INTRODUCTION

The term electrostatics contains two words such as electro and statics. The electro has come from Greek word “elektron” . The meaning of elektron is amber.

Definition: - Electrostatics is the branch of Physics which deals with charges at rest.

Frictional electricity

The charges at rest are produce due to friction between two insulating body which are rubbed against each other that electrostatic is also called frictional electricity.

Ex: - 1. When glass rubbed is against silk. Glass becomes + ve charge and silk becomes – ve charge.

2. When glass is rubbed with flannel .it is found that glass become –ve charge and flannel +ve charge.

3. When amber is rubbed with wool amber becomes –ve charge and wool becomes +ve charged.

TWO TYPES OF CHARGES

+ Ve charge	- ve charge
Glass	Silk
flannel	glass
Wool	amber
Wool	rubber
Mica	Wool
Fur	ebonite
Catskin	ebonite

ELECTRIC CHARGE’S

Atom: - (i) Matter is composed of atoms the word atom has come from the Greek word which means individual.

(ii) Each atom has a nuclear part and extra nuclear part.

(iii) Nucleus like protons and neutron present in nuclei while electrons are revolving around the nucleus.

2. Proton (+ve)

i. Protons are found to be +ve charges particles.

ii. Each proton has mass 1.6×10^{-27}k.g.

iii. Magnitude of charge on proton 1.6×10^{-19}c.

3. Neutron

i. Neutrons are found to be a neutral.

ii. Each neutron has a mass 1.67×10^{-27}k.g

iii. Charge of Neutron is zero that is neutron carries no charge.

4. Electron (-ve)

i. Electrons are found to be –ve charge.

ii. Each electron has mass 9.11×10^{-31} k.g.

Quantization of charge

Electric charge of anybody in nature is always an integral multiple of amount of charge ‘e’.

$$q = \pm ne$$

Where n is an integer and e is the charge of the electron.

Electric charge

i. Electric charge is an intrinsic property of elementary particles of matter which give rise to electric force between various objects.

ii. Electric charge is a scalar quantity.

iii. Its SI unit is coulomb.

Properties of electric charge

i. Like charges repel each other and unlike charges attract each other.
ii. The magnitude of elementary negative or positive charge is same and is equal to 1.6×10^{-19} C.
iii. The electric charge is additive in nature. It implies that total charge on an object is algebraic sum of the charge located at different points in the object.
iv. The charge is quantized i. e charge carried by a charged object is equal to $\pm$ ne, where n is an integer.
v. The electric charge of a system is always conserved.
vi. Unlike mass, the electric charge on an object is not affected by the motion of the object.

COULOMB'S LAW: - In 1785 coulomb's obtained a law by measuring the force electricity between two stationary charges known as coulombs law. This law is really electrical along of Newton's law of gravitation.

Statement: - This law states that the electrostatics force of attraction (or) repulsion between two stationary charges is directly proportional to the product of magnitude of the charges and inversely proportional to the square of distance between them.

Let us consider two charges q_1 & q_2 are separated by distance r,then the force acting between them is given by

$$F \propto q_1 q_2 \text{..........1}$$

$$F \propto \frac{1}{r^2} \text{..........2}$$

Combing above two equation

$$F \propto \frac{q_1 q_2}{r^2}$$

$$\mathbf{F = \beta \frac{q_1 q_2}{r^2}} \quad \text{.......3}$$

Where β=proportionality constant.

VALUE OF β

1. In M. K. S system (or) S. I unit, when charge are taken in free space.

Hence the value of $\beta = \frac{1}{4\pi\varepsilon_0}$

$$\therefore \mathbf{F = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2}}$$

Where ε_0 is called permittivity of the free space.

Note: - permittivity of a medium is defined as the response of that medium to the present of electrified inside it.

2. In C. G. S. system, when charge's are taken in free space.

Here value of $\beta = \frac{1}{k}$ for free space k = 1

Form coulomb's law $F = \frac{q_1 q_2}{r^2}$

3. In M. K. S system (or) S.I unit, when charges are taken in dielectric medium.

In this case $\beta = \frac{1}{4\pi\varepsilon}$

$\therefore$ Form the coulomb's law $F = \frac{1}{4\pi\varepsilon} \frac{q_1 q_2}{r^2}$

Where ε= permittivity of dielectric medium

4. In C.G.S system when charges are taken in dielectric medium.

Here the value of $\beta = \frac{1}{k}$

$\therefore$ Form of coulomb's law F = $\frac{1}{k}\ \frac{q_1 q_2}{r^2}$

Where k is called as dielectric constant of the medium.

Note: - 1. Permittivity of air is 1.0005 times of permittivity of vacuum.

2. Dielectric is constant for free space is 1.

Unit of ε_0 F = $\frac{1}{4\pi\varepsilon_0}\ \frac{q_1 q_2}{r^2}$

$$q_1 q_2 = 4\pi\varepsilon_0 \times F \times r^2$$

$$\varepsilon_0 = \frac{q_1 q_2}{4\pi \times F \times r^2} = \frac{c \times c}{Nm^2}$$

Unit of ε_0 = $\frac{c^2}{Nm^2}$ (or) $C^2N^{-1}m^{-2}$

Value of ε_0 :- Experimentally it has been found that the value of ε_0 is given by

$$\boldsymbol{\varepsilon_0 = 8.854 \times 10^{-12}\ C^2N^{-1}m^{-2}}$$

Similarly the value of $\frac{1}{4\pi\varepsilon_0} = \mathbf{9 \times 10^9\ C^{-2}N^1m^2}$ (8.9875×10^9)

Dimensional formula of ε_0

$$\varepsilon_0 = \frac{q_1 q_2}{4\pi \times F \times r^2}$$

$$= [AT][AT] / [M^1L^1T^{-2}][L^{-2}]$$

$$= [A^2T^2]/[M^1L^3T^{-2}]$$

$$[\boldsymbol{\varepsilon_0}] = [\boldsymbol{M^{-1}L^{-3}T^4A^2}]$$

Relative permittivity (ε_r) /Dielectric Constant (k)

Consider two charge's $q_1$and q_2 are taken at distance 'r' in free space (or) vacuum.

$$\therefore F_{vac} = \frac{1}{4\pi\varepsilon_0}\ \frac{q_1 q_2}{r^2} \quad \ldots\ldots\ldots\ldots.1$$

Where ε_0 = permittivity of free space (or) vacuum.

When charge $q_1$and q_2 are taken in dielectric medium force between them.

$$\therefore F_{med} = \frac{1}{4\pi\varepsilon}\ \frac{q_1 q_2}{r^2} \quad \ldots\ldots\ldots\ldots.2$$

Where ε = permittivity of the medium.

Dividing equation (1) by (2)

$$\therefore \frac{F_{vac}}{F_{med}} = \frac{\frac{1}{4\pi\varepsilon_0}}{\frac{1}{4\pi\varepsilon}} = \frac{\frac{1}{\varepsilon_0}}{\frac{1}{\varepsilon}} = \frac{\varepsilon}{\varepsilon_0}$$

$$\frac{F_{vac}}{F_{med}} = k \quad \text{(Or)} \quad \frac{\varepsilon}{\varepsilon_0} = \varepsilon_r$$

Where k is called dielectric constant and ε_r is called relative permittivity of the medium

Definition of 'k':- Dielectric constant (k) is defined as the ratio of force between two charge's in vacuum to the force between same to charge's placed in dielectric medium.

Definition of ε_r :- Relative permittivity is defined as the ratio between permittivity of medium to the permittivity of free space. It has no unit.

We have $\varepsilon_r = \frac{\varepsilon}{\varepsilon_0} \Longrightarrow \varepsilon = \varepsilon_r . \varepsilon_0$

$$\Longrightarrow F = \frac{1}{4\pi\varepsilon} \frac{q_1 q_2}{r^2}$$

$$F = \frac{1}{4\pi\varepsilon_0\, \varepsilon_r} \frac{q_1 q_2}{r^2}$$

COULOMB'S LAW IN VECTOR FORM

We have coulomb's law

$$F = \beta \frac{q_1 q_2}{r^2}$$

$$F\hat{F} = \beta \frac{q_1 q_2}{r^2} \hat{r}$$

$$\vec{F} = \beta \frac{q_1 q_2}{r^2} \hat{r} \qquad (\hat{r} = \frac{\vec{r}}{r})$$

$$\vec{F} = \beta \frac{q_1 q_2}{r^2} \frac{\vec{r}}{r}$$

$$\vec{\mathbf{F}} = \boldsymbol{\beta} \frac{\mathbf{q_1 q_2}}{\mathbf{r^3}} \vec{\boldsymbol{r}}$$

Case: - 1 when charge q_1 and q_2 are opposite nature.

Let $\overrightarrow{F_{12}}$ be the force exerted on charge q_1 by charge q_2 and $\overrightarrow{F_{21}}$ be the force exerted on charge q_2 by charge q_1. $\overrightarrow{r_{12}}$ position vector of q_1 to q_2 and $\overrightarrow{r_{21}}$ position vector of q_2 to q_1.

Charges are dissimilarly in a nature of force between them is attraction. $\overrightarrow{F_{12}}$ *and* $\overrightarrow{r_{12}}$ are in same direction.

$$\overrightarrow{F_{12}} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{{r_{12}}^3} \overrightarrow{r_{12}} \quad \ldots\ldots..1$$

Similarly $\overrightarrow{F_{21}}$ *and* $\overrightarrow{r_{21}}$ are in same direction.

$$\overrightarrow{F_{21}} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{{r_{21}}^3} \overrightarrow{r_{21}} \ldots\ldots..2$$

Since $\overrightarrow{r_{12}} = -\overrightarrow{r_{21}}$

Substituting in equation (1)

$$\overrightarrow{F_{12}} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{|-r_{21}|^3} (-\overrightarrow{r_{21}})$$

$$\overrightarrow{F_{12}} = -\frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{{r_{21}}^3} \overrightarrow{r_{21}} \ldots\ldots\ldots..3$$

Comparing equation (2) and (3)

$$\boldsymbol{\overrightarrow{F_{12}} = -\overrightarrow{F_{21}}}$$

Case: - 2 when charge q$_1$ and q$_2$ are similar nature.

Since charges are similarly the nature of force between them is repulsion.

$$\overrightarrow{F_{12}} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{{r_{21}}^3} \overrightarrow{r_{21}} \ldots\ldots.1$$

Similarly $\overrightarrow{F_{21}} and\ \overrightarrow{r_{12}}$ are in same direction.

$$\overrightarrow{F_{21}} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{{r_{12}}^3} \overrightarrow{r_{12}} \ldots\ldots..2$$

Since $\overrightarrow{r_{12}} = -\overrightarrow{r_{21}}$

Substituting on equation (1)

$$\overrightarrow{F_{12}} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{|-r_{12}|^3} (\overrightarrow{-r_{12}})$$

$$\overrightarrow{F_{12}} = -\frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{{r_{12}}^3} \overrightarrow{r_{12}} \ldots\ldots\ldots..3$$

Comparing equation (2) and (3) we get,

$$\boldsymbol{\overrightarrow{F_{12}} = -\overrightarrow{F_{21}}}$$

Case: - 3 coulomb's law in terms of position vectors

Let r_A=position vector of point A

r_B =position vector of point B

Since $\overrightarrow{F_{12}}\ and\ \overrightarrow{r_{21}}$ are in same direction.

$$\overrightarrow{F_{12}} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{{r_{21}}^3}\overrightarrow{r_{21}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Applying triangle law of vector addition in the Δ AOB

$$\overrightarrow{OA}=\overrightarrow{OB}+\overrightarrow{BA}$$

$$\overrightarrow{r_A}=\overrightarrow{r_B}+\overrightarrow{r_{21}}$$

$$\overrightarrow{r_A}-\overrightarrow{r_B}=\overrightarrow{r_{21}}$$

Substituting for equation (1)

$$\overrightarrow{F_{12}} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{|\overrightarrow{r_A}-\overrightarrow{r_B}|^3}(\overrightarrow{r_A}-\overrightarrow{r_B}) \ldots\ldots (2)$$

Similarly force and $q_2$due to q_1

$$\overrightarrow{F_{21}} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{|\overrightarrow{r_B}-\overrightarrow{r_A}|^3}(\overrightarrow{r_B}-\overrightarrow{r_A}) \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Again rewriting equation (2)

$$\overrightarrow{F_{12}} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{|-(\overrightarrow{r_B}-\overrightarrow{r_A})|^3}\{-(\overrightarrow{r_B}-\overrightarrow{r_A})\}$$

$$\overrightarrow{F_{12}} = -\left\{\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{|\overrightarrow{r_B}-\overrightarrow{r_A}|^3}\{(\overrightarrow{r_B}-\overrightarrow{r_A})\}\right\} \ldots\ldots\ldots\ldots (4)$$

Comparing (3) and (4)

$$\overrightarrow{\boldsymbol{F_{12}}} = -\overrightarrow{\boldsymbol{F_{21}}}$$

UNIT OF ELECTRIC CHARGE

1. M.K.S system or S.I unit

In M. K. S. system or S.I system, the unit of electric charge is **coulomb.**

<u>Definition of coulomb</u>

We have electric force F = $\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}$

If $F = 9\times 10^9 N$

r = 1m

$q_1=q_2=q$ (say)

$$9\times 10^9 N = 9\times 10^9 N \times \frac{q^2}{1^2}$$

$$q^2 = 1$$

$$q = \pm 1c$$

1c is that amount of charge which when taken near another similar charge at 1m apart in free space and expression's a force of $9\times 10^9 N$.

2. In C.G.S .system

- **E.S.U (Electro Static Unit) of charge / stat coulomb.**

  1coulomb = 3×10^9 stat coulomb.

- **E.M.U (Electro Magnetic Unit) of charge / ab coulomb**

  1coulomb = $\frac{1}{10}$ ab coulomb

CHARGE DISTRIBUTION

Charge distribution is of following 2 types.

1. Continuous charge distribution

2. Discrete charge distribution.

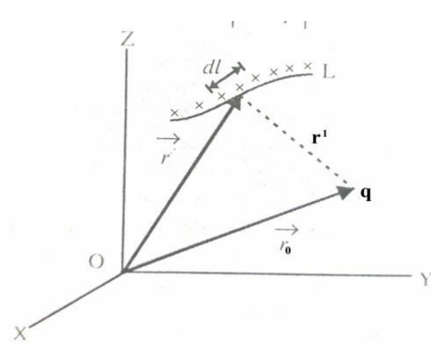
Continuous charge distribution is further dividing 3 types.

1. Liner charge distribution (charge distribution along a line)
2. Surface charge distribution (charge distribution in a surface)
3. Volume charge distribution (charge distribution in a cube)

Electro static force due to a continuous charge distribution

1. Linear charge distribution

Consider a line of length (l) having a continuous charge distribution of charge along its length.



dl = small elementary length of the line .

q_0= at a test charge placed at p.

r = position vector of the element dl.

r_0= position vector of the point.

λ= linear charge density (amount of charge per unit length)

Charge contained in small length dl is given by dq = λdl

Electrostatic force between λdl and q_0

$$\overrightarrow{dF}=\frac{1}{4\pi\varepsilon_0}\ \frac{\lambda \mathrm{dl} q_0}{{r_1}^3}\ \overrightarrow{r_1}$$

Applying triangle law of vector addition $\overrightarrow{r_0}+\overrightarrow{r_1}=\vec{r}$

$$\overrightarrow{r_1}=(\vec{r}-\overrightarrow{r_0})$$

$$\overrightarrow{dF}=\frac{1}{4\pi\varepsilon_0}\ \frac{\lambda \mathrm{dl} q_0}{(\vec{r}-\overrightarrow{r_0})^3}\ (\vec{r}-\overrightarrow{r_0})$$

Integrating on both sides

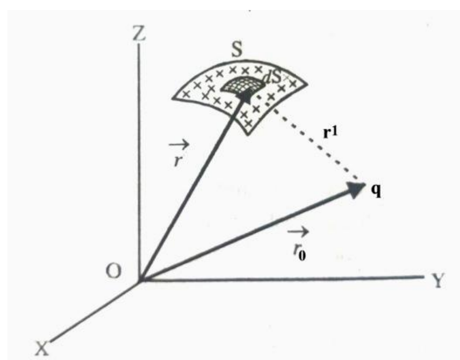
$$\int\overrightarrow{dF}=\int\frac{1}{4\pi\varepsilon_0}\ \frac{\lambda \mathrm{dl} q_0}{(\vec{r}-\overrightarrow{r_0})^3}\ (\vec{r}-\overrightarrow{r_0})$$

$$\mathbf{F}=\frac{1}{4\pi\varepsilon_0}\int\frac{\lambda \mathrm{dl} q_0}{(\vec{r}-\overrightarrow{r_0})^3}\ (\vec{r}-\overrightarrow{r_0})$$

- Its SI unit is coulomb/metre.
- Its Dimensional formula is $[M^0L^{-1}T^1A^1]$

2. Surface charge distribution

Consider a surface S having a continuous charge distribution of charge along its surface.



ds = small area of the surface .

q_0= at a test charge placed at p.

r = position vector of the element ds .

r_0= position vector of the point p.

σ= surface charge density (amount of charge per unit area)

Charge on the surface ds is given by dq = σds

Electrostatic force between σds and q_0

$$\overrightarrow{dF}=\frac{1}{4\pi\varepsilon_0}\ \frac{\sigma \mathrm{ds} q_0}{{r_1}^3}\ \overrightarrow{r_1}$$

Applying triangle law of vector addition $\overrightarrow{r_0}+\overrightarrow{r_1}=\vec{r}$

$$\vec{r_1} = (\vec{r} - \vec{r_0})$$

$$\overrightarrow{dF} = \frac{1}{4\pi\varepsilon_0} \frac{\sigma ds q_0}{(\vec{r} - \vec{r_0})^3} (\vec{r} - \vec{r_0})$$

Integrating on both sides

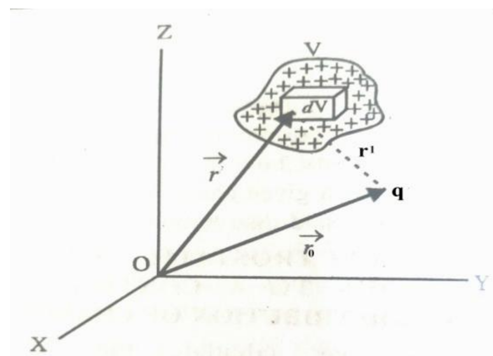
$$\int \overrightarrow{dF} = \int \frac{1}{4\pi\varepsilon_0} \frac{\sigma ds q_0}{(\vec{r} - \vec{r_0})^3} (\vec{r} - \vec{r_0})$$

$$\mathbf{F} = \frac{1}{4\pi\varepsilon_0} \iint \frac{\sigma ds q_0}{(\vec{r} - \vec{r_0})^3} (\vec{r} - \vec{r_0})$$

- Its SI unit is coulomb/(metre)^2.
- Its Dimensional formula is $[M^0L^{-2}T^1A^1]$

3. Volume charge distribution

Consider a cube of volume (v) having a continuous charge distribution.



q_0= at a test charge placed at p.

r = position vector of dv.

r_0= position vector of p.

ρ= volume of charge density (the amount of charge per unit volume).

Charge contained in small volume dv is dq = ρdv

Let $\overrightarrow{dF}$ = Electrostatic force between ρdv and q_0

$$\overrightarrow{dF} = \frac{1}{4\pi\varepsilon_0} \frac{\rho dv q_0}{r_1{}^3} \vec{r_1}$$

Applying triangle law of vector addition $\vec{r_0} + \vec{r_1} = \vec{r}$

$$\vec{r_1} = (\vec{r} - \vec{r_0})$$

$$\overrightarrow{dF} = \frac{1}{4\pi\varepsilon_0} \frac{\rho dv q_0}{(\vec{r} - \vec{r_0})^3} (\vec{r} - \vec{r_0})$$

Integrating on both sides

$$\int \overrightarrow{dF} = \int \frac{1}{4\pi\varepsilon_0} \frac{\rho dv q_0}{(\vec{r} - \vec{r_0})^3} (\vec{r} - \vec{r_0})$$

$$F=\frac{1}{4\pi\varepsilon_0}\iiint \frac{\rho dv q_0}{(\vec{r}-\vec{r_0})^3}(\vec{r}-\vec{r_0})$$

- Its SI unit is coulomb/(metre)^3.
- Its Dimensional formula is $[M^0L^{-3}T^1A^1]$

PRINCIPLE OF SUPER POSITION (OR) FORCE BETWEEN MULTIPLE CHARGES

Definition: - The principle of super position states that when a number of charges placed near each other behave in depended of each other and the net force on one charge due to the vector sum of all force's produced by individual charges.

Mathematically, net force $\vec{F}=\vec{F_1}+\vec{F_2}+\vec{F_3}+\cdots+\vec{F_i}+\cdots\vec{F_n}$

Consider a 'n' number of charges q_1, q_2 ,q_3..............q_i, ...q_n are distribution in space .

$\vec{r_1}$ = Position vector of charge q_1 .

$\vec{r_2}$= position vector of charge q_2 .

$\vec{r_3}$= position vector of charge q_3

$\vec{r_i}$= position vector of charge q_i

$\vec{r_n}$= position vector of charge q_n

$\vec{F_1}$= Force on charge q_o due toq_1

$$\vec{F_1}=\frac{1}{4\pi\varepsilon_0}\frac{q_0\ q_1}{(\vec{r_0}-\vec{r_1})^3}(\vec{r_0}-\vec{r_1})$$

$\vec{F_2}$= Force on charge $q_o$due toq_2

$$\vec{F_2}=\frac{1}{4\pi\varepsilon_0}\frac{q_0\ q_2}{(\vec{r_0}-\vec{r_2})^3}(\vec{r_0}-\vec{r_2})$$

$\vec{F_3}$ = Force on charge $q_o$due toq_3

$$\vec{F_3}=\frac{1}{4\pi\varepsilon_0}\frac{q_0\ q_3}{(\vec{r_0}-\vec{r_3})^3}(\vec{r_0}-\vec{r_3})$$

....

$\vec{F_i}$= Force on charge q_o due to q_i

$$\vec{F_i}=\frac{1}{4\pi\varepsilon_0}\frac{q_0\ q_i}{(\vec{r_0}-\vec{r_i})^3}(\vec{r_0}-\vec{r_i})$$

....

$\vec{F_n}$ = Force on charge q_o due to q_n

$$\vec{F_n} = \frac{1}{4\pi\varepsilon_0} \frac{q_0\, q_n}{(\vec{r_0}-\vec{r_n})^3} (\vec{r_0} - \vec{r_n})$$

According to principle of super position theorem

$$\vec{F} = \vec{F_1} + \vec{F_2} + \vec{F_3} + \cdots + \vec{F_i} + \cdots \vec{F_n}$$

$$\vec{F} = \frac{1}{4\pi\varepsilon_0} \frac{q_0\, q_1}{(\vec{r_0}-\vec{r_1})^3} (\vec{r_0} - \vec{r_1}) + \frac{1}{4\pi\varepsilon_0} \frac{q_0\, q_2}{(\vec{r_0}-\vec{r_2})^3} (\vec{r_0} - \vec{r_2}) + \frac{1}{4\pi\varepsilon_0} \frac{q_0\, q_3}{(\vec{r_0}-\vec{r_3})^3} (\vec{r_0} - \vec{r_3}) + \ldots\ldots\ldots\ldots \frac{1}{4\pi\varepsilon_0} \frac{q_0\, q_i}{(\vec{r_0}-\vec{r_i})^3} (\vec{r_0} - \vec{r_i}) + \frac{1}{4\pi\varepsilon_0} \frac{q_0\, q_n}{(\vec{r_0}-\vec{r_n})^3} (\vec{r_0} - \vec{r_n})$$

$$\boldsymbol{\vec{F}} = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_0\, q_i}{(\vec{r_0}-\vec{r_i})^3} (\vec{r_0} - \vec{r_i})$$

EXTRANOTE

Conservation of electric charge

Electric charges can neither be created nor destroyed. According to the law of conservation of electric charge, the total charge in an isolated system always remains constant. But the charges can be transferred from one part of the system to another, such that the total charge always remains conserved.

Example:-Uranium (${}_{92}U^{238}$) can decay by emitting an alpha particle (${}_{2}He^{4}$ nucleus) and transforming to thorium (${}_{90}Th^{234}$).

$${}_{92}U^{238} \longrightarrow {}_{90}Th^{234} + {}_{2}He^{4}$$

Total charge before decay = +92e, total charge after decay = 90e + 2e.Hence, the total charge is conserved. i.e. it remains constant.

Additive nature of charge

The total electric charge of a system is equal to the algebraic sum of electric charges located in the system.

Example:- if two charged bodies of charges +3q, −7q are brought in contact, the total charge of the system is –2q.