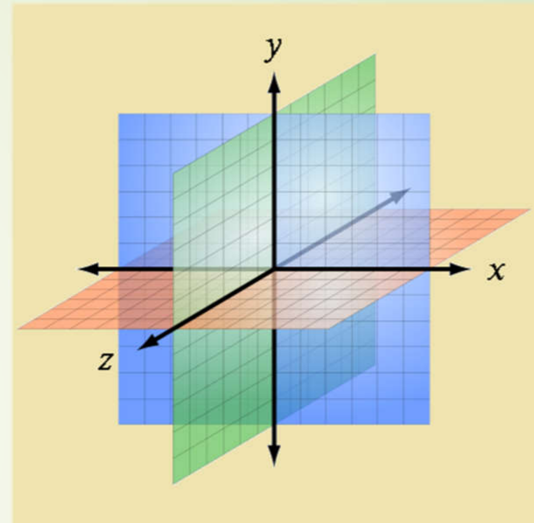
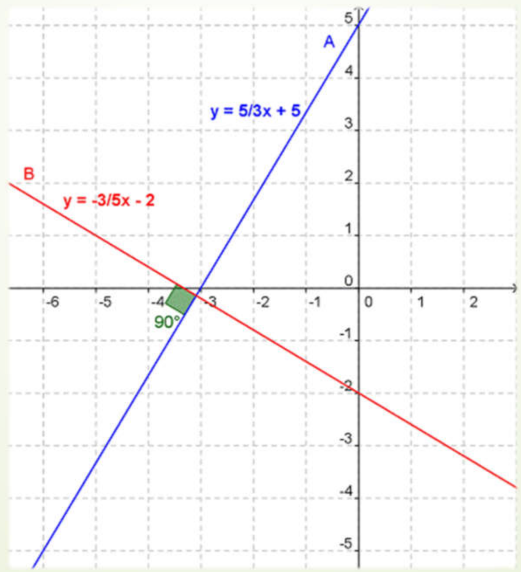
ARRANGEMENT

3. Matrices

- Introduction
- Matrix
- Types of Matrices
- Operations on Matrices
- Transpose of a Matrix
- Symmetric and Skew Symmetric Matrices
- Elementary Operation (Transformation) of a Matrix
- Invertible Matrices

Why to study matrices





What are matrices

- It is a mode of representing data to ease calculation.
- Let us now suppose that we wish to express the information of possession of notebooks and pens by Radha and her two friends Fauzia and Simran which is as follows:

Radha	has	15	notebooks	and	6	pens
Fauzia	has	10	notebooks	and	2	pens
Simran	has	13	notebooks	and	5	pens

	Notebooks	Pens
Radha	15	6
Fauzia	10	2
Simran	13	5

$$\begin{array}{c} \\ \\ \\ \text{First column} \end{array} \begin{array}{c} \begin{bmatrix} 15 & 6 \\ 10 & 2 \\ 13 & 5 \end{bmatrix} \\ \end{array} \begin{array}{l} \leftarrow \text{First row} \\ \leftarrow \text{Second row} \\ \leftarrow \text{Third row} \\ \end{array}$$

Cont.

$$\begin{array}{cc}
\left[\begin{array}{ccc} 15 & 10 & 13 \\ 6 & 2 & 5 \end{array}\right] & \begin{array}{l} \leftarrow \text{First row} \\ \leftarrow \text{Second row} \end{array} \\
\begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ \text{First} & \text{Second} & \text{Third} \\ \text{column} & \text{column} & \text{column} \end{array} &
\end{array}$$

A *matrix* is an ordered rectangular array of numbers or functions.

The numbers or functions are called the elements or the entries of the matrix

The horizontal lines of elements are said to constitute the *Rows* of the matrix and the vertical lines of elements are said to constitute the *Columns* of the matrix.

Order of a matrix

- A matrix having '*m*' rows and '*n*' columns is called a matrix of *order* $m \times n$ or simply $m \times n$ matrix (read as an m by n matrix).

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1j} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2j} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & & \vdots & & \vdots \\ a_{i1} & a_{i2} & a_{i3} & \cdots & a_{ij} & \cdots & a_{in} \\ \vdots & \vdots & \vdots & & \vdots & & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mj} & \cdots & a_{mn} \end{bmatrix}_{m \times n}$$

- $A = [a_{ij}]_{m \times n}$, $1 \le i \le m$, $1 \le j \le n$ & $i, j \in N$

Types of Matrices

(i) Column matrix

(ii) Row matrix

(iii) Square matrix

(iv) Diagonal matrix

(v) Scalar matrix

(vi) Identity matrix

(vii) Zero matrix

i. Column matrix

- A matrix is said to be a *column matrix* if it has only one column.

$$A = \begin{bmatrix} 3 \\ 7 \\ 11 \\ 13 \end{bmatrix} \text{ is a column matrix of order } 4 \times 1$$

- In general, $A = [a_{ij}]_{m \times 1}$ is a column matrix of order $m \times 1$

ii. Row matrix

- A matrix is said to be a *row matrix* if it has only one row.

$$A = \begin{bmatrix} 3 & 5 & 7 & 11 \end{bmatrix} \quad \text{is a column matrix of order } 1 \times 4$$

- In general, $A = [a_{ij}]_{1 \times n}$ is a column matrix of order $1 \times n$

iii. Square matrix

- A matrix in which the number of rows are equal to the number of columns, is said to be a *Square matrix.*
- An $m \times n$ matrix is said to be a square matrix if $m = n$ and is known as a square matrix of order ‘m’ .

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$ is a square matrix of order 3.

- In general, $A = [a_{ij}]_{m \times \mathrm{m}}$ is a column matrix of order m .

iv. Diagonal matrix

- A square matrix B = $[b_{ij}]_{m \times m}$ is said to be a *diagonal matrix* if all its non-diagonal elements are zero.
- A matrix B = $[b_{ij}]_{m \times m}$ is said to be a diagonal matrix if $b_{ij} = 0$, when $i \neq j$.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$B = \begin{bmatrix} 4 & 0 \\ 0 & 5 \end{bmatrix} \qquad C = [\ 2\ ]$$

v. Scalar matrix

- A diagonal matrix is said to be a *scalar matrix* if its diagonal elements are equal.
- A square matrix B = $[b_{ij}]_{m \times m}$ is said to be a scalar matrix if

$$b_{ij} = 0, \text{ when } i \neq j$$

$$b_{ij} = k, \text{ when } i = j, \text{ for some constant } k.$$

$$A = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$B = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$

vi. Identity matrix

- A square matrix in which elements in the diagonal are all ‘1’ and rest are all zero is called an *identity matrix*.
- We denote the identity matrix of order n by I_n .

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

vii. Zero matrix

- A matrix is said to be *zero matrix* or *null matrix* if all its elements are zero.

$$A = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

- We denote zero matrix by O. Its order will be clear from the context.

Equality of matrices

- Two matrices A = [a_{ij}] and B = [b_{ij}] are said to be equal if
  (i) they are of the same order
  (ii) each element of A is equal to the corresponding element of B, that is $a_{ij} = b_{ij}$ for all i and j.
- Symbolically, if two matrices A and B are equal, we write A = B

E.g. Find the values of a, b, c, and d from the following equation:

$$\begin{bmatrix} 2a+b & a-2b \\ 5c-d & 4c+3d \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 11 & 24 \end{bmatrix}$$

Operations on Matrices

I. Addition

II. Subtraction

III. Multiplication

- Scalar multiplication
- Negative of a matrix
- Multiplication of 2 matrices

i. Addition

- If A = [a_{ij}] and B = [b_{ij}] are two matrices of the same order, say $m \times n$.
- Then, the sum of the two matrices A and B is *defined* as a matrix C = [c_{ij}]$_{m \times n}$, where $c_{ij} = a_{ij} + b_{ij}$, for all possible values of i and j .

Properties of matrix addition:

1. Commutative Law - A+B = B+A
2. Associative Law - (A + B) + C = A + (B + C)
3. Existence of additive identity -
   Let A = [a_{ij}] be an $m \times n$ matrix and O be an $m \times n$ zero matrix, then A + O = O + A = A. In other words, O is the additive identity for matrix addition.
4. The existence of additive inverse -
   Let A = [a_{ij}]$_{m \times n}$ be any matrix, then we have another matrix as – A = [– a_{ij}]$_{m \times n}$ such that A + (– A) = (– A) + A= O. So –A is the additive inverse of A or negative of A.

iii. Multiplication

Scalar multiplication -

- If $A = [a_{ij}]_{m \times n}$ is a matrix and k is a scalar, then kA is another matrix which is obtained by multiplying each element of A by the scalar k.

Negative of a matrix –

- The negative of a matrix is denoted by – A. We define $-A = (-1)A$.

Properties of scalar multiplication of a matrix:

(i) $k(A + B) = kA + kB$

(ii) $(k + l)A = kA + lA$

multiplication of two matrices

Let A = [a_{ij}] be an $m \times n$ matrix and B = [b_{jk}] be an $n \times p$ matrix. Then the product of the matrices A and B is the matrix C of order $m \times p$.

$$CD = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & 4 \end{bmatrix} \times \begin{bmatrix} 2 & 7 \\ -1 & 1 \\ 5 & -4 \end{bmatrix}$$

CD = Entry in first row first column

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & 4 \end{bmatrix} \begin{bmatrix} 2 & 7 \\ -1 & 1 \\ 5 & -4 \end{bmatrix} = \begin{bmatrix} (1)(2)+(-1)(-1)+(2)(5) & ? \\ ? & ? \end{bmatrix}$$

Entry in first row second column

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & 4 \end{bmatrix} \begin{bmatrix} 2 & 7 \\ -1 & 1 \\ 5 & -4 \end{bmatrix} = \begin{bmatrix} 13 & (1)(7)+(-1)(1)+2(-4) \\ ? & ? \end{bmatrix}$$

Entry in second row first column

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & 4 \end{bmatrix} \begin{bmatrix} 2 & 7 \\ -1 & 1 \\ 5 & -4 \end{bmatrix} = \begin{bmatrix} 13 & -2 \\ 0(2) + 3(-1) + 4(5) & ? \end{bmatrix}$$

Entry in second row second column

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & 4 \end{bmatrix} \begin{bmatrix} 2 & 7 \\ -1 & 1 \\ 5 & -4 \end{bmatrix} = \begin{bmatrix} 13 & -2 \\ 17 & 0(7) + 3(1) + 4(-4) \end{bmatrix}$$

$$CD = \begin{bmatrix} 13 & -2 \\ 17 & -13 \end{bmatrix}$$

Properties of multiplication of matrices

1. A x B ≠ B x A
2. The associative law - (AB) C = A (BC)
3. The distributive law -
   (i) A(B+C) = AB + AC
   (ii) (A+B)C = AC + BC
4. The existence of multiplicative identity -
   For every square matrix A, there exists an identity matrix of same order such that IA = AI = A

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