INTEGRATION

Integrate the following expressions:

1. $\int 5\,dx$

2. $\int (3x^2 - 5)\,dx$

3. $\int (x - 7)\,dx$

4. $\int (x+1)(2-x)\,dx$

5. $\int (3x-2)^2\,dx$

6. $\int (x^8 - 4x^3 - x)\,dx$

7. $\int \frac{dx}{x^3}$

8. $\int (1 - x^{-1/3})\,dx$

9. $\int \frac{3+2x^2}{x^2}\,dx$

10. $\int x\sqrt{x}\,dx$

11. $\int \left(1-\frac{1}{z}\right)^2 dz$

12. $\int (\sqrt{x}+\sqrt{a})^2\,dx$

13. $\int \frac{dx}{x\sqrt{2x}}$

14. $\int (x\sqrt{x}-5)^2\,dx$

15. $\int \frac{x^3-1}{x-1}\,dx$

16. $\int_0^1 (2-x)\,dx$

17. $\int_{-2}^{2} (2x+x^2)\,dx$

18. $\int_{-3}^{-2} x(x+1)^2\,dx$

19. $\int_0^a (\sqrt{a}-\sqrt{x})^2\,dx$

20. $\int_2^5 \left(x^2+\frac{1}{x^2}\right)dx$

Evaluate the following integrals :

21. $\int \sqrt{3x}\,dx$

22. $\int x(x^2-2)^2\,dx$

23. $\int \frac{dx}{5-2x}$

24. $\int \frac{\sqrt{x}\,dx}{1+x\sqrt{x}}$

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ANSWER KEY

1. $[5x + C]$

2. $[x^3 - 5x + C]$

3. $[\frac{x^2}{2} - 7x + C]$

4. $[-\frac{x^3}{3} + \frac{x^2}{2} + 2x + C]$

5. $[3x^3 - 6x^2 + 4x + C]$

6. $[\frac{x^9}{9} - x^4 - \frac{1}{2}x^2 + C]$

7. $[-\frac{1}{2x^2} + C]$

8. $[x - \frac{3}{2}x^{2/3} + C]$

9. $[\frac{2x^2-3}{x} + C]$

10. $[\frac{2}{5}x^2\sqrt{x} + C]$

11. $[z - \frac{1}{z} - 2\ln z + C]$

12. $[\frac{x^2}{2} + \frac{4}{3}x\sqrt{ax} + ax + C]$

13. $[-\sqrt{\frac{2}{x}} + C]$

14. $[\frac{x^4}{4} - 4x^{5/2} + 25x + C]$

15. $[\frac{x^3}{3} + \frac{x^2}{2} + x + C]$

16. $[\frac{3}{2}]$

17. $[\frac{16}{3}]$

18. $[-\frac{73}{12}]$

19. $[\frac{1}{6}a^2]$

20. [39.3]

21. $[\frac{2}{3}\sqrt{3}x^{3/2} + C]$

22. $[\frac{1}{6}(x^2-2)^3 + C]$

23. $[-\frac{1}{2}\ln(5-2x) + C]$

24. $[(\frac{2}{3}\ln(1+x^{3/2}) + C]$

INTEGRATION

Evaluate the following integrals :

1. $\int \cos^3\theta \sin\theta \, d\theta$

2. $\int \frac{e^t \, dt}{\sqrt{1-e^t}}$

3. $\int \frac{y+4}{y-4} dy$

4. $\int \frac{\sec^2\theta}{\tan\theta} d\theta$

5. $\int \frac{\ln x}{x} dx$

6. $\int \frac{1+\cos y}{y+\sin y} dy$

7. $\int_{-1/2}^{1/2} (2y+1)^7 \, dy$

8. $\int e^{4x} \, dx$

9. $\int 3^{2y} \, dy$

10. $\int x^{-2} e^{1/x} \, dx$

11. $\int e^{\sin\theta} \cos\theta \, d\theta$

12. $\int \frac{2+e^x}{e^x} dx$

13. $\int_1^4 \frac{e^{\sqrt{x}} \, dx}{\sqrt{x}}$

14. $\int_0^1 (e^x + x^e) \, dx$

15. $\int_0^1 (e^x + e^{-x})^2 \, dx$

16. $\int_0^{\pi/3} \frac{\sec\theta \tan\theta}{\sqrt{e^{\sec\theta}}} d\theta$

17. $\int_{-1}^{0} \frac{dy}{1+e^y}$

18. $\int \frac{1}{x^2} \sin\frac{\pi}{x} dx$

19. $\int \sec 4\theta \tan 4\theta \, d\theta$

20. $\int \cot\frac{x}{2} dx$

21. $\int \sin^3 x \cos^3 x \, dx$

22. $\int \cos^2\left(\frac{y}{2}\right) dy$

23. $\int \sec^n\theta \tan\theta \, d\theta$

24. $\int \tan^2\left(\frac{3}{4}x\right) dx$

25. $\int \sin 3x \cos 5x \, dx$

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ANSWER KEY

1. $[-\frac{1}{4}\cos^4\theta + C]$ **2.** $[-2\sqrt{1-e^t} + C]$ **3.** $[y + 8\ln y - 4) + C]$ **4.** $[\ln\tan\theta + C]$

5. $[\frac{1}{2}(\ln x)^2 + C]$ **6.** $[\ln(y + \sin y) + C]$ **7.** [16] **8.** $[\frac{1}{4}e^{4x} + C]$

9. $[\frac{3^{2y}}{\ln 9} + C]$ **10.** $[-e^{1/x} + C]$ **11.** $[e^{\sin\theta} + C]$ **12.** $[x - 2e^{-x} + C]$

13. [9.342] **14.** [1.987] **15** [5.627] **16.** [0.477]

17. [0.62] **18.** $[\frac{1}{\pi}\cos\frac{\pi}{x} + C]$ **19.** $[\frac{1}{4}\sec 4\theta + C]$ **20.** $[2\ln\sin\frac{x}{2} + C]$

21. $[\frac{1}{4}\sin^4 x - \frac{1}{6}\sin^6 x + C]$ **22.** $[\frac{y}{2} + \frac{\sin y}{2} + C]$ **23.** $[\frac{1}{n}\sec^n\theta + C]$ **24.** $[\frac{4}{3}\tan\left(\frac{3x}{4}\right) x + C]$

25. $[\frac{1}{4}\cos 2x - \frac{1}{16}\cos 8x + C]$

INTEGRATION

Integrate by using the substitution suggested in bracket.

1. $\int \sin 3x\, dx$, (use, $u = 3x$)

2. $\int x \sin(2x^2) dx$, (use, $u = 2x^2$)

3. $\int \sec 2t \tan 2t\, dt$, (use, $u = 2t$)

4. $\int \left(1 - \cos\frac{t}{2}\right)^2 \sin\frac{t}{2}\, dt$,

(use, $u = 1 - \cos\frac{1}{2}$)

5. $\int x^3(x^4 - 1)^2\, dx$, (use, $u = x^4 - 1$)

6. $\int \frac{9r^2}{\sqrt{1-r^3}}\, dr$, (use, $u = 1 - r^3$)

7. $\int \frac{1}{x^2}\cos^2\left(\frac{1}{x}\right) dx$, (use, $u = -\frac{1}{x}$)

Integrate by using a suitable substitution.

8. $\int (2x+1)^3\, dx$

9. $\int \frac{3}{(2-x)^2}\, dx$

10. $\int \frac{4y}{\sqrt{2y^2+1}}\, dy$

11. $\int \cos(3z+4)\, dz$

12. $\int \sin(8z-5)\, dz$

13. $\int \frac{1}{\sqrt{t}} \cos(\sqrt{t} + 3)\, dt$

Definite Integration.

14. $\int_{-2}^{1} 5\, dx$

15. $\int_{-4}^{-1} \frac{\pi}{2}\, d\theta$

16. $\int_{-2}^{4} \left(\frac{x}{2} + 3\right) dx$

17. $\int_{\sqrt{2}}^{5\sqrt{2}} r\, dr$

18. $\int_{0}^{2\pi} \sin\theta\, d\theta$

19. $\int_{0}^{1} e^x dx$

Calculation of area

Use a definite integral to find the area of the region between the given curve and the x-axis on the interval $[0, b]$

20. $y = 2x$

21. $y = \frac{x}{2} + 1$

Use a definite integral to find the area of the region between the given curve and the x-axis on the interval $[0, \pi]$

22. $y = \sin x$

23. $y = \sin^2 x$

Find integrals of given functions.

24. $\int (2x^3 - 5x + 7)\, dx$

25. $\int \left(\frac{1}{5} - \frac{2}{x^3} + 2x\right) dx$

26. $\int (\sqrt{x} + \sqrt[3]{x})\, dx$

27. $\int x^{-3}(x+1) dx$

28. $\int \frac{t\sqrt{t} + \sqrt{t}}{t^2}\, dt$

29. $\int \frac{4 + \sqrt{t}}{t^3}\, dt$

30. $\int \cos^2 x\, dx$

(Hint $1 + \cot^2 x = \operatorname{cosec}^2 x$)

31. $\int (1 - \cot^2 x)\, dx$

32. $\int \cos\theta(\tan\theta + \sec\theta)\, d\theta$

Integrate by using the substitution suggested in bracket.

33. $\int 28(7x-2)^{-5}\, dx$, (use, $u = 7x - 2$)

34. $\int 12(y^4 + 4y^2 + 1)^2(y^3 + 2y)\, dy$,

(use, $u = y^4 + 4y^2 + 1$)

35. $\int \sqrt{x} \sin^2(x^{3/2} - 1)\, dx$

(use, $u = x^{3/2} - 1$)

36. $\int \operatorname{cosec}^2 2\theta \cot 2\theta\, d\theta$

(a) Using $u = \cot 2\theta$

(b) Using $u = \operatorname{cosec} 2\theta$

37. $\int \frac{dx}{\sqrt{5x+8}}$

(a) Using $u = 5x + 8$

(b) Using $u = \sqrt{5x+8}$

Integrate by using suitable substitution.

38. $\int \sqrt{3-2s}\, ds$

39. $\int \theta \sqrt[4]{(1-\theta^2)}\, d\theta$

40. $\int 8\theta \sqrt[3]{(\theta^2 - 1)}\, d\theta$

41. $\int \frac{1}{\sqrt{x}(1+\sqrt{x})^2}\, dx$

42. $\int \frac{(1+\sqrt{x})^3}{\sqrt{x}}\, dx$

43. $\int \sec^2(3x+2)\,dx$

44. $\int \tan^2 x \sec^2 x\,dx$

45. $\int \sec\left(\upsilon+\frac{\pi}{2}\right)\tan\left(\upsilon+\frac{\pi}{2}\right)d\upsilon$

46. $\int \text{cosec}\left(\frac{v-\pi}{2}\right)\cot\left(\frac{v-\pi}{2}\right)dv$

47. $\int \frac{\sin(2t+1)}{\cos^2(2t+1)}dt$

48. $\int \frac{6\cos t}{(2+\sin t)}dt$

49. $\int_{\pi}^{2\pi} \theta\, d\theta$

50. $\int_{0}^{\sqrt[3]{7}} X^2 dx$

51. $\int_{0}^{\pi} \cos x dx$

52. $\int_{0}^{\sqrt{\pi}} x\sin x^2 dx$

53. $\int_{0}^{1} \frac{dx}{3x+2}$

Use a definite integral to find the area of the region between the given curve and the *x*-axis on the interval [0, *b*].

54. $y = 3x^2$

55. $y = \sqrt{b^2 - x^2}$

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ANSWER KEY

1. $-\frac{1}{3}\cos 3x + C$ **2.** $-\frac{1}{4}\cos(2x^2) + C$ **3.** $\frac{1}{2}\sec 2t + C$ **4.** $\frac{2}{3}\left(1-\cos\frac{1}{2}\right)^3 + C$

5. $\frac{1}{12}(x^4-1)^3 + C$ **6.** $-6(1-r^3)^{1/2} + C$ **7.** $-\frac{1}{2x} - \frac{1}{4}\sin\frac{2}{x} + C$ **8.** $\frac{(2x+1)^4}{8} + C$

9. $\frac{3}{2-x} + C$ **10.** $2\sqrt{2y^2+1} + C$ **11.** $\frac{1}{3}\sin(3z+4) + C$ **12.** $-\frac{\cos(8z-5)}{8} + C$

13. $2\sin(\sqrt{t}+3) + C$ **14.** 15 **15.** $\frac{3\pi}{2}$ **16.** Area = 21 square units

17. 24 **18.** 0 **19.** $e-1$

20. Using n subintervals of length $\Delta x = \frac{b}{n}$ and right endpoint values : Area = $\int_{0}^{b} 2x\,dx = b^2$

21. $\frac{b^2}{4} + b = \frac{b(4+b)}{4}$ **22.** 2 **23.** $\frac{\pi}{2}$ **24.** $\frac{x^4}{2} - \frac{5x^2}{2} + 7x + C$

25. $\frac{x}{5} + \frac{1}{x^2} + x^2 + C$ **26.** $\frac{2}{3}x^{3/2} + \frac{3}{4}x^{4/3} + C$ **27.** $-\frac{1}{x} - \frac{1}{2x^2} + C$ **28.** $2\sqrt{t} - \frac{2}{\sqrt{t}} + C$

29. $-2t^{-2} - \frac{2}{3}t^{-3/2} + C$ **30.** $-\cot x - x + C$ **31.** $2x + \cot x + C$ **32.** $-\cos\theta + \theta + C$

33. $-(7x-2)^{-4} + C$ **34.** $(y^4 + 4y^2 + 1)^3 + C$ **35.** $\frac{1}{3}(x^{3/2}-1) - \frac{1}{6}\sin(2x^{3/2}-2) + C$

36. (a) $-\frac{1}{4}(\cot^2 2\theta) + C$; (b) $-\frac{1}{4}(\text{cosec}^2 2\theta) + C$ **37.** $\left[\frac{2}{5}\sqrt{5x+8}\right] + C$ **38.** $-\frac{1}{3}(3-2s)^{3/2} + C$

39. $-\frac{2}{5}(1-\theta^2)^{5/4} + C$ **40.** $3(\theta^2-1)^{4/3} + C$ **41.** $\frac{(-2)}{(1+\sqrt{x})} + C$ **42.** $\frac{(1+\sqrt{x})^4}{2} + C$

43. $\frac{1}{3}\tan(3x+2) + C$ **44.** $\frac{\tan^3 x}{3} + C$ **45.** $\sec\left(\upsilon+\frac{\pi}{2}\right) + C$ **46.** $-2\,\text{cosec}\left(\frac{\upsilon-\pi}{2}\right) + C$

47. $\frac{1}{2\cos(2t+1)} + C$ **48** $\frac{-3}{(2+\sin t)^2} + C$ **49.** $\frac{3\pi^2}{2}$ **50.** $\frac{7}{3}$ **51.** 0 **52.** 1

53. $\frac{1}{3}\ln\frac{5}{2} = \ln\left(\frac{5}{2}\right)^{1/3}$ **54.** b^3 **55.** $\frac{\pi b^2}{4}$