Question-7

On comparing the ratios $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the lines representing the following pairs of linear equations are consistent, or inconsistent.

(i) $3x + 2y = 5$
$2x - 3y = 17$

(ii) $2x - 3y = 8$
$4x - 6y = 9$

(iii) $\frac{3}{2}x + \frac{5}{3}y = 7$
$9x - 10y = 14$

(iv) $5x - 3y = 11$
$-10x + 6y = -22$

(v) $\frac{4}{3}x + 2y = 8$
$2x + 3y = 12$

Solution:

(i) $3x + 2y = 5$; $2x - 3y = 17$

$\frac{a_1}{a_2} = \frac{3}{2}$

$\frac{b_1}{b_2} = \frac{2}{-3}$

Since $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, equations are consistent.

(ii) $2x - 3y = 8$; $4x - 6y = 9$

$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$

$\frac{b_1}{b_2} = \frac{-3}{-6} = \frac{1}{2}$

$\frac{c_1}{c_2} = \frac{-8}{-9}$

Here $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\therefore$ The equations are inconsistent.

Question-27

A man walks a certain distance at a certain speed. Had he walked (1/2) km/hr faster, he would have taken 3hours longer. Find the distance.

Solution:

Let the original speed be x km/hr and time taken be y hrs.

Then, distance covered = xy km.

Speed = $\left(x+\frac{1}{2}\right)$km/hr,

Time taken = (y - 1)hrs.

Distance = $\left(x+\frac{1}{2}\right)$(y - 1) km.

Therefore xy = $\left(x+\frac{1}{2}\right)$(y - 1)

$xy = xy - x + \frac{1}{2}y - \frac{1}{2}$

$-x + \frac{1}{2}y - \frac{1}{2} = 0$

$y - 2x - 1 = 0$

$y - 2x = 1$ .....(i)

New speed = (x - 1) km/hr, time taken = (y + 3) hrs.

Therefore distance = (x - 1)(y + 3) km.

Therefore xy = (x - 1)(y + 3)

$3x - y = 3$ .....(ii)

Adding (i) and (ii), we get x = 4

Substituting x = 4 in (i), we get: y =9

Therefore speed = 4 km/hr, time taken = 9 hrs

Hence, distance= 4 × 9 km = 36 km.

Question-21

Solve the following system of equations:

$\frac{x+y}{xy}=2;\ \frac{x-y}{xy}=6.$

Solution:

$\frac{x+y}{xy}=2;\ \frac{x-y}{xy}=6$

Putting 1/x = u and 1/y = v, we get

v + u = 2 ----(i)

v - u = 6 -----(ii)

Subtracting (i) and (ii),

2u = -4 or u = -2;

Hence x = $-\frac{1}{2}$.

Substituting u = -2 in (i),

v = 4.

Hence y = $\frac{1}{4}$.

The solution set is $\{-\frac{1}{2}, \frac{1}{4}\}$.

Question-19

Solve the following system of equations:

$$\frac{x+y-8}{2} = \frac{x+2y-14}{8} = \frac{3x+y-12}{11}$$

Solution:

$$\frac{x+y-8}{2} = \frac{x+2y-14}{8}$$

8(x+y-8) = 2(x+2y-14)

6x + 4y - 36 = 0

3x + 2y - 18 = 0 ------(i)

$$\frac{x+2y-14}{8} = \frac{3x+y-12}{11}$$

11(x + 2y -14) = 8(3x + y - 12)

13x - 14y + 58 = 0 -----(ii)

Multiplying (i) by 7 and subtracting from (ii),

34x - 68 = 0 or x = 2

Substituting x = 2 in (i),

3(2) + 2 y - 18 = 0

6 + 2y -18 = 0

2y -12 = 0

y = 6

The solution set is {2, 6}.

Question-31

A two-digit number is 4 more than 6 times the sum of its digits. If 18 is subtracted from the number, the digits are reversed. Find the number.

Solution:

Let the units digit be x. Then the tens digit is y.

The number is 10x + y.

Condition I:

$10x + y = 6(x + y) + 4$

$10x + y = 6x + 6y + 4$

$4x - 5y = 4$ .....(i)

Condition II:

$10x + y - 18 = 10y + x$

$9x - 9y = 18$

$x - y = 2$.....(ii)

(i) $\Rightarrow 4x - 5y = 4$ (-)

(ii) × 5 $\Rightarrow \underline{5x - 5y = 10}$

$-x = -6$

$\therefore x = 6$

Substitute x = 6 in (i)

$4(6) - 5y = 4$

$24 - 5y = 4$

$-5y = 4 - 24$

$-5y = -20$

$y = 4$

Therefore the required number is 10(6) + 4 = 60 + 4 = 64.

Solve the equation and check your solution: $\frac{1-9y}{19-3y} = \frac{5}{8}$

Solution:

We have; $\frac{1-9y}{19-3y} = \frac{5}{8}$

Cross multiplying , we have:

$8 \times (1 - 9y) = 5 \times (19 - 3y)$

$8 - 72y = 95 - 15y$

$-72y + 15y = 95 - 8$ (Transposing $-15y$ and 8)

$-57y = 87$

$\frac{-57y}{-57} = \frac{87}{-57}$ (Dividing both sides by -57)

$y = \frac{87}{-57}$

$y = -\frac{29}{19}$

Thus, $y = -\frac{29}{19}$ is the solution of the given equation.

Check: At $y = -\frac{29}{19}$

$L.H.S = \frac{1-9y}{19-3y}$

$= \frac{1-9\left(-\frac{29}{19}\right)}{19-3\left(\frac{-29}{19}\right)} = \frac{1+\frac{261}{19}}{19+\frac{87}{19}}$

$= \frac{\frac{19+261}{19}}{\frac{361+87}{19}} = \frac{\frac{280}{19}}{\frac{448}{19}} = \frac{280}{448}$

$= \frac{5}{8} = R.H.S$

Mathematics

(www.olympiadsuccess.com)

Chapter 3: Pair of Linear Equations in Two Variables

Class: X

Exercise number 3.3

Question 1

Solve the following pair of linear equations by the substitution method.

(i) $x+y=14$

$x-y=4$

(ii) $s-t=3$

$\frac{s}{3}+\frac{t}{2}=6$

(iii) $3x-y=3$

$9x-3y=9$

(iv) $0.2x+0.3y=1.3$

$0.4x+0.5y=2.3$

(v) $\sqrt{2}x+\sqrt{3}y=0$

$\sqrt{3}x-\sqrt{8}y=0$

(vi) $\frac{3x}{2}-\frac{5y}{3}=-2$

$\frac{x}{3}+\frac{y}{2}=\frac{13}{6}$

Answer 1

(i) $x + y = 14$ .....(1)

$x - y = 4$

.....(2) From (1),

we obtain $x = 14 - y$ (3)

Substituting this value in equation (2), we obtain

$(14-y)-y=4$

$14-2y=4$

$10=2y$

$y=5 \qquad (4)$

Substituting this in equation (3), we obtain

$x=9$

$\therefore x=9, y=5$

(ii) $s-t=3 \qquad (1)$

$\frac{s}{3}+\frac{t}{2}=6 \qquad (2)$

From (1), we obtain

$s=t+3 \qquad (3)$

Substituting this value in equation (2), we obtain

$\frac{t+3}{3}+\frac{t}{2}=6$

$2t+6+3t=36$

$5t=30$

$t=6 \qquad (4)$

Substituting in equation (3), we obtain s