RAY OPTICS - II

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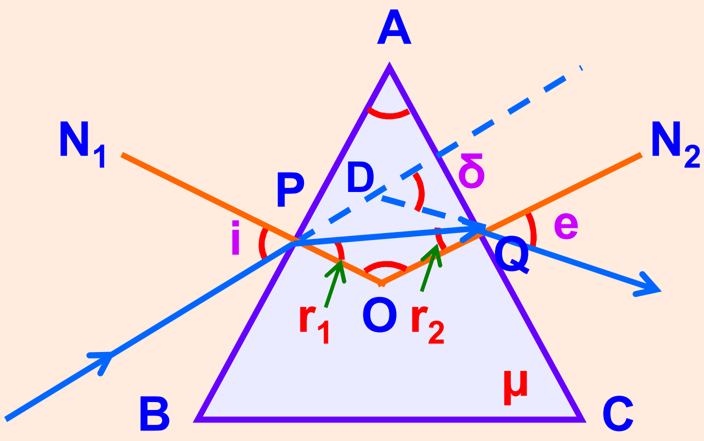
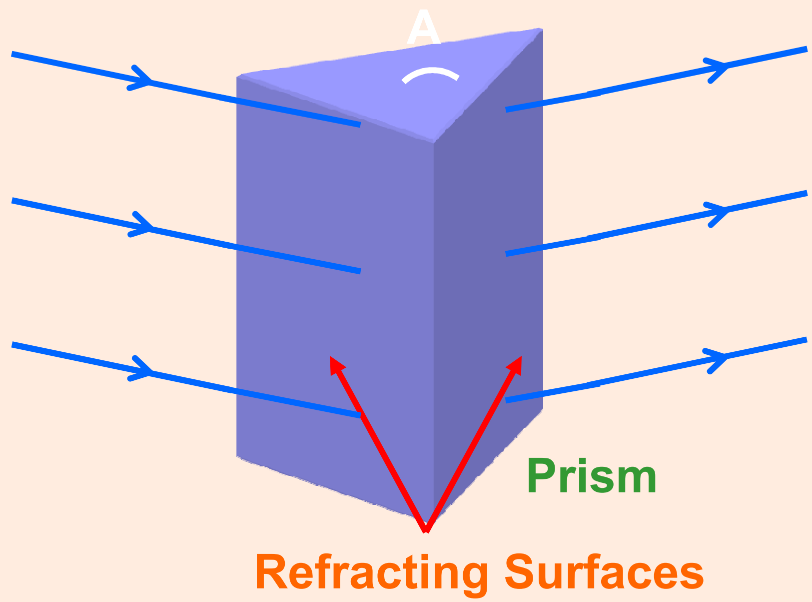
Refraction of Light through Prism:





In quadrilateral APOQ,

$A + O = 180°$ …….(1)

(since N_1 and N_2 are normal)

In triangle OPQ,

$r_1 + r_2 + O = 180°$ …….(2)

In triangle DPQ,

$\delta = (i - r_1) + (e - r_2)$

$\delta = (i + e) - (r_1 + r_2)$ …….(3)

From (1) and (2),

$A = r_1 + r_2$

From (3),

$\delta = (i + e) - (A)$

or $i + e = A + \delta$

Sum of angle of incidence and angle of emergence is equal to the sum of angle of prism and angle of deviation.

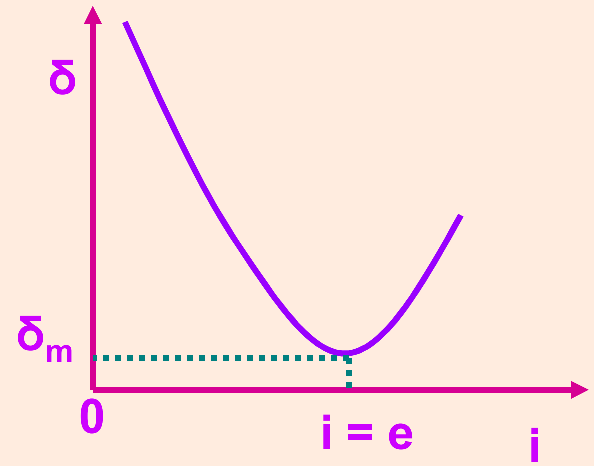
Variation of angle of deviation with angle of incidence:

When angle of incidence increases,
the angle of deviation decreases.

At a particular value of angle of incidence the angle of deviation becomes minimum and is called 'angle of minimum deviation'.

At δ_m, $i = e$ and $r_1 = r_2 = r$ (say)

After minimum deviation, angle of deviation increases with angle of incidence.



Refractive Index of Material of Prism:

$A = r_1 + r_2$

$A = 2r$

$r = A / 2$

$i + e = A + \delta$

$2i = A + \delta_m$

$i = (A + \delta_m) / 2$

According to Snell's law,

$$\mu = \frac{\sin i}{\sin r_1} = \frac{\sin i}{\sin r}$$

$$\therefore \quad \mu = \frac{\sin \frac{(A + \delta_m)}{2}}{\sin \frac{A}{2}}$$

Refraction by a Small-angled Prism for Small angle of Incidence:

$$\mu = \frac{\sin i}{\sin r_1} \quad \text{and} \quad \mu = \frac{\sin e}{\sin r_2}$$

If i is assumed to be small, then r_1, r_2 and e will also be very small.
So, replacing sines of the angles by angles themselves, we get

$$\mu = \frac{i}{r_1} \quad \text{and} \quad \mu = \frac{e}{r_2}$$

$$i + e = \mu\,(r_1 + r_2) = \mu\,A$$

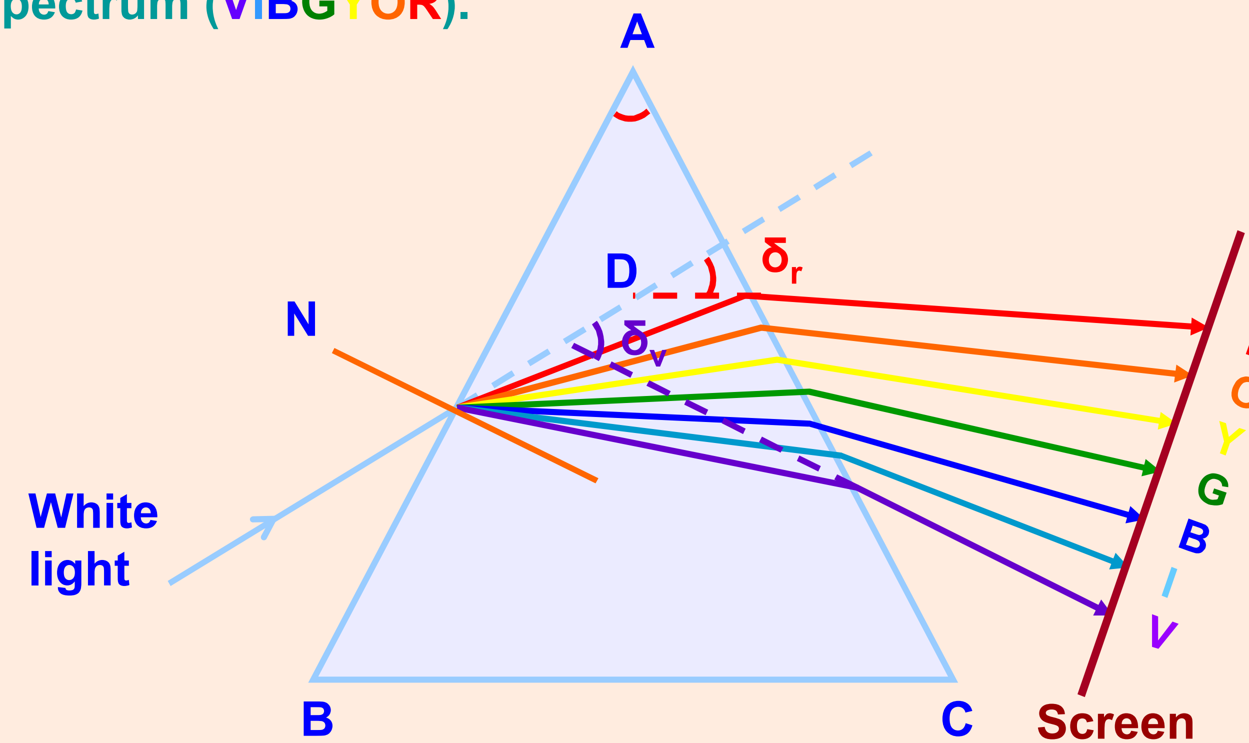
But $i + e = A + \delta$

So, $A + \delta = \mu\,A$

or $$\boxed{\delta = A\,(\mu - 1)}$$

Dispersion of White Light through Prism:

The phenomenon of splitting a ray of white light into its constituent colours (wavelengths) is called dispersion and the band of colours from violet to red is called spectrum (VIBGYOR).



Cause of Dispersion:

$$\mu_v = \frac{\sin i}{\sin r_v} \quad \text{and} \quad \mu_r = \frac{\sin i}{\sin r_r}$$

Since $\mu_v > \mu_r$, $r_r > r_v$

So, the colours are refracted at different angles and hence get separated.

Dispersion can also be explained on the basis of Cauchy's equation.

$$\mu = a + \frac{b}{\lambda^2} + \frac{c}{\lambda^4}$$ (where a, b and c are constants for the material)

Since $\lambda_v < \lambda_r$, $\mu_v > \mu_r$

But $\delta = A(\mu - 1)$

Therefore, $\delta_v > \delta_r$

So, the colours get separated with different angles of deviation.
Violet is most deviated and Red is least deviated.

Angular Dispersion:

1. The difference in the deviations suffered by two colours in passing through a prism gives the angular dispersion for those colours.
2. The angle between the emergent rays of any two colours is called angular dispersion between those colours.
3. It is the rate of change of angle of deviation with wavelength. ($\Phi = d\delta / d\lambda$)

$\Phi = \delta_v - \delta_r$ or $\boxed{\Phi = (\mu_v - \mu_r) A}$

Dispersive Power:

The dispersive power of the material of a prism for any two colours is defined as the ratio of the angular dispersion for those two colours to the mean deviation produced by the prism.

It may also be defined as dispersion per unit deviation.

$$\omega = \frac{\Phi}{\delta}$$

where δ is the mean deviation and $\delta = \frac{\delta_v + \delta_r}{2}$

Also $\omega = \frac{\delta_v - \delta_r}{\delta}$ **or** $\omega = \frac{(\mu_v - \mu_r) A}{(\mu_y - 1) A}$ **or** $\omega = \frac{(\mu_v - \mu_r)}{(\mu_y - 1)}$

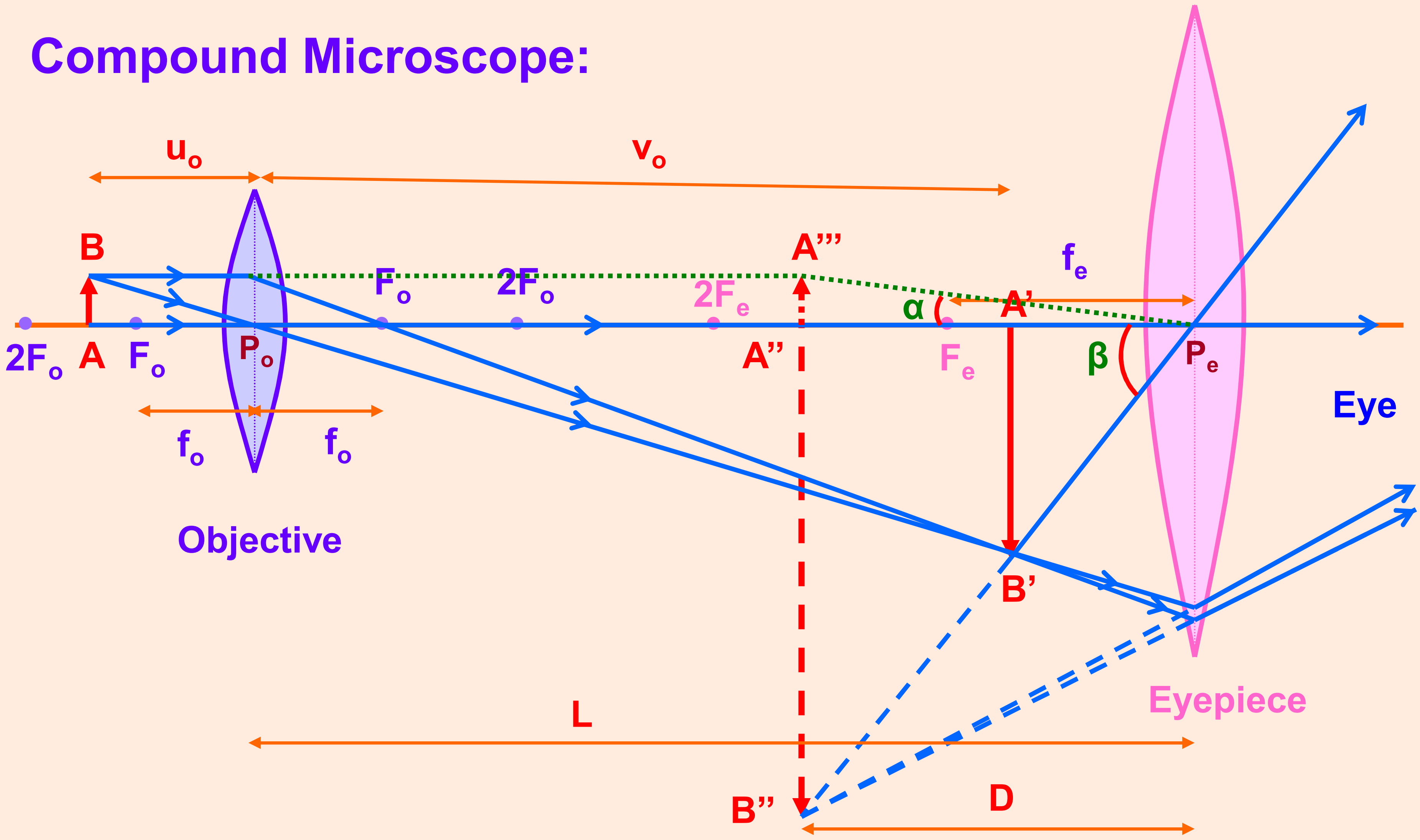
Scattering of Light – Blue colour of the sky and Reddish appearance of the Sun at Sun-rise and Sun-set:

The molecules of the atmosphere and other particles that are smaller than the longest wavelength of visible light are more effective in scattering light of shorter wavelengths than light of longer wavelengths. The amount of scattering is inversely proportional to the fourth power of the wavelength. (Rayleigh Effect)

Light from the Sun near the horizon passes through a greater distance in the Earth's atmosphere than does the light received when the Sun is overhead. The correspondingly greater scattering of short wavelengths accounts for the reddish appearance of the Sun at rising and at setting.

When looking at the sky in a direction away from the Sun, we receive scattered sunlight in which short wavelengths predominate giving the sky its characteristic bluish colour.

Compound Microscope:



Objective: The converging lens nearer to the object.

Eyepiece: The converging lens through which the final image is seen.

Both are of short focal length. Focal length of eyepiece is slightly greater than that of the objective.

Angular Magnification or Magnifying Power (M):

Angular magnification or magnifying power of a compound microscope is defined as the ratio of the angle β subtended by the final image at the eye to the angle α subtended by the object seen directly, when both are placed at the least distance of distinct vision.

$$M = \frac{\beta}{\alpha}$$

Since angles are small,
$\alpha = \tan\alpha$ and $\beta = \tan\beta$

$$M = \frac{\tan\beta}{\tan\alpha}$$

$$M = \frac{A''B''}{D} \times \frac{D}{A''A'''}$$

$$M = \frac{A''B''}{D} \times \frac{D}{AB}$$

$$M = \frac{A''B''}{AB}$$

$$M = \frac{A''B''}{A'B'} \times \frac{A'B'}{AB}$$

$$M = M_e \times M_o$$

$$M_e = 1 - \frac{v_e}{f_e} \quad \text{or} \quad M_e = 1 + \frac{D}{f_e}$$

($v_e = -D$
$= -25$ cm)

and $M_o = \dfrac{v_o}{-u_o}$ ∴

$$M = \frac{v_o}{-u_o}\left(1 + \frac{D}{f_e}\right)$$

Since the object is placed very close to the principal focus of the objective and the image is formed very close to the eyepiece,
$u_o \approx f_o$ and $v_o \approx L$

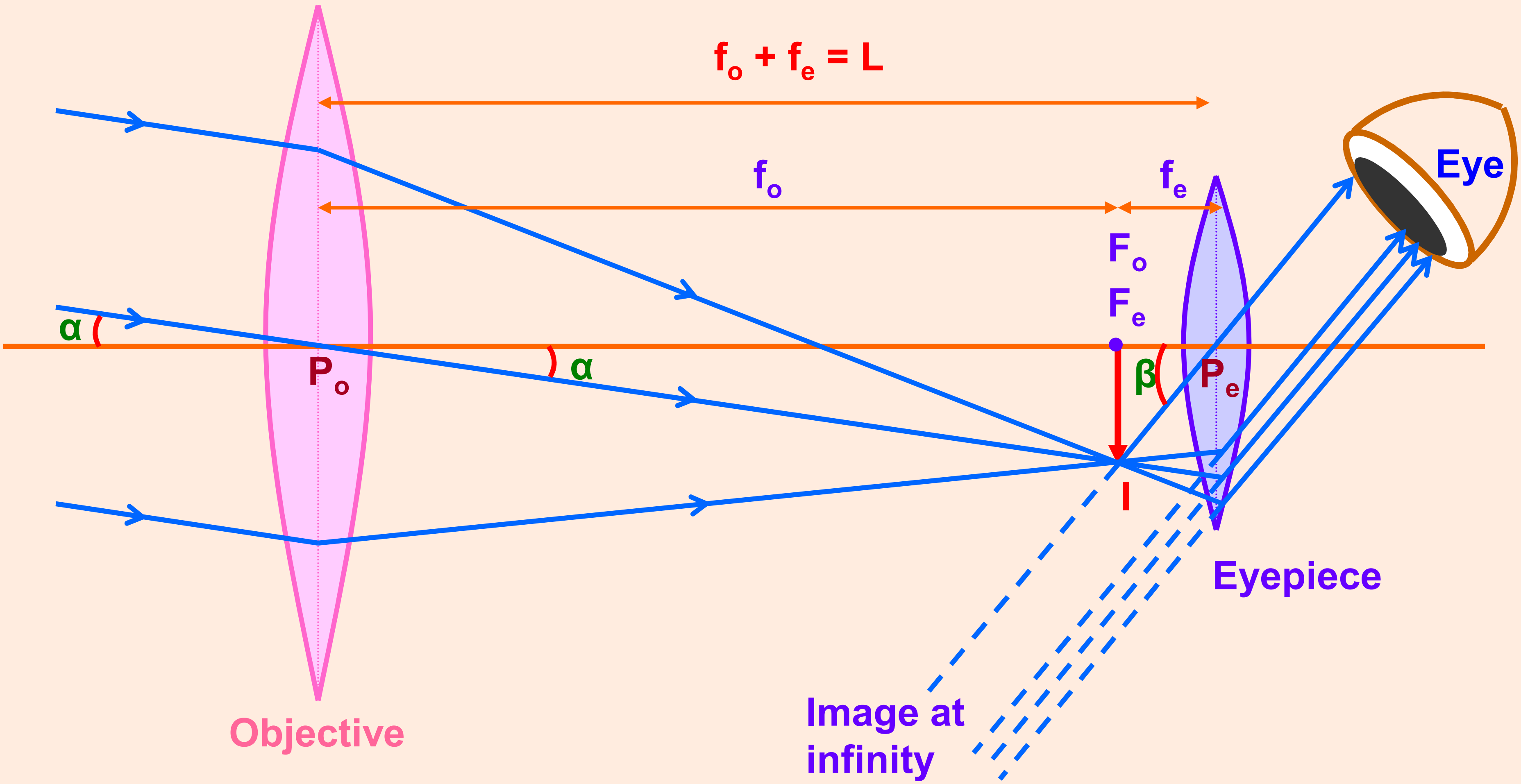
$$M = \frac{-L}{f_o}\left(1 + \frac{D}{f_e}\right)$$

or

$$M \approx \frac{-L}{f_o} \times \frac{D}{f_e}$$

(Normal adjustment
i.e. image at infinity)

Astronomical Telescope: (Image formed at infinity – Normal Adjustment)



Focal length of the objective is much greater than that of the eyepiece.

Aperture of the objective is also large to allow more light to pass through it.

Angular magnification or Magnifying power of a telescope in normal adjustment is the ratio of the angle subtended by the image at the eye as seen through the telescope to the angle subtended by the object as seen directly, when both the object and the image are at infinity.

$$M = \frac{\beta}{\alpha}$$

Since angles are small, α = tan α and β = tan β

$$M = \frac{\tan\beta}{\tan\alpha}$$

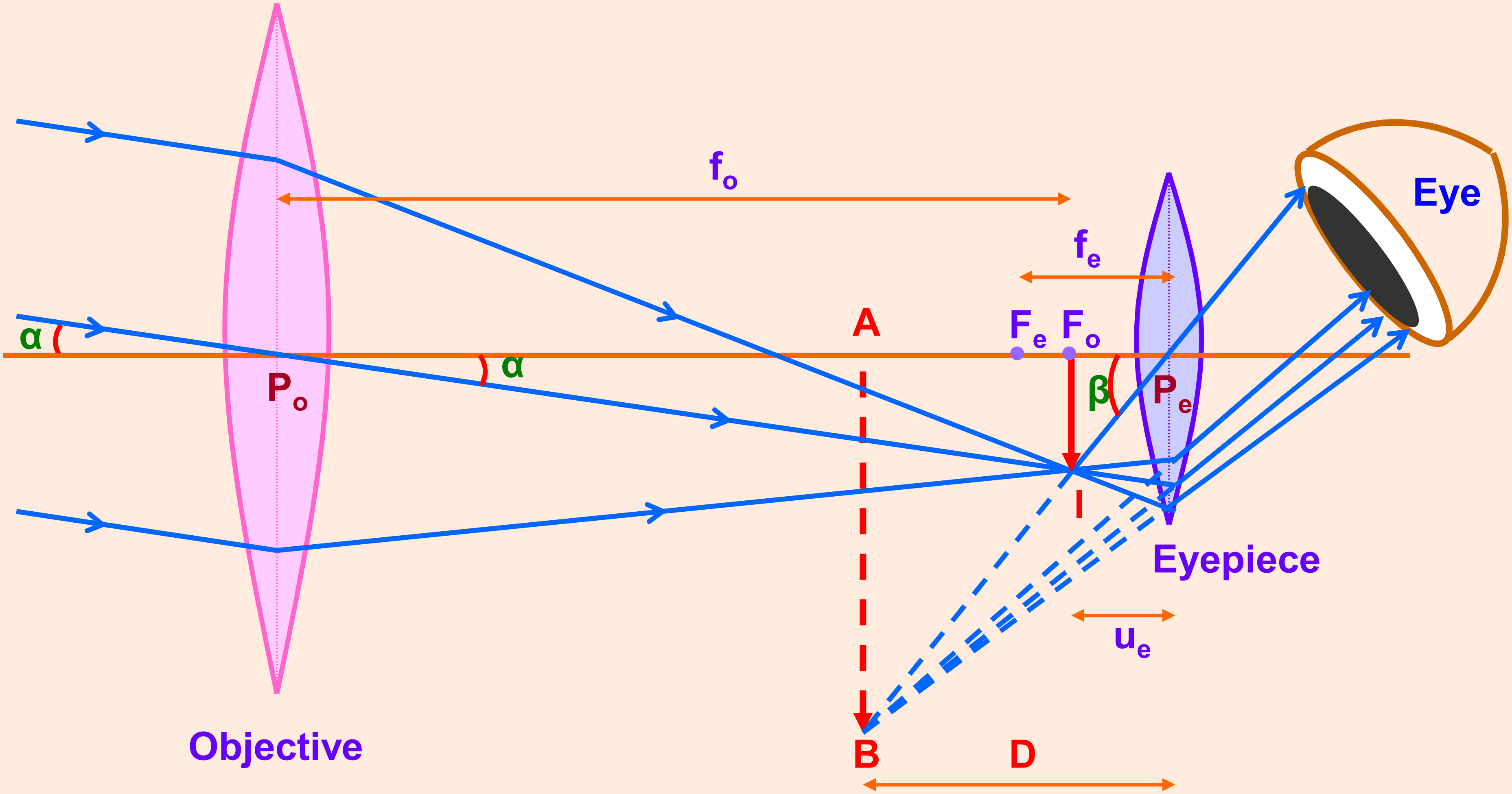
$$M = \frac{F_e I}{P_e F_e} \;/\; \frac{F_e I}{P_o F_e}$$

$$M = \frac{-I}{-f_e} \;/\; \frac{-I}{f_o}$$

$$M = \frac{-f_o}{f_e}$$

($f_o + f_e = L$ is called the length of the telescope in normal adjustment).

Astronomical Telescope: (Image formed at LDDV)



Angular magnification or magnifying power of a telescope in this case is defined as the ratio of the angle β subtended at the eye by the final image formed at the least distance of distinct vision to the angle α subtended at the eye by the object lying at infinity when seen directly.

$$M = \frac{\beta}{\alpha}$$

Since angles are small,
$\alpha = \tan\alpha$ and $\beta = \tan\beta$

$$M = \frac{\tan\beta}{\tan\alpha}$$

$$M = \frac{F_o I}{P_e F_o} \Big/ \frac{F_o I}{P_o F_o}$$

$$M = \frac{P_o F_o}{P_e F_o} \quad \text{or} \quad M = \frac{+f_o}{-u_e}$$

Lens Equation

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \quad \text{becomes}$$

$$\frac{1}{-D} - \frac{1}{-u_e} = \frac{1}{f_e}$$

$$\text{or} \quad \frac{1}{u_e} = \frac{1}{f_e} + \frac{1}{D}$$

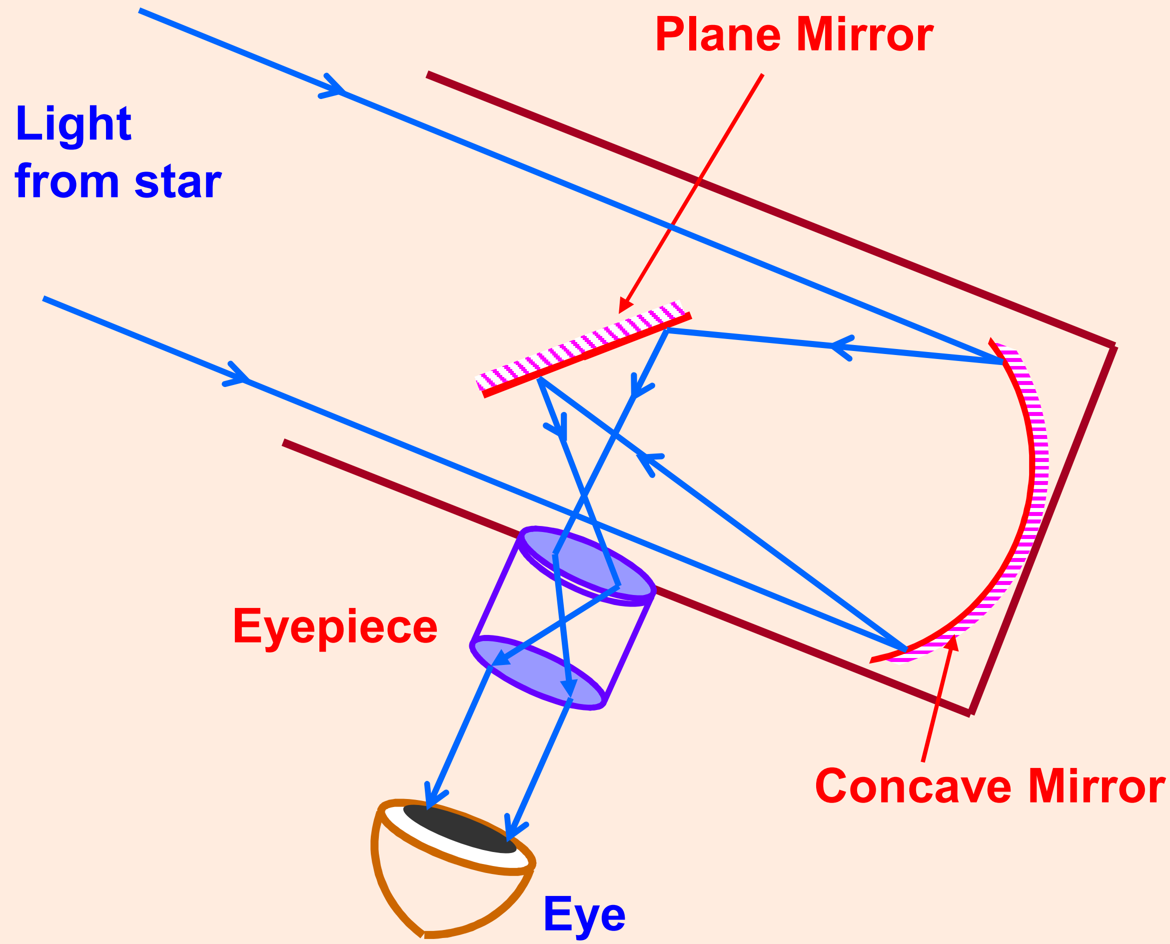
Multiplying by f_o on both sides and rearranging, we get

$$M = \frac{-f_o}{f_e}\left(1 + \frac{f_e}{D}\right)$$

Clearly focal length of objective must be greater than that of the eyepiece for larger magnifying power.

Also, it is to be noted that in this case M is larger than that in normal adjustment position.

Newtonian Telescope: (Reflecting Type)



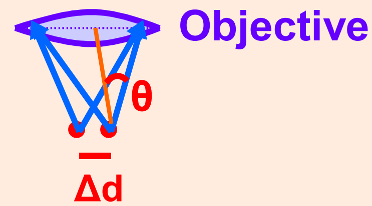
Magnifying Power:

$$M = \frac{f_o}{f_e}$$

Resolving Power of a Microscope:

The resolving power of a microscope is defined as the reciprocal of the distance between two objects which can be just resolved when seen through the microscope.

$$\text{Resolving Power} = \frac{1}{\Delta d} = \boxed{\frac{2\,\mu \sin\theta}{\lambda}}$$

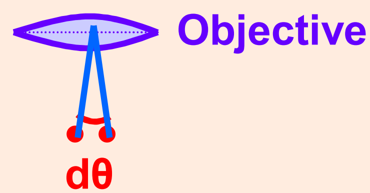


Resolving power depends on i) wavelength λ, ii) refractive index of the medium between the object and the objective and iii) half angle of the cone of light from one of the objects θ.

Resolving Power of a Telescope:

The resolving power of a telescope is defined as the reciprocal of the smallest angular separation between two distant objects whose images are seen separately.

$$\text{Resolving Power} = \frac{1}{d\theta} = \boxed{\frac{a}{1.22\,\lambda}}$$



Resolving power depends on i) wavelength λ, ii) diameter of the objective a.

End of Ray Optics - II