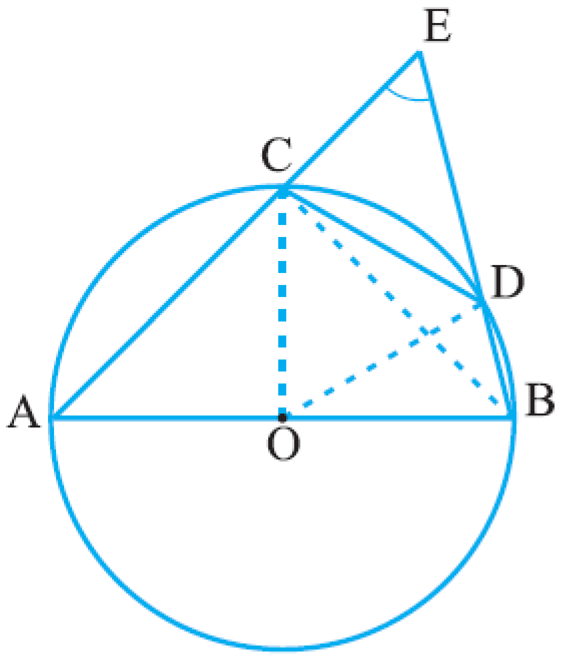
Therefore, Δ OLE $\cong \Delta$ OME (Why ?)

This gives OL = OM (CPCT)

So, AB = CD (Why ?)



AB is a diameter of the circle, CD is a chord equal to the radius of the circle. AC and BD when extended intersect at a point E. Prove that ∠AEB = 60°.



Solution :

Join OC, OD and BC.

- Triangle ODC is equilateral (Why?) Therefore, ∠COD = 60°
- Now, ∠CBD =1/2 ∠COD Reason This gives ∠CBD = 30°
- Again, ∠ ACB = 90° (Why ?)

So, ∠BCE = 180° – ∠ACB = 90°

- Which gives ∠CEB = 180-(90° + 30°) = 60°,
- i.e. ∠AEB = 60°

ABCD is a cyclic quadrilateral in which AC and BD are its diagonals.If ∠DBC = 55° and ∠BAC = 45°, find ∠BCD.

Solution :

Given ∠CAD = ∠DBC = 55° (Angles in the same segment)

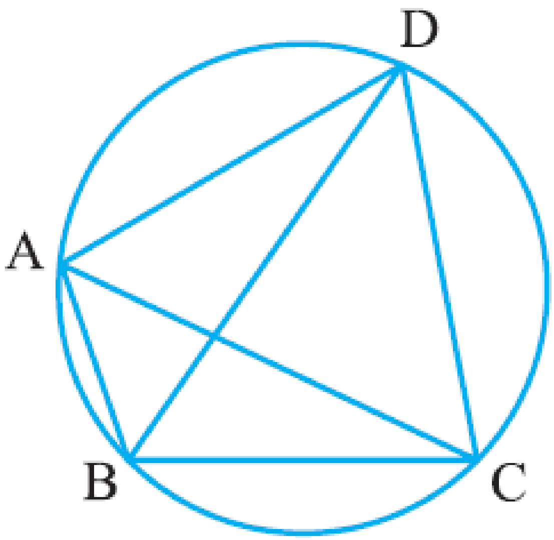
Therefore, ∠DAB = ∠CAD + ∠BAC

= 55° + 45° = 100°

But ∠DAB + ∠BCD = 180°

(Opposite angles of a cyclic quadrilateral)

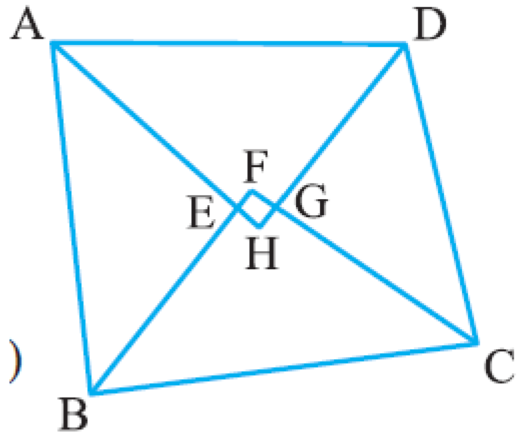
So, ∠BCD = 180° – 100° = 80°



Prove that the quadrilateral formed (if possible) by the internal angle bisectors of any quadrilateral is cyclic.

Solution :

ABCD is a quadrilateral in which the angle bisectors AH, BF, CF and DH of internal angles A, B, C and D respectively form a quadrilateral EFGH.



Now,

$\angle FEH = \angle AEB = 180^\circ - \angle EAB - \angle EBA$ (Why?)

$= 180^\circ - 1/2(\angle A + \angle B)$

and

$\angle FGH = \angle CGD = 180^\circ - \angle GCD - \angle GDC$ (Why?)

$= 180^\circ - 1/2(\angle C + \angle D)$

Therefore, $\angle FEH + \angle FGH = 180^\circ - 1/2(\angle A + \angle B) + 180^\circ - 1/2(\angle C + \angle D)$

$= 360^\circ - 1/2 (\angle A + \angle B + \angle C + \angle D)$

$= 360^\circ - 1/2 \times 360^\circ$

$= 360^\circ - 180^\circ = 180^\circ$

Therefore quadrilateral EFGH is cyclic REASON

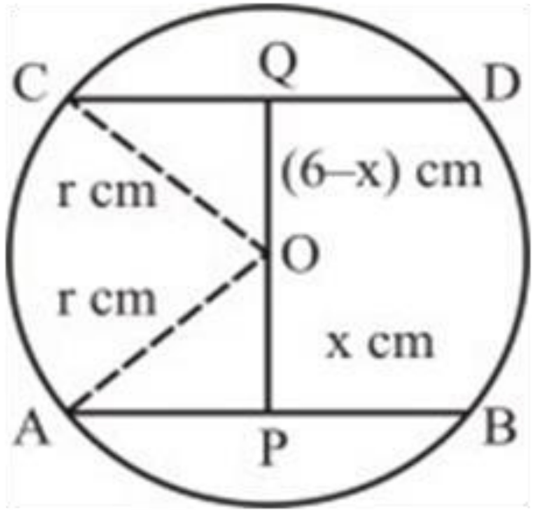
Two chords AB and CD of lengths 5 cm and 11 respectively of a circle are parallel to each other and are on opposite sides of its centre. If the distance between AB and CD is 6 cm, find the radius of the circle.

Sol.

Let O be the centre of the given circle and let its radius be r cm. Draw OP⊥AB and OQ⊥CD. Since OP⊥AB, OQ⊥CD and AB || CD. Therefore, points P, O and Q are collinear. So PQ = 6 cm.

Let OP = x. Then, OQ = (6 – x) cm. Join OA and OC. Then, OA = OC = r.

Since the perpendicular from the centre to a chord of the circle bisects the chord.



Therefore AP = PB = 2.5 cm and CQ = QD = 5.5 cm. In right ∆s OAP and OCQ, we have

$OA^2=OP^2+AP^2$ and $OC^2=OQ^2+CQ^2$

$\Rightarrow\ r^2=x^2+(2.5)^2$ .... (1)

and $r^2=(6-x)2+(5.5)^2$ .... (2)

$\Rightarrow\ x^2+(2.5)^2=(6-x)^2+(5.5)^2 \Rightarrow\ x^2+6.25 = 36-12x + x^2 + 30.25$

$\Rightarrow\ 12x = 60 \Rightarrow\ x = 5$

Putting x = 5 in (1), we get

$r^2=52+(2.5)^2=25 + 6.25 = 31.25$

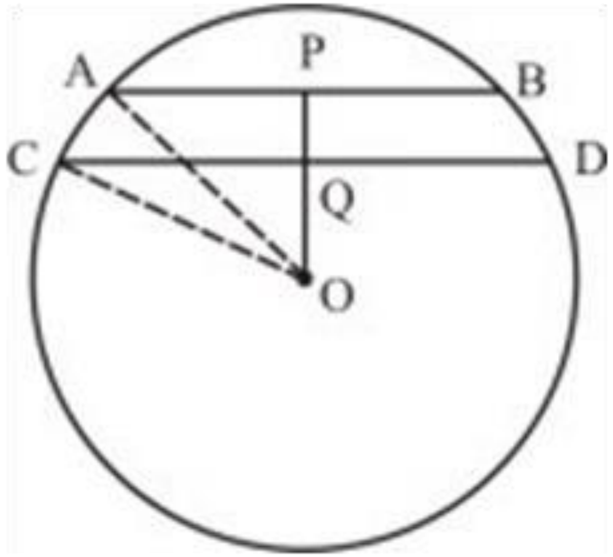
$\Rightarrow\ r=\sqrt{31.25} = 5.6$ (approx.)

Hence, the radius of the circle is 5.6 cm (approx.)

The lengths of two parallel chords of a circle are 6 cm and 8 cm. If the smaller chord is at distance 4 cm from the centre, what is the distance of the other chord from the centre.

Sol.

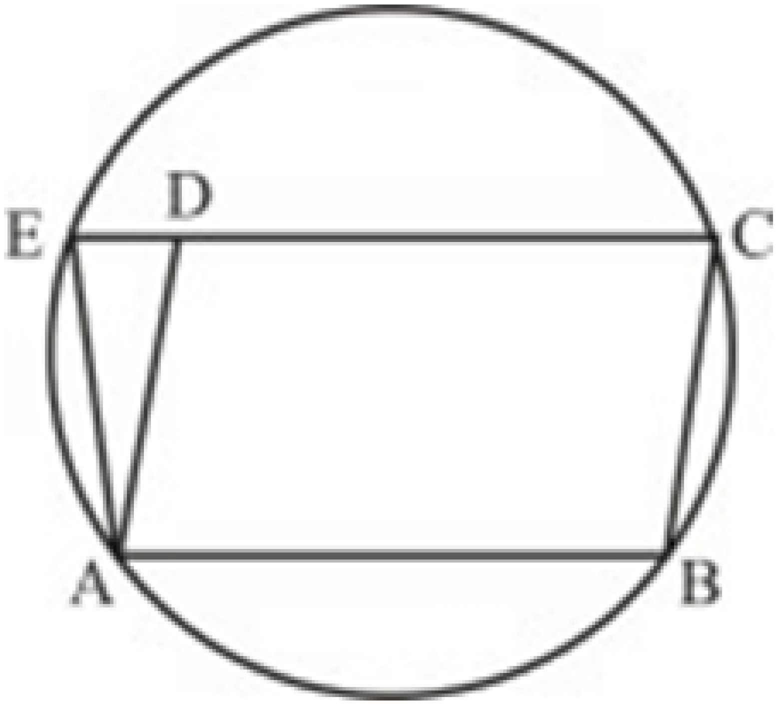
Let AB and CD be two parallel chords of a circle with centre O such that AB = 6 cm and CD = 8 cm.
Let the radius of the circle be r cm.
Draw OP⊥AB and OQ⊥CD. Since AB || CD and OP⊥AB, OQ⊥CD. Therefore, points O, Q and P are collinear. Clearly OP = 4 cm, and P, Q are mid-points of AB and CD respectively.
Therefore AP=PB=1/2(AB)=3 cm
and, CQ=QD=1/2(CD)=4 cm
In rt. ∠dΔOAP, we have
$OA^2=OP^2+AP^2$
$\Rightarrow r^2=4^2+3^2=16+9=25$
$\Rightarrow r = 5$
In rt. ∠dΔ OCQ, we have
$OC^2=OQ^2+CQ^2$
$\Rightarrow r^2=OQ^2+4^2$
$\Rightarrow 25=OQ^2+16$
$\Rightarrow OQ^2=9$
$\Rightarrow OQ = 3$
Hence, the distance of chord CD from the centre is 3 cm.



ABCD is a parallelogram. The circle through A, B and C intersect CD (produced if necessary) at E. Prove that AE = AD.

To prove: AE = AD

(ΔAED is an isosceles triangle it is sufficient to prove that ∠AED=∠ADE)



Proof:

Since ABCE is a cyclic quadrilateral.

Therefore ∠AED+∠ABC=180° .... (1)

Now, CDE is a straight line.

⇒ ∠ADE+∠ADC=180° .... (2)

[Since ∠ADC and ∠ABC are opposite angles of a parallellogram i.e. ∠ADC=∠ABC

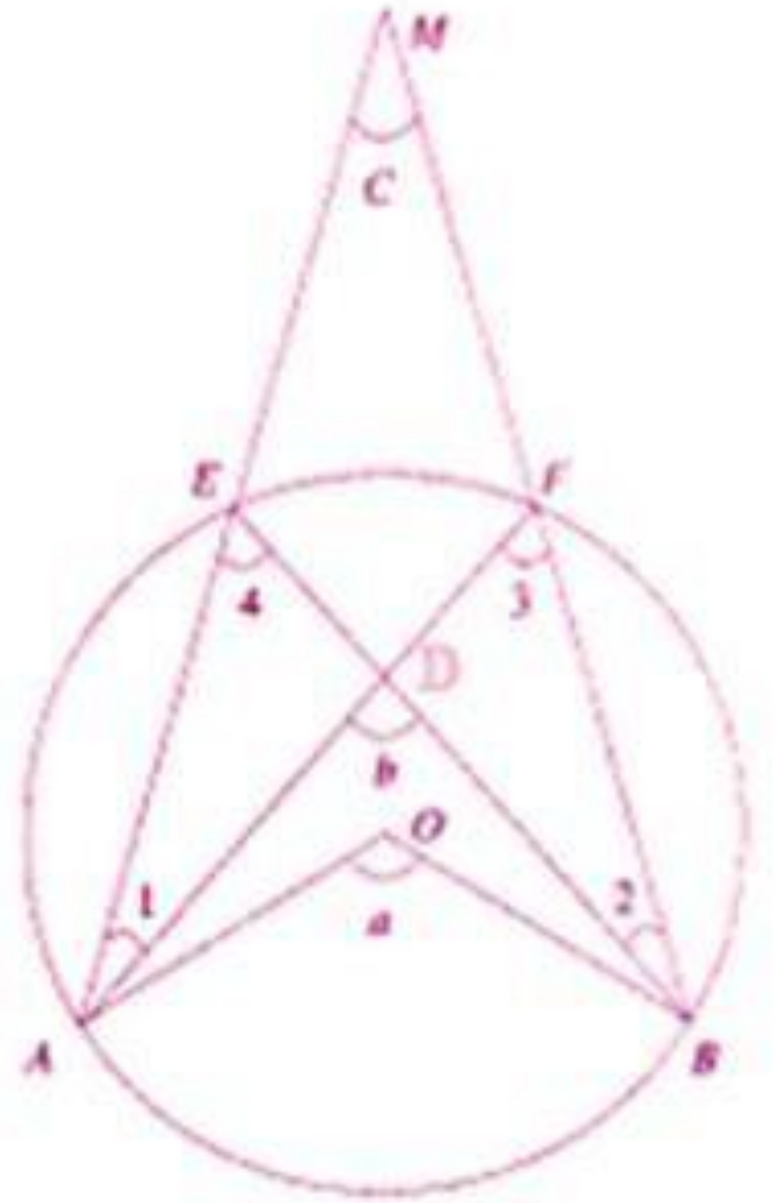
From (1) and (2), we get

∠AED+∠ABC=∠ADE+∠ABC

⇒ ∠AED=∠ADE

Therefore In ΔAED, we have ∠AED=∠ADE ⇒ AD = AE.

In the Fig, O is the centre of the circle, prove that, ∠a = ∠b + ∠c



Solution:

∠AOB = 2∠AEB = 2∠AFB

∴ ∠AOB = ∠AEB + ∠AFB

∴ ∠a = ∠4 + ∠3 = 2∠4 .... (i)

Now ∠1 = ∠2 (In same segment) .... (ii)

∠b = ∠4 + ∠1 (Exterior angles equal to sum of two equal interior opposite angles)

⇒ ∠b = ∠4+∠2 .... (iii) [from (ii)]

∠c = ∠4 - ∠2 .... (iv) (Since ∠c +∠2 = ∠4)

Adding equations (iii) and (iv) we get,

∠b + ∠c = 2∠4 = ∠a [from(i)]

or ∠a = ∠b + ∠c